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Thesis

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Comparison of Alternative Policy Rules
in a Structural Model of the Czech
Economy

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Declaration

Hereby I confirm that this thesis represents my own work. The contribution of my supervisors and others to the research and to the thesis was consistent with usual supervisory practice. External contributions to the research are acknowledged.

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1 Introduction

The main goal of this thesis is to analyze the dynamic properties of alternative policy rules in a calibrated small-open-economy model that captures the main channels of the monetary transmission mechanism in the Czech Republic. The work will focus on some of the policy issues that are currently in the center of central banks' forecasters and researchers. For this purpose a simple structural model will be presented and calibrated on Czech data. The approach in terms of model implementation shall represent one of the currently used strategies applied in a growing number of central banks. The model is based on forward-looking model-consistent expectations, endogenous policy reaction function and it is brought to data through model-consistent filtering based on the Kalman filter (KF).

In order to understand the wider context of the empirical part of the thesis, first an overview is presented covering all important aspects of the modeling work, presented later in the text. The emphasis is put on the discussion of those structural macroeconomic models that play important role in the development of forecasting and policy analysis models used in central banks. Since the examination of policy rules is in the centerpiece of the thesis and the literature provides a rich source for understanding the role of monetary policy rules in central banks' model development, the related literature will be covered both in terms of theoretic and empiric dimensions.

The historical overview capturing the progress that has been achieved in the area of model development and policy rules will be followed by the introduction of a structural model that will be used for the analysis in the subsequent sections of the thesis. The model is a small-open-economy model where agents form model-consistent forward-looking expectations and monetary policy is endogenous. The key model equations, besides identities, include the IS curve, quantifying aggregate demand, the Phillips-curve, the uncovered interest rate parity condition, interest rate arbitrage condition (yield curve) and the policy rule. The supply side of the model, approximated by equilibrium values for real variables, is assumed to be exogenous. The equilibrium values on the historic data sample are obtained through model consistent filtering, on the forecast horizon they are assumed to be exogenous.

Since the model will be brought to data through calibration as opposed to econometric estimation, some of the reasons for this choice will be mentioned in the next section. This will be followed by the definition of the domestic and external data and by the short description of the data transformation being carried out (seasonal adjustment, log-transformation, etc.) for the applied empirical work. The detailed discussion of the calibration of the model and its verification through in-sample model simulations will close this section on calibrating the model to Czech data.

The model described in the previous section includes a standard monetary policy rule, where the central bank targets the consumer price inflation. The main goal of the thesis, however, is to analyze the implications of alternative policy rules, obtained by changing the targeted price index. First, the motivation for such alternative rules is explained, including the comparative advantages and disadvantages of these alternative reaction functions. The policy rules are then exactly defined and subsequently tested through impulse response exercises. The impulse response analysis is useful for illustrating the transmission mechanism implied by the model and for understanding the reaction of the model for various shocks. These shocks, in turn, will be selected in order to support the motivation for the selected policy rules.

The exact specification of alternative monetary policy reaction functions discussed above enables the comparison of these rules in the calibrated model on real data. This is the final goal of the thesis. First alternative parameterizations of the considered simple forward-looking rules will be introduced, for CPI and domestic inflation targeting. That will be followed by finding optimized coefficients for these rules with respect to three alternative loss functions. Finally these optimized rules will be used for understanding their stabilization properties within the presented model. This is done by evaluating the implied variance of inflation, output and short-term interest rates and minimizing the corresponding loss function based on these variances.

The structure of the thesis is as follows. The introduction is followed by the *second chapter* providing the historic overview in the economic theory that has led to the use of current macroeconomic models. Within this chapter we will concentrate not only on the history of macroeconomic modeling but

also on main achievements that formed micro end macro- and micro-theory, including empirical and implementation aspects. Given the importance of the policy rule literature for central bank policy models, we will shortly shed a light on the progress in research of this particular area. The chapter will be closed with the overview of the most important developments in the area of central bank modeling, that shaped the direction of applied work in recent years. The *third chapter* will be devoted to the specification of the model used subsequently for the empirical analysis. In order to illustrate the links between the individual model equations, the model's transmission mechanism is described here as well. The *fourth chapter* specifies the data that has been used, provides all necessary details regarding the data transformations necessary for the modeling work as well as the calibration and the solution of the model. This will involve the description of the filtering results - the determination of unobservable variables - and the verification of the model by means of shock decomposition and in sample simulations. The *fifth chapter* is concerned with the specification and evaluation of policy rules by analyzing impulse response functions for alternative targeting regimes (CPI vs. domestic inflation targeting) as well as analysis based on the magnitude of standard errors of the shocks specified in the model. The last *sixth chapter* provides the summary of the results and concludes. *Appendix 6* is focused on the Blanchard-Kahn model solution algorithm for linear rational expectation models, *Appendix 7* describes the Kalman-filter, *Appendices 8-10* contain the loss function values for three alternative loss functions and a grid-search range of monetary policy rule coefficients.

2 The evolution of central bank models and policy rules

2.1 Central bank models

The development of central bank models has achieved a breathtaking progress in terms of their applied policy work over the last two decades. The gap between central bankers and academicians, in terms of the models they rely

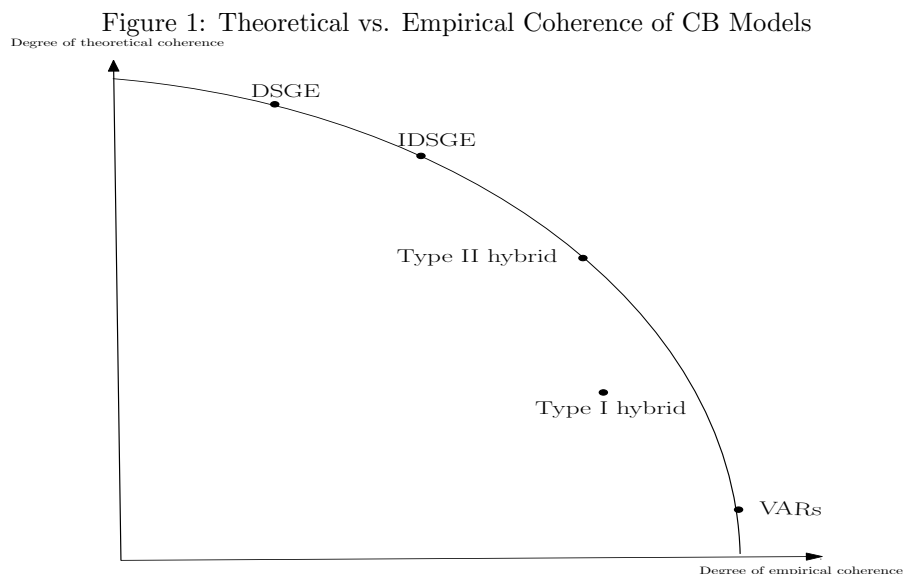
on, narrowed dramatically over time. Both sides contributed to this convergence. Academic researchers made a significant progress in terms of making their models more realistic, especially by incorporating nominal rigidities into their theoretically sound real business cycle models. In turn, central banks improved the theoretical coherence of their models and tried to apply the academic state-of-the-art modeling methodology when building their forecasting models. As a result of this, we witnessed much closer cooperation and stronger mutual feedback between central banks and academic institutions in recent years.

In order to put the central banks' modeling effort into some unifying framework, that enables classifying the currently used central bank models, we rely on the Pagan's report classification, described in Pagan (2003)¹. Pagan classifies central bank models according to their degree of *empirical* as well as *theoretical* coherence.

Figure 1 below depicts the tradeoff between theoretical and empirical coherence of economic models, including the efficiency frontier. Pagan thought of the two extreme cases in the following way. On the vertical axis he considers all theoretically sound models that have never been exposed to data. On the horizontal axes he puts empirical models fitting perfectly the data but whose outcomes are difficult to interpret structurally. On the efficiency frontier we have got models with combinations of various degree of empirical vs. theoretical coherence.

The first class of models, with fairly low theoretical but high empirical coherence, are *VAR models*. VAR models are used practically in all central banks. They are simultaneous, data-driven models. Even some theoretical restrictions can be imposed on them, but their results are often difficult to interpret in terms of an economic story. They are, however, frequently used as a benchmark for evaluating the empirical properties of structural mod-

¹The Pagan report was published on 30 January 2003. Adrian Pagan wrote the report on the basis of the invitation of the BoE's Court, who asked prof. Pagan to evaluate whether the BoE's modeling and forecasting work is sophisticated enough, measured by world standards. In particular, A. Pagan was asked to focus on the technical aspects of the modeling and forecasting process. The report can be found on <http://www.bankofengland.co.uk/publications/news/2003/011.htm>



els. The next class of models, reflecting already economic theory, are the so called *hybrid models*. Hybrid models were based on a two-stage approach to modeling the economy. The first stage rested on the assumption, that the economy is evolving alongside an explicitly or implicitly given equilibrium path. The second stage focused on the specification of the nature of adjustment to the equilibrium path. This two-stage approach also brought into the focus of policymakers the concept of "gaps", percentage deviations of variables from their corresponding equilibrium values. Within the class of hybrid models Pagan distinguished between Type I and Type II models. The former class denotes models with implicit, the latter with explicit long-term equilibrium part specification. Let's describe a little bit more in detail the differences between the two classes of models. The equilibrium relationship among some model variables in type I hybrid models were often assumed to be based on a stable functional form. The econometric support for identifying these stable relationships came from the literature on *co-integration*. Models, that identified both the long-term relationships as well as the process (speed) of adjustment from the disequilibrium back to equilibrium were

modeled by means of *error correction models*. The methodology of applied co-integration theory resp. error-correction models (ECM) is well captured in Hendry (1993) resp. Davidson, Hendry, Srba and Yeo (1978). As mentioned above, the more widespread use of ECMs in policy making institutions directed policy discussions towards the extent of disequilibrium and the speed by which the corresponding "gap" should be closed. Type II models took a further step in deepening the identification of equilibrium path of variables by making them fully explicit. In addition, these models also incorporate forward-looking model consistent expectations ², therefore they work more realistically with the expectation channel ³ The Type II models, compared with the Type I models, had some other advantages. The results of Type II models are more easy to interpret due to their strong theoretical foundations as well as due to the correct treatment of with stock-flow equilibrium. They also modeled the steady-state in a similar manner as it was the case in academic models, but they did not reflect the economic theory in capturing ad-hoc short-term dynamics. The next class of models, that Pagan entitled as *incomplete dynamic stochastic general equilibrium* (IDSGE) models, addressed exactly this weakness of Type II models. IDSGE models were based on the recognition, that economic theory should be applied not only for determining the equilibrium paths but also for describing the adjustment dynamics to the equilibrium. Since the incorporation of short-term nominal and real rigidities into theory-based models was not as developed as it is the case nowadays, rule-of-thumb short-term adjustment terms were used in IDSGE models to make their dynamic properties more suited for real-life policy work. Including these ad-hoc adjustment terms into the IDSGE models is also the only difference from DSGE models, that are based on a fully optimizing behavior. The DSGE models are important in recent history of macroeconomic modeling in many respects. Their microeconomic foundations provide a theoretical framework for the structure of the model that is being estimated, which may be of particular importance in those cases where the data themselves are short or not very informative. (This is especially true for emerging markets

²The importance of forward-looking rational expectations was recognized already with Muth (1961).

³The well-defined steady-state was also important in terms of computing expectations for future variables.

and transition economies.) The "deep" structural parameters make the use of the model for policy analysis more appropriate, i.e. less subject to the Lucas critique (see Lucas (1976)), since the structural parameters are less likely to change in response to changes in policy regime. Finally, micro-founded models may provide a more suitable framework for analyzing the optimality of various policy strategies as the utility of the agents in the economy can be taken as a measure of welfare.

Given Pagan's classification of models, let's turn our attention to the evolution of modeling strategies at those central banks, whose reputation in macroeconomic modeling is among the best. We start our discussion with VAR models, that rely the least on economic theory from the variety of models we will consider of. Subsequently we'll move towards theoretically more coherent modeling strategies.

There is a large number of central banks, who rely on VAR models for analytic or forecasting purposes. A very good description of how VAR models are used in central banks can be found in Quinn (2000). The Bank of England, as of during 1990's, used VAR models not only for general forecasting purposes, but also for examining models, exploiting the leading indicator properties of monetary and credit aggregates. These results provided the Bank with stylized fact about the short-run correlations between monetary variables and activity indicators. Another VAR application at the BoE, elaborated in contributions Astley and Garratt (1996) and Astley and Garratt (1998), included the analysis of forecast-error variance decomposition to quantify the contribution of main shocks ⁴ to the change of the nominal exchange rate. Large number of papers were also produced at the BoE for analyzing the transmission mechanism. Studies Quinn (2000) and Daley and Haldane (1995) analyzed the speed with which monetary policy changes transmitted into corporate and personal sectors of the UK economy. Further VAR-related research of the BoE focused on assessing the impact of permanent vs. temporary monetary policy shocks on the economy. Besides the BoE there are a number of other examples of published research conducted by CBs in the area of VARs. Research at the ECB conducted by Mojon and Peersman (2001) concentrated on the

⁴The shocks were the following: the bilateral XR, UK consumer prices and UK GDP, all in relative terms with respect to their foreign counterpart.

quantification of the effects of monetary policy on the individual countries of the Euro area. Similarly as in the case of the BoE, the quantification of the main monetary policy channels of the Czech transmission mechanism was in the focus of Arnoštová and Hurník (2005). The authors focus in their paper on the reaction of the Czech economy to a monetary policy shock. They conclude, that conclude that an unexpected rise in policy rates is followed by a fall in output, although they warn, that the short data sample brings a faster and less persistent output response.

The next class models, classified by Pagan as Type I, represent the results of a huge modeling effort that characterized central bank's modeling strategies since the 70s of the last century. Let's start our exposition again with the BoE's medium-term macro model (MM). The MM is based on an error-correction structure, although, as Pagan notes, "this is not always obvious" and even the steady-state solution is not given explicitly. The long-run structure of the model is pinned down by few equations. Output is modeled as Cobb-Douglas function of labor, capital and technology, demand for labor depends on output and real wage, the demand for capital on output and real cost of capital. Real unit labor costs depend on a set of structural variables, including the unemployment rate. The model also includes short-run disequilibrium dynamic responses of prices and quantities to economic shocks. These short-run dynamic responses are ensuring realistic real and nominal rigidities of the model, such as delayed response of trade volumes to the change in the real XR or staggered interaction between wages and prices. The short-term dynamic terms were included to achieve good fit of the model for the short-run, but all of this was done at an expense of breaking the theoretical coherence of the model. Another good example of an Type I model is that of the Bank of Finland (BoF), described in more detail in Tarkka (1985). The BOF3 is a medium-sized quarterly model, consisting of 198 equations, of which 88 may be considered as behavioral. The model is basically set up in a Keynesian income-expenditure tradition, in fact, it is a standard IS-LM framework. The model relies on adaptive expectation formation. The components of aggregate demand are modeled in detail. Aggregate demand less imports is converted into value added of four production sectors. This is done through a compact input-output system, reflecting the structure of the Finnish economy at the

time. The four sectors are: (i) agriculture; (ii) services and government; (iii) forestry; and (iv) mining and manufacturing. Pricing, employment and incomes are also analyzed at this level of aggregation. The effects of financial markets on the rest of the economy is captured in terms of supply of and demand for money. The supply side is modeled by a Cobb-Douglas production function of the value added in each sector. This has been used to derive employment, investment and pricing equations. The model also reflected some special features of the Finnish economy, such as strong trade links with the former Soviet-Union. The next model of this category, to be discussed within the Type I class of models, is the Area Wide Model (AWM) of the ECB, described in Fagan, Henry and Mestre (January 2001) as well as Fagan and Morgan (2005). The AWM uses data with quarterly frequency, allowing for a richer treatment of short-term dynamic adjustment. Most of the equations are estimated on historical data from 1970, rather than calibrated. The model treats the euro area as a single economy. It is a medium sized model with approximately 84 equations, of which 15 are estimated. The model is detailed enough for analytic and forecasting purposes, nonetheless, sufficiently small to be manageable for real time applications. The AWM is designed to have a long run equilibrium consistent with neoclassical economic theory, while its short run dynamics are demand driven. Expectation formation, similarly as in the BOF3 case, is backward-looking, i.e. expectations are reflected via the inclusion of lagged variables. For simulation purposes the model is run with endogenous fiscal and monetary policy rules. In a forecasting mode, however, the projections are based on exogenously determined assumptions on the future path of monetary and fiscal policies ⁵. The production function of the model is based on a Cobb-Douglas specification. The investment equation comprises of a long-term component, in which capital stock is a function of output and the real user cost of capital. The short-term equations, in turn, ensure significant effect of the interest rate on aggregate demand. Employment depends on real wages and output growth in the short-run. In long-run it is derived from the inverted production function. Aggregate demand is determined by the expenditure items of GDP and the specification of these equations is

⁵This is given by the practice of the ECB to generate forecasts based on the assumption of unchanged short-term interest rates.

fairly standard. The key price index used in the model is the deflator for real GDP at factor costs ⁶. The deflator is modeled as a function of trend unit labor cost, import prices have also some short-term effects. Wages are modeled as a Phillips curve relationship, where wage growth depends on productivity, lagged inflation and on deviation of the unemployment rate from its NAIRU level. The fiscal block is relatively simple, with transfers being a function of the unemployment rate, most of the other relationships being modeled as ratios to GDP. Finally, the monetary and financial sector are captured through money demand resp. yield curve equations. Besides the model development in the above mentioned European central banks ⁷ also serious effort was put into policy-related model building in the US. The so called MIT-Penn-SSRC (MPS) model (in more detail see Brayton and Mauskopf (July 1985)) of the US economy was developed in the late 1960s, it became operational in 1970. The model contains 332 equations of which 124 are behavioral and 208 are identities. There are 197 exogenous variables. The model's core was based on a simplified growth model, that assumed a closed economy characterized by perfect competition, Cobb-Douglas production technology and intertemporal utility maximization by consumers. A government sector purchases goods, taxes income and issues money and bonds. Taxable income consists of net output (wages plus the net return to capital) and nominal interest income on government debt. The real wage equals the marginal product of labour, and the real cost of capital equals the net marginal product of capital. Private saving is the product of the saving rate, which depends on the real cost of capital, and income. Net investment equals net (public and private) saving. The real quantity of money demanded is held for transactions purposes and is a function of the nominal rate of interest and real output. Because the government taxes nominal interest income, the real rate of return on government bonds is set equal to the nominal return less the rate of inflation. The final equation is the government budget constraint expressed in real terms ⁸ The dynamic

⁶Excluding the effect of indirect taxes and subsidies.

⁷Of course, there were many other European central banks who developed Type I models. A good example is the Nederlandsche Bank, MORKMON model developed for the Dutch economy has been developed during the 1980s (see Bikker, Boeschoten and Fase (1986)). The Bank later also developed a multi-country model called EUROMON, that has been developed during the 1990s as a reaction of the European integration process. The description of the model's structure can be found in Demertzis, van Els and Peeters (2002).

⁸The change in real debt equals government purchases plus interest on the debt, less the

part of the model has been estimated, expectation formation was specified as backward-looking. The lag structure ensures, that the behavior of the model is Keynesian in the short run: output and employment are mainly determined by the level of demand because of wage and price rigidities. Both monetary and fiscal policy have significant effects on real activity in the short-run. The MPS model was later replaced by the FRB model (for details see Brayton and P. (1996)).

The next class of models in the Pagan classification are Type II models. Since Type II models are in many respects very similar to IDGSE models, the differences between them are subtle and are related to the level of their theoretical coherence, we will discuss them altogether. Probably the most famous representative of these classes of models, having a significant impact on the modeling effort in central banks all over the world is the QPM model of the Bank of Canada (BoC). The model has got an extensive documentation. The introduction to the model is described in Poloz, Rose and Tetlow (1994), the steady-state part, based on neo-classical economic theory in Black, Laxton, Rose and Tetlow (1994), the details related to the solution of the model in Armstrong, Black, Laxton and Rose (1995) and the dynamic part of the model in Coletti, Hunt, Rose and Tetlow (1996). The steady-state part of the model is based on optimizing microeconomic behavior. This includes the behavior of households, firms, foreigners, a government and a central bank. The decisions of these agents interact to determine the ultimate levels of four key stock variables: household financial wealth, capital, government debt, and net foreign assets. These stock levels in turn are key determinants of the associated flows ⁹. Therefore, the model is consistent with a full stock-flow equilibrium among all variables both in long-run as well as along the dynamic adjustment path. The dynamic part of the QPM specifies the gradual adjustment from out-of-equilibrium state to the steady-state of the model. Besides the effect of expectation formation on the speed of adjustment, the nominal and real rigidities in the economy are given due labour market contracts, the fixed costs associated with changing investment or consumption behavior. More gener-

sum of tax receipts, real money creation, and the rate of inflation times the stock of money and bonds.

⁹Such as consumption spending, saving, investment spending, government spending and revenues, and the external balance.

ally, the QPM incorporates costs of adjustment, which causes all agents in the economy to choose adjust step-by-step to disturbances. QPM incorporated rigidities into the model also through the way how expectations were modeled. In order to produce a dynamic response of the model to shocks, that seems to replicate the properties of time series data reasonably well, the expectations in QPM were modeled as a mixture of backward- and forward-looking components. The change in the relative weights on the two components enabled to the model builders to generate the sort of stylized facts that were in line with empirical evidence and judgment. The BoC with its QPM model represented one of the most progressive modeling strategies during mid 1990s. By relying on sophisticated numerical solution methods, including the Troll software package, the BoC pushed the frontiers of central bank modeling the closest to the academic state-of-the-art modeling at the time. By the use of advanced macroeconomic theory based on microeconomic optimization, incorporation forward-looking model consistent expectations, replacing estimation by calibration and generating realistic short-term dynamic properties of the model, they created a benchmark for central bank modeling for years. At the same time, their modeling strategy got quite close to the current practice of the new DSGE modeling. The BoC's QPM model had also a direct impact. A very similar model structure, derived from the Blanchard-Buiter-Weil model of overlapping generations, and modeling strategy was adopted with the assistance of the BoC, at the Reserve Bank of New Zealand (RBNZ)¹⁰. Many Type II or IDSGE models were built also in Europe. Let's mention Finland again, where at the BoF there was a continued effort to improve their BoF3 and later BOF4 models and push them higher up to the direction of a "neoclassic-core with forward-looking model-consistent expectations" class. The BoF achieved this goal by constructing a BOF5 model, that is described in detail in Kortelainen, L., M. and A. (2000). The BOF5's theoretical core structure, with approx. 400 equations. As it is usual within this class of models, it is based on the neoclassical synthesis. In the case of BOF5 it means, that given higher rigidity of wages compared with prices, production, income and employment are determined by aggregate demand. In the short-run, therefore, the model is Keynesian. In the long-run, however, the wages and prices respond to excess

¹⁰For more detail see Black, Cassino, Drew, Hunt, Rose and Scott (1997).

supply or demand and markets converge to full employment and purchasing power parity between domestic and foreign prices. As mentioned earlier, the BOF5 model, as opposed to BOF3 and 4 versions, works with forward-looking expectations with respect to prices, wages, labor demand and money. The BoF created an aggregate version of the BOF5 model called BOFMINI (with "only" about 240 equations.). This version of the model has been used for generating a quarterly forecast as well as sensitivity analyses or alternative scenarios. The BoF developed another structural model within the considered class of Type II models. The model, called EDGE, was created by M. Kortelainen (for more detail see Kortelainen (2002)) to capture the euro area economy and thus contributing to the policy discussions at the ECB level¹¹. The EDGE model reflected quite substantially the model-building experience of the BoF modeling staff, especially with that of the BOF5. The most important features of EDGE include consumption-saving decisions according to Blanchard's stochastic lifetime approach, the valuation of private wealth according to the present value of capital income, Calvo-type wage contracts, Rotemberg-type sticky prices and neoclassical supply side with Cobb-Douglas production technology. The exchange rate is determined by UIP condition, the policy rule is specified as a classic Taylor-rule. Some of the limitations of the EDGE model include the exogenous labor supply specification, the absence of population growth, or the somewhat unrealistic assumption of perfect competition at a representative firm level. The model was used mainly for carrying out policy simulation at the euro area level, regular forecasts were not produced. Last, but not least, within the Type II or IDSGE models we can mention a number of small-scale small open economy models, based on forward-looking model consistent expectations, endogenous policy rules and short-term nominal rigidities with well defined steady-state. They, however, miss stock-flow equilibrium or deep structural parameters that would be a result of microeconomic optimization. In order to mention examples of such models, at the BoE a model of this sort is the Batini-Haldane model, specified in Batini and Haldane (1999). At the CNB a similar small open economy model has been created, called QPM (see Coats, Laxton and Rose

¹¹The EDGE model has been slightly modified and calibrated for the Czech economy by Hlédik (2003a)

(2003). In the academic literature Lars Svensson concentrated on these types of small, rational-expectation models (see for instance Svensson (2000)). The structural model, specified later in this thesis, belongs into the same class of models.

