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Job Information Networks and Game Theory

Autor: **Bc. Anita Benešová, MSc.**

Katedra: **Katedra ekonometrie**

Obor: **Ekonometrie a operační výzkum**

Vedoucí práce: **Doc. Ing. Mgr. Martin Dlouhý, Dr., MSc.**

Prohlášení:

Prohlašuji, že jsem diplomovou práci na téma „Job Information Networks and Game Theory" zpracovala samostatně. Veškerou použitou literaturu a další podkladové materiály uvádím v seznamu použité literatury.

V Praze dne 12. prosince 2011 ....

Anita Benešová

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Abstract

Title: Job Information Networks and Game Theory
Author: Bc. Anita Benešová, MSc.
Department: Department of Econometrics
Supervisor: Doc. Ing. Mgr. Martin Dlouhý, Dr., MSc.

The use of personal contacts and the role of education as a signal of the Worker's productivity are two important aspects of the job search process. The aim of this thesis is to develop a model that combines both approaches. We distinguish between random and strategic models of job information networks. In the former case the structure of the network is given, while in the latter it depends on the strategic decision of the Workers. We present a strategic model of network formation with two types of Workers who are able to signal their productivity by the level of their education. When applying for a job they have two possibilities of contacting the Employer: a direct application and an indirect application through a friend who currently works for the Employer.

Keywords: Signaling Games, Job Information Networks, Network Formation

Abstrakt:

Název práce: Informační sítě na trhu práce a teorie her
Autor: Bc. Anita Benešová, MSc.
Katedra: Katedra ekonometrie
Vedoucí práce: Doc. Ing. Mgr. Martin Dlouhý, Dr., MSc.

Využívání osobních kontaktů a signalizování produktivity pracovníka pomocí vzdělání jsou dva důležité aspekty procesu hledání uplatnění na trhu práce. Cílem této diplomové práce je vytvořit model, který spojuje oba tyto přístupy. Modely informačních sítí na trhu práce mohou být náhodné nebo strategické. V případě náhodných modelů je struktura sítě dána, naopak u strategických modelů je tvořena na základě rozhodnutí hráčů. Model představený v této práci je strategický model se dvěma typy pracovníků, kteří mohou signalizovat svoji produktivitu pomocí svého vzdělání. Když se uchází o pracovní místo, mají dvě možnosti, jak kontaktovat případného zaměstnavatele: buď přímo, nebo nepřímo přes svého přítele, který pracuje pro tohoto zaměstnavatele.

Klíčová slova: Signální hry, Informační sítě na trhu práce, Teorie sítí

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Introduction

Social networks play a central role in the transmission of information about job opportunities. Many Workers obtain their job through friends or relatives. Networks of personal contacts constitute a widely used alternative to more formal methods of finding about job openings such as employment agencies.

Another important aspect of the job search process is the role of education. The Employer offers to Workers who have reached some given level of education a higher wage, because he believes that they have high productivity. The education therefore serves as a signal of the Worker's productivity.

Although there are many models in the recent literature that describe the role of social networks in the job search process, none of them allows for signaling. The aim of this thesis is to develop a model of the job market that combines both the signaling games theory and the theory of network formation. We will try to answer the question if Employers should prefer the use of referrals of their current employees when looking for new Workers.

The structure of this thesis is as follows. In the first chapter we present some facts about the job market and some characteristics shared by social networks. The second chapter describes the way the networks are modeled, while the third chapter presents the theory of signaling games with focusing on signaling on the job market. Chapter four provides an overview of models of job information networks that have already been described in the literature, before developing our own model of job information network that allows for signaling.

1 Job Information Networks

Social relationships play an important role in various aspects of life. They affect the options we hold and the information we obtain. One of the best-studied roles of social networks concerns obtaining employment, which is the problem this theses focuses on. In this chapter, we present some facts and results of empirical research concerning the job market and we discuss some characteristics shared by socially generated networks.

1.1 Job Market

The empirical research about the way individuals collect information for the purpose of finding a job has revealed many interesting facts about the job market. The most important is the widespread use of friends, relatives, and other social and professional acquaintances to search for jobs. The structure of the network of contacts of an individual therefore influences his access to information about job openings. Ioannides and Datcher Loury (2004) offer a summary of the empirical literature on job information networks and present some general facts about job market the research has provided.

One of the most influential studies of the role of social networks in finding jobs was conducted by Granovetter (1973), (1995). He distinguishes three means of finding about job openings: formal means (public and private employment agencies, newspaper advertisements, school and college placement services), personal contacts (friends and relatives) and direct applications. Direct application means that the Worker has contacted directly the Employer without having heard about a specific job opening. He found out that personal contact is the predominant method of finding about jobs. In his sample 56% of respondents used personal contact, 18,8% formal means and 18,8% direct applications. Moreover, most respondents preferred the use of personal contacts, because they believed the information about the job is of better quality. Employers expressed a similar preference for hiring methods.

He studied also the strength of the social relationships that were used to find a job. He measured the strength of a relationship as the "amount of time, the emotional intensity, the intimacy (mutual confiding), and the reciprocal services which characterize the tie" (Granovetter, 1973). He found out that a surprising proportion of jobs that were found through social contacts were obtained through weak ties. Weak ties are more likely to form bridges across groups and therefore play a crucial role in transferring information among groups.