The newest advances in macroeconomic modeling are linked to the development and implementation of Dynamic Stochastic General Equilibrium (DSGE) models within a forecasting and policy analysis framework of central banks. Similarly, as the BoC's QPM model within the previous class of Type II models, the most famous DSGE model developed in a central bank is connected to economists working for the European System of Central Banks (ESCB) F. Smets and R. Wouters. In their contribution Smets and Wouters (2002) they estimated a closed economy DSGE model by Bayesian estimation technique and created a work-horse model for central bankers (as well as academicians) for years ahead. In fact, the model they relied on is based on a paper Christiano, Eichenbaum and Evans (2005). The Christiano-Eichenbaum-Evans model incorporates features such as habit formation, costs of adjustment in capital accumulation and variable capacity utilization. Prices and wages are modeled based on the basis of Calvo specification. These features are important, since they allow for working with nominal and real rigidities that are necessary for producing realistic model properties and consistency of the model with the observed stylized facts. The model assumes, that households derive their utility from consumption relative to their habit formation and disutility from work. Each household monopolistically competitively provides labor. The various kinds of labor are used to produce differentiated intermediate goods. Their production requires labor and capital. These goods produce a single final good, based on the assumption that each intermediate good producer, who supplies their products for final good production, is monopolistically competitive. As mentioned, the model is estimated with Bayesian techniques¹² using seven key macro-economic variables: GDP, consumption, investment, prices, real wages, employment and the nominal interest rate. Using the estimated model, the paper also analyses the output (real interest rate) gap, defined as the difference between the actual

¹² For the further exposition it is important to note, that the authors detrend the data before estimation and measure all variables as deviation from trend.

and model-based potential output (real interest rate). The Smets-Wouters paper has been a big success in terms of bringing a DSGE model close to data ¹³. regarding other central banks, the BoF is again among those central banks ¹⁴, who achieved a significant success in building and implementing DSGE models in a policy analysis and forecasting framework by creating the AINO model for the Finnish economy ¹⁵. The core of the model fully in line with modern dynamic macroeconomics, with special features around the neoclassical core. The model works with heterogeneous population consisting of workers and retirees. Therefore it is capable of examining the fiscal and macroeconomic implications of aging. In addition, the social security system of the model is in line with the Finnish system. Labour and goods markets are characterized by imperfect competition, in wage and price adjustment there is inertia, in order to be consistent with price and wage rigidity present in the data. Real rigidities are modeled through costly investment adjustment. The model has been partially estimated, as well as calibrated. The model provides a common platform for policy analysis and research on monetary policy at the BoF. The AINO model has been used for examining the impact of labour and product market competition in the Finnish economy in Kilponen and Ripatti (2006a). When discussing the most sophisticated modeling strategy, we should mention the ECB, who developed a sophisticated DSGE model for the euro area. The New Area Wide Model (NAWM) ¹⁶ is a clear attempt of the ECB to create a new, state-of-the-art model for the eurozone, that builds on the tradition established by F. Smets and R. Wouters. The NAWM is neo-classical model in nature. It is derived from an intertemporal optimization of households and firms which maximize their expected life-time utility and the expected stream of profits, respectively. As a result, forward-looking expectations play a key role in influencing the adjustment dynamics of both quantities and prices. Changes in the supply-side have, therefore, a signifi-

¹³Besides the positive reactions there were also critical reactions, regarding the large number of shocks that enables the good empirical fit of the model, see for instance Chari, Kehoe and McGrattan (2008)

¹⁴At this point we also should mention a very strong modeling tradition at the Sveriges Riksbank. The Bank's new DSGE model, called RAMSES, that is documented in Laséen, Lindé and Villani (2007).

¹⁵A detailed description regarding the use of the Aino model can be found in Kilponen, Kontulainen, Ripatti and Vilmunen (2004) and Kilponen and Ripatti (2006b)

¹⁶For detailed description see Christoffel, Coenen and Warne (2008).

cant impact already in the short-run. At the same time, the NAWM includes a number of nominal and real frictions that have been identified as empirically important, such as sticky prices and wages a la Calvo (so that some Keynesian features prevail in the short-run), habit persistence in consumption and adjustment costs in investment. Moreover, it incorporates analogous frictions relevant in an open-economy setting, including local-currency pricing, that gives rise to imperfect exchange-rate pass-through in the short-run, and adjustment costs related to trade flows. The authors employed Bayesian estimation methods, that focused on eighteen key macroeconomic variables, including real GDP, private consumption, total investment, government consumption, exports and imports, a number of deflators, employment and wages, and the short-term nominal interest rate. The model's expected use is to serve within the Broad Macroeconomic Projection Exercises regularly undertaken by ECB/Eurosystem staff. Let's mention another two central banks, who have chosen a slightly different strategy to implement their DSGE model to serve in a forecasting regime. The first is the BoE, the second is the CNB. Let's start with the BoE's BEQM model (for details see Harrison, Nikolov, Quinn, Ramsay, Scott and Thomas (2005)). BEQM describes the behaviour of the UK economy at a relatively aggregated level that is closely related to the incomes and expenditures recorded in the UK national accounts. Households consume imported and domestically produced goods. Households are assumed to borrow and save using a range of financial assets. In addition, in the short-run, households levels of consumption can be influenced by short-term fluctuations in their income and their confidence about the future. Firms maximize their profits by hiring labour and buying capital in order to produce output. Firms and workers bargain over wages and, given the outcome, firms are assumed to choose the labour they wish to employ so that the costs of any extra workers are compensated for by the higher revenues they generate. Similarly, firms desired level of capital is determined by the cost of capital and the return to extra investment. The output that firms produce is sold in markets for domestic consumption, investment and government procurement, as well as in housing and export markets. Firms are assumed to face varying degrees of competition in these markets. Firms face competition from importers for consumption and investment goods, and have to price their products in ex-

port markets so as to achieve maximum profits. The government buys output from domestic firms and labour from households, financed by raising taxes and selling debt. Monetary policy anchors the nominal side of the economy by targeting the annual inflation rate of the CPI of 2 %, using the short nominal interest rate as its instrument. The most discuss feature of BEQM is its core-noncore structure. The core model consists of the exactly derived DSGE model, the non-core part adds extra dynamics to the core part designed, in part, to facilitate judgmental adjustments. The CNB decided to create a DSGE model that has not got a non-core part for incorporating judgment. At the same time due to the demand for incorporating out-of-model information into a model forecast, a lots of effort was put into creating procedures that through structural shock adjustment enables to work flexibly with the model. The g3 model of the CNB ¹⁷. has been motivated by several stylized facts that are important when modeling the Czech economy. The model, therefore, had to account for trends ¹⁸ in sectoral relative prices and evolution of nominal expenditure shares, the high import intensity of exports and increase in trade openness. At the same time nominal and real rigidities, present in the data, had to be reflected by the model equations. In terms of the main agents of the model, households , who optimize their lifetime utility, consume all varieties of the consumption final good, rent capital services to intermediate goods sector and monopolistically supply differentiated unit of labor. Wage settings follow Calvo contracts. Households also own and accumulate the stock of capital goods. The model has got a simple production structure. The economy consists of two intermediate goods sectors (domestic and imported) and four final good (consumption, investment, government and export goods) producers. Sectors are monopolistically competitive in order to introduce price rigidities into the model. Monetary authority in the model targets a deviation of year-on-year CPI inflation from its target four periods ahead. Government collects taxes and fees (transaction costs), distribute lump-sum

¹⁷This new DSGE model of the CNB was introduced into the shadow forecasting regime in 2007 and in July 2008 the first official forecast of the CNB was produced. The evolution of the model is documented in the following publications: Beneš, Hlédik, Kumhof and Vávra (2005), Andrlé (2007), Andrlé, Hlédik, Kameník and Vlček (2009)

¹⁸The data are not detrended prior to transforming them for the purposes of the model, as in the case of most currently used DSGE models brought to data. Therefore trends in variables are important for both in the short- and long-term.

transfers and consumes public-spending goods. An important feature of the new structural model is model-consistent (national) accounting of all stock and flows. The model is calibrated in a very broad sense, by relying on variety of tools to achieve model properties consistent with stylized facts and observed data dynamics. The implementation of the model, the identification of structural shocks, has been achieved by relying on the Kalman-filter. The two-years experience with the model in a forecasting mode suggest, that the framework is robust enough to satisfy the challenge of real-time forecasting.

2.2 Policy rules

The literature on monetary policy rules is very rich and it steadily grows over time. The research has been motivated by the increasing number of central bank in the world relying on models incorporating rules that approximate the systematic inflation stabilizing behavior of monetary authorities.

Probably the most famous contribution to the monetary policy rule literature is that of Taylor (1993) entitled "*Discretion versus policy rules in practice*". In his article John Taylor describes the systematic behavior of the US Federal Reserve Bank (FED) by the following simple policy rule ¹⁹:

$$r_t = \pi_t + 0.5y_t + 0.5(\pi_t - 2) + 2$$

where:

r_t = federal fund rate;

π_t = the rate of inflation over the four previous quarters;

y_t = is percentage deviation of real GDP from trend;

The Taylor-rule states, that the central bank increases its short-term policy rate as soon as the y-o-y inflation is above target or output above trend. Otherwise the short-term nominal policy rate is a function of the "equilibrium" real interest rate and expected inflation ²⁰.

The paper concentrates on three main issues: the design of policy rules,

¹⁹Policy rules specified in this functional form are called in the economic literature *Taylor-rules*.

²⁰The expected inflation according to the original Taylor specification is approximated by inflation over the four previous quarters and not (model-consistent) future inflation.

the problem of transition from an old to a new policy rule and finally on making the policy rule operational in a policy environment. Taylor evaluates the specification of policy rules on the basis of their ability to stabilize inflation around target as well as output around potential output. He stresses, that policy rules in monetary policy conduct cannot be used automatically. Discretion of the central bank is necessary either in terms of deciding to move to a new policy rule or in deviating from the actual rule in situations when policy analysis supports such action. In his opinion the use of policy rules combined with the set of macroeconomic (leading) indicators could lead to better results of MP than solely relying on either a policy rule or indicators only.

The research of the effectiveness of policy rules, motivated by Taylor's work, intensified during the second half of 1990's. In Great Britain, where at that time the BoE pursued an IT regime, many authors concentrated on the usefulness of policy rules in the conduct of forward-looking monetary policy. At the famous think-tank of the National Institute of Economic and Social Research (NIESR) Blake and Westaway analyzed in their paper Blake and Westaway (1996) the credibility and effectiveness of various policy rules within an inflation targeting regime. Their work has been a motivation for N. Batini and A. Haldane at the BoE, who wrote a paper contributing to the next important milestone in the policy rule literature, by focusing on *inflation-forecast targeting rules*. In their famous paper Batini and Haldane (1999) the authors stress the importance of a *forward-looking* monetary policy rules. They modify the original Taylor rule specified above as:

$$r_t = \gamma r_{t-1} + (1 - \gamma)r_t^* + \theta(E_t(\pi_{t+j}) - \pi^*)$$

where:

$$r_t = i_t - E_t(\pi_{t+1})$$

r_t = short-term ex-ante real interest rate;

i_t = short-term nominal interest rate;

r_t^* = short-term equilibrium real interest;

$\pi^* =$ inflation target;

$E_t(\pi_{t+j}) =$ expected inflation j -quarters ahead, based on the information set available at time t ;

N. Batini and A. Haldane focused on the frontiers of inflation resp. output volatility around the inflation target resp. potential output ²¹ depending on the lead of targeted inflation j periods ahead, as well as the share of forward-looking agents in the Phillips curve. By changing the MP feedback horizon j in the interval of 0-14 quarters, they arrive to a conclusion that is currently widely accepted in central bank circles. Namely, when the feedback horizon for monetary policy is very short ($j=0,1$), or too long ($j=12-14$), both inflation and output volatility is quite high. This result is quite intuitive if one takes into account the lags in the monetary transmission mechanism: inflation in the current quarter can be affected by MP only through the direct exchange rate and expectation channels. The change in the policy instrument, however, should be substantial to appreciate the exchange rate in the current quarter significantly and this would subsequently lead to high volatility of the real output as well as inflation. It is thus straightforward, that the more MP looks ahead, the stronger will be control over the economy. This is true, however, only to a certain extent, until j becomes too large. Although by growing forward horizon j output and inflation volatility decreases, above certain value of j it starts to increase again, since the control of inflation too far ahead anchors inflation expectations as well as the real economy too loosely in the short-run.

The Batini-Haldane paper had an impact in two dimensions. First, it provided a rationale to the rhetoric of the BoE related to the forward-looking 2-2.5 years MP feedback horizon. Second, it drew attention to simple forward-looking policy rules and their *robustness* in a wide range of policy models.

Forward-looking monetary policy rules are more than an interesting academic concept. They are widely used in central banks. Empirical studies support the hypothesis, that many of the leading central banks in the world behave in a forward-looking manner while conducting MP. Clarida, Galí and Gertler (June 1998) conclude, that Germany, Japan and the US since 1980's

²¹Within a class of simple inflation-forecast-based rules specified above.

responded to anticipated as opposed to contemporaneous or lagged inflation.

One of the next popular topics emerged in the policy rule literature is the notion of *optimal policy rules*. One of the most well-known economist, who not only enriched the literature by a large number of important papers in the area of optimal policy but also encouraged the use of optimal MP rules in practice, is Lars E. Svensson. He proposed, that the central bank should derive and follow an optimal MP rule that would be based on the minimization of a loss function, that has been agreed upon by the policymakers. He showed, how such optimal policy rules can be derived in structural small open economy models. One of his first papers on this topic is Svensson (2000), where he analyses the implications of alternative policy rules, depending on the targeted index as well as the loss function of policy makers. Svensson's papers, among others, shed a light on the key role of the exchange rate in small open economy models and quantified the implications of alternative inflation targeting strategies, through the optics of optimal MP.

Besides the large positive impact that Svensson exerted on the development of MP rules in policymaking institutions, he also provoked critical reactions with respect to the practical usefulness of optimal MP rules. The most frequent critique of optimal policy is related to the model-specific functional form of optimal MP rules combined with the absence of their robustness in a wide range of macroeconomic models. Orphanides and Williams (October 2008) show, that optimal policy rules perform poorly in circumstances when agents, who have imperfect knowledge about the structure of the economy²², gradually learn about its structure. In addition the authors compare the stabilization properties of simple rules with that of optimal rules. They show, that in the presence of learning, plausibly calibrated simple rules perform as well as optimal rules and they are more robust in a wide range of models.

The economic literature related to the Czech economy also includes papers concerned with optimal and optimized simple rules. Optimal policy rules in the Czech economy, specified for alternative loss functions, were examined by Hlédik (2003b) in a small calibrated structural model. Optimized simple rules,

²²Parameter and model uncertainty is one of the key issues in applied forecasting and policy analysis work. In his famous contribution Brainard (1967) warned that parameter uncertainty can lead to less activist (aggressive) policy.

examined in the QPM model of the CNB for alternative feedback horizons, were analyzed in Stráský (2005).

3 Model Description

The simple structural model, that shall be described below in more detail, is a New-Keynesian small open-economy model with forward-looking model-consistent expectations. The model is very similar in structure to the models presented in Blake and Westaway (1996), Batini and Haldane (1999) and Svensson (2000). It has been calibrated on quarterly seasonally adjusted data and specified in "gap" form. This means that all variables are defined as deviations from their trends, in the case of inflation, as deviations from the inflation target. Let's start to introduce the model, equation by equation. Our first equation describes aggregate demand (IS curve).

$$y_t^{gap} = a_1 y_{t-1}^{gap} + a_2 r_{t-1}^{gap} + a_3 q_t^{gap} + a_4 y_t^{*gap} + res_t^y \quad (1)$$

where:

y^{gap} = output gap, percentage deviation of real GDP from trend;

r^{gap} = deviation of one year real interest rate from equilibrium;

q^{gap} = percentage deviation of real exchange rate from trend;

y^{*gap} = foreign demand gap, percentage deviation of real effective foreign GDP from trend;

res_t^y = output gap shock;

Equation (1) specifies a standard open-economy backward-looking IS curve²³. The lagged output term captures the inertia in aggregate GDP present in the Czech national accounts data. In models with microeconomic foundations rigidity in real GDP is most often modeled through introducing habit formation in consumption, time-to-build capital formation/capital adjustment costs and/or real rigidities in foreign trade. Higher real interest rate curb, ceteris

²³The presence of a forward-looking output gap term in the IS curve assures more pronounced reaction of current GDP to anticipated shocks. This effect, however, can be for the purposes of this paper, neglected without significant impact on the empirical properties of the model.

paribus, real output through their dampening effect consumption and investment. The competitive position of exporters is quantified by the deviation of real exchange rate - defined by relative prices measured by domestic and foreign domestic prices - from its trend. Given the specification, that domestic prices are specified as constant mark-ups over wages, the real exchange rate are implicitly closely linked to the wage competitiveness of domestic producers against their foreign competitors. Foreign demand positively feeds into domestic output by boosting - most importantly - net exports. The residual term res_t^y captures demand shocks, those influences that are directly not modeled an example could be fiscal policy generated demand pressures.

$$\pi_t^{cpi} = (\alpha + \beta)\pi_t^{net} + (1 - \alpha - \beta)(\pi_t^{adm} + \pi_t^{tax}) + res_t^\pi \quad (2)$$

where:

π_t^{cpi} = quarterly, seasonally adjusted, annualized CPI inflation rate;

π_t^{net} = quarterly, seasonally adjusted, annualized net inflation;

π_t^{imp} = quarterly, seasonally adjusted, annualized imported inflation rate, in CZK;

π_t^{adm} = quarterly, seasonally adjusted, annualized administered inflation rate;

π_t^{tax} = quarterly, seasonally adjusted, annualized change in indirect taxes;

res_t^π = shock to the CPI identity, capturing approximation error stemming from linearization;

Equation (2) captures the desegregation of the consumer price index (CPI) identity into its subgroups, carried out on the basis of individual representants of the CPI and the knowledge of indirect tax changes ²⁴. The equation reflects the division of the CPI inflation into net inflation ²⁵, regulated price inflation and the contribution of indirect tax changes to total inflation. Regu-

²⁴The consumer price index in the Czech Republic measures the price level of a consumption basket consisting of approximately 700 representative products and services. The current data collection practice of the Czech Statistical Office does not make it possible to differentiate between domestically produced and imported goods or services in the CPI. The methodology of dividing CPI inflation into its subcomponents is described in more detail in section 3.1.

²⁵Net inflation is defined as a CPI inflation without the contribution of administered price inflation and indirect taxes.

lated prices are partially imported energy related commodities as well as state controlled services.

$$\begin{aligned}
\pi_t^d &= \frac{\chi(1-\alpha\lambda)}{(1-\chi)+\alpha(\chi-\lambda)}\pi_{t+1}^d + \frac{\alpha(1-\chi)(1-\lambda)}{(1-\chi)+\alpha(\chi-\lambda)}\pi_{t-1}^d \\
&- \frac{\beta\chi\lambda}{(1-\chi)+\alpha(\chi-\lambda)}\pi_{t+1}^{imp} + \frac{\beta(\lambda-\chi)}{(1-\chi)+\alpha(\chi-\lambda)}\pi_t^{imp} \\
&- \frac{(1-\alpha-\beta)\chi\lambda}{(1-\chi)+\alpha(\chi-\lambda)}\pi_{t+1}^{adm} + \frac{(1-\alpha-\beta)(\lambda-\chi)}{(1-\chi)+\alpha(\chi-\lambda)}\pi_t^{adm} \\
&+ \frac{\lambda\omega}{(1-\chi)+\alpha(\chi-\lambda)}y_t^{gap} + \frac{(1-\lambda)\omega}{(1-\chi)+\alpha(\chi-\lambda)}y_{t-1}^{gap} \\
&+ res_t^{\pi^d}
\end{aligned} \tag{3}$$

where:

$res_t^{\pi^d}$ = shock to domestic inflation;

Equation (3) is a Phillips curve for domestic inflation derived from a FuhrerMoore-type wage-contracting specification (FM), see Fuhrer and Moore (1995), that has been modified for a small open economy. This modification is based on the assumption that wage setters do not derive their nominal wage demand from a real product wage, as it is the case in the FM, but rather from their real consumer wage. This assumption is very much in line with the past and current practice of wage negotiations in the Czech Republic ²⁶ Indeed, trade unions always communicate their nominal wage demands in terms of some "plausible" future real wage growth increased by expected CPI based inflation. Equation (2), as will be shown later, gives rise to second-round effects of some selected supply-side shocks or nominal exchange rate shocks on domestic inflation via the wage contracting channel. Besides equation (2), which defines the CPI, the following two equations served for deriving the Phillips curve specified by equation (3) for domestic inflation:

$$p_t^d = \lambda w_t + (1-\lambda)w_{t-1} \tag{4}$$

²⁶ Compared with some other European countries (such as France, Italy), the trade unions are relatively weak in terms of wage bargaining power.

$$w_t - p_t^{cpi} = \chi(w_{t+1} - p_{t+1}^{cpi}) + (1 - \chi)(w_{t-1} - p_{t-1}^{cpi}) + \omega y_t^{gap} \quad (5)$$

where:

p_t^d = GDP deflator;

w = average wage in the national economy (national accounts concept);

p_t^{cpi} = CPI index;

Equation (4) is a standard mark-up equation based on the assumption that prices are determined as a weighted average of current and past nominal contract wages. The mark-up was set to zero, and the lagged term captures rigidities in price adjustment to reflect nominal marginal costs of production. In models with microeconomic foundations price rigidities are often modeled a la Calvo (see Calvo (1983)) or Rotemberg-type (see Rotemberg (1982)) wage or price setting. Our specification, therefore, could be interpreted as a shortcut to this microeconomically based specification. The next equation is based on the assumption that real wages today depend on the weighted average of expected real consumer wages one period ahead and lagged real wages. Real wages are, most importantly, positively related to the cyclical position of the domestic economy, approximated by the output gap. The overheated economy, approximated by the positive output gap, drives up real wages, in recession real wages fall. Equation (3) is derived by summing up equations (5) for time indices t resp. $t-1$ multiplied by λ resp. $1 - \lambda$ and by substituting out w and p^{cpi} by using equations (2) and (4).

$$\pi_t^{imp} = \Delta s_t + \pi_t^* + res_t^{\pi^{imp}} \quad (6)$$

$$\pi_t^* = b_1 \pi_t^{cpi*} + b_2 \pi_t^{ppi*} + (1 - b_1 - b_2)(\pi_t^{oil} - \Delta s_t^{cr}) + res_t^{\pi^*} \quad (7)$$

where:

Δs = quarterly, annualized change in the nominal exchange rate of the Czech koruna against the EUR;

π^* = quarterly, seasonally adjusted, annualized foreign inflation rate, in EUR;

π^{ppi*} = quarterly, seasonally adjusted, annualized PPI inflation, in EUR;

π^{cpi*} = quarterly, seasonally adjusted, annualized CPI inflation, in EUR;

π^{oil} = quarterly, seasonally adjusted, annualized oil price inflation, in USD;

Δs^{cr} = quarterly, seasonally adjusted, annualized change in the USD against the euro;

$res_t^{\pi^{imp}}$ = shock to import price inflation;

$res_t^{\pi^*}$ = shock to the annualized foreign inflation rate;

Equations (6) and (7) provide the explanation for the main determinants of imported inflation. The first relationship states, that imported inflation is determined by the change in the nominal exchange rate against the EUR as well as by the direct effect of imported inflation, measured by prices denominated in EUR.²⁷ Equation (7) simply states, that foreign prices are transmitted into Czech CPI inflation through two main channels. The first channel is an import of foreign final goods, approximated by π^{cpi*} . The second channel captures the use of imported intermediate goods in the production of domestic final goods, that is approximated by the prices of foreign industrial producers prices π^{ppi*} as well as oil prices expressed in Czech koruna terms $\pi^{oil} - \Delta s^{cr}$.²⁸

$$\pi_t^{net} = \frac{\alpha}{1 - \alpha - \beta} \pi_t^d + \frac{\beta}{1 - \alpha - \beta} \pi_t^{imp} - \pi_t^{tax^{net}} \quad (8)$$

where:

π_t^{net} = quarterly, annualized change in CPI inflation without the direct effect of administered prices and indirect taxes;

²⁷Notice that no staggered transmission of the nominal exchange rate or foreign prices into imported prices is assumed here, the pass-through is immediate. The more gradual transmission of foreign price shocks into the import prices, however, could be easily introduced by introducing lagged terms of the exchange rate and foreign prices in equation (4).

²⁸Oil prices are usually quoted in USD. Therefore, the (exogenous) forecast of the change in the USD/EUR exchange rate Δs^{cr} is necessary for obtaining the forecast of oil prices in local currency (koruna) terms.