The role of the Internet for the purpose of job search is increasing. The use of personal contacts to find a job is also the main purpose of the social networking site LinkedIn. Individuals can create on LinkedIn professional profiles which can be viewed by other users in their network. A network of an individual consists not only of people to which he is connected directly (known as *connections*), but also of his connections' connections (2^{nd} degree connections), as well as of his 2^{nd} degree's connections (called 3^{rd} degree connections). It allows both employers and job seekers to find information about each other. The network of LinkedIn comprises already over 100 million nodes (registered users)[1].

1.2 Characteristics of Socially Generated Networks

Social and economic networks have been studied through a large number of case studies. One of the earliest experiments was conducted by Stanley Milgram (1967) who studied the average path length between two people. During the experiment people had to send a letter to a person they did not know directly. Recent studies involve web pages, coauthorship, email and citation networks. An overview can be found for example in Barabási (2002).

These studies have revealed the following regularities and stylized facts about socially generated networks. They are summarized in Jackson and Rogers (2007) and described in detail in Jackson (2008).

1.2.1 Diameter and Small Worlds

Social networks tend to have *small diameter* (maximum distance between any pair of nodes) and *small average path lengths.* This stylized fact is also known as *small worlds* or *six degrees of separation.* The theory was first proposed in 1929 by the Hungarian writer Frigyes Karinthy in a short story called "Chains". It was popularized in John Gaure's play "Six degrees of separation". The research of this topic started with Milgram (1967) and an overview can be found in Watts (2003).

1.2.2 Clustering

A *clustering coefficient* measures the tendency of linked nodes to have common neighbors. In particular, if node *i* has relationship with both nodes *j* and *k,* it expresses how likely on average it is that *j* and *k* are also linked. Social networks tend to have high clustering coefficients compared to networks where links are generated by an independent random process.

1.2.3 Degree Distributions

The *degree* of a node is the number of links that the node has. The *degree distribution* of a network is a description of the relative frequencies of nodes that have different degrees. The distribution of degrees of the nodes in social networks tends to exhibit "fat tails", so that

[1] Data from March 2011. Source: http://press.linkedin.com/about

there are more nodes with relatively high and low degrees, and fewer nodes with medium degrees, than one would find in a network where links are formed uniformly at random.

1.2.4 Correlations and Assortativity

Beyond the degree distribution of a network, we can also ask questions about the correlation patterns in the degrees of connected nodes. The degrees of linked nodes tend to be positively correlated, so that higher degree nodes are more likely to be linked to other high degree nodes, and lower degree nodes are more likely to be linked to other lower degree nodes. This is referred to as *(positive) assortativity.*

1.2.5 Patterns of Clustering

Another indication which helps to characterize the networks is how clustering is distributed across a network. A pattern observed in at least some social networks is that the *clustering* among the neighbors of a given node is *inversely related to the node's degree.* That is, the neighbors of a higher degree node are less likely to be linked to each other compared to neighbors of a lower degree node.

2 Theory of Networks

2.1 Models of Network Formation

A network is a system of agents, in our case Workers and Employers, who may be connected to allow communication or interaction between them. A network can be modeled as a graph (directed or undirected), where the nodes correspond to agents and the edges to links between them. Agents can benefit from the interaction with other agents, but can only interact if they are connected by a link or a series of links. In social networks the links correspond to active relationships between individuals. However, forming links is costly, so agents must compare the costs and benefits of forming links.

Benefits that arise from social networks may include not only party invitations, but also information about business opportunities, scientific collaboration among academics or sharing information about job openings. The outcome of an agent depends on his position in the network and on the structure of the network. The process of social link formation and the resulting networks have been studied extensively in the economic analysis and game theoretic literature. Jackson (2008) provides a synthesis of this work.

The literature contains a large number of different models of networks formation which can be roughly split into two categories: *random-graph models* and *models of strategic network formation*.

2.1.1 Random-Graph Models

In random-graphs the creation of links is not influenced by strategies of the player but only by some random process. Random-graph models can be *static* or *growing*. In static models all nodes are created at the same time and then links between them are drawn according to some probabilistic rule. In growing models new nodes are introduced over time and form link with existing nodes as they enter the network. Jackson and Rogers (2007) present a growing random-graph model where new links are formed in two ways: uniformly at random or searching locally through the current structure of the network (e.g., meeting friends of friends).

2.1.2 Strategic Network Formation

When not only chance but also the choice of the players influences the network formation, strategic models are used. In strategic network formation models the evolution of the network depends on strategies of the players who are maximizing their payoffs. The payoffs are given by costs and benefits that arise from the link formation. We distinguish two main methods for modeling strategic network formation: the concept of *pairwise stability* and the concept of *Nash equilibrium* used in directed networks.

A *Nash equilibrium* is a choice of action by each player, such that no player would benefit by changing his or her action, given the actions of the other player(s). The concept of Nash equilibrium is used in directed networks where the link formation is one-sided and a player cannot refuse to form a connection offered by another player. This model was presented by Bala and Goyal (2000) and is discussed in detail in chapter 2.4.

The concept of *pairwise stability* was introduced by Jackson and Wolinsky (1996). They argue that the concept of Nash equilibrium is not very useful when the consent of both players is needed to form a link. Instead they define a pairwise stable network as a set of links such that no two individuals would choose to create a link if there is no link between them, and no individual would chose to sever any existing link. Thus both players must cooperate to form links, but an individual can remove a link.

The same approach is used by Gilles and Sarangi (2004), who study both one-way and two-way link formation costs. In both cases the consent of both players is needed to form a link and the difference is if just the player who initiates the link or both players bear the cost of forming link.

Johnson and Gilles (2000) extend the Jackson-Wolinsky framework by introducing a spatial cost topology. They use the main hypothesis from Debreu (1969) which says that it is less costly to form links between neighbors than to form them with agents located further. They use the term neighbors to describe agents with similar individual characteristics. The more similar the agents are, the less costly it is for them to form links with each other.

2.2 Representing Networks

Networks are usually represented as directed or undirected graphs. A *graph* (N, g) consists of a set of nodes $N = \{1, \dots, n\}$ and a real-valued $n \times n$ matrix g. The nodes represent the players and the entries of the matrix g the links between them. The matrix g is called the *adjacency matrix* and the value $g_{ij} = 1$ indicates the presence of a directed link between players i and j, while $g_{ij} = 0$ indicates the absence of such a link. In this thesis we assume that $g_{ii} = 0$ for all i.

If $g_{ij} = g_{ji}$ for all nodes i and j, the network is *undirected* and the fact that i has a relationship with j automatically means that j has the same relationship with i. If it is possible that $g_{ij} \neq g_{ji}$, the network is *directed*. In case we want to track the intensity level of relationships we can allow the entries of g to take on other values than just 0 and 1. We then talk about a *weighted* graph.

For any node $i \in N$, the *neighborhood* of node i is the set of all nodes that i is linked to, that is $N_i(g) = \{j \in N | g_{ij} = 1\}$. So the neighborhood is a set of all j such that there is a link from i to j. Note that $i \notin N_i(g)$ if we assume that $g_{ii} = 0$ for all i.[2] If we consider undirected graphs, $i \in N_j(g)$ is equivalent to $j \in N_i(g)$. However, if in case of a directed graph $g_{ji} = 1$ and $g_{ij} = 0$, then $i \in N_j(g)$ but $j \notin N_i(g)$.

The cardinality of $N_i(g)$, denoted by $n_i(g) = |\{j \in N | g_{ij} = 1\}|$, is the number of nodes i is linked to. In case of directed networks this calculation corresponds to the *out-degree* of node i. The *in-degree* of node i, denoted by $d_i(g) = |\{j \in N | g_{ji} = 1\}|$, is the number of links that connect to i from other nodes.

2.3 One-sided Model of Network Formation

We consider the model of network formation that was presented in Bala and Goyal (2000). In this model the cost of forming a link is only borne by the agent who initiates the link. We say that the agent who initiates the link *sponsors* it. He does not need the permission of the other player to form a link with him. The process of network formation can be modeled as a non-cooperative game. The strategy of a player consists of the set of agents with whom he forms links.

Bala and Goyal (2000) study two possibilities of flow of benefits. In the one-way flow model only the agent who sponsors the link gets benefits from it. In the two-way flow model both agents get benefits from the connection. We say that two players are *connected* if there exists a path between them and that they are *linked* if there is a direct link between them. A *path* between nodes i and j is a sequence of links $i_1 i_2, i_2 i_3, \dots, i_{K-1} i_K$ such that $i_k i_{k+1} \in g$ (that is $g_{i_k i_{k+1}} = 1$) for each $k \in \{1, \dots, K-1\}$, with $i_1 = i$ and $i_K = j$, and such that each node in the sequence $i_1, \dots, i_K$ is distinct.[3] As we assume that $g_{ii} = 0$ for all i, we consider that i is not connected to himself.

The benefits may or may not depend on the length of the path. In the former case we measure the level of decay by a parameter $\delta \in (0,1]$. If the shortest path between two players has q links, then the benefit b from this connection is discounted by δ^q. The case where the benefits are insensitive to the number of intermediaries corresponds to $\delta = 1$.

In case the benefits do not depend on the length of the path, each connection gives the same benefit. So the total benefits to a player depend only on the number of players to which he is connected. The payoff to a player is thus equal to $Kb - Lc$, where K is the number of

[2] The approach that $g_{ii} = 1$ and $i \in N_i(g)$ is also possible and used for example by Bala and Goyal (2000).

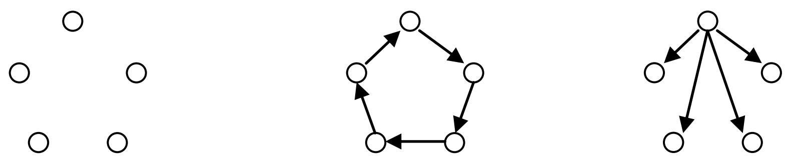
[3] Definition from Jackson (2008).

players to which he is connected, b is the benefit per connection (homogenous for every connection), L is the number of links sponsored by the player and c is the cost of forming a link (also homogenous for every link).

In case of one-way flow of benefits, the player gets benefits only from the links he sponsors. So the formula reduces to $L(b-c)$. The player will form links only if the benefit per connection b is larger than the cost of forming a link c.

Bala and Goyal (2000) show that Nash networks are either connected or empty. A network is connected if there is a path between every pair of agents and empty if there are no links in the network. A network is called a Nash network if it is a Nash equilibrium, *i.e.* no agent is better off if he unilaterally selects another strategy, so each agent plays the best reply to this network. A Nash equilibrium is called *strict* if every agent gets a strictly higher payoff with his current strategy than he would with any other strategy.