$\pi^{tax^{net}}$ = quarterly, annualized change in the indirect taxes contributing to net inflation;

Equation (8) states, that net inflation consists of domestically produced goods and imported goods ²⁹. Of course, this identity is an approximation. In reality most domestically produced goods are composite goods "blended" from domestic value added and imported component. The CPI also contains of imported final goods as reflected in the specification of import prices above.

$$\pi_t^{tax} = (\alpha + \beta)\pi_t^{tax^{net}} + (1 - \alpha - \beta)\pi_t^{tax^{adm}} \quad (9)$$

where: $\pi^{tax^{adm}}$ = quarterly, annualized change in the indirect taxes contributing to administered inflation;

Equation (9) reflects the analytic division of the overall contribution of indirect taxes into tax changes affecting net inflation resp. administered price inflation.

$$\pi_t^{mp} = \pi_t^{cpi} - \pi_t^{tax} \quad (10)$$

π^{mp} = quarterly, seasonally adjusted, annualized monetary policy inflation rate;

Equation (10) defines so called monetary-policy inflation *MP inflation* as CPI inflation without the direct effect of indirect tax changes on aggregate inflation. This index is closely linked to the implementation of the inflation targeting (IT) regime in the Czech Republic. When setting short-term interest rates, the CNB targets MP inflation instead of CPI inflation, in order to

²⁹In other words, it is assumed, that *net inflation* - CPI inflation without the effect of administered prices and the direct effect of indirect taxes - is composed of domestic value added and imported goods.

prevent suppressing economic activity due to direct affects of changes on the price level. Any potential second-round effects of such tax changes, however, are monitored and eliminated should they emerge.

$$s_t = E_t^p(s_{t+1}) - (i_t^{3m}/400 - i_t^*/400) + prem_t/400 + res_t^s/400 \quad (11)$$

where:

$$E_t^p(s_{t+1}) = c_1 s_{t-1} + (2\Delta q_t^{*ss} - 2\pi_t^{*ss} + 2\pi_t^{tar})/400 + (1 - c_1)s_{t+1} \quad (12)$$

$E_t^p(s_{t+1})$ = private sector's exchange rate expectations;

s = nominal exchange rate against the EUR;

Δq^{*ss} = annualized growth rate of the equilibrium real exchange rate;

π^{*ss} = foreign annualized quarterly inflation rate in the steady-state (inflation target);

π^{tar} = domestic annualized quarterly change in the inflation target;

i^{3m} = 3M PRIBOR, interbank market interest rate;

i^* = 3M EURIBOR, interbank European market interest rate;

$prem$ = annualized exchange rate risk premium;

$res_t^s/400$ = UIP shock;

Equation (11) is a standard uncovered interest-rate parity arbitrage condition. The expectation formation of the private sector is not assumed to be fully forward-looking model-consistent, as it is apparent from the specification of the equation. Private agents exchange rate expectations are modeled as a weighted average of forward-looking rational and backward-looking expectations. This modification in the UIP condition makes the reaction of the nominal exchange rate, and consequently the models dynamics to shocks more gradual, consequently more in line with observed exchange rate behavior. The $(2\Delta q_t^{*ss} - 2\pi_t^{*ss} + 2\pi_t^{tar})/400$ term might at first look slightly complicated and somewhat non-intuitive. The main reason for including this term is to be

consistent with the equilibrium appreciation path of the real exchange rate after modifying the fully forward-looking model consistent expectations as described above.

$$\begin{aligned} i_t^{3m} = & d_1 i_{t-1}^{3m} + (1 - d_1)(r_t^{eq} + \pi_4^{tar} + d_2(d_3(\pi_4^{mp} - \pi_4^{tar}) \\ & + (1 - d_3)(\pi_4^d - \pi_4^{tar}))) + d_4 y_t^{gap} + res_t^{i^{3m}} \end{aligned} \quad (13)$$

where:

$\pi_4^\#$ = year-on-year inflation rate measured by price index #, where # = cpi,

d, tar;

r^{eq} = real equilibrium interest rate;

$res_t^{i^{3m}}$ = short-term interest rate (policy) shock;

The next behavioral equation (13) is the central bank's policy rule for setting short-term interest rates (3M Pribor).³⁰ The reaction function is Taylor-type forward-looking policy rule. The specification assumes that the central bank smoothes short-term interest rates³¹, as well as positively reacts to any deviation of the inflation forecast from target and output from its potential. The specification of the rule is general enough to examine the model properties under the assumption of inflation targeting for CPI based inflation ($c_3 = 1$) as well as domestic inflation ($c_3 = 0$).

$$i_t^{1y} = (i_t^{3m} + i_{t+1}^{3m} + i_{t+2}^{3m} + i_{t+3}^{3m})/4 + res_t^{i^{1y}} \quad (14)$$

where:

³⁰We deliberately work with a simplified assumption here. In reality the CNB sets the 2W repo rate, the 3M Pribor is determined on the interbank market. Most of the time the 2W repo is highly correlated with the 3M Pribor, therefore the simplifying assumption that the CNB sets the 3M Pribor, is quite realistic, shall we abstract from the period of the ongoing global financial crises. The risk premium, that created a wedge between the repo and interbank market rates during the end of 2008, would show up in the model as a monetary policy shock.

³¹A more detailed discussion on the topic of interest smoothing by central banks see Goodhart (1997)

i^{1y} = one-year PRIBOR, interbank market interest rate;
 $res_t^{i^{1y}}$ = shock to one-year PRIBOR;

Equation (14) is a one year money market interest rate based on a risk-neutral arbitrage condition. We assume that private agents form forward-looking model-consistent expectations. We abstract from any risk or liquidity premium related to uncertainty related to the forecast of short-term interest rates. Including this yield-curve equation makes it possible to examine the impact of a change in interest rates with longer maturity on aggregate demand. It also enables implicitly take into account expectational yield-curve effects on the dynamic properties of the model.

Identities:

$$r_t = i_t^{1y} - E_t(pi_{t+1}^d) \quad (15)$$

$$E_t(\pi_{t+1}^p) = e_3(e_1\pi_t^d + (1 - e_1)\pi_{t+1}^d) + (1 - e_3)(e_2\pi_t^{cpi} + (1 - e_2)\pi_{t+1}^{cpi}) \quad (16)$$

$$\Delta s_t = 400(s_t - s_{t-1}) \quad (17)$$

$$p_t^d = p_{t-1}^d + \pi_t^d/400 \quad (18)$$

$$p_t^{cpi} = p_{t-1}^{cpi} + \pi_t^{cpi}/400 \quad (19)$$

$$p_t^* = p_{t-1}^* + \pi_t^*/400 \quad (20)$$

$$q_t = s_t - p_t^d + p_t^* \quad (21)$$

$$y_t^{gap} = (y_t - y_t^{eq})100 \quad (22)$$

$$q_t^{gap} = (q_t - q_t^{eq})100 \quad (23)$$

$$r_t^{gap} = (r_t - r_t^{eq})100 \quad (24)$$

$$q_t^{eq} = q_{t-1}^{eq} + \Delta q_t^{eq}/400 \quad (25)$$

where:

$E_t(pi_{t+1}^p)$ = private sector's inflation expectations;

p^d = level of domestic prices measured by the GDP deflator;

p^{cpi} = consumer price index;

y^{eq} = potential output;

q^{eq} = real equilibrium exchange rate;

p^* = foreign consumer price index;

Equations (15)-(25) are simple identities, therefore we shall describe them briefly. Equation (15) defines the real interest rate. The weighted average of CPI resp. domestic inflation approximate the real interest rates for households resp. firms. Equation (17) defines the quarterly annualized percentage change in the nominal exchange rate. Equations (18)-(20) are based on the definitions of domestic, CPI and foreign inflation rates and link inflation rates with corresponding price levels. Equation (21) defines the real exchange rate

³² The following three equations define the gaps for output, real exchange rate and real interest rate as a percentage deviation of these variables from their trend or long term average. ³³ Equation (25) defines the percentage quarterly annualized change in the equilibrium real exchange rate.

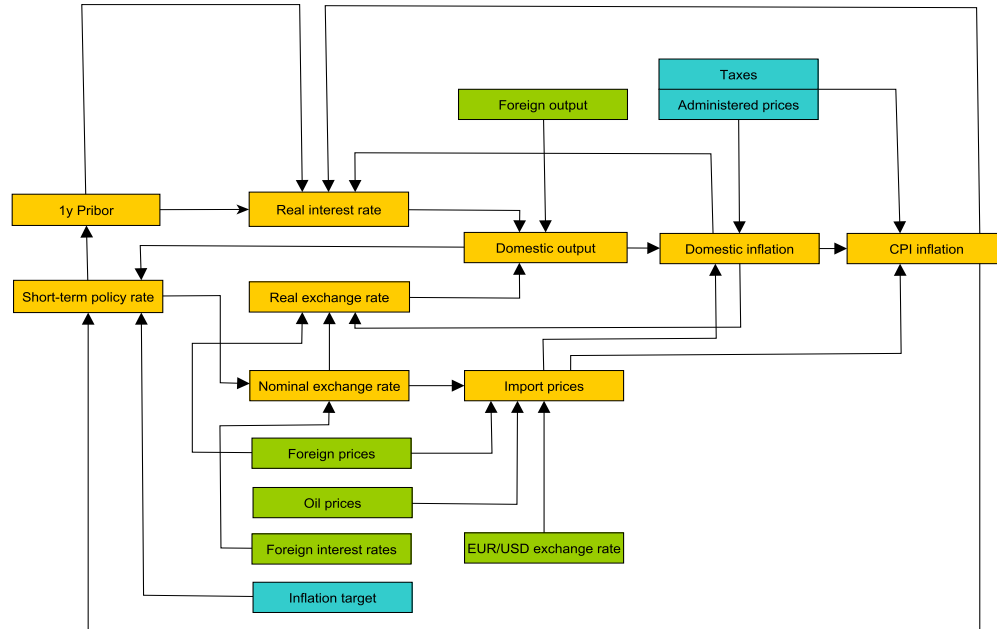
3.1 The implied transmission mechanism of the model

Before turning to the implementation part, we would like to highlight the main channels of the transmission mechanism implied by the model. The following simple scheme captures the structure of the model and the main behavioral links between the key model variables. The endogenous variables are depicted with a yellow-, the domestic exogenous variables with blue- and the foreign exogenous variables with green rectangle. The arrows illustrate the behavioral links between the variables, obviously without the expectation channel.

³²Notice that the domestic price level is defined in the real exchange rate in terms of domestic prices, not CPI. Domestic prices are more closely linked to the competitive position of firms and do not reflect the short term fluctuations of the nominal exchange rate. For the foreign price index, given the plausible large closed economy assumption, we use foreign CPI, since its external forecast is more readily available.

³³The estimation of these equilibrium values will be made by means of the Kalman filter. The methodology will be discussed later in the text.

The scheme of the transmission mechanism



One of the important features of the model - a necessary condition for using it in a monetary policy environment - is that agents form model-consistent forward-looking expectations and monetary policy provides a nominal anchor to the economy. Short-term interest rates affect the economy through a number of channels. In broad terms, MP influences demand conditions in the economy through the changing the output gap as well as by changing import prices via the uncovered interest rate parity condition.

Let's start with the channel influencing the real GDP gap. The rise in short-term nominal interest rates will result in an increase of one-year interest rates through the yield curve equation (14). Given the relatively high inertia in inflation, the change in the nominal rates will induce a positive change in the real-interest rate and this will exert, *ceteris paribus*, a dampening effect on the output gap. The hike in the policy rate also results in an appreciation of both the nominal and real exchange rate through the UIP condition. The appreciation in the real exchange rate opens the exchange rate gap and subsequently more deepens the already negative output gap. The excess supply

in the economy will lead to lower wage demands (equation (5)), that will mitigate domestic cost of production and through constant mark-ups the prices of domestically produced goods.

The hike in nominal interest rates will lead, *ceteris paribus*, to an appreciation of the nominal exchange rate. The strengthening of the domestic currency, in turn, will feed into import prices (equation (6)) and curb CPI inflation. Lower import prices will have impact in subsequent periods through the wage negotiation process (equation (5)). The fall in import prices will lower CPI inflation and subsequently wage growth, that will curb domestic price pressures.

The transmission cycle is closed by the reaction of central bank in the next period. The overall fall in both future inflation and the output gap feeds back into a policy rule itself and the level of short-term interest rates, caused by their initial rise, will return to the policy neutral level.³⁴ The fall in interest rates will lead to a depreciation of the exchange rate and result in falling real interest rate in the medium-term to neutral level (until closing all gaps). Let's note, that it is exactly the central bank's reaction function, that ensures that the economy, *ceteris paribus*, will converge into the steady-state.

There is one aspect of the transmission mechanism, that we did not discussed yet but it is very important for the model's dynamics. That is the expectation channel.³⁵ The rise in the policy rate will have a strong impact on the current nominal exchange rate, that depends on the *expected* nominal exchange rate. The exchange rate is the most forward-looking variable in the model and its change will influence the real economy (through the IS curve equation) as well as will generate cost-push effects by the change in import prices. Simi-

³⁴The policy neutral level of the nominal interest rate is a sum of the real equilibrium interest rate and the inflation target.

³⁵We stressed in the model description section of this thesis the importance of forward-looking, model-consistent expectations. They assure that for any shock hitting the economy expectation of agents will be adjusted according to the anticipated effect of the shock at hand on all model variables. These anticipations will be formed on the basis of currently available information set at the time and the knowledge of the whole model, including behavioral equations and their parameterization. In fact, the agents forming forward-looking expectations are assumed to be able to solve the model and calculate the its dynamic response to the shock. This might seem to be a strong assumption for policy models aiming at forecasting real-life economic behavior. Yes it is. It is important to be aware of the fact, that model-consistent forward-looking expectations are just a mainstream benchmark for modeling the expectation channel and some other complementary theories, such as *learning*, might address some of the weak points of the forward-looking model consistent expectations assumption.

larly strong expectation effect is present in the determination of the one-year nominal interest rate, that is a function of expected three months short-term rates. Domestic inflation is also strongly affected by the expectation channel. The 1Q-ahead expected inflation term feeds back from all predetermined variables and will reflect the dampening effect of the negative output gap and fall in import prices in the next period. Finally, short-term interest rates are, among others, the function of expected MP-inflation 4Q-ahead, that is in fact the MP-inflation forecast.

4 Data, calibration and solution of the model

The focus of this section is bringing the model to data by relying on calibration rather than estimation. The term calibration is meant in a very broad sense, since we use a variety of tools to achieve model behavior consistent with our priors, stylized facts and observed data dynamics. The use of Czech data in the model calibration serves a key and a limited role at the same time. There are two reasons for this. First, macroeconomic time series at quarterly frequency in the Czech Republic are quite short. Second, not all available data can be viewed as supportive for a relatively simple and standard macroeconomic model. The reason is that the economy is still in its transition or post-transition period. Some pieces of data cannot be viewed as reliable. Still, we devote utmost care to data analysis. In our model calibration we follow the minimal econometric approach to DSGE models of Geweke (1999). We are aware of models highly aggregated nature and its inherent misspecification. We focus mainly on the story-telling potential of the model. Our ongoing exploration of identification issues confirm conclusions of Canova and Sala (2006) that one must be cautious when formal estimation techniques are used. A well-known fact is that mapping from deep to reduced form parameters may cause serious identification problems itself even in case when ideal population data were available.

First we describe the data we are going to use in this text. This will include all data sources and data transformation (seasonal adjustment, log transformation, creating proxies, etc.) that has been necessary for analyzing the

business cycle development of the Czech economy. Although some considerations regarding data issues were already touched upon in the previous section, here however, we will discuss data mainly in a statistical and not so much in economic sense. It will be followed by a description of the calibration of the model. The calibration heavily relies on the practical experience of the author with calibrating a number of small economy models in recent years as well as on the gradually growing Czech literature on this topic. The third subsection will be devoted to the first (of the two) verification test of the calibration. We will use the model to forecast in-sample and compare the results with actual outcome. The last part of this section will use shock decomposition to assess, whether the residuals identified by the model are in line with the intuition given the knowledge of well-distinguished periods of the business cycle in the Czech economy ³⁶.

4.1 Data and data transformation

The database relies on three main data sources. The national accounts data and data on domestic prices are taken from the Czech Statistical Office (CZSO). The financial data (interest rates, exchange rates) for the Czech variables are published by the Czech National Bank (CNB). The data source of foreign variables for both the historical sample as well as for the exogenous two years-ahead forecast, is based on the Consensus Forecast (CF). This is a commercial product and it is published by the Consensus Economics Inc. We use their data for G7 and Western Europe, Eastern Europe and Foreign Exchange outlook ³⁷.

Let's start our data description with variables produced by the CZSO. The original data source for the real GDP and corresponding GDP deflator are seasonally *not adjusted* data with the base year of 2000=100. These data from Czech national accounts are available for the period of 1996Q1-2008Q3.

The primary data on the prices of cca. 730 items ³⁸, aggregated into the

³⁶We have in mind periods of recent economic history of the Czech Republic, when sudden changes in economic growth, the exchange rate or food prices should be detectable, with quantitative implications to the size of residuals.

³⁷For more detail on this data source see <http://www.consensuseconomics.com/>.

³⁸The detailed structure of the CPI is available on: <http://www.czso.cz>.

CPI, are also generated by the CZSO. These time series, similarly as GDP, are available in a seasonally unadjusted form. In order to separate the direct effect of indirect taxes on inflation, these highly disaggregated data are used by the CNB's Monetary Department's for the construction of analytic variables of higher aggregation, that are "cleaned" from tax change effects on prices as well as jumps in data due to the change in the weights of individual CPI representatives. By summing up all these tax-change contributions at a representative level with the corresponding weight will determine the overall direct contribution of taxes to inflation ³⁹ in total CPI. The potential jumps in inflation due to the change in the definition of CPI are eliminated by "bridging" the old and new basket by assuming a gradual - often linear - transition in the "constant" weights of CPI components from old to new weights. This highly disaggregated work with CPI representatives makes it possible to obtain price indices that enter into the model specified above. The CPI data in this disaggregated structure are available for the period of 1993Q1-2008Q3.

The data source for the domestic Pribor with three-months and one-year maturity and nominal exchange rate of the koruna against the EUR is the CNB (period 1993Q1-2008Q3). The foreign short-term interest rate - the Euribor with three months maturity - as well as the EUR/USD exchange rate is obtained from the CF. The foreign demand indicator is a real effective GDP calculated for the Eurozone. The weights of individual countries in the effective indicators, however, are not weighted according to their share in the Eurozone but according to their share of trade with the Czech Republic. All of these data are available for the 1993Q1-2008Q3 period, besides the foreign effective PPI, that is available for 1995Q1-2008Q3.

The data transformation necessary for being consistent with the model structure is made automatically within a MATLAB/IRIS routine. ⁴⁰ First all data that exhibit seasonal pattern (domestic and foreign GDP, prices) are seasonally adjusted. Financial market variables (exchange rate, interest rates) and prices of commodities (oil) are left seasonally not adjusted. The seasonal ad-

[//www.czso.cz/csu/redakce.nsf/i/spotrebni_kosod_lidna2008/File/spot_kos08.xls](http://www.czso.cz/csu/redakce.nsf/i/spotrebni_kosod_lidna2008/File/spot_kos08.xls)

³⁹Constant weights of representative goods and services of the year of 1995 are used.

⁴⁰In other words, it is sufficient to update the primary database (raw data), all other data related work is done reliably within seconds. This is a necessary condition for any replicable empirical work, that must be executed within tight time constraints.

justment is followed by a logarithmic transformation ⁴¹ for all variables in levels, except nominal interest rates, given the log-linear specification of the model. The quarterly percentage changes are expressed in annualized terms, multiplied by 100 ⁴². Since the determination of equilibrium values is not done by univariate filters (such as the Band-Pass or Hodrick-Prescott Filters), the estimation results for equilibrium values will be discussed in Section 4.3.

4.2 The calibration and solution of the model

The model has been calibrated not only in terms of coefficients of equations (1)-(25), but also with respect to the Kalman-filter hyper-parameters. Those, together with the steady-state parameters determine the model's equilibrium values on the historic sample as well identify the shocks. ⁴³

The parameters influencing dynamic properties of the model were set as follows:

The IS curve, equation (1):

$$a_1 = 0.70 \quad a_2 = -0.20 \quad a_3 = 0.10 \quad a_4 = 0.4$$

The share of domestic and imported goods in the net inflation basket, equation (2): $\alpha = 0.73 \quad \beta = 0.10$

Costs and wage contracting relationship, equations (4), (5) and Phillips curve (3):

$$\chi = 0.40 \quad \lambda = 0.48 \quad \omega = 0.20$$

Composition of imported inflation, equation (6):

$$b_1 = 0.45 \quad b_2 = 0.40$$

Rigidity in the UIP, equation (11):

$$c_1 = 0.50$$

Policy rule, equation (13):

$$d_1 = 0.65 \quad d_2 = 3.0 \quad d_3 = 1.00 \quad d_4 = 0.50$$

One year real-interest rate, equation (14):

⁴¹The growth rates based on a logarithmic transformation are of course approximations of the true values. They are exact for small values and exhibit increasing error for high growth rates.

⁴²This notation is the main reason for the multiplication by 100 or 400 in some of the model equations above.

⁴³The methodology of applying the Kalman filter for obtaining unobservable variables and the model's residuals will be described later in the text.

$$e_1 = 0.50 \quad e_2 = 0.50 \quad e_3 = 0.65$$

The size of IS curve elasticities are very similar to those in the literature that are focusing on the empirical modeling aspects of the Czech economy, see for example Coats et al. (2003), Hlédik (2003b) or Kotlán (2002). The 0.7 autoregressive coefficient reflects fairly high inertia characterizing the Czech real GDP data, explained mainly by observed real rigidities in consumption and investment behavior. The elasticity is, however, lower than previous calibration. The lowering of this coefficient improved the in-sample forecasting properties of the model and might potentially reflect some positive structural changes caused by large volume of FDI, making the economy more flexible. The elasticities with respect to real interest- and exchange rates are only slightly lower than calibrations in the above cited literature. Their values are motivated with overall model properties. Finally the calibration of the foreign demand gap coefficient is consistent with a 0.4 % rise in domestic real GDP for any 1 % increase in foreign demand. This relatively high elasticity reflects the significant trade openness and high dependence of the Czech economy on export orders.

The share of domestically produced and imported goods in the net-inflation basket are calibrated exactly according to Hlédik (2003b). The calibration values of the parameters was obtained by applying a numeric non-linear optimizing algorithm. The algorithm minimized the square of the residuals in equation (3) with respect to linear restrictions that all parameters in equations (4)(5) are positive and the share of administered prices in the CPI basket is known (0.17).

The main reasons for calibrating equation (3) by means of constrained optimization were the following:

- to be able derive the parameters of the wage-contracting equation implicitly without relying too much on labor market data, which are very short and subject to changes in data collection methodology over time;
- to obtain plausible parameter values that are consistent with equations(2), (4) and (5) as well as available data.

Parameters $\chi = 0.40$ and $\lambda = 0.48$ of equations (22)(23), which determine the

Phillips-curve (equation (3)), were set to obtain plausible impulse response functions for key shocks (to be captured by Figures 2 - 5.) as well as improve in-sample simulation results. The calibration of nominal rigidities also reflects the author's experience with practical forecasting during the last 3-4 years, namely, the revision of nominal rigidities in the Czech Republic downwards over time ⁴⁴.

The weights of foreign CPI, PPI and oil prices entering into the model's import price proxy π^* - that approximates the import price content of the CPI basket - are calibrated to reflect the share of imported final goods and imported intermediate goods used for the production of domestically produced goods. The calibration values are based on the analysis of imports, grouping SITC import groups into goods intended for consumption, investment and intermediate goods production.

The UIP condition requires only the calibration of the forward- and backward-looking share in the expected nominal exchange rate. The higher is the forward-looking share of agents in the expectation formation on the foreign exchange market, the most significant is the initial reaction of the exchange rate to shocks. The 0.5 coefficient is plausible in terms of the size of the exchange rate jump for most of the shocks and it also leads to acceptable overall model properties and implied macroeconomic "story". For comparison, this calibration value is the same as in Coats et al. (2003) and only marginally higher than the calibration in Hlédik (2003b) (0.4).

The calibration of the forward-looking Taylor-type policy rule is quite standard. The value $d_1 = 0.65$ implies fairly high level of instrument smoothing, higher than the first-pass calibration of the QPM model at 0.5, see again Coats et al. (2003). The higher value is more realistic within the conditions of a small open economy, in which shocks to the nominal exchange rate during volatile times could lead to the reversion in the direction of short-term interest rates from one quarter to another ⁴⁵ The closer look on the other parame-

⁴⁴There might be different reasons for this. One of the explanations might be the under-shooting of the inflation target by the CNB in recent years and the possibly the improved credibility of the inflation target - or low inflation, being below the target for years

⁴⁵Notice, that the high level of interest rate smoothing might also generate costs in terms of too sluggish elimination of shocks in the economy (too little, too late). For instance the lagged term for the short-term interest rates, entering into optimal rules in Hlédik (2003b) ranges from 0.43 to 0.52. Of course, caution is necessary in interpreting these

ters, however, reveals that the higher inertia is reflected in relatively lower elasticities for expected inflation and output gap, at least compared with the "benchmark" QPM calibration. This is quite logical. The cumulative effect of a one basis point shock to expected inflation resp. output be approximated by $\frac{d_2}{d_1}$ resp. $\frac{d_4}{d_1} = 2$. The comparison of these elasticities, adjusted for the corresponding AR(1) coefficients, show that the calibrations of the two interest rate rules is quite close (measured by the adjusted coefficients). Finally, parameter d_3 determines whether the central bank targets CPI (for $d_3 = 1$) or domestic (for $d_3 = 0$) inflation. The benchmark calibration is $d_3 = 1$.