The result proved by Bala and Goyal (2000) is that in the one-way flow model the only strict Nash networks are the wheel and the empty network. In the two-way flow model the strict Nash network is either a center-sponsored star or an empty network, depending on the parameters. An empty network (a), wheel (b) and center-sponsored star (c) are shown in Figure 2.1. The links are represented by arrows from the player who initiates the link[4]. In a wheel network each agent forms exactly one link. A center-sponsored star is a network in which one player (the center) sponsors links with all other players.



(a) Empty network (b) Wheel network (c) Center-sponsored star

Figure 2.1 Three types of networks

Bala and Goyal (2000) consider homogenous costs and benefits, *i.e.* cost and benefits that are the same for every link and every player. Galeotti, Goyal and Kamphorst (2006) generalize the two-way flow model with no decay of Bala and Goyal (2000) for heterogeneity among players. In particular, they consider that costs and the benefits may depend on the player. They show that in this case a non-empty strict Nash network may not be connected.

[4] This notation is used for example in Kamphorst (2005). In Bala and Goyal (2000) the arrows are depicted pointing in the other direction: from the recipient to the sponsor.

First, they consider homogeneous costs and heterogeneous benefits and show that then a strict Nash network may consist of several center-sponsored star components. If the costs are heterogeneous any minimal network may arise as a strict Nash network. A *component* of a network is a connected subset of players which becomes unconnected when adding any other player to the subset. A set of players is *connected* if all the players in the set are connected, *i.e.* there exists a path between each of them. A network is *minimal* if deleting any link would break a connection between two players and the number of components of the network would increase.

Galeotti, Goyal and Kamphorst (2006) conclude that both costs and benefits heterogeneity are important in determining the level of connectedness of the network and that cost heterogeneity is important for the architecture of the network. They develop a model with homogeneous and two-sided benefits and heterogeneous costs. In particular, they consider a society divided into groups, where the costs of forming links increase with the distance between the groups. The links within a group are called internal links and forming them is cheaper than forming links outside the group (called external links). Therefore they name this model the *Insider-Outsider model.* The same approach is used in Kamphorst (2005) and further developed in Kamphorst and van der Laan (2007), where the model is rather called the *Multiple Group model.* The term *Insider-Outsider model* is used when there are only two groups concerned.

They show that in this model a strict Nash network may be a single center-sponsored star, a collection of center-sponsored stars, a *generalized center-sponsored star* or an empty network. The generalized center-sponsored star architecture has a central player i with the property that if we move along a path with players $i_1, i_2, \dots, i_k$ then the link from i_j to i_{j+1} is sponsored by $i_j, j = 1, \dots, k-1$. Furthermore, the group to which i belongs constitutes a center-sponsored star and all other groups are completely fragmented (there is no direct link between any members of such a group). Figure 2.2 shows an example of a generalized center-sponsored star with three groups, each having four members. The structures of strict Nash networks that arise for different values of the costs of internal links (denoted $f(O)$) and of external links of length one (denoted $f(1)$) are summarized in Table 2.1.

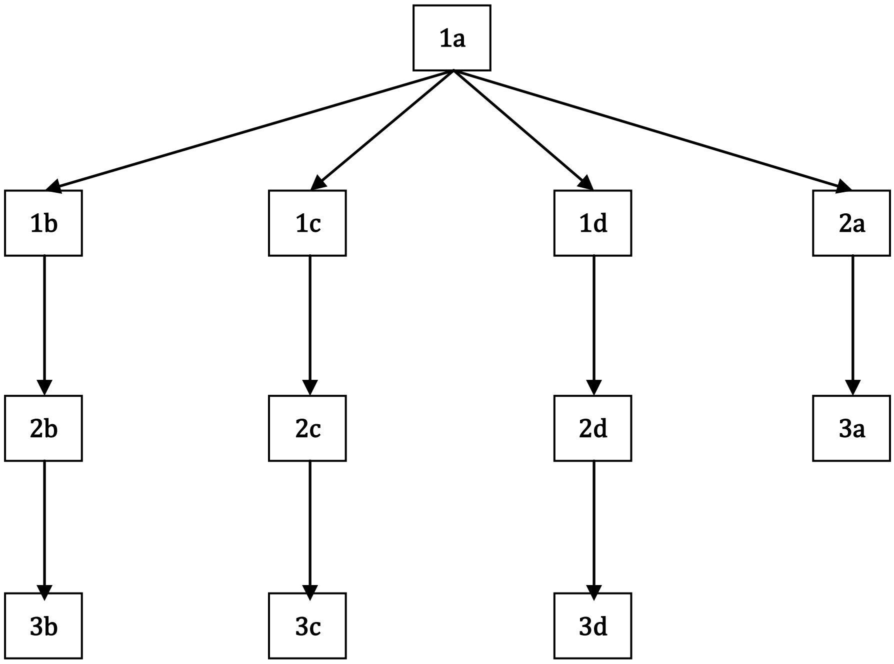


Figure 2.2 A generalized center-sponsored star architecture (Galeotti, Goyal, Kamphorst, 2003)

Table 2.1 Strict Nash networks for different values of $\boldsymbol{f(0)}$ and $\boldsymbol{f(1)}$

Values of $f(0)$ **and** $f(1)$	**Resulting strict Nash network**
$f(0)$ high	Empty network
$f(0)$ low, $f(1)$ high	Each group forms a center-sponsored star component
$f(0)$ low, $f(1)$ low	Generalized center-sponsored star
$f(0)$ low, $f(1)$ moderate	Strict Nash network does not exist

3 Signaling Games

In this chapter we present the theory of signaling games used in the later chapters. The main references here are Gibbons (1992) and Bierman and Fernandez (1998).