Let's turn our attention to the steady-state parameters of the model, which were set as follows:

$$\begin{aligned}\pi^{*ss} &= 2.0 \quad \pi^{tar} = 2.0 \\ \Delta y^{ss} &= 4.5 \quad \Delta y^{*ss} = 2.0 \\ \Delta q^{eq^{ss}} &= -4.5 \\ \pi^{oil^{ss}} &= \pi^{cpi^{*ss}} = \pi^{ppi^{*ss}} = 2.0 \\ r^{eq} &= 0.75 \quad r^{*eq} = 0.75\end{aligned}$$

The steady-state of the model is very simple. Both domestic and foreign inflation targets π^{*ss} and π^{tar} are assumed to be 2 %. The domestic steady/state growth rate of the economy is set to 4.5 %, the foreign growth rate is 2.0 %. The real equilibrium appreciation of the Czech koruna Δq^{ss} is set to be -4.5 %, which is broadly in line with the rate of trend appreciation. Foreign prices that approximate imported inflation, $\pi^{oil^{ss}}$, $\pi^{cpi^{*ss}}$ and $\pi^{ppi^{*ss}}$ are calibrated to grow at 2 %, to be consistent with the steady-state growth of prices in the rest of the model ⁴⁶ Domestic and foreign real equilibrium interest rates are 0.75 %, the implied exchange rate risk premium in the long run is $prem^{ss} = 4.5\%$ ⁴⁷.

The calibration of the model being completed, let's technically describe the way how the model is solved. We rely on the Blanchard-Kahn solution, that

results too strictly. The change in the loss function of the central bank, as well as the model parameterization or its structure, can easily change these estimates. The level of interest rate smoothing, however, is limited by the Blanchard-Kahn (B-K) stability region - too much smoothing might prevent the central bank to stabilize the economy at all.

⁴⁶The fact, that these prices grow in the long run at 2 % do not mean that they cannot deviate substantially from this value for a longer period of time.

⁴⁷This is a direct implication of an identity arising from the real uncovered interest rate condition in the steady-state.

is described in detail in (7). After this technical discussion related to the B-K solution of the model, let's progress to the description of the methodology explaining the filtration process. The application of the Kalman-filter on the one hand determines the equilibrium values for the model, on the other hand it identifies the model residuals.

4.3 The determination of unobserved variables

The unobserved variables were determined in a model-consistent manner, by relying on multivariate KF. The following unobserved variables play a key role in terms of the interpretation of historic shocks: : equilibrium real interest rate r^{eq} , equilibrium real exchange rate q^{eq} and potential output y^{eq} . Of course, the identified equilibrium values, besides they determine the "gaps" in the model, also contribute to the quality of in-sample forecast results. In addition, the KF identifies the residuals of all equations of the model too.

Let's remind the key parameters of the KF by the steady-state values of the model from the previous section. The steady-state value of the real interest rate (both that of domestic and foreign IR) was set to 0.75 %, the rate of real exchange rate appreciation to -4.5 % and potential output growth to 4.5 %. Both domestic and foreign inflation targets are assumed to be 2 %. The steady-state risk premium, is determined by the UIP condition (11). Its implied value equals to 4.5. These steady-state values are important in a sense the equilibrium values of all variables will fluctuate around the corresponding steady-state values (or growth rates in the case of the real XR). The equilibrium values of all variables are modeled as AR(1) processes. The AR(1) coefficients are calibrated to broadly reflect the inertia present in these variables. The *relative magnitude of standard errors of the shocks* determining the equilibrium values, relative to s.e. other shocks in the model, will decide about how much the equilibrium values will differ from the steady-state.

The calibration of standard errors is the following:

$$\begin{aligned}\sigma^{y^{gap}} &= 0.015; \sigma^{\pi^{cpi}} = 0.01; \sigma^s = 0.05; \\ \sigma^{i^{3m}} &= 0.05; \sigma^{i^{1y}} = 0.01; \sigma^{r^{eq}} = 0.02; \\ \sigma^{\Delta q^{eq}} &= 0.015; \sigma^{\Delta y^{eq}} = 0.015; \sigma^{\Delta \pi^{oil}} = 0.01; \\ \sigma^{i^*} &= 0.025; \sigma^{r^{*eq}} = 0.01; \sigma^{\pi^{*cpi}} = 0.01;\end{aligned}$$

$$\begin{aligned}\sigma^{\pi^{*ppi}} &= 0.01; \sigma^{prem} = 0.05; \sigma^{\pi^{tar}} = 0.00001; \\ \sigma^{y^{gap}} &= 0.01; \sigma^{\pi^{adm}} = 0.01; \sigma^{\pi^d} = 0.01; \\ \sigma^{s^{cr}} &= 0.01; \sigma^{\pi^{imp}} = 0.02;\end{aligned}$$

Let's describe the way how the calibration of the standard errors of residuals were obtained. First, it is important to note, that there are probably more "plausible" calibrations of the model than the one specified above. In other words, the results of the KF are not "objective" estimation results, not the only "correct" interpretations of the recent economic development. They reflect judgment of the author about the timing, magnitude and implications some of the shocks that hit the Czech economy.

We believe, that due to the short time series available for this empirical analysis, the estimation of all parameters of the model (elasticities as well as standard errors) is not very realistic. Of course, formalized Bayesian estimation is relatively easy to conduct even on this short sample, due to the freely accessible toolboxes, being able to carry out such estimation ⁴⁸. The disadvantages of following the formalized, "mechanical" way of estimation are twofold. *First*, most of these Bayesian estimation results reveal, that data have not got enough information to move with the priors. *Second*, the pure estimation as opposed to "iterative calibration" do not enhance learning and might end up with results that are not in line with the intuition of policymakers or forecasters. This is the main reason for concentrating on the story telling implications ⁴⁹ of the calibration of standard errors: the identified equilibrium values for real output, exchange rate and interest rate and corresponding gaps as well as the identified demand and supply-side shocks should somehow intuitively correspond with the episodes of recent economic history of the Czech Republic.

The first step of the calibration has been setting all standard errors uniformly to the value of 0.01 ⁵⁰. The *relative magnitude of standard errors* will decide about the equilibrium values as well as the shocks identified in the system

⁴⁸Such a widely applied tool-box for solving forward-looking models and estimating them by Bayesian methods is for instance *DYNARE* developed by S. Adjemian, M. Juillard and O. Kamenik. For more information on Dynare see [http : //www.cepremap.cnrs.fr/dynare/](http://www.cepremap.cnrs.fr/dynare/)

⁴⁹This approach is followed also in Andrlé et al. (2009).

⁵⁰The absolute value of the standard error itself is not important, we could have set the initial value also to for instance 1, instead of 0.01.

of equations. The next step how we proceeded was to decide about those equations that should hold "tightly" or "loosely" ⁵¹. In our case we required the inflation target to hold tightly, therefore $\sigma^{\pi^{tar}}$ was set to 0.00001 to fit the inflation target exactly to the preset exogenous value. The standard errors of the nominal exchange rate equation residual σ^s , XR premium σ^{prem} as well as that of the policy rule $\sigma^{i^{3m}}$ were considerably relaxed relative to the uniformly set initial value. As far as the XR is concerned, the relatively large standard errors for UIP shock and premium reflects the judgment about the large volatility of XR shocks. The fact, that the s.e. of the interest rate rule residual is relatively large, is based on the fact, that our model and the corresponding policy rule is different from the CNB's QPM model specification that has been used for policy decisions for the most of the considered period. Therefore we do not require tight fit here.

The "fine-tuning" of other relative s.e. is somewhat more complicated. Let's illustrate this process on the example of the following three standard errors σ^{π^d} , $\sigma^{y^{gap}}$, $\sigma^{\Delta y^{eq}}$, to show how the simultaneous links between equations can be used for obtaining a results that can reflect intuition or judgment. The link between the relative s.e. of the Phillips-curve, IS-curve and potential output residuals is key for estimating the non-inflationary output ⁵². Setting the magnitude of the s.e. of potential output and output gap, relative to that of the Phillips-curve's, decides about how strongly will inflation influence the estimate of potential output (and output gap). The smaller the s.e. of the Phillips curve relative to the other two standard errors, the more demand shocks as well as potential output will explain from inflation and vice versa ⁵³. The actual values set for the standard errors above reflect the somewhat stronger feedback from inflation to the determination of the output gap. Of course, the Kalman filtration above is more than 3-dimensional. The final results were obtained by an iterative approach, through experimenting with many alternative parameterization of the filter, that were assessed on the basis of shock decompositions and in-sample simulations, to be discussed later.

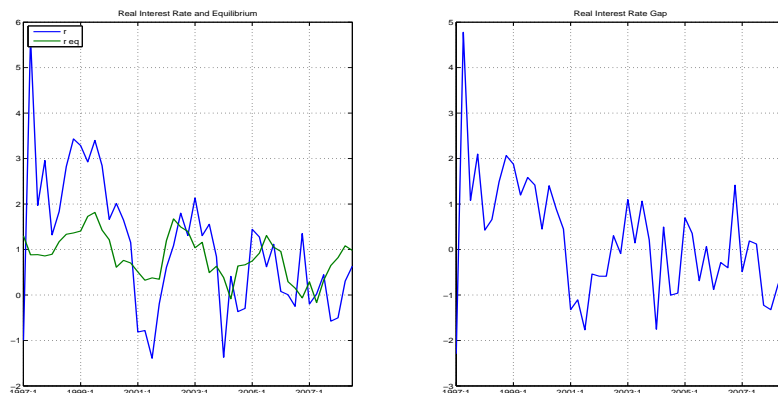
Let's take a closer look on the final filtration results. The graphs, that we

⁵¹"Tight" means small, "loose" large standard errors, relative to other s.e..

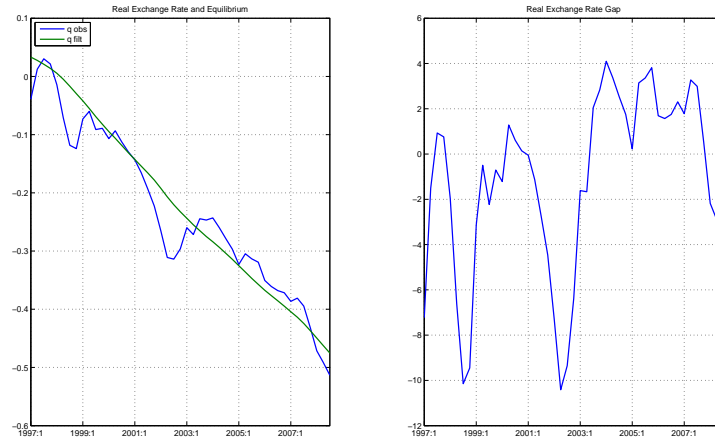
⁵²This KF link has been crucial in the interpretation of the potential output in the CNB's QPM model that has also been reflected in the communication of the CNB's forecast.

⁵³This is in sharp contrast with the approach that relies on HP filter equilibrium estimates.

shall gradually describe below, capture the actual and filtered values for the real interest rate, real exchange rate and real output, including the percentage change of their actual value from equilibrium (gap).

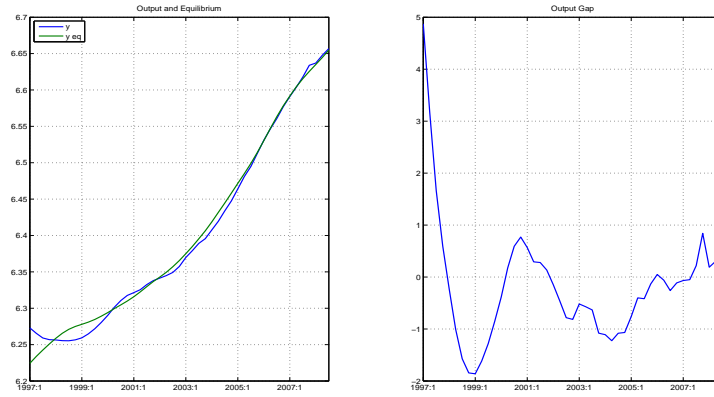


Eye-balling the *real interest rate* above and the estimated equilibrium real interest rate reveal few, intuitively acceptable findings. First, the real interest rate is probably the most volatile variable from the three variables examined above. For certain periods it is even not straightforward to determine, whether real interest rates are accommodative or restrictive. This is especially true for the period of mid 2003 - mid 2006, during which real interest rates could be interpreted at given volatility as broadly neutral. The restrictive stance of real interest rates immediately after the foreign exchange crises in spring 1997 is self-evident. That was the period, when the CNB defended the exchange rate after exiting from the fixed XR regime. This restrictive stance lasted until the beginning of 2001, when real interest rates became accommodative. This accommodative stance is closely linked to the next very strong appreciation period, when the CNB tried to curb the appreciation by cutting interest rates quite aggressively. After the period of relatively neutral stance real interest rates become more relaxed again after mid 2007. This period is linked again with an appreciation of the nominal exchange rate and stabilizing reaction of the Czech central bank.

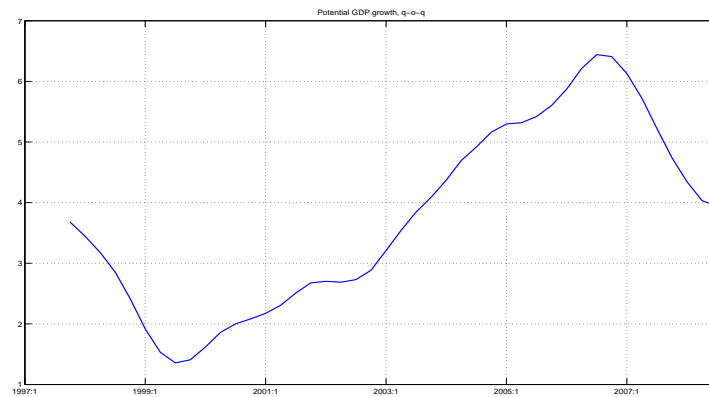


The development of the *real exchange rate* is key for understanding the recent Czech economic history. Given the significant appreciation and depreciation periods and the high trade openness of the Czech economy, the real exchange rate is one of the key contributors to real monetary conditions in the IS-curve⁵⁴ By looking on the real exchange rate and its equilibrium, the periods of strong resp. weak real exchange rate is more straightforward than in the case of the real interest rate. There are two clearly distinguishable periods when the real exchange rate was over-valued. The first period is linked to the sharp appreciation of the exchange rate after the 1997 crisis, the second the already mentioned end 2001 - beginning 2002 rapid XR appreciation, the causes of which are still not entirely clear. The recent appreciation period at the end of 2008 is also signal overvalued XR, but in terms of equilibrium appreciation it is less restrictive stance than the other two episodes.

⁵⁴The *real monetary conditions index* (MCI) is defined as a weighted average of the real exchange rate and interest rate, where the weights correspond with the corresponding elasticities in the IS curve.

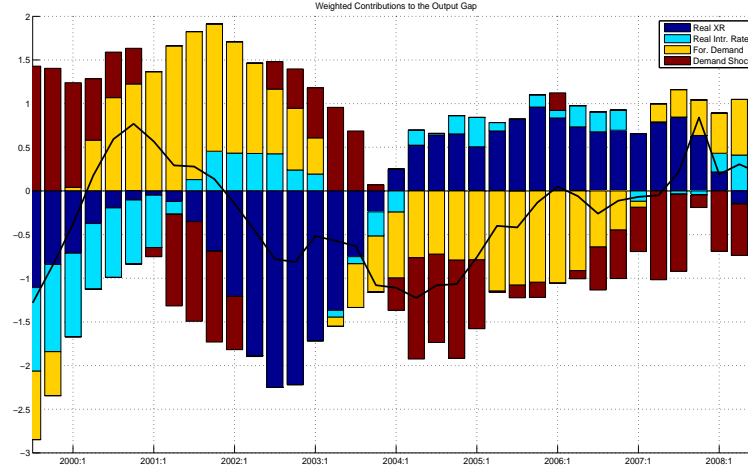


The actual and filtered value of *real output* is consistent with two distinct periods of negative output gap. The first is the after- the-crises recession, that hit the Czech economy in 1997. The economy, that was strongly overheated at the beginning of 1997, decelerated rapidly and subsequently hit its bottom at the beginning of 1999. The second, even longer period of negative output gap starts at the beginning of 2002 and lasts until end 2006. The already mentioned two sharp appreciation periods of the real exchange rate is one of the main reasons for the negative output gaps for these periods. The more detailed analysis of the main contributors to the negative output gap, however, will be analyzed below.



The next graph above captures the growth of potential output. It implies an increasing growth rate of the potential growth of the economy since mid 1999 until the beginning of 2007. This rapid growth in the growth rate of potential

output is line with ample FDI characterizing this period inflows, especially in manufacturing, resulting in a high growth of real exports.



The decomposition graph above, that is based on the quantification of the contribution of individual factors explaining the dynamics of the output gap. It helps to shed light on the strength of real exchange rate, interest rate foreign demand channels as well as demand shocks on the domestic output gap ⁵⁵. The decomposition results seem to be quite intuitive. There are two distinct periods of strong resp. weak cumulative foreign demand effect on the Czech business cycle during the 3Q 1999 -3Q 2008 period. The first period starts in mid 2000 until the end of 2002, the second, negative period begins in mid 2003 until end 2006. These periods, of course with some delay, correspond with the periods of the positive resp. negative foreign effective output gap. The other most important factor, causing significant fluctuations in the Czech business cycle, is the deviation of real exchange rate from its equilibrium level. The shock decomposition implies, that the most important factor, explaining the significant economic slowdown starting from mid 2002, was the sharp negative deviation of the real exchange rate from equilibrium at the end of 2001 and the beginning of 2002. The graph also reflects the attempt of the CNB

⁵⁵It has been calculated very simply. We start with the IS curve equation (1). By applying a lag operator for equation (1), we express y_{t-1}^{gap} and substitute out this expression from the original IS curve. We will repeat this as long as we obtain y_t^{gap} as an infinite sum of lagged right-hand-side variables. The graph of course uses a finite sum, by summing up the cumulative effect of all variables for the past 10 quarters.

to mitigate the negative impact of the real XR appreciation on growth by creating pushing the real interest rates below equilibrium level. The graph reveals too, that monetary policy has been only partially successful in doing that. From the beginning of 2004, almost at the same time when the foreign output gap had a negative impact on Czech economic growth, the exchange rate, together with slightly accommodative MP, quite successfully dampened the effect of low foreign demand on the Czech output gap.⁵⁶ Without the negative demand shock⁵⁷ over the course of 2004 the output gap would be close to neutral even during 2004.

4.4 Verification of the calibration ad 1: shock decomposition

Similarly as in the case of unobserved variables, the shocks corresponding with the model equations are determined by the Kalman filter. The shocks decomposition determines how the deviation of every endogenous variable from its steady state value in the model is determined by the current and past shocks that were identified by the Kalman filter during the filtering process⁵⁸. In order to keep the quantification of the effect of shocks focused, we created group of shocks, whose cumulative effect will add up to the value of the variable under consideration. The groups we created are the following:

1. Demand= $\{ res_t^y \}$
2. UIP= $\{ prem_t, res_t^s \}$
3. MP Vars.= $\{ res_t^{i^{1y}}, res_t^{i^{3m}}, res_t^{\pi^{tar}} \}$
4. World= $\{ res_t^{\pi^*}, res_t^{\pi^{oil}}, res_t^{\pi^{*ppi}}, res_t^{\pi^{*cpi}}, \}$
5. DomPr= $\{ res_t^{\pi^r}, res_t^{\pi^d} \}$
6. ImpPr= $\{ res_t^{\pi^{imp}} \}$
7. AdmPr= $\{ res_t^{\pi^{adm}} \}$
8. Eq= $\{ res_t^{r^{eq}}, res_t^{\Delta q^{eq}}, res_t^{\Delta y^{eq}} \}$

⁵⁶In small open economy models the fall in foreign demand often results in a depreciation of the exchange rate, mainly through the deterioration of net foreign assets (NFA).

⁵⁷Negative demand shocks often capture fiscal policy, that is directly not modeled here or some other out of model information, respectively misspecification of the model.

⁵⁸The impact of past shock on current variables will be a function of the AR(1) coefficient, that determines the dynamics of the model after being solved, in our case by the Blanchard-Kahn algorithm.

The first group quantifies the demand shock, the second the shock to the policy rate and market interest rates, the third the impact of foreign variables, including oil prices, foreign PPI and CPI and shocks to import prices. The next group involves the residuals effecting domestic prices. This is followed by groups quantifying the effects of import- and administered prices. We close the line with all shocks determining the equilibrium values for the real interest rate, exchange rate and real output.

Figures 6 - 9, capturing the decomposition of monetary policy inflation, output gap, change in the nominal exchange rate and the policy rate are included mainly to identify the factors that potentially explain the deviations of in-sample simulation results from actual outcome. We will use these graphs to refer to them in Section 4.5. Before we switch to the actual results of in-sample simulations, let's spell out some observations related to the significance of shock decomposition results in general and to the obtained empirical results themselves.

The shock decomposition of a kind, that is presented in Figures 6 - 9 helps to assess the impact of unobservables (structural shocks or shocks that contribute to the determination of equilibrium values, etc.) or exogenous variables, that are often modeled as AR(1) processes. They enable to check the "economic story" and reveal, whether the shocks empirically identified are correlated with some episodes, when out-of-model information could explain these shocks.⁵⁹

We would like to present, how certain episodes during the period of the examined Czech economic history can be supported by the presented shock decomposition results. Eye balling MP inflation on Figure 6 reveals, that the above mentioned nominal appreciation episode of end-2001, 2002 nicely show up since the beginning of 2003 in form of the UIP shock and gradually diminishes over 2003. It also can be seen, that MP during this period eliminated some of the appreciation effect on MP inflation. The recent significant 2008 1Q increase in regulated prices is an important factor behind the jump of CPI inflation in 2008. Figure 7, that depicts the output gap. The real

⁵⁹A good example might be fiscal policy. In our model, for instance, there is no fiscal block. Therefore we would expect any discretionary fiscal spending to show up as an IS curve residual. When the identified shock fits such episodes, the shock identification process might help to implicitly verify the model results.

exchange rate appreciation, induced by the identified UIP shocks show up in full strength during 2002. Word demand has been positive all over 2000 and beginning 2001. The demand shock curbs the output gap quite significantly for the same period. The shocks to domestic prices, that exerts a positive effect on the output gap over 2008-9, might be an effect of decreasing price mark-ups over wages. The shock decomposition of the change in the nominal XR in Figure 8 explains the sudden appreciation of the XR at the end of 2001 beginning 2002 by a strong negative UIP shock. The UIP shock changed its direction since the second half of 2003 and the foreign variables as well as MP contributed to the correction. The UIP shocks also explain the strong appreciation period at the end 2007 beginning 2008. The last graph, Figure 9, depicts the factors shaping the development of the policy rate. The stabilizing effect of MP on the UIP shocks is evident for the 2001-2 appreciation period. External sectors pushed interest rates up all over 2007-8. MP variables pushes interest rates up as well, partially due to the change in the inflation target.

4.5 Verification of the calibration ad 2: in-sample simulations

Running in-sample simulations is one of the frequently applied methods for evaluating the quality of the model's empirical relevance. Let's first explain how these in-sample simulations have been executed. First the model's equilibrium values have been obtained by means of a the KF for the entire sample period available (2006Q1-2008Q3) ⁶⁰. We also assume, that exogenous domestic and foreign variables are known for the whole historic sample and the forecast of exogenous variables from the latest CF for the period of 2008Q4-2010Q3 is available too.

Based on the knowledge of the model, its parameters and equilibrium values that were derived from the KF based on the 2006Q1-2008Q3 period, we produce a two-years-ahead forecast for every single quarter, starting from 1Q 1998 and ending 3Q 2008. It is important to stress, that the simulations are carried out under the assumptions, that the shocks, that are consistent with

⁶⁰The filtering exercise has been described in more detail above.

the forecast of exogenous variables are *unanticipated*⁶¹. We believe, that the assumption of unexpected shocks is closer to a real-life situation, in which the agents learn about the realization of shocks only gradually. The fully anticipated shocks might have relative strong implications at the beginning of the forecast horizon, given forward-looking expectations. Therefore, when we assume fully anticipated shocks in situations when the shocks in reality were not expected, we might deviate with the model forecast from reality.

The in-sample model simulations are depicted below. The first look on the results reveals, that there are periods of the examined sample when the two-years-ahead model forecast is very close to actual data as well as periods when the forecast error is quite significant. We will make an attempt to find the causes for the forecast errors and show the links between the presented macroeconomic variables.