3.1 Incomplete and imperfect information

In order to present the theory of signaling games, we first need to explain the difference between the terms incomplete and imperfect information.

If every player has information about all moves other players have done before, we talk about *games of perfect information*. Only dynamic games can be games with perfect information. In dynamic games the players do not move simultaneously but in a fixed sequence, strategies are formed by sequences of moves (Maňas, 1991). Dynamic games are usually modeled as games in extensive form using a game tree. Game tree is a graph where all possible situations in the game are identified with nodes of the tree. A game starts in the root (=initial node) of the game tree, moves along a sequence of edges and ends in an end node (Maňas, 2009).

We talk about *games of imperfect information* if the player could not observe some of the moves the players have done before him. In that situation he cannot uniquely specify in which node of the game tree he makes his strategic decision (Dlouhý, 2009).

Games of complete information are games in which information about every player is available to all other players in the game. On the contrary, in *games of incomplete information* some players do not know the value the others place on the possible outcomes, so the payoffs are not common knowledge. Signaling games are dynamic games of incomplete information.

3.2 Basic Concept of Signaling Games

A signaling game generally involves two players: a Sender (*S*) and a Receiver (*R*). The Sender has private information about his *type.* The Receiver only has some *prior beliefs* concerning the type of *S*, which are given by a probability distribution. These prior beliefs are common knowledge. The Sender then sends a message (called a signal) and the Receiver responds with an action. The key idea is that communication can occur if one type of the informed player is willing to send a signal that would be too expensive for another type to send.

3.3 Perfect Bayesian Equilibrium in Signaling Games

In this section we introduce the main features of a *perfect Bayesian equilibrium* – an equilibrium concept used for dynamic games of incomplete information.

To be able to study games of incomplete information we need to transform them into games of complete but *imperfect* information. This process is called the *Harsanyi transformation.* The idea is that Nature moves first and chooses the player's *type*. The player then knows his own type, but does not know the types of his competitors. Players share a common *belief* about the way Nature makes its probabilistic choice. Because some players do not observe the move Nature did, we talk about games of imperfect information. However, the payoff functions of all players are now common knowledge, so it is a game of complete information.

We define a signaling game as follows (Gibbons, 1992):

1. Nature draws a type t_i for the Sender from a set of feasible types $T = \{t_1, \dots, t_I\}$ according to a probability distribution $p(t_i)$, where $p(t_i) > 0$ for every i and $p(t_1) + \dots + p(t_I) = 1$.
2. The Sender observes t_i and then chooses a message m_j from a set of feasible messages $M = \{m_1, \dots, m_J\}$.
3. The Receiver observes m_j (but not t_i) and then chooses an action a_k from a set of feasible actions $A = \{a_1, \dots, a_K\}$.
4. Payoffs are given by $U_S(t_i, m_j, a_k)$ and $U_R(t_i, m_j, a_k)$.

A player's strategy is a complete plan of action for every situation that might happen during the game. Therefore, a pure strategy for the Sender is a function $m(t_i)$ specifying which message will be chosen for each type that Nature might draw, and a pure strategy for the Receiver is a function $a(m_j)$ specifying which action will be chosen for each message that the Sender might send. We call the Sender's strategy *pooling* if every type sends the same message, and *separating* if every type sends a different message. A strategy can be *partially pooling* as well if some types send the same message and some send different messages.

Analogously to mixed strategies, we talk about *hybrid* strategies when a type of player chooses the message he sends randomly. But in the following text we will restrict our attention to pure strategies.

In order to define a perfect Bayesian equilibrium in a signaling game Gibbons (1992) introduces the following requirements:

Signaling Requirement 1 After observing any message m_j from M, the Receiver must have a belief about which type could have sent m_j. Denote this belief by the probability distribution $\mu(t_i|m_j)$, where $\mu(t_i|m_j) \geq 0$ for each t_i in T, and

$$\sum_{t_i \in T} \mu(t_i|m_j) = 1.$$

Signaling Requirement 2R For each m_j in M, the Receiver's action $a^*(m_j)$ must maximize the Receiver's expected utility, given the belief $\mu(t_i|m_j)$ about which types could have sent m_j. That is, $a^*(m_j)$ solves

$$\max_{a_k \in A} \sum_{t_i \in T} \mu(t_i|m_j) U_R(t_i, m_j, a_k).$$

Signaling Requirement 2S For each t_i in T, the Sender's message $m^*(t_i)$ must maximize the Sender's utility, given the Receiver's strategy $a^*(m_j)$. That is, $m^*(t_i)$ solves

$$\max_{m_j \in M} U_S\left(t_i, m_j, a^*(m_j)\right).$$

Signaling Requirement 3 For each m_j in M, if there exists t_i in T such that $m^*(t_i) = m_j$, then the Receiver's belief at the information set corresponding to m_j must follow from Bayes' rule and the Sender's strategy:

$$\mu(t_i|m_j) = \frac{p(t_i)}{\sum_{t_i \in T_j} p(t_i)}$$

where T_j denotes the set of types that send the message m_j. In other words, $t_i \in T_j$ if $m^*(t_i) = m_j$.

Definition (Gibbons, 1992):
A pure-strategy *perfect Bayesian equilibrium* in a signaling game is a pair of strategies $m^*(t_i)$ and $a^*(m_j)$ and a belief $\mu(t_i|m_j)$ satisfying Signaling Requirements (1), (2R), (2S) and (3).

To summarize the previous comments, we note that a perfect Bayesian equilibrium has the following properties:

1. Each player strategy is an optimal response to other players' strategies and the players' beliefs about the game.
2. Each player's belief can be derived from the other players' strategies using Bayes' theorem.

3.4 Job-Market Signaling

We will now restrict our attention to one application of signaling games, namely on signaling on a job market. This model was introduced by Michael Spence (1973) in his essay Job Market Signaling, where he first described the procedure of signaling.

We define the job-market signaling game analogously to the general case described in the previous section:

1. Nature chooses a type for the Worker corresponding to his productivity θ, which can be *high* (H) or *low* (L). The probability that $\theta = H$ is q.
2. The Worker observes his productivity and then chooses a level of education $e \geq 0$.
3. The Employer observes the Worker's education (but not his productivity) and offers him a wage w.
4. The Worker decides if he accepts the job with the offered wage w or not.

An important feature of the model is that obtaining an education e is associated with cost $c(\theta, e)$ depending on the productivity of the Worker. A crucial assumption is that high-productivity Workers find it less costly to obtain education than low-productivity Workers. In other words, the signaling costs are negatively correlated with productivity. Signaling costs may include psychic costs, time and other costs, as well as direct monetary costs. We can imagine that it is more difficult (costly) for students of lower ability to receive education than for high-ability students.

This condition can be rewritten more precisely as:

$$c_e(L, e) > c_e(H, e), \tag{3.1}$$

which states that for every level of education e the marginal cost of education for a low-productivity Worker is higher than the marginal cost for a high-productivity Worker. This implies that a low-productivity Worker will require a larger increase in wage to compensate him for an extra unit of education. The graphical result is that low-productivity Workers have steeper indifference curves than high-productivity Workers as shown in Figure 3.1. Because the indifference curves intersect exactly once, the equation (3.1) is often referred to as *single-crossing condition.*

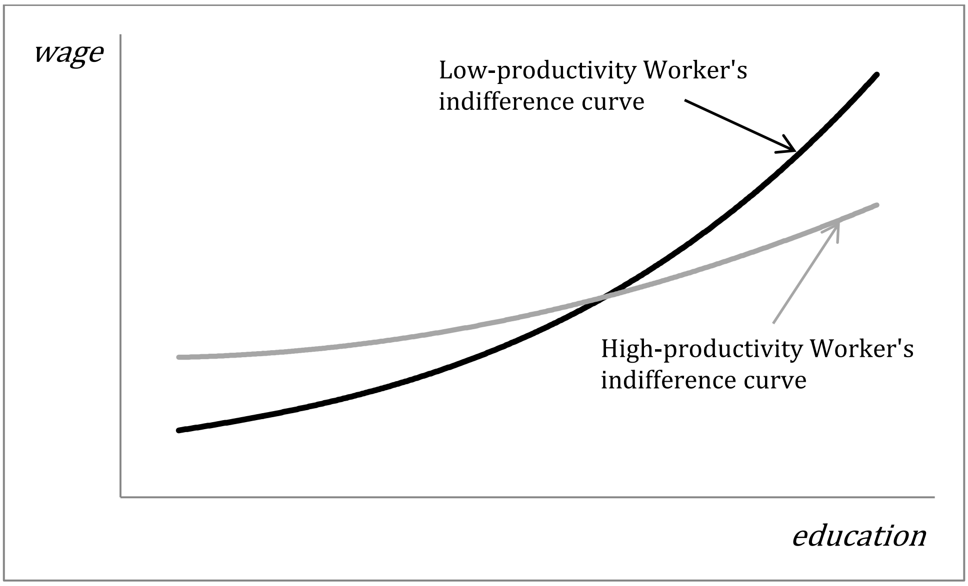


Figure 3.1 Single-crossing condition

The payoff of the Worker is then $w - c(\theta, e)$ if he accepts the job and $-c(\theta, e)$ if he does not, because the education is a sunk cost. The payoff of the Employer is $y(\theta, e) - w$ when the Worker is hired and 0 when he is not, where $y(\theta, e)$ is the output of a Worker with productivity θ and education e. We assume that at a given level of education high-productivity Workers are more productive that low-productivity Workers, *i.e.* $y(H, e) > y(L, e)$ for every e.

We will now discuss a specific numerical example proposed by Spence (1973). Suppose that the output of the Worker does not depend on his education but only on his productivity and that

$$y(H, e) = 2, \forall e$$

$$y(L, e) = 1, \forall e.$$

Further suppose that the costs of obtaining education for a low-productivity Worker are two times higher than for a high productivity Worker and that

$$c(H, e) = \tfrac{1}{2}e$$

$$c(L, e) = e,$$

as can be seen in Figure 3.2.