Let's start our discussion with the forecast of *CPI inflation*. We identify two clearly distinguishable periods, when the model forecasts are consistent with higher inflation rates than what is present in the data. The first is the end-2008, 2009 period when the Czech National Bank defended the domestic currency after the foreign exchange crises after May 1997. The implied high interest rates, that resulted in a sharp appreciation of the XR, fall in import prices, the output gap and subsequently also a fast deterioration in the inflation rate. The second period is the episode of the unexpected 2001-2 appreciation. Let's note, that for that period we also identify significant exchange rate shocks, as it can be verified on the exchange rate shock decomposition, captured in Figure 8. Due to the strong exchange rate we identify the same anti-inflationary factors as in the previous case: import prices and the output gap. In the subsequent periods the model forecast relatively well fits the reality, with some outliers from time to time. The *domestic inflation* predictions are somewhat less impressive as in the previous case, although

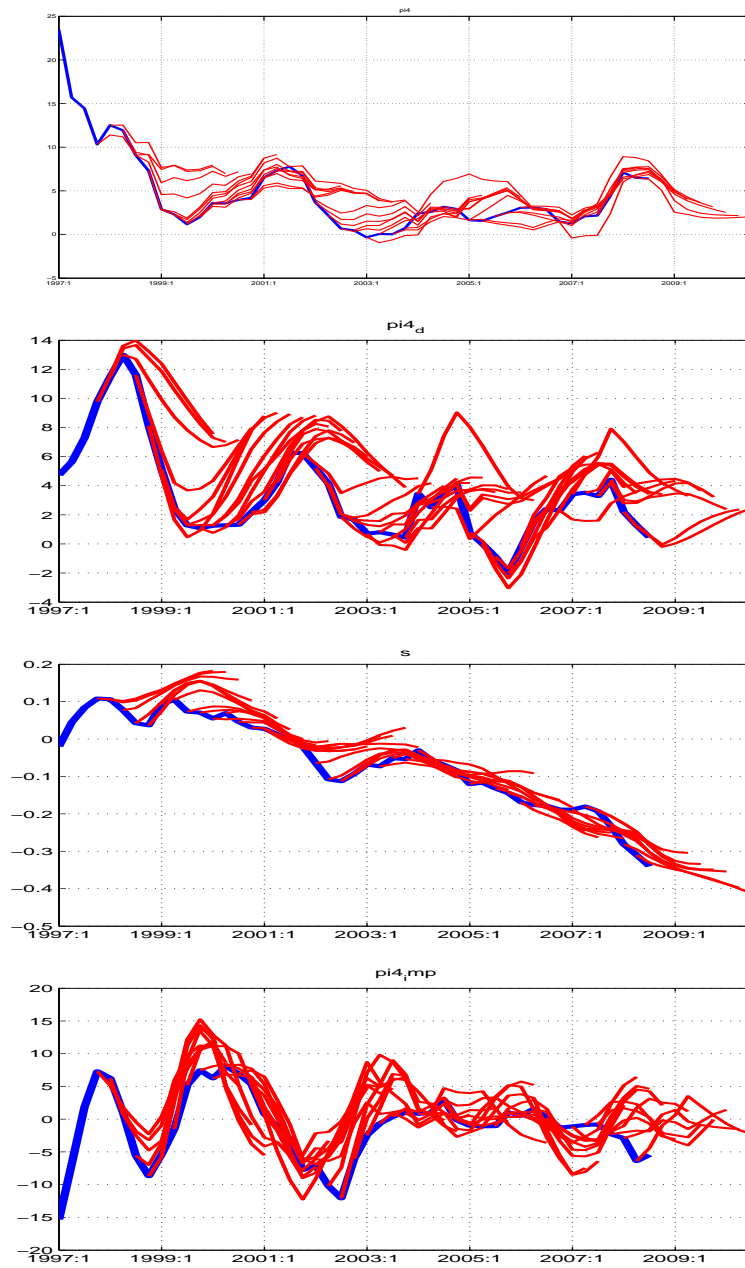
⁶¹The exogenous variables are modeled as AR(1) processes. The knowledge of the forecast of a given exogenous variable is equivalent with the identification of the shocks that will generate the desired forecast trajectory. The difference between *anticipated* and *unanticipated* shocks is quite substantial. When the shocks are anticipated, the agents in the model know - at the time when the forecast is made - the exact realization of the shock for the whole forecast horizon. In the opposite case, for every period they are "surprised" with the new shock and for the subsequent periods they expect a return to a steady-state, based on the AR(1) process driving the given exogenous variable.

there are some quite prolonged periods, when the model forecasts reasonably well (for instance 2005-6). The forecasts generated during 2000-2001 expected faster increase in domestic inflation than the actual outcome. This might indicate still too little inertia in the Phillips curve calibration or the presence of some unexpected shocks, that were hitting the economy. Another low quality forecast is the one, that has its peak at the end of 2004. This indicates, that the lagged q-o-q domestic inflation, that is very high, has a strong positive impact through inertia on the forecast and it takes a year for that base effect to disappear. The forecast of the *nominal exchange rate* is surprisingly good overall, but this, of course does not mean that we have got a good model for forecasting the nominal exchange rate. The satisfactory model performance obviously reflects the fact, that the calibration has been done on the basis of the knowledge of the whole depicted history. As a consequence, the trend appreciation of the real XR, for instance, could well be calibrated ⁶². Nonetheless, the two sudden appreciation periods, mentioned previously, are not captured with the model. That can also be seen in the y-o-y import price forecast on the graph below. The deviation of *import price* predictions from actual outcome are a reflection of what we observed for the exchange rate forecast. The model forecast exhibit a little bit of less inertia than actual data. This also can be attributable to the assumption, that the exchange rate and foreign inflation feed into import prices without any delay, immediately. The *output gap* projection is quite close to the actual outcome, although there are some periods again when the fit is relatively loose, especially for the 2000-2002 period. This overshooting, however cannot be understood without analyzing the real XR and IR development, important determinants of the output gap. The model forecast, despite the fact that expected higher inflation for the 1999-2000 period was consistent with lower nominal interest rates for all over 1998-2000. The low nominal short-term rates were consistent with very negative real interest rates as well as for certain periods weaker real XR than it was the case in reality. Consequently real monetary conditions were looser than the data suggest and this more accommodative policy in the model simulations resulted in a higher output gap for the considered period.

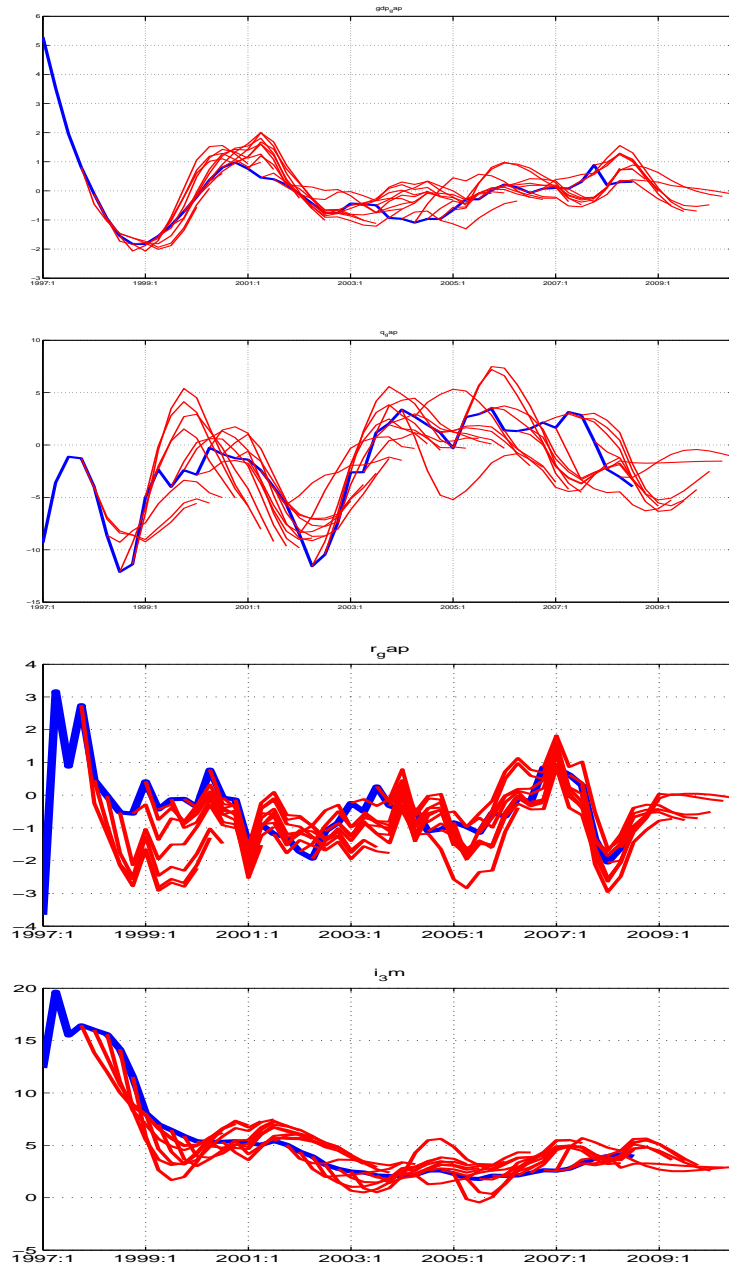
⁶²In practical policy making at the CNB there were at those times many discussions and existed large uncertainty related to the rate of equilibrium real XR appreciation as well as the right level of XR risk premium

The end-2003, 2004 period also reflects a relatively significant positive forecast error, that is partially a result of relaxed real interest rate setting prior to the considered period. To understand these forecast errors better, it is also useful to look into shock decomposition Figure 6 produced for the output gap. For the considered period we identify negative demand shocks, that could be part of an explanation for lower than forecasted output gap development. The *real exchange rate gap* forecast errors are very much the reflections of the previously discussed errors with respect to domestic inflation and the nominal exchange rate. The nominal exchange rate overshootings after 1999 and the periods of domestic inflation overshootings during the 1999-2000 period are the most significant explanations for the errors. Finally, we shall discuss the *real interest rate gap- and nominal interest rate* forecast together, since they are closely linked to each other, through expected inflation. As mentioned earlier, the mid-2007, 2008 period was consistent with higher nominal and real rates than the forecast would suggest, mainly due to the strategy of the CNB to defend the Czech koruna. The somewhat higher interest, rates generated by the model for the early 2007 period, are attributable to higher inflation forecasts generated one year earlier. To sum up the results altogether, we consider the model's in-sample forecasting performance fairly good, the results suggest that the calibration is able to explain some of the important episodes in the recent economic history of the Czech Republic.

In-sample simulation results I.



In-sample simulation results II.



5 Policy rules and their evaluation

5.1 The motivation of alternative policy rules and their specification

We will consider two classes of rules. The first class will be based on targeting CPI inflation, the second class will be concerned with domestic prices. The question that naturally arises, why should we examine the properties of alternative policy rules? What could be the motivation of the CB to target domestic inflation instead of CPI inflation? Considering CPI inflation is straightforward. This is the most frequently targeted index among IT countries, therefore it is a natural benchmark. The good reason for considering domestic inflation instead of CPI inflation is the presence of highly volatile import and administered prices in the CPI basket (see equation (2)). When the CB targets domestic inflation instead of CPI inflation, it ignores the direct effects of import and administered prices on inflation. Consequently it will not try to change the short-term interest rate due to the sudden change in these volatile items, to bring inflation faster back to target. The central bank, however, will react to any second-round effects of changes in import or administered prices that will feed into wages and subsequently to domestic inflation. Thus these volatile items in the CPI basket have got the potential through the wage channel to influence domestic prices.

As mentioned earlier, we will evaluate two classes of simple inflation forecast based policy rules. Both classes are incorporated in the functional form of the reaction function (13). For the first class of rules we will assume, that the central bank targets CPI inflation without the direct effect of indirect tax changes. This, currently most widely applied MP strategy among IT central banks, is captured by the reaction function, where d_3 is set to 1. For the second class of rules the central bank reaction function will feed back from domestic as opposed to CPI inflation. In this case the parameterization of the reaction function (13) will be consistent with d_3 to be set to 0. The way of obtaining the optimized weights of the reaction function is described in section

5.2 Comparison of policy rules by impulse response results

In order to compare the difference between domestic vs. CPI inflation targeting, the first straightforward task is to analyze the impulse response results. They were produced for the most important shocks in the model; we considered the following ones:

- Demand shock
- Shock to domestic inflation
- Shock to the exchange rate risk premium
- Shock to the policy rate

(i) Demand Shock

The positive demand shock ⁶³ will induce an immediate rise in the policy rate, through various channels. It generates excess demand and through the positive output gap feeds into the policy rule. Moreover, the positive output gap leads to higher domestic inflation, both via the higher output gap in the Phillips curve as well as through an increase in domestic inflation expectations. Note, that in the case of domestic inflation targeting (D-IT) nominal interest rates will be higher in comparison with CPI targeting (CPI-IT). The reason for that is the fact, that the immediate inflationary impact - that must be curbed by monetary policy - is higher for domestic than CPI inflation, due to the anti-inflationary effect of appreciating exchange rate that feeds into import prices. The appreciation of the nominal exchange rate itself is generated via the UIP condition, when the reaction of the central bank to the shock opens the interest rate differential ⁶⁴. In case of D-IT the initial appreciation is slightly deeper, explained by the relatively stronger initial interest rate reaction to the shock. Due to slightly higher real interest rates as well as the more appreciated real exchange rate in the D-IT case the output gap is a little

⁶³Notice, that the positive 1 b.p. shock to the output gap results in a less than 1 b.p. effect on the output gap on impact. The main reason lies in the specification of the IS curve. Since real exchange rate q_t^{gap} enters into the output gap equation with a zero lag, the appreciation of the real exchange rate immediately puts a downward pressure on real output.

⁶⁴Foreign interest rates are assumed to be at the steady-state level.

bit more negative for the initial few quarters. After 3-4Q, however, the impact of the nominal exchange rate on CPI inflation as well as domestic prices dominates the other effects. The results for the two targeting regimes are very much in line with the intuition: the targeted index is always converging faster to the target, since the policy instrument is set with respect to the deviation of the targeted variable 4Q ahead from inflation target. The difference in the two scenarios is very much given by the share of imported prices in total CPI. As soon as the share of import prices in the CPI would converge to zero, the difference between the two targeting scenarios would diminish accordingly.

(ii) Shock to domestic inflation

The 1 b.p. supply shock to the Phillips curve generates a *more than a 1 b.p.* immediate contribution to domestic price inflation. This is because of the presence of an expected inflation term π_{t+1}^d in the Phillips curve. This term reflects the expectations of the model's forward-looking agents. They understand that the shock will be transmitted into the next quarter, due to the model's nominal rigidities and despite the stabilizing interest rate reaction of the central bank. The shock, obviously, induces a simultaneous dynamic response of the model. All variables, that are a function of forward-looking expectations - inflation, exchange rate and interest rates - will jump to the trajectory determined by the model-consistent forward-expectation solution. The policy rate, similarly as in the case of the demand shock, will be more pronounced in the case of D-IT, compared with CPI-IT. The nominal exchange rate, consistent with this IR reaction, will slightly be more appreciated for the first quarter in the D-IT scenario. The difference in the interest-rate reaction between the two scenarios emerges from the comparison between real interest rate and exchange rate gaps. In the D-IT case the slightly higher inflation and lower interest rates during the 3Q to 6Q period results in more relaxed real interest rates and more restrictive real exchange rates in comparison with CPI-IT, and vice versa. The difference between the two scenarios is, however, relatively small. Only for a large shock to domestic inflation, potentially hitting the mark-up or wages, could cause measurable difference in the response of the economy depending on the IT strategy.

(iii) Shock to the exchange rate risk premium

The shock to the XR risk premium by 1 b.p. generates an immediate effect on the nominal XR by more than 1 b.p. The reason for the more than 1 b.p. jump in the nominal XR, similarly as in the other two simulations described above, is the presence of forward-looking XR expectations 1Q ahead in the UIP equation and the fact, that the risk premium is modeled as an AR(1) process, with a coefficient of 0.5 ⁶⁵. Despite the fact, that short-term policy interest rates react immediately to the shock, the nominal and real inertia in the model will prevent the model variables to jump in the next period to the steady-state. For D-IT the interest rate reaction to the shock is higher than for CPI-IT. This is caused by the higher initial jump in 4Q-ahead domestic inflation compared with CPI inflation ⁶⁶ at the same level of the output gap for both targeting regimes. The higher interest rates in the two considered targeting regimes will generate the same pattern that we have seen in the previous simulations: the real interest rate dominates the real exchange rate. As a result the output gap falls more in the D-IT targeting regime compared with the alternative.

(iv) Shock to the policy rate

The shock to the policy rate results in an increase of the real interest rate - due to a jump in the nominal interest rate with one year maturity (through the yield curve equation) and the fall in both domestic and CPI inflation on the forecast horizon ⁶⁷. The nominal exchange rate jumps more than the magnitude of an after-the-shock fall in the domestic price level, therefore, the real exchange rate appreciates. The appreciation, accompanied by the rise in real interest rates, opens a negative output gap. The excess supply puts a down-

⁶⁵In other words, the shock will live for a while.

⁶⁶Although this is obvious, but the 4Q lead comes from the specification of the reaction function.

⁶⁷The falling inflation expectations reflect the anti-inflationary effect of positive real interest rates and exchange rate on the output gap as well as the nominal appreciation of the exchange rate resulting from the jump in the interest rate differential.

ward pressure on wages and subsequently on domestic prices. The nominal exchange rate appreciation, through the direct exchange rate channel, lowers import price inflation. Consequently CPI-inflation falls below target due to both low demand pressures as well as imported inflation. The output gap is negative for the first three quarters, therefore inflation starts to return to the target after one year. The main difference between D-IT and CPI-IT stems again from the reaction of monetary policy to import price inflation within the CPI index and the absence of it in the D-IT case. The fast switch of the nominal exchange rate from appreciation to depreciation from the 2Q of the simulation results in a slightly higher CPI inflation than domestic inflation 4Q ahead ⁶⁸ at comparable level of initial output gap in the two targeting regimes. The higher real interest rates as well as real exchange rate in the initial quarters results in a deeper negative output gap in the CPI-IT case compared with the D-IT. Therefore the policy shock generates higher volatility for the output gap and inflation for the CPI-IT case.

5.3 Alternative simple policy rules with optimized weights and their evaluation

This section focuses on the evaluation of alternative policy rules. Let's start our discussion by explaining how we obtain the optimized coefficients of the forward looking simple rules that we motivated and described in Section 5.1. Since the baseline parameters of the reaction function 13 were calibrated to achieve plausible empirical properties of the model overall, we decided to search for optimized values of some of the coefficients of the reaction function (13) by simple *grid search* method. In order to keep the dimensionality of the optimization problem relatively low ⁶⁹ we have decided to keep the coefficient of the lagged short term interest rate d_1 fixed at 0.65. At the same time search the inflation resp. output gap coefficients of the policy rule for values $d_2 = 0.40 + i * 0.75$; resp. $d_4 = 0 + j * 0.5$; where $i, j = 0 \dots 10$. The range of

⁶⁸Four quarters is the assumed targeting horizon.

⁶⁹We search for optimal reaction function coefficients with respect to inflation and output gap. Since we evaluate the properties of the resulting policy rules by minimizing the loss function of the CB (to be specified later), the problem is already three dimensional. In order to depict the corresponding graphs, we decided only for changing two coefficient of the policy rule.

$\langle 0.40 \ 7.90 \rangle$ resp. $\langle 0 \ 5.0 \rangle$ seem to be sufficient for obtaining optimized weights for the considered coefficients. At the same time for d_2 the value lower than 0.4 is close to the B-K (in)stability region.

We will optimize the performance of the two above mentioned classes of policy rules with respect to the perceived *loss function* of the central bank. We define the loss function in terms of unconditional variances of the deviation of inflation from target, real output from equilibrium and short-term interest rates from their equilibrium level. To put it into a formalized framework, the loss function is defined by the following equation:

$$\begin{aligned} L_t = & w^i(i_t^{3m} - i_t^{eq3m})^2 + w^\pi(\pi_t^{mp} - \pi_t^{tar})^2 \\ & + (1 - w^i - w^\pi)(y_t - y_t^{eq})^2 \end{aligned} \quad (26)$$

where: w^i, w^π are the weights put on corresponding policy variables in the loss function by the policymakers.

The loss function of the central bank, the sum of unconditional variances of the short-term interest rates, inflation and output, is, therefore, defined as follows:

$$E(L_t) = w^i var(i_t^{3m}) + w^\pi var(\pi_t^{mp}) + (1 - w^i - w^\pi) var(y_t) \quad (27)$$

where: $var(X)$ denotes the variance of variable X , for $X = i_t^{3m}, \pi_t^{mp}, y_t$.

Before turning our attention to the specification of the discussion about the chosen weights w^i, w^π of the loss function as well as the results of the optimization of the simple rules, let's briefly describe how the unconditional variances are calculated within our model framework ⁷⁰.

In line with the description in chapter 4.2 first we solve the model by the Blanchard-Kahn methodology. The solution enables us to write the model in the following general form:

$$\xi_t = \mathbf{F}\xi_{t-1} + \nu_t \quad (28)$$

⁷⁰Our exposition will closely follow that of Hamilton (1994) p. 259, similarly as it has also been done in Stráský (2005).

where ξ_t is the vector of endogenous variables, expressed in terms of deviation from their mean. Matrix \mathbf{F} is a function of all parameters of the model, determined also by the B-K solution of the model for forward-looking variables⁷¹. Vector ν_t consists of the model residuals. For calculating the unconditional variances for all model variables, we rely on the following recursive relationship:

$$\Sigma \equiv \mathbf{E}(\xi_t \xi_t') = \mathbf{F} \mathbf{E}(\xi_{t-1} \xi_{t-1}') \mathbf{F}' + \mathbf{E}(\nu_t \nu_t') \quad (29)$$

where Σ is the unconditional variance, \mathbf{F} matrix determining the dynamics of the model, based on the B-K solution, $\mathbf{Q} \equiv \mathbf{E}(\nu_t \nu_t')$ is the variance-covariance matrix of the stochastic shocks entering into the model. Based on the knowledge of all model parameters and the variance-covariance matrix \mathbf{Q} , we can calculate the unconditional variances for all endogenous variables of the model, based on the following relationship, that is derived in Hamilton (1994):

$$\text{vec}(\Sigma) = [\mathbf{I}_{r^2} - \mathbf{F} \otimes \mathbf{F}]^{-1} \text{vec}(\mathbf{Q}) \quad (30)$$

where $\text{vec}(X)$ denotes matrix vectorization for $X = \Sigma, \mathbf{Q}$. \mathbf{I}_{r^2} is an identity matrix with r^2 rows ($r = np$, where n is a number of variables, p the number of lags in structural specification of the model). Matrix operation \otimes denotes the Kronecker product.

After pinning down some of the technical details of the methodology of calculating the unconditional variances of the model, let's turn our attention to the actual evaluation of the two classes of policy rules specified above. As mentioned previously, we will grid-search for the coefficients of the policy rule (equation 13) to minimize the loss function, specified by equation 27.⁷²

To keep the analysis relatively simple, we decided to examine 3 alternative loss functions of the central bank. Since we consider interest rate smoothing

⁷¹It is useful to be aware of the fact, that any linear model with forward-looking model consistent expectations can be transformed into form 28 after being solved, in our case by the B-K method.