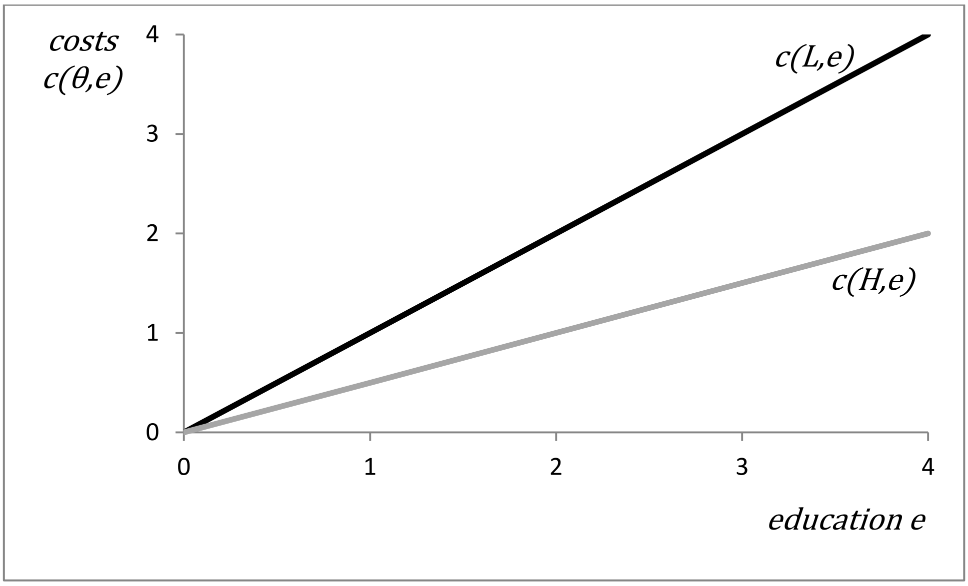


Figure 3.2 Costs of education

Suppose that the Employer beliefs that there is some level of education, say e^* such that if $e < e^*$, then productivity of the Worker is low with probability one, and if $e \geq e^*$, then productivity is high with probability one. Then he will offer the Worker a wage corresponding to his expected output, so a wage equal to one to every Worker with education $e < e^*$ and a wage equal to two to every Worker with $e \geq e^*$, as depicted in Figure 3.3.

The Worker chooses his optimal education according to this wage schedule. If he chooses $e < e^*$, his optimal choice is $e = 0$, because education is costly and there are no benefits of increasing his education until he reaches e^*, given the Employer's beliefs. Similarly, if he chooses $e \geq e^*$, he will in fact choose $e = e^*$, since increasing his education would not bring him any benefits. Therefore the Worker will either chose $e = 0$ or $e = e^*$.

The Worker is trying to maximize his payoff, which is the difference between his wage and cost of education. In Figure 3.4 the maximal possible payoffs for a low(L)- and high(H)-productivity Worker are shown with the corresponding optimal choices of education. This obviously depends on the level of e^* chosen, but in the case depicted in Figure 3.4 low-productivity Workers will chose $e = 0$ and high-productivity Workers will chose $e = e^*$. This confirms the Employer's beliefs and we have a signaling equilibrium.

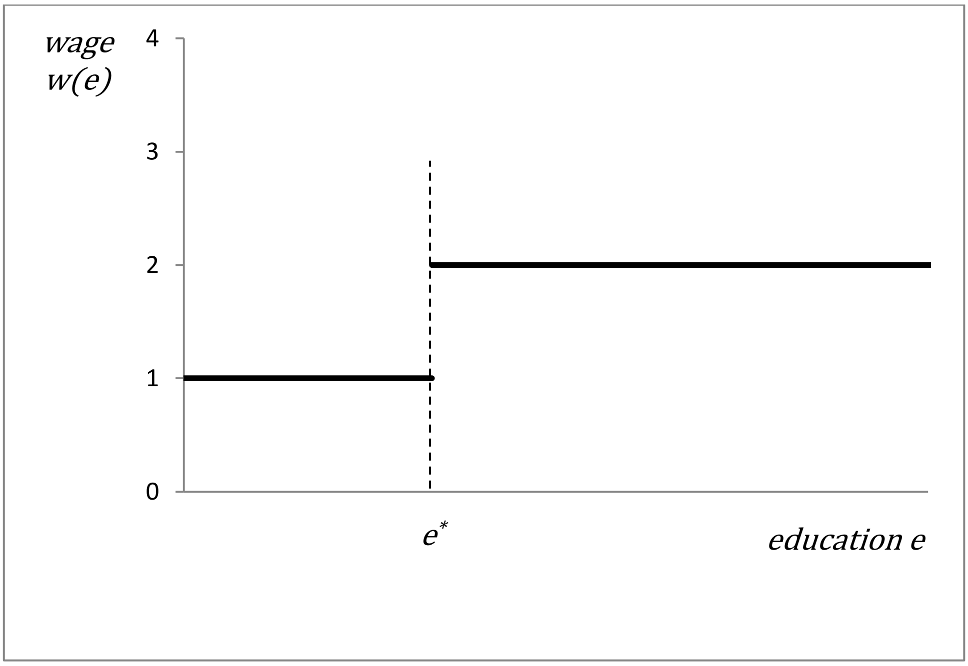


Figure 3.3 Offered wages as function of level of education (Spence, 1973)