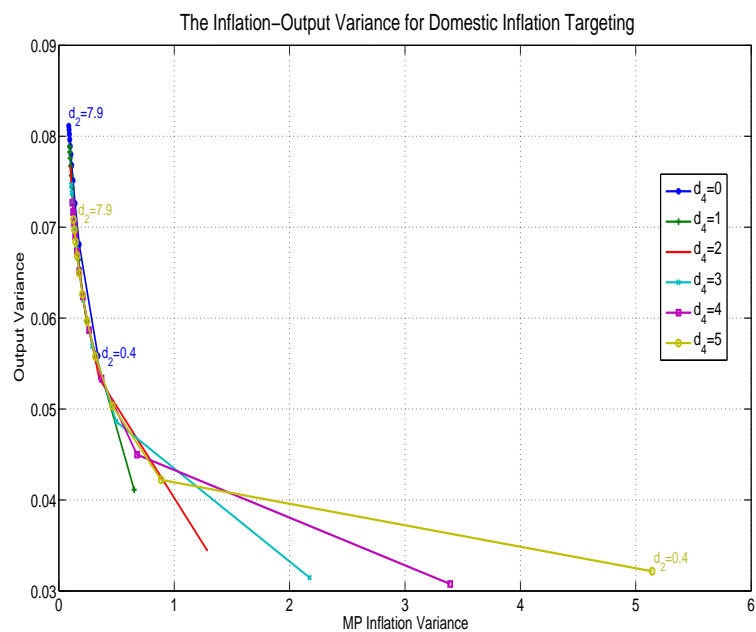
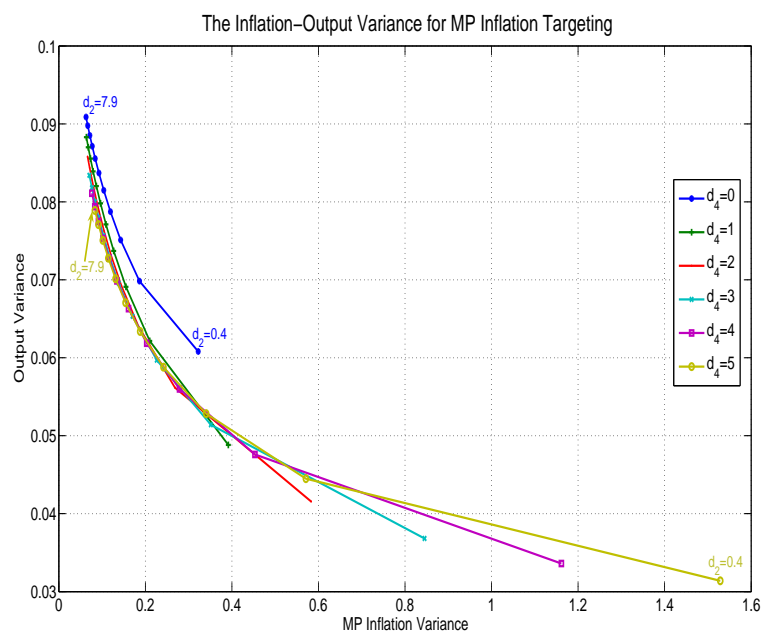
⁷²The alternative to optimized simple forward looking rules are optimal rules, as described in section 2.2. The author of the thesis deliberately decided not to go in this direction. The first reason is that he had already derived optimal rules in a structural model framework (see Hlédik (2003b)). The second reason is that optimal rules are not applied often in practical policy due to the lack of robustness and complexity of optimal rules.

as an inevitable characteristics of CB's behavior - due to its communication and uncertainty related rationale ⁷³ - but not as a primary goal of MP as such, we will keep both the loss function, we will fix both the weight of short term interest rates in the loss function, similarly the coefficient of interest rate smoothing in the policy rule. We have decided for three alternative parameterizations. In the first case the central bank puts zero weight on the variance of output and 90 % weight on the variance of inflation. In the second case the weight on output and inflation variance is equally 45 %. In the last case the CB's emphasis is put on output variance with a weight of 90 %. The weight on short term interest rate smoothing is in all cases 10 %. To summarize:

weights	w^i	w^π	w^y
$w^\pi > w^y$	0.10	0.90	0.00
$w^\pi = w^y$	0.10	0.45	0.45
$w^\pi < w^y$	0.10	0.00	0.90

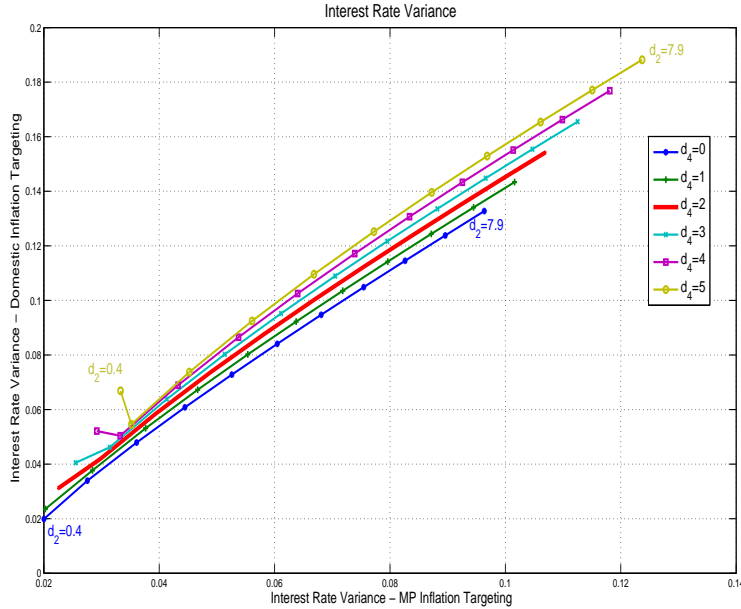
The loss functions being specified, let's start examining the two classes of policy rules within the already declared parameter space for the inflation resp. output gap coefficients in the policy rule $d_2 \in \langle 0.40 \ 7.90 \rangle$ resp. $d_4 \in \langle 0 \ 5.0 \rangle$, first in a two dimensional space, determined by the variance of inflation and output. The following two graphs capture the variance of MP inflation and output, for all possible combinations of the policy rule within the parameter space:

⁷³Central banks often argue, that a sudden change in interest rates, especially when it is accompanied with fast change in the direction of interest rates, can destabilize expectations and complicate the signal extraction of economic agents. That is why most of the CBs will be unwilling to increase the flexibility of interest rate reaction above a certain threshold, despite the fact that in deterministic model simulations the higher flexibility of interest rate reaction will be consistent with more powerful control of the CB over inflation stabilization.



The results for both targeting regimes confirm, that there is a nonlinear relationship between inflation and output variance depending on the parameterization of the policy rule. The higher is the weight on the output gap in the policy rule, the lower values for output variance are attainable. This is true, however, only for the MP inflation targeting case. For domestic inflation targeting this holds only to a certain threshold. The graphs above, capturing the D-IT case show, that the output variance is lower for the combination of $(d_2, d_4) = (0.4, 4)$ than $(d_2, d_4) = (0.4, 5)$. The output variance can even further be lowered by putting lower weight on the inflation gap (= deviation of inflation forecast from target). It is similar with inflation variance. The more "aggressive" is the CB in terms of the feedback from the inflation gap, the lower is the volatility of inflation. This is due to a more aggressive use of the direct exchange rate channel in the transmission mechanism through the UIP condition but more generally, the CB more devoted to inflation stabilization anchors better inflation expectations.

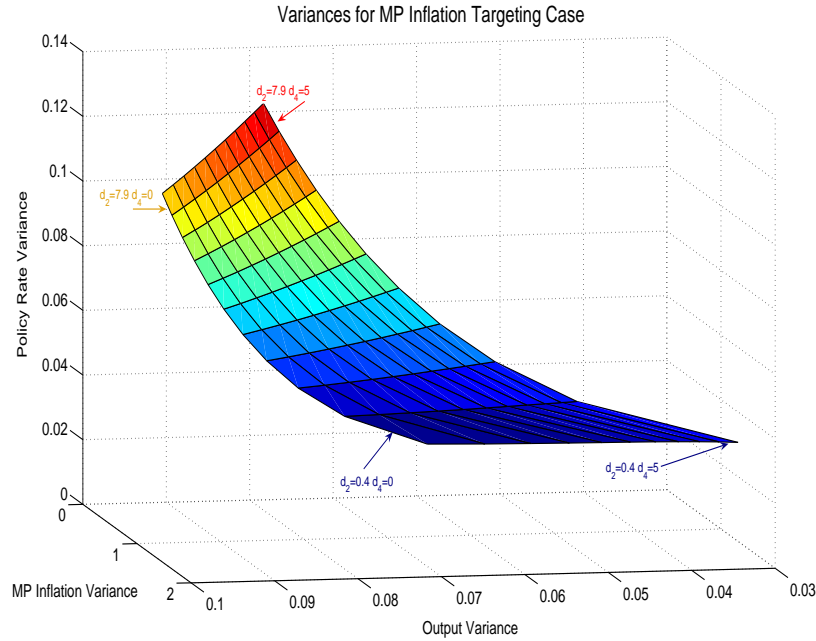
The following graph depicts the short term interest rate variance in the two targeting regimes (CPI-IT, D-IT).



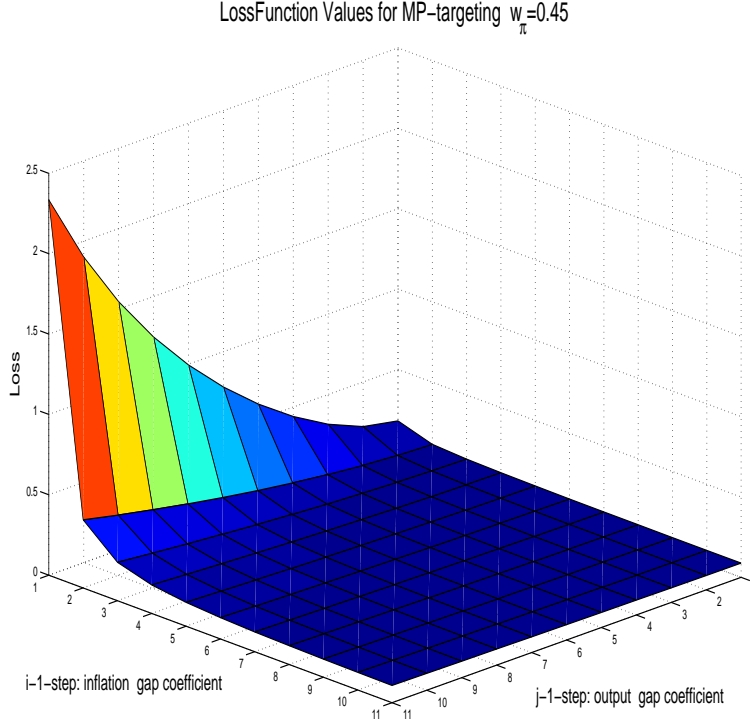
The conclusions are in line with the results related to output and inflation

variances. The lower are the inflation and output gap coefficients in the reaction function, the lower is the variance of the policy rate. This holds for both targeting regimes. The higher variance with more "activist" policy make economic sense: interest rates have to be adjusted more aggressively to any shock when policy react with higher weight than in the opposite case. The slope of the almost linear relationship between the variances of the policy rate corresponding with the two targeting regimes is somewhat higher than 45 degree. This means, that for the same weights of inflation and output gaps in the policy rule domestic D-IT requires higher variance of short interest rates than CPI-IT. Some "nonlinearity", however, can be detected only for the D-IT case, when the feedback is high from the output gap and low for inflation. This could be a result of the weak stabilizing properties of those policy rules for that particular calibration.

Let's turn our attention now to the values of the three alternative loss functions, for both CPI-IT and D-IT targeting cases. Let's consider now the case when $w_\pi = 0.45$. Before analyzing the values of the loss function itself, we depict all of its three determinants on the following graph, depicting the variances of the output gap, inflation and the policy rate for the case of CPI-IT:



For the parameter values $d_2 = 0.4, d_4 = 0$ the policy rule generates the lowest variance for the short-term interest rate. This is the least activist rule, both in terms of inflation and output. By increasing the coefficient on the inflation gap, we successfully cut the variance of inflation, but we definitely increase the variance of the MP. instrument, until we are in point $d_2 = 7.9, d_4 = 0$. Along this way we increase the variance of the output gap. When we leave $d_2 = 7.9$ and start to increase the coefficient of the output gap incrementally to 5, we move in the direction lower output-, higher interest rate instrument- and almost unchanged inflation variance. By leaving $d_4 = 5$ and gradually cutting the reactivity on inflation, we arrive at point $d_2 = 0.4, d_4 = 5$, where we have got the highest inflation and almost the lowest output variance. By the gradual decrease of the inflation coefficient we lower the instrument volatility significantly. To close the circle, the move from the point $d_2 = 0.4, d_4 = 5$ to $d_2 = 0.4, d_4 = 0$ is consistent with incrementally growing output variance and rapidly decreasing inflation volatility. The variance of interest rates falls too. To sum up, the increasing coefficient on the inflation gap lowers the variance of inflation at the price of a slight increase in the volatility of output. The growing coefficient of the output gap rapidly worsens the volatility of inflation variance. The growing coefficients of either inflation or output leads in most cases to higher volatility of the MP instrument. Now we can turn our attention to the loss function with the weight of $w_\pi = 0.45$. That is depicted, for the alternative parameterizations of the policy rule, on the following graph:



The 3D representation of the loss function, captured for all parameters of the reaction function, reveals few simple facts. For $d_2 = 0.4$, the growing coefficient on the output gap significantly increases inflation variance and subsequently the overall loss. With the increasing coefficient of the inflation gap in the MP rule, the role of the output gap coefficient diminishes and the whole surface flattens. It is quite clear, that the optimal simple rule will be the one with relatively high coefficient on inflation and low coefficient on the output gap ⁷⁴.

The tables in Appendices 9- 11 below contain the loss function values for $w_\pi = 0.9, 0.45$ and 0, including the CPI-IT and D-IT strategies. They tables reveal, that with the increasing coefficient of the inflation gap in the reaction function we minimize the loss very fast ⁷⁵. At the same time, the increase of

⁷⁴The graph is very similar for the D-IT case as well, the only difference is that the maximal losses in the extreme cases higher than in the CPI-IT case. Therefore the graph equivalent of the D-IT case will not be included.

⁷⁵We will measure the loss only by three digit numbers and we will use the smallest of

the output gap coefficient in the rule increases the inflation variance related loss rapidly, especially when the inflation gap coefficient is very low. The most important summary statistics for the optimized rules with respect to the three loss functions under consideration are included into the table below.

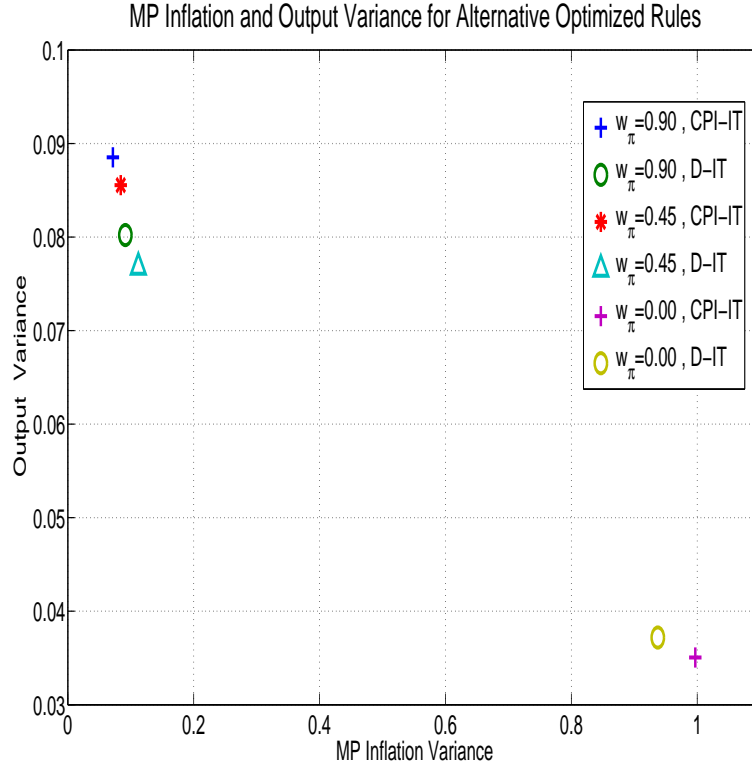
MP/Param.	d_2	d_4	loss	$var(i_t^{3m})$	$var(\pi_t^{mp})$	$var(y_t)$
$w_\pi = 0.90, CPI - IT$	6.4	0.0	0.07	0.08	0.07	0.09
$w_\pi = 0.90, D - IT$	6.4	0.0	0.09	0.12	0.09	0.08
$w_\pi = 0.45, CPI - IT$	4.9	0.0	0.08	0.07	0.08	0.09
$w_\pi = 0.45, D - IT$	3.4	0.0	0.09	0.07	0.11	0.08
$w_\pi = 0.00, CPI - IT$	0.4	3.50	0.03	0.03	1.00	0.04
$w_\pi = 0.00, D - IT$	0.4	2.00	0.03	0.03	0.93	0.04

The main findings can be summarized as follows:

- The differences between optimized rules for CPI-IT and D-IT are relatively small. The corresponding losses are comparable for all considered loss functions.
- The output gap coefficient of the MP rule is non-zero only for the loss function where $w_\pi = 0.00$. For all other specifications of the loss function $d_4 = 0$.
- More aggressive reaction to the inflation gap in the policy rule increases the variance of the MP instrument.
- The minimization of the loss function with a 0.9 weight on the output gap although lowers the variance of the output gap but at an expense of very rapid increase of inflation volatility.

The following graph shows how the CPI-IT and D-IT strategies perform with the optimized rules corresponding with our loss functions in an inflation-output variance space.

those coefficients of the rule that first minimize the loss function.



In all cases, when the weight of inflation in the loss function is non-zero, there is almost balance between inflation-output variance. In the opposite case the inflation variance is much larger than that of output's. The difference between CPI-IT and D-IT strategies is always the same: the CPI-IT strategy always results in smaller inflation and larger output variance relative to D-IT. The differences, however, are not significant. Some changes in the calibration of the model (the share of import prices in the CPI, the standard error of the XR, etc.) could however change these results.

6 Summary

The main goal of this thesis has been a study of alternative policy rules in a small structural model calibrated to capture the Czech economy. After the overview of the historic development of economic theory and structural

modeling we have specified a small open economy model that has served as a main technical tool for the analysis. The model represents a framework, where forward-looking model-consistent expectations are formed with respect to the development of the exchange rate and interest rates. Inflation expectations are forward looking too with some nominal rigidities in inflation dynamics.

The model's structure is relatively simple. The IS curve captures the dynamics of real GDP, that exhibits real rigidity, motivated by habit formation or investment adjustment costs. In our specification the real GDP is a function of (the deviation of) real XR, real IR and foreign demand (from corresponding equilibrium levels). The Phillips-curve is based on the F-M type wage setting behavior, therefore it enables to consider domestic prices, that are modeled as mark-ups over wages. CPI inflation then consists of domestic, imported and administered inflation, including the effect of any indirect taxes changes. The exchange rate is modeled by the UIP arbitrage condition. Exchange rate expectations are forward-looking, but with some inertia in expectation formation. Interest rates with one year maturity are also modeled as an arbitrage condition on the money market, they are fully model-consistently forward looking. The model is closed by a Taylor-type forward-looking policy rule. The interest rate exhibits some inertia and feeds back from deviation of inflation from target and output from its equilibrium. The specification (parameterization) of the rule is general enough to examine CPI and domestic inflation targeting.

The model specification has been followed by empirical work leading towards the implementation of the previously specified model on Czech data. Based on the sources of the Czech Statistical Office, Czech National Bank, Consensus Economics Inc., we first processed the data by executing seasonal adjustment and other transformations necessary for being consistent with the definition of model variables. The database has been created by an automatic MATLAB based routine, therefore the calculations were relatively easy to update. The database being completed, we have set up a Kalman-filter for determining equilibrium values for the real interest rate, exchange rate and output. At the same time through Kalman filtering we identified all model residuals. We paid special attention to the decomposition of the output gap and discussing

In order to assess the overall dynamic properties of the model and judge how well the model fits the data, we conducted several exercises. First we decomposed some of the important endogenous variables of the model to shocks to see, whether the identified shocks are in line with our intuition and episodes of the recent Czech economic history. We found, that the shocks are not in contrast with some of the clearly distinguishable episodes. After the shock decomposition we run in-sample simulations to see, how well the model is able to fit the reality two years ahead. We found the overall results quite encouraging. We were able to fit quite well the output gap as well as MP inflation. Domestic inflation has been slightly more inertial in model simulations than in reality, but even in this case the results were acceptable. The model was not able to fit the 2001-2 appreciation of the nominal XR ⁷⁶, which is not a big surprise. The model calibration part of the thesis concludes, that the model fits the data and economic story reasonably well.

The analysis heading towards the empirical model calibration is followed by the section contributing the main focus of the thesis, the specification and evaluation of alternative policy rules. The main question we ask is, what is the difference between CPI vs. domestic inflation targeting outcome when we optimize the reaction function coefficients ⁷⁷ of the central bank within a class of simple optimal rules. First we analyze impulse response simulation results to judge the difference between the two policy rules. The most significant differences between the two MP strategies we identified for demand shock and exchange rate risk premium shock. The impulse response analysis is followed by determining optimized weights for the two classes (CPI=IT, D=IT) of simple forward-looking rules, for three alternative loss functions with weight on inflation volatility $w_\pi = 0.00, 0.45$ and 0.9 and asymmetrically for the weight of output stabilization $w_y = 0.9, 0.45$ and 0 . For $w_\pi = 0.90$ and 0.45 the policy rules with optimized weights are consistent with relatively high coefficients on inflation in the policy rule and zero coefficient on the output gap. On contrary, for $w_\pi = 0$ the optimized output gap coefficient is relatively high and the inflation gap coefficient is the lowest from the considered options. The results suggest, that in terms of the loss functions there are no significant

⁷⁶ Understandably it neither forecasted well the fast fall in inflation after the appreciation period.

⁷⁷ With respect to inflation and output gaps.

differences between the considered two MP strategies, they approximately results in the same stabilizing properties, with slightly better results for CPI-IT. The optimized rules for CPI-IT and D-IT, however differ in a sense, that the CPI-IT rule is more efficient in inflation stabilizing, less in terms of output variance and it holds vice versa for D-IT.

7 Appendix - The Blanchard-Kahn Solution

The main goal of this Appendix is shortly described the Blanchard-Kahn algorithm, that is the most widespread solution method for solving linear rational expectation models. Let's assume, that the model's matrix representation is expressed in the following form:

$$AE_t x_{t+1} = Bx_t + C\epsilon_t \quad (31)$$

where A , B are matrices of coefficients of vectors x_{t+1} a x_t . C is a matrix of coefficients of exogenous variables. Operator E_t denotes expectations formed at time t . To put it differently, the system of equations (31) is possible to transform into a following form:

$$A \begin{bmatrix} x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix} = B \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + C \begin{bmatrix} \epsilon_t \\ \mathbf{0} \end{bmatrix} \quad (32)$$

where x_{1t} is a vector of predetermined variables ⁷⁸. Before proceeding further, let's show a simple example of the transformation into the state space form.

1. Example:

$$\pi_t = \alpha\pi_{t-1} + \beta E_t \pi_{t+1} + \epsilon_t \quad (33)$$

We transform the model into the form of (31)

$$\begin{bmatrix} 0 & -\beta \\ 1 & 0 \end{bmatrix} \begin{bmatrix} E_t \pi_t \\ E_t \pi_{t+1} \end{bmatrix} = \begin{bmatrix} \alpha & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ \pi_t \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \epsilon_t \\ 0 \end{bmatrix} \quad (34)$$

At the same time holds $\pi_t = E_t \pi_t$.

⁷⁸For the predetermined variables always holds $E_t x_{t+1} = x_{t+1}$, This means, that at time $t+1$ the variable is a function of other variables that are known until time t . The method of identifying non-predetermined variables in fact means pinning down state vector at time t and deciding whether the variable is known or not.

After this simple example, let's follow our exposition and assume, that matrix A is regular⁷⁹. We transform equation (32) into the following form:

$$\begin{bmatrix} x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix} = A^{-1}B \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + A^{-1}C \begin{bmatrix} \epsilon_t \\ \mathbf{0} \end{bmatrix} \quad (35)$$

or, to put it differently:

$$\begin{bmatrix} x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix} = \tilde{B} \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + \tilde{C} \begin{bmatrix} \epsilon_t \\ \mathbf{0} \end{bmatrix} \quad (36)$$

Let's apply operator E_t for both sides of equation (32) resp. let's apply the expectation operator on equation (32) based on the information set at time t .

$$E_t \begin{bmatrix} x_{1t+1} \\ x_{2t+1} \end{bmatrix} = \tilde{B} \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} \quad (37)$$

First we find a solution for the deterministic system of (37), then we put the shocks ϵ_t back.

In order to do that, we diagonalize matrix \tilde{B} to

$$\tilde{B} = PVP^{-1} \quad (38)$$

where P is transformation matrix and V is a matrix with eigenvalues on its diagonal (Eigenvalues determine the dynamics of the system.). It is much more suitable to rely on the Schur decomposition, which is numerically more stable. For the Schur decomposition see Klein (2000).

Let's linearly transform vector x_t

$$Pz_t = x_t \quad (39)$$

From this equation it is straightforward, that all elements of vector z_t include

⁷⁹If A is singular matrix, it is necessary to apply general Schur decomposition algorithm. For more on the Schur decomposition see Klein (2000)

information, which affect x_t . We substitute this transformation into equation 37).

$$Pz_{t+1} = \tilde{B}Pz_t \quad (40)$$

and therefore

$$z_{t+1} = P^{-1}\tilde{B}Pz_t = Vz_t \quad (41)$$

As a result we obtain

$$z_{t+1} = Vz_t \quad (42)$$

where $V = P^{-1}\tilde{B}P$

We are searching for unique solution. According to the Blanchard-Kahn condition must be true the following:

- the number of predetermined variables equal to the number of stable eigenvalues
- the number of non-predetermined variables equal to the number of non-stable eigenvalues

If the Blanchard-Kahn condition is met, there might occur two cases:

- the number of unstable eigenvalues is larger than the number of predetermined variables \Rightarrow the system has not got any solution
- the number of unstable eigenvalues is smaller than the number of predetermined variables \Rightarrow we get infinite number of solutions.

We reorder equation (42) in such a way, that in matrix V we first include the stable eigenvalues ($|\lambda| \leq 1$) in matrix V_{11} and afterwards unstable eigenvalues in matrix denoted by V_{22} . They correspond with the elements of z , where

$$z = \begin{bmatrix} z^s \\ z^u \end{bmatrix} \text{ and } V = \begin{bmatrix} V_{11} & 0 \\ 0 & V_{22} \end{bmatrix}$$

Now let's first solve the system for the stable part. We decompose equation

$$z_{t+1}^u = V_{22} z_t^u \quad (43)$$

For $t + 2$ we get

$$z_{t+2}^u = V_{22} z_{t+1}^u \quad (44)$$

It follows, that should there exist a stable solution for $\forall x$ then V_{22} with the elements on the diagonal larger than 1 must be $z_0 = 0$ and therefore also $z_\infty = 0$. From (43) and subsequently from (44) therefore we get

$$z_t^u = V_{22}^{-1} z_{t+1}^u = V_{22}^{-1} (V_{22}^{-1} z_{t+2}^u) = \dots = (V_{22}^{-1})^\infty z_{t+\infty}^u = 0 \quad (45)$$

Similarly, in order the process (43) not to be divergent, it is necessary to have $z_0 = 0$. Every stable solution requires $z_t = 0$ for t .

Let's turn to the solution of the stable part.

$$z_{t+1}^s = V_{11} z_t^s \quad (46)$$

In order to solve this difference equation we need to know the initial condition

That should be known by means of x_{10} . Vector x $x_t = \begin{bmatrix} x_t^{pred} \\ x_t^{unpred} \end{bmatrix}$ Since $Pz_t = X_t$ we get

$$P \begin{bmatrix} z_t^s \\ z_t^u \end{bmatrix} = \begin{bmatrix} x_t^{pred} \\ x_t^{unpred} \end{bmatrix} \quad (47)$$

Let's denote $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$ Since $z_t^u = 0$, we obtain

$$P_{11} z_t^s = x_t^{pred} \quad (48)$$

where x_t^{pred} in time t is known. Therefore $z_t^s = P_{11}^{-1} x_t^{pred}$ resp. $z_0^s = P_{11}^{-1} x_0^{pred}$

We substitute back the residuals. According to (32) it holds:

$$x_{1t+1} - E_t x_{1t+1} = \epsilon_t \quad (49)$$

Shall we use equation $Pz_t = x_t$ from the previous step

$$P_{11}(z_{t+1}^s - E_t z_{t+1}^s) = \epsilon_t \quad (50)$$

and then

$$z_{t+1}^s = E_t z_{t+1}^s + P_{11}^{-1} \epsilon_t \quad (51)$$

and according to (46)

$$z_{t+1}^s = V_{11} z_t^s + P_{11}^{-1} \epsilon_t \quad (52)$$

Let's calculate $x_{1t}(=x^{pred})$ and $x_{2t}(x^{unpred})$ by means of backward transformation, where $x_t = Pz_t$. By relying on $z_t^s = P_{11}^{-1} x_{1t}$ from (48) in (52) we get

$$P_{11}^{-1} x_{1t} = V_{11} P_{11}^{-1} x_{1t} + P_{11}^{-1} \epsilon_t \quad (53)$$

By adjusting (52)

$$x_{1t} = P_{11} V_{11} P_{11}^{-1} x_{1t} + \epsilon_t \quad (54)$$

Similarly from $x_{2t} = P_{21} z_t^s$ where $z_t^s = P_{11}^{-1} x_{1t}$ we get $x_{2t} = P_{21} P_{11}^{-1} x_{1t}$.

8 Appendix - The Kalman Filter

The main goal of this Appendix is to briefly describe the main steps of the Kalman filter recursion. We will strictly follow the exposition a la Harvey (1993).

8.1 State Space Form

Let's assume, that the $N \times 1$ vector of observed variables at time t , \mathbf{y}_t , is related to an $m \times 1$ vector, α_t , known as a *state vector*, via a *measurement equation*

$$\mathbf{y}_t = \mathbf{Z}_t \alpha_t + \mathbf{d}_t + \epsilon_t, t = 1, \dots, T$$

where \mathbf{Z}_t is an $N \times m$ matrix, \mathbf{d}_t is an $N \times 1$ vector and ϵ_t is an $N \times 1$ vector of serially uncorrelated disturbances with zero mean and covariance matrix \mathbf{H}_t , that is

$$E(\epsilon_t) = \mathbf{0} \text{ and } Var(\epsilon_t) = \mathbf{H}_t$$

We assume, that the elements of α_t are not observable, but they are generated by a first-order Markov process of the following form, called *transition equation* :

$$\alpha_t = \mathbf{T}_t \alpha_{t-1} + \mathbf{c}_t + \mathbf{R}_t \eta_t, t = 1, \dots, T$$

where \mathbf{T}_t is an $m \times m$ matrix, \mathbf{c}_t is an $m \times 1$ vector, \mathbf{R}_t is an $m \times g$ matrix and η_t is a $g \times 1$ vector of serially uncorrelated disturbances with zero mean and covariance matrix, \mathbf{Q}_t , that is

$$E(\eta_t) = \mathbf{0} \text{ and } Var(\eta_t) = \mathbf{Q}_t$$

The specification of the state-space form is completed by the following two assumptions:

- The initial state vector, α_0 , has a mean of \mathbf{a}_0 and covariance matrix \mathbf{P}_0 , that is $E(\alpha_0)=\mathbf{a}_0$ and $Var(\alpha_0)=\mathbf{P}_0$
- The disturbances, ϵ_t , and η_t are uncorrelated with each other in all periods and uncorrelated with the initial state, that is $E(\epsilon_t \eta_s')=\mathbf{0}$ for all $s, t = 1, \dots, T$ and $E(\epsilon_t \alpha_0')=\mathbf{0}$ and $E(\eta_t \alpha_0')=\mathbf{0}$

8.2 Filtering

Let \mathbf{a}_t denote the optimal estimator of the state vector, α_t , based on all the observations up to, including, \mathbf{y}_t . Let \mathbf{P}_t denote the $m \times m$ covariance matrix of the associated estimation error, that is

$$\mathbf{P}_t = \mathbf{E}[(\alpha_t - \mathbf{a}_t)(\alpha_t - \mathbf{a}_t)']$$

This may also be referred as the mean square error (MSE) matrix of \mathbf{a}_t . Suppose that we are in time $t-1$ and that \mathbf{a}_{t-1} and \mathbf{P}_{t-1} are given. The optimal estimator of α_t is given by the *prediction equations*

$$\mathbf{a}_{t,t-1} = \mathbf{T}_t \mathbf{a}_{t-1} + \mathbf{c}_t$$

and

$$\mathbf{P}_{t,t-1} = \mathbf{T}_t \mathbf{P}_{t-1} \mathbf{T}_t' + \mathbf{R}_t \mathbf{Q}_t \mathbf{R}_t' \quad t = 1, \dots, T$$

while corresponding estimator of \mathbf{y}_t is

$$\tilde{\mathbf{y}}_{t,t-1} = \mathbf{Z}_t \mathbf{a}_{t,t-1} + \mathbf{d}_t, \quad t = 1, \dots, T$$

The MSE of the prediction error, or *innovation*, vector

$$\mathbf{v}_t = \mathbf{y}_t - \tilde{\mathbf{y}}_{t,t-1} = \mathbf{Z}_t(\alpha_t - \mathbf{a}_{t,t-1}) + \epsilon_t, \quad t = 1, \dots, T$$

is

$$\mathbf{F}_t = \mathbf{Z}_t \mathbf{P}_{t,t-1} \mathbf{Z}_t' + \mathbf{H}_t$$

Once the new observation becomes available, the estimator of the state can be updated. The *updating equations* are

$$\mathbf{a}_t = \mathbf{a}_{t,t-1} + \mathbf{P}_{t,t-1} \mathbf{Z}_t' \mathbf{F}_t^{-1} (\mathbf{y}_t - \mathbf{Z}_t \mathbf{a}_{t,t-1} - \mathbf{d}_t)$$

and

$$\mathbf{P}_t = \mathbf{P}_{t,t-1} - \mathbf{P}_{t,t-1} \mathbf{Z}_t' \mathbf{F}_t^{-1} \mathbf{Z}_t \mathbf{P}_{t,t-1}, \quad t = 1, \dots, T$$

It will be observed that the prediction error, \mathbf{v}_t , plays a key role in the updating. The more the predictor of the observation deviates from its realized value, the bigger the change made to the estimator of the state. Given the initial conditions \mathbf{a}_0 and \mathbf{P}_0 , the KF delivers optimal estimator of the state as each new observation becomes available.

8.3 Prediction

The more-than-one ahead prediction equation is given by simply by-passing the updating equation. The optimal estimator of the state vector at times $T + 1$, based on information at time T is given by

$$\mathbf{a}_{T+1,T} = \mathbf{T}_{T+1} \mathbf{a}_{T,T} + \mathbf{c}_{T+1}, \quad l = 1, 2, \dots$$

with $\mathbf{a}_{T,T} = \mathbf{a}_T$, while the associated MSE matrix is obtained from

$$\mathbf{P}_{T+1,T} = \mathbf{T}_{T+1} \mathbf{P}_{T,T} \mathbf{T}_{T+1}' + \mathbf{R}_{T+1} \mathbf{Q}_{T+1} \mathbf{R}_{T+1}' \quad l = 1, 2, \dots$$

with $\mathbf{P}_{T,T} = \mathbf{P}_T$. The predictor of \mathbf{y}_{T+1} is

$$\tilde{\mathbf{y}}_{T+1,T} = \mathbf{Z}_{T+1} \mathbf{a}_{T+1,T} + \mathbf{d}_{T+1}, \quad l = 1, 2, \dots$$

with prediction MSE

$$MSE(\tilde{\mathbf{y}}_{T+1,T}) = \mathbf{Z}_{T+1}\mathbf{P}_{T+1,T}\mathbf{Z}_{T+1}' + \mathbf{H}_{T+1}$$

8.4 Smoothing

The main aim of the KF is to estimate a state vector, α_t , conditional on the information available at time t . The aim of the smoothing is to take into account of the information *made available after time t* . The smoothed estimator, denoted as, $\mathbf{a}_{t,T}$, is based on more information than filtered estimator, and therefore it will have an MSE matrix, $\mathbf{P}_{t,T}$, which, in general, is smaller than that of the filtered estimator. We will show the *fixed interval* smoothing estimates, that is concerned with computing the full set of smoothed estimators for fixed span of data.

The fixed-interval smoothing algorithm consists of a set of recursions, which starts with the final quantities, \mathbf{a}_T , and \mathbf{P}_T , given by the KF and works backwards. The equations are

$$\mathbf{a}_{t,T} = \mathbf{a}_t + \mathbf{P}_t^*(\mathbf{a}_{t+1,T} - \mathbf{T}_{t+1}\mathbf{a}_t + \mathbf{c}_{t+1})$$

and

$$\mathbf{P}_{t,T} = \mathbf{P}_t + \mathbf{P}_t^*(\mathbf{P}_{t+1,T} - \mathbf{P}_{t+1,t})\mathbf{P}_t^{*'}.$$

where

$$\mathbf{P}_t^* = \mathbf{P}_t\mathbf{T}_{t+1}'\mathbf{P}_{t+1,t}^{-1} \quad t = T-1, \dots, 1$$

with $\mathbf{a}_{T,T} = \mathbf{a}_T$ and $\mathbf{P}_{T,T} = \mathbf{P}_T$. The algorithm therefore requires that \mathbf{a}_t and \mathbf{P}_t be stored for all t so that they can be combined with $\mathbf{a}_{t+1,T}$ and $\mathbf{P}_{t+1,T}$.

9 Appendix - PR with loss function $w_\pi = 0.90$,

CPI-IT case

coef.	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
0.40	0.29	0.30	0.35	0.43	0.53	0.64	0.76	0.90	1.05	1.21	1.38
1.15	0.17	0.18	0.19	0.22	0.24	0.28	0.32	0.36	0.41	0.46	0.52
1.90	0.13	0.14	0.14	0.15	0.17	0.19	0.21	0.23	0.26	0.28	0.31
2.65	0.11	0.11	0.12	0.13	0.13	0.15	0.16	0.17	0.19	0.21	0.22
3.40	0.10	0.10	0.10	0.11	0.11	0.12	0.13	0.14	0.15	0.16	0.18
4.15	0.09	0.09	0.09	0.10	0.10	0.11	0.11	0.12	0.13	0.14	0.15
4.90	0.08	0.08	0.09	0.09	0.09	0.10	0.10	0.11	0.11	0.12	0.13
5.65	0.08	0.08	0.08	0.08	0.08	0.09	0.09	0.10	0.10	0.11	0.11
6.40	0.07	0.07	0.07	0.08	0.08	0.08	0.08	0.09	0.09	0.10	0.10
7.15	0.07	0.07	0.07	0.07	0.07	0.08	0.08	0.08	0.09	0.09	0.09
7.90	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.08	0.08	0.08	0.09

D-IT case

coef.	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
0.40	0.31	0.40	0.59	0.85	1.16	1.53	1.96	2.46	3.06	3.77	4.63
1.15	0.16	0.18	0.22	0.26	0.32	0.38	0.45	0.53	0.62	0.71	0.80
1.90	0.13	0.14	0.16	0.18	0.21	0.23	0.27	0.30	0.34	0.38	0.42
2.65	0.12	0.12	0.13	0.15	0.16	0.18	0.20	0.22	0.24	0.27	0.29
3.40	0.11	0.11	0.12	0.13	0.14	0.15	0.17	0.18	0.20	0.21	0.23
4.15	0.10	0.11	0.11	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.20
4.90	0.10	0.10	0.11	0.11	0.12	0.13	0.14	0.14	0.15	0.16	0.17
5.65	0.10	0.10	0.10	0.11	0.11	0.12	0.13	0.13	0.14	0.15	0.16
6.40	0.09	0.10	0.10	0.10	0.11	0.12	0.12	0.13	0.13	0.14	0.15
7.15	0.09	0.10	0.10	0.10	0.11	0.11	0.12	0.12	0.13	0.13	0.14
7.90	0.09	0.09	0.10	0.10	0.10	0.11	0.11	0.12	0.12	0.13	0.13

10 Appendix - PR with loss function $w_\pi = 0.45$,

CPI-IT case

coef.	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
0.40	0.17	0.18	0.20	0.24	0.28	0.34	0.40	0.47	0.54	0.62	0.71
1.15	0.12	0.12	0.13	0.14	0.15	0.17	0.18	0.21	0.23	0.25	0.28
1.90	0.10	0.10	0.10	0.11	0.12	0.12	0.13	0.14	0.16	0.17	0.18
2.65	0.09	0.09	0.09	0.10	0.10	0.11	0.11	0.12	0.12	0.13	0.14
3.40	0.09	0.09	0.09	0.09	0.09	0.10	0.10	0.10	0.11	0.11	0.12
4.15	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.10	0.10	0.10	0.11
4.90	0.08	0.08	0.08	0.08	0.08	0.09	0.09	0.09	0.09	0.10	0.10
5.65	0.08	0.08	0.08	0.08	0.08	0.08	0.09	0.09	0.09	0.09	0.09
6.40	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.09	0.09	0.09
7.15	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.09	0.09
7.90	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.09

D-IT case

coef.	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4	4.5	5.0
0.4	0.18	0.22	0.31	0.44	0.60	0.78	1.00	1.25	1.55	1.90	2.34
1.15	0.11	0.12	0.14	0.16	0.19	0.21	0.25	0.29	0.33	0.38	0.42
1.9	0.10	0.11	0.11	0.12	0.13	0.14	0.16	0.18	0.20	0.21	0.24
2.65	0.10	0.10	0.10	0.11	0.12	0.12	0.13	0.14	0.15	0.16	0.18
3.4	0.09	0.09	0.10	0.10	0.10	0.11	0.12	0.12	0.13	0.14	0.15
4.15	0.09	0.09	0.09	0.10	0.10	0.11	0.11	0.12	0.12	0.13	0.13
4.9	0.09	0.09	0.09	0.10	0.10	0.10	0.10	0.11	0.11	0.12	0.12
5.65	0.09	0.09	0.09	0.10	0.10	0.10	0.10	0.11	0.11	0.11	0.12
6.4	0.09	0.09	0.09	0.10	0.10	0.10	0.10	0.10	0.11	0.11	0.11
7.15	0.09	0.09	0.09	0.10	0.10	0.10	0.10	0.10	0.10	0.11	0.11
7.9	0.09	0.09	0.09	0.09	0.09	0.10	0.10	0.10	0.10	0.11	0.11

11 Appendix - PR with loss function $w_\pi = 0$,

CPI-IT case

coef.	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
0.40	0.06	0.05	0.05	0.04	0.04	0.04	0.04	0.03	0.03	0.03	0.03
1.15	0.07	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.04	0.04
1.90	0.07	0.07	0.07	0.06	0.06	0.06	0.06	0.06	0.05	0.05	0.05
2.65	0.08	0.07	0.07	0.07	0.07	0.07	0.06	0.06	0.06	0.06	0.06
3.40	0.08	0.08	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.06	0.06
4.15	0.08	0.08	0.08	0.08	0.08	0.07	0.07	0.07	0.07	0.07	0.07
4.90	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.07	0.07	0.07	0.07
5.65	0.09	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
6.40	0.09	0.09	0.09	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
7.15	0.09	0.09	0.09	0.09	0.09	0.09	0.08	0.08	0.08	0.08	0.08
7.90	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.08	0.08	0.08

D-IT case

coef.	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
0.40	0.05	0.04	0.04	0.04	0.03	0.03	0.03	0.03	0.03	0.03	0.04
1.15	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.05	0.04	0.04
1.90	0.07	0.07	0.07	0.06	0.06	0.06	0.06	0.06	0.05	0.05	0.05
2.65	0.07	0.07	0.07	0.07	0.07	0.07	0.06	0.06	0.06	0.06	0.06
3.40	0.08	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.06
4.15	0.08	0.08	0.08	0.08	0.07	0.07	0.07	0.07	0.07	0.07	0.07
4.90	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.07	0.07	0.07	0.07
5.65	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
6.40	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
7.15	0.09	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08
7.90	0.09	0.09	0.09	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08

Figure 2: Impulse response results - Demand shock

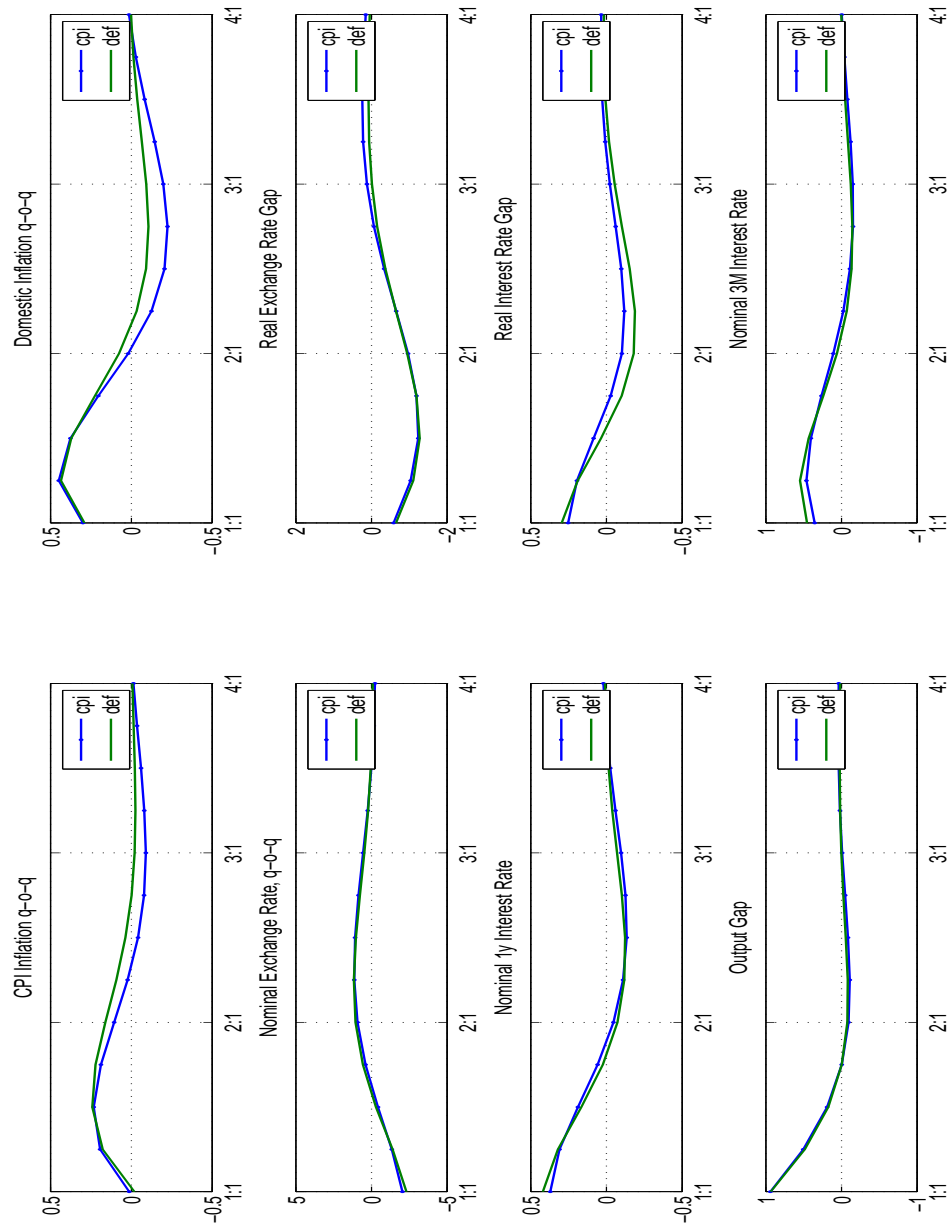


Figure 3: Impulse response results - Shock to domestic inflation

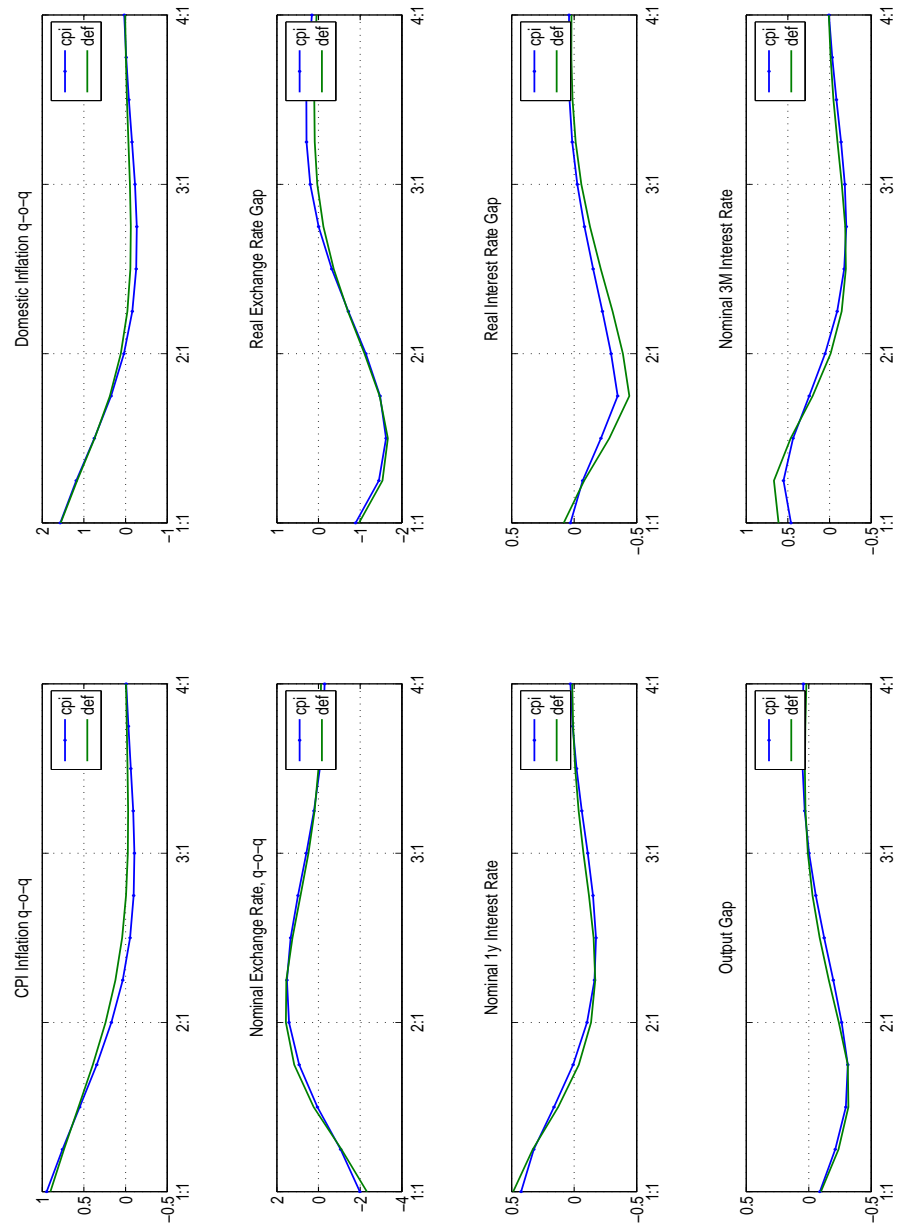


Figure 4: Impulse response results - Shock to the exchange rate risk premium

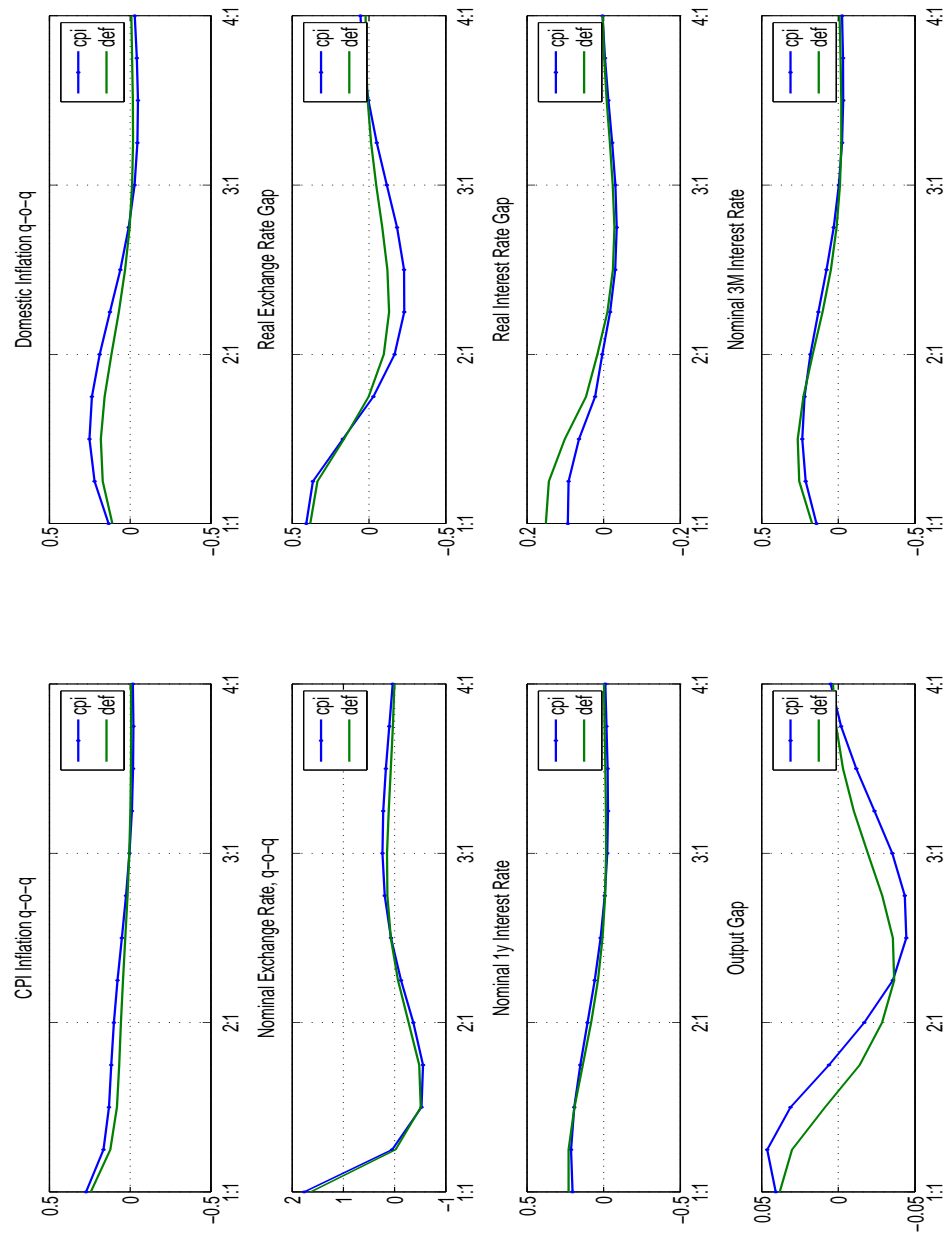


Figure 5: Impulse response results - Shock to the policy rate

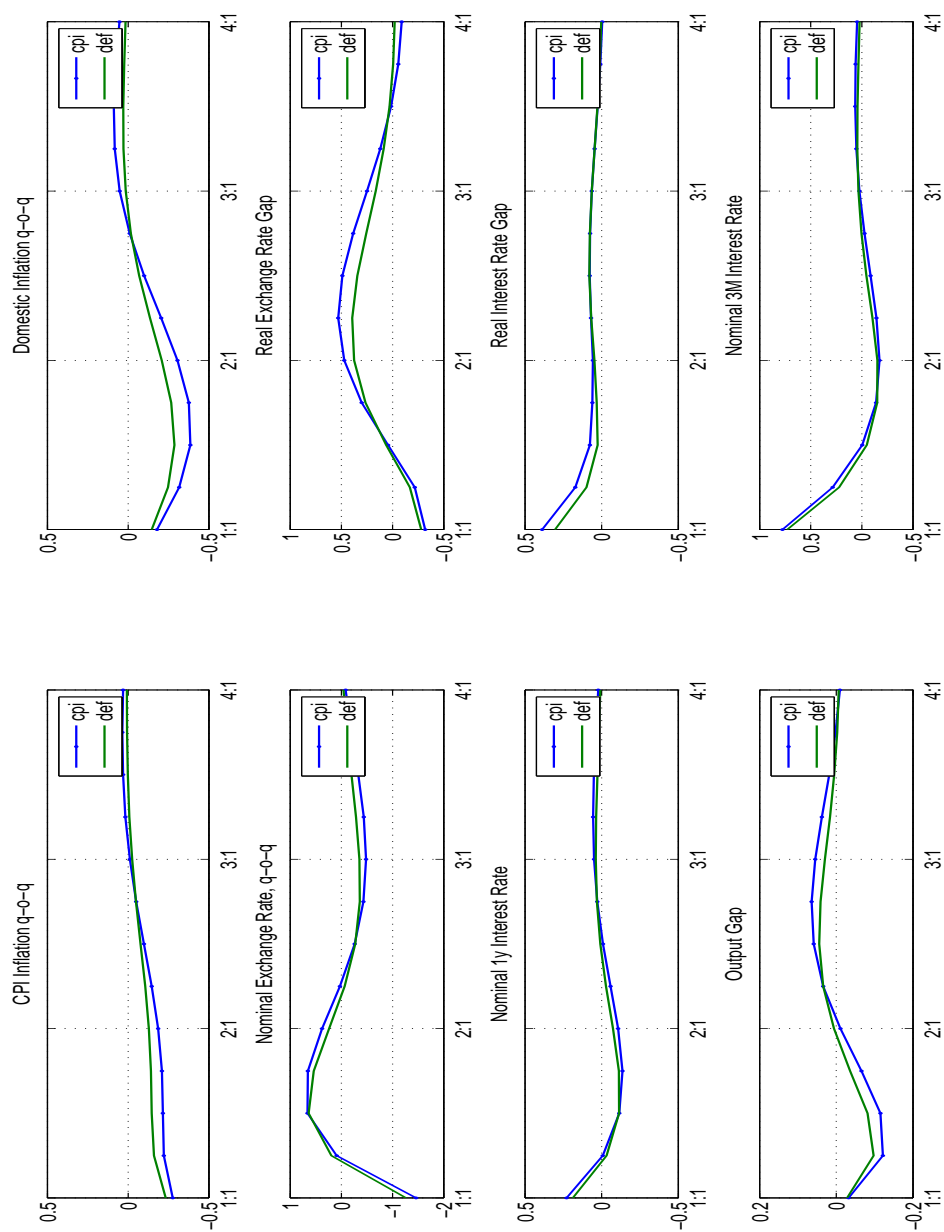


Figure 6: Shock decomposition results - Monetary policy inflation

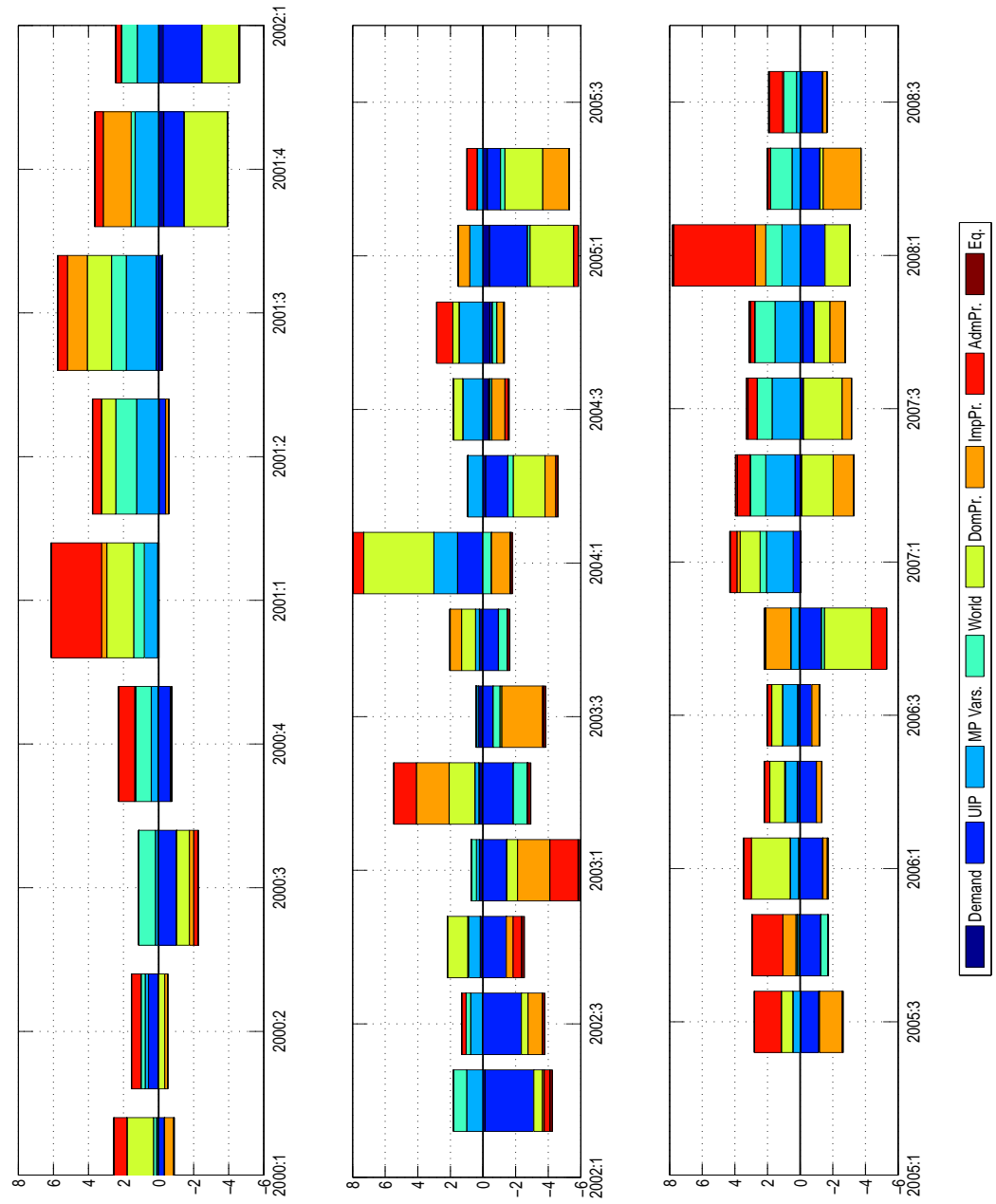


Figure 7: Shock decomposition results - Output gap

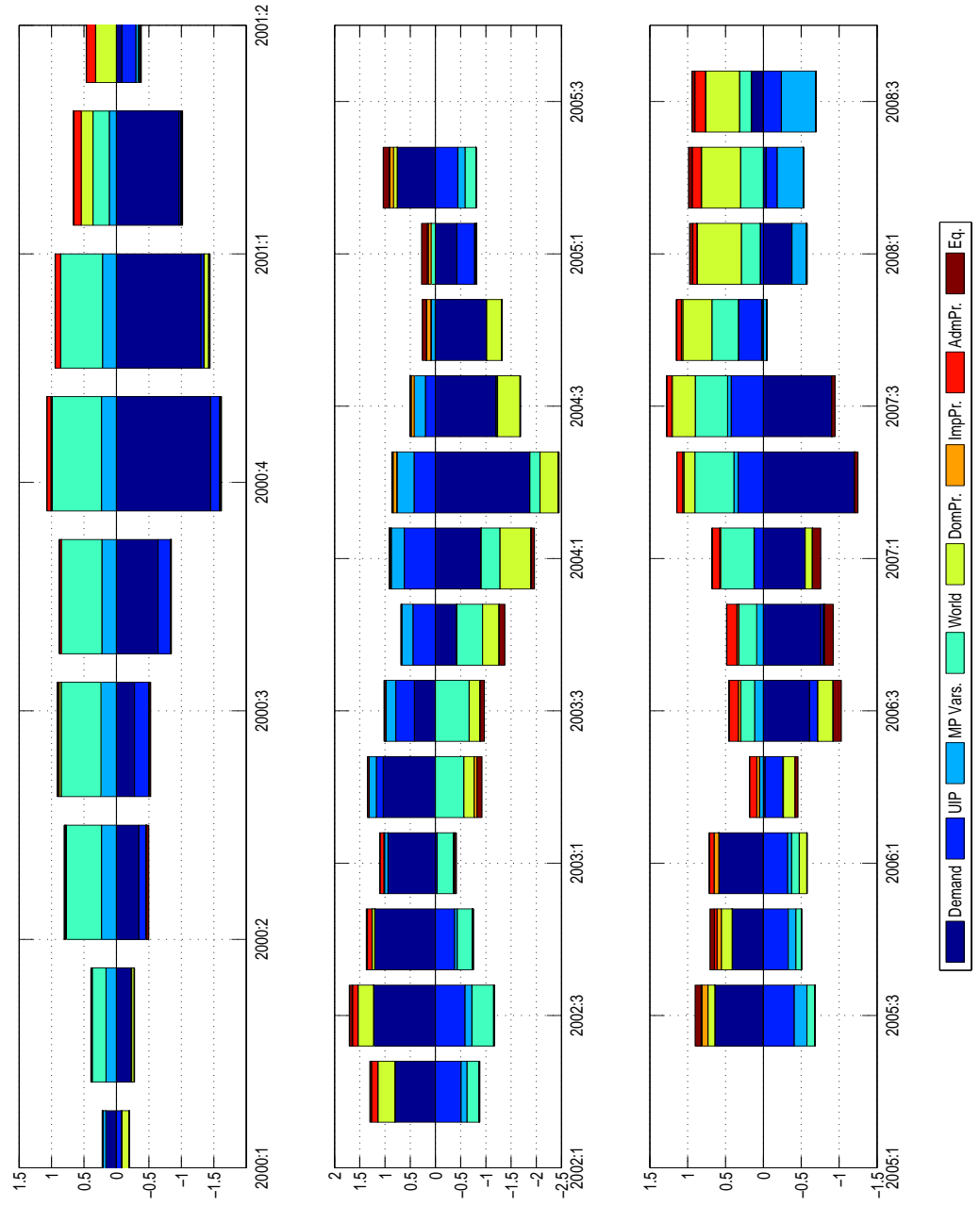


Figure 8: Shock decomposition results - Change in the nominal exchange rate

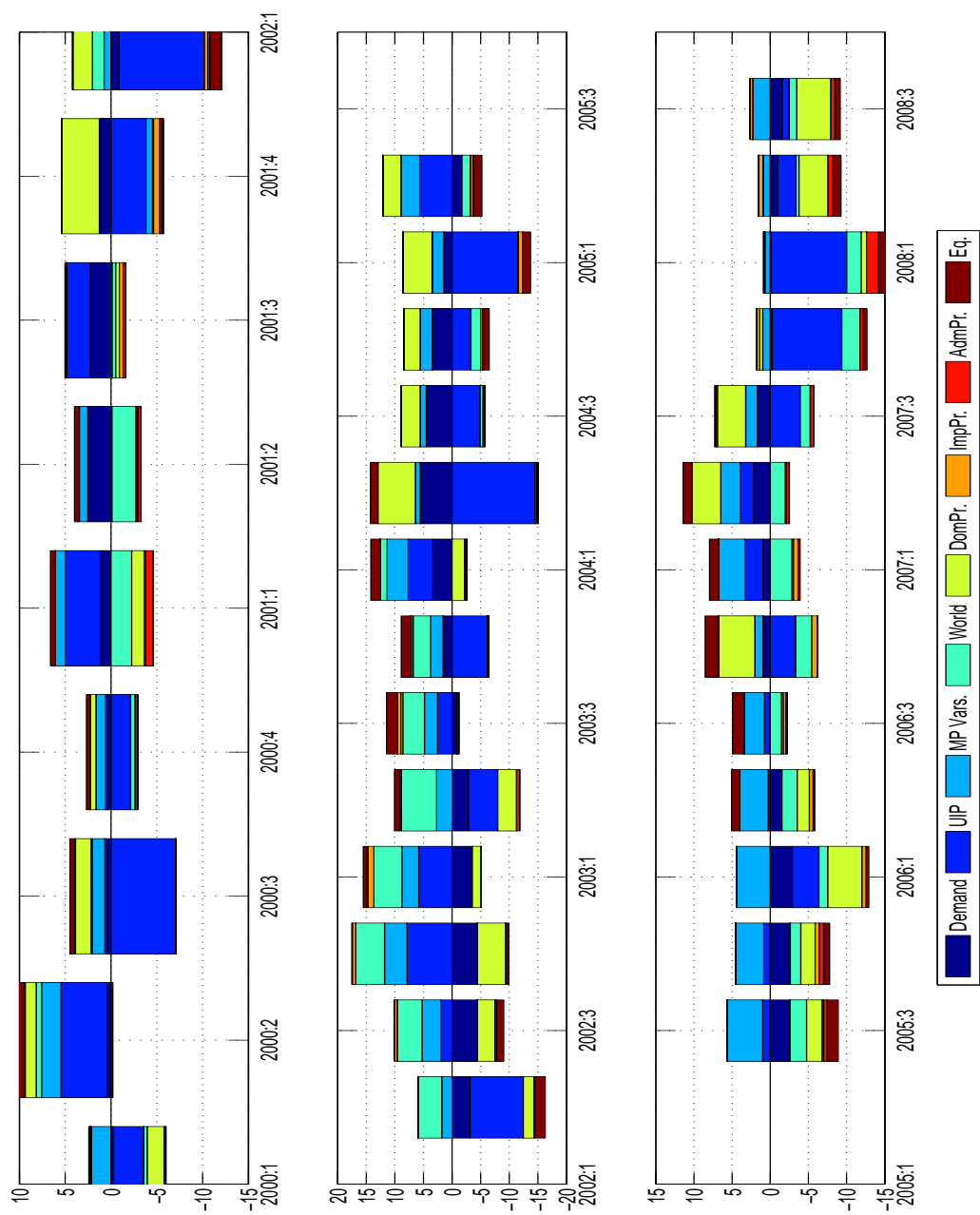
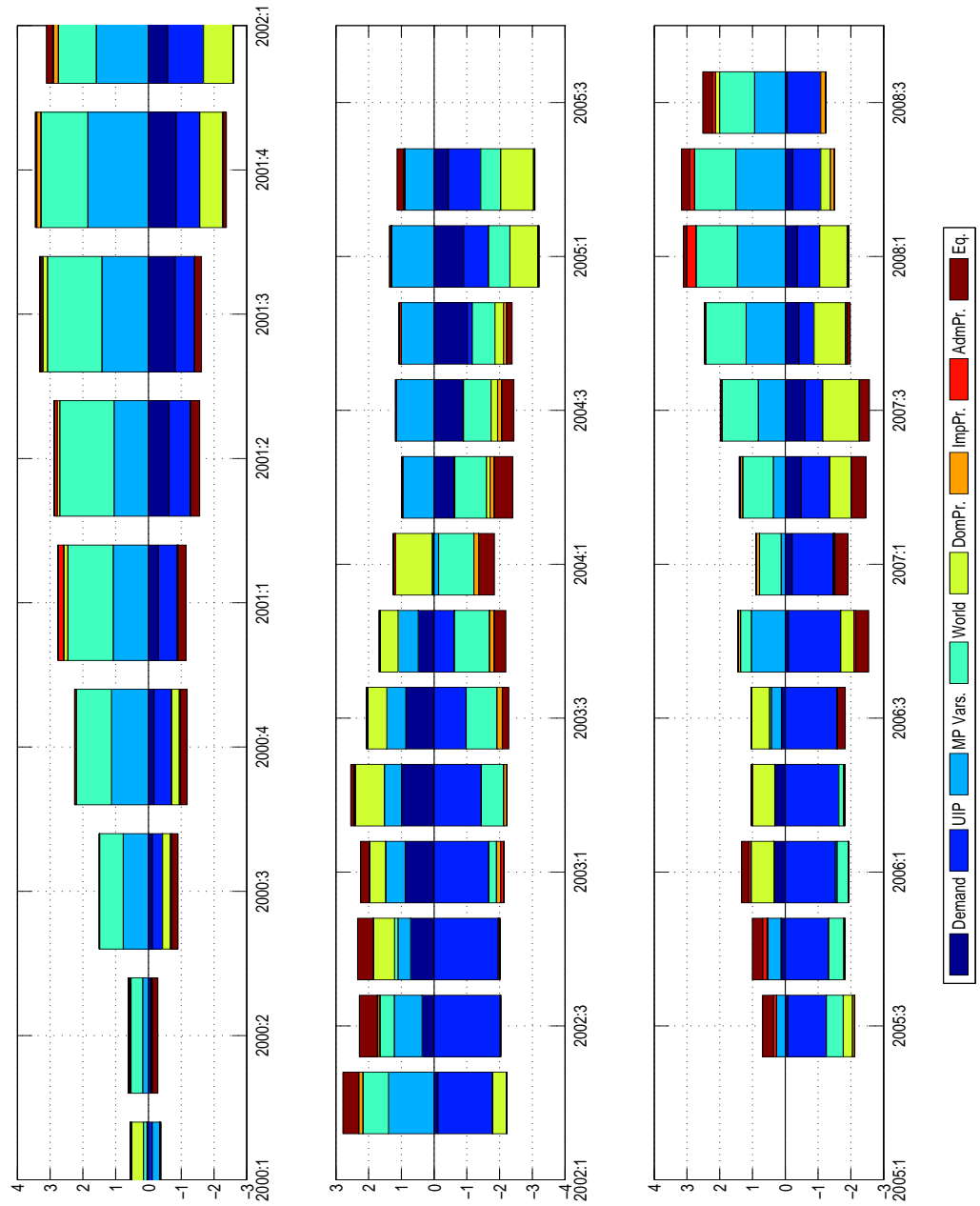


Figure 9: Shock decomposition results - Policy rate



Abbreviations

3D = three dimensional;

$\#Q = \#quarter(s)$, where $\# = 1, 2, 3 \dots$

AWM = Area-Wide Model of the ECB;

B-K = Blanchard-Kahn;

BEQM = the Bank of England Quarterly Model;

BoC = Bank of Canada;

BoE = Bank of England;

BoF = Bank of Finland;

BOF5 = the BOF5 model of the Bank of Finland;

BOFMINI = aggregate version of the Bank of Finland's BOF5 model;

b.p. = basis point;

CB = central bank;

CF = Consensus Forecast;

CNB = Czech National Bank;

CZSO = Czech Statistical Office;

CPI = consumer price index;

CPI-IT = CPI inflation targeting;

D-IT = domestic inflation targeting;

DSGE = Dynamic Stochastic General Equilibrium;

ECB = European Central Bank;

EDGE = Euro area Dynamic General Equilibrium model of the BoF;

EUR = euro nominal exchange rate;

FDI = foreign direct investment

FED = The US Federal Reserve Bank;

FRB/US = The US Federal Reserve Bank's econometric model of the domestic economy;

g3 = The first DSGE model of the CNB introduced in a forecasting context (2008);

F-M = Fuhrer-Moore type wage-contracting specification;

GDP = gross domestic product

IDSGE = Incomplete Dynamic Stochastic General Equilibrium, Pagan's classification;

IR = interest rate;
 IRIS = Matlab-based modeling toolbox developed by Jaromír Beneš;
 IT = inflation targeting;
 KF = Kalman filter;
 MCI = monetary conditions index;
 MM = The Macroeconomic Model of the Bank of England;
 MP = monetary policy;
 MPS = MIT-Penn-SSRC model of the US Federal Reserve Bank;
 MSE = mean square error;
 NAIRU = non-accelerating inflation rate of unemployment;
 NAWM = New Area-Wide Model of the ECB;
 NFA = net foreign assets
 NIESR = National Institute of Economic and Social Research, London, Great Britain;
 PR = policy rules;
 RAMSES = the Riksbank Aggregate Macromodel for Studies of the Economy of Sweden;
 RBNZ = Reserve Bank of New Zealand;
 UIP = uncovered interest rate parity;
 US = United States of America;
 USD = US dolar;
 QPM = the quarterly projection model of the CNB;
 SSRC = Social Science Research Council, US;
 s.e. = standard error;
 XR = exchange rate;

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