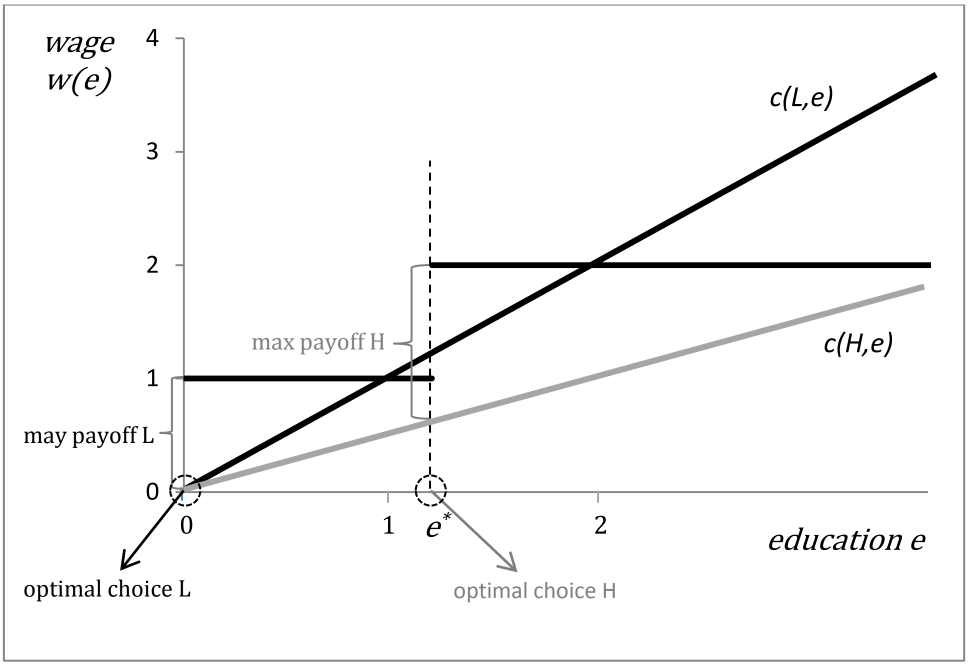


Figure 3.4 Optimal choice of education

We will now analyze for which levels of e^* the Employer's beliefs will be confirmed and we will have a signaling equilibrium. The optimal choice for L is $e = 0$ if:

$$w(0) - c(L, 0) > w(e^*) - c(L, e^*)$$

$$1 > 2 - e^*$$

and the optimal choice for H is $e = e^*$ if:

$$w(e^*) - c(H, e^*) > w(0) - c(H, 0)$$

$$2 - \frac{e^*}{2} > 1.$$

Putting these two conditions together, we obtain the inequality:

$$1 < e^* < 2.$$

So for each e^* that satisfies the condition $1 < e^* < 2$ we have a signaling equilibrium[5], given the Employer's beliefs used above. So there is an infinite number of such equilibria defined by the level of e^*. In any of those equilibria the Employer is able to make perfect point predictions about the productivity of the Worker, having observed his level of education.

[5] For $e^* = 1$ the low-productivity Worker is indifferent between $e = 0$ and $e = e^*$, because both choices bring him a payoff equal to one. Similarly, the high-productivity Worker is indifferent between $e = 0$ and $e = e^*$ for $e^* = 2$.

To summarize we will state the strategies and beliefs that form a perfect Bayesian equilibrium for this numerical example:

Worker's strategy: Not acquire education when he is a low-productivity type and choose $e = e^*$ when he is a high-productivity type.

Employer's strategy: Offer the Worker a wage $w = 1$ when his education $e < e^*$ and a wage $w = 2$ when his education $e \geq e^*$.

Employer's beliefs: The Worker is certainly a low-productivity type when his education $e < e^*$ and a high-productivity type if $e \geq e^*$.

Table 3.1 gives general formulas for payoffs of the Employer and the Worker in case he accepts or rejects the job offer. Calculations of the payoffs for the 16 possible situations which may arise are presented in Table 3.2. Figure 3.5 represents the game tree for this Signaling game. It has 16 end nodes corresponding to the 16 possible situations. One of them corresponds to the equilibrium when the Worker is a low-productivity type and another to the equilibrium when he is a high-productivity type.

Table 3.1 Payoffs for the Signaling game (general formulas)

	Worker	Employer
Accepts	$w - c(\theta, e)$	$y(\theta, e) - w$
Rejects	$-c(\theta, e)$	0

Table 3.2 Payoffs for the Signaling game (calculations)

Type	Educ	Wage	A/R		Payoffs: Worker	Payoffs: Employer
H	0	1	A		$1-0=1$	$2-1=1$
H	0	1	R		0	0
H	0	2	A		$2-0=2$	$2-2=0$
H	0	2	R		0	0
L	0	1	A		$1-0=1$	$1-1=0$
L	0	1	R		0	0
L	0	2	A		$2-0=2$	$1-2=-1$
L	0	2	R		0	0
H	e^*	1	A		$1-\frac{e^*}{2}$	$2-1=1$
H	e^*	1	R		$-\frac{e^*}{2}$	0
H	e^*	2	A		$2-\frac{e^*}{2}$	$2-2=0$
H	e^*	2	R		$-\frac{e^*}{2}$	0
L	e^*	1	A		$1-e^*$	$1-1=0$
L	e^*	1	R		$-e^*$	0
L	e^*	2	A		$2-e^*$	$1-2=-1$
L	e^*	2	R		$-e^*$	0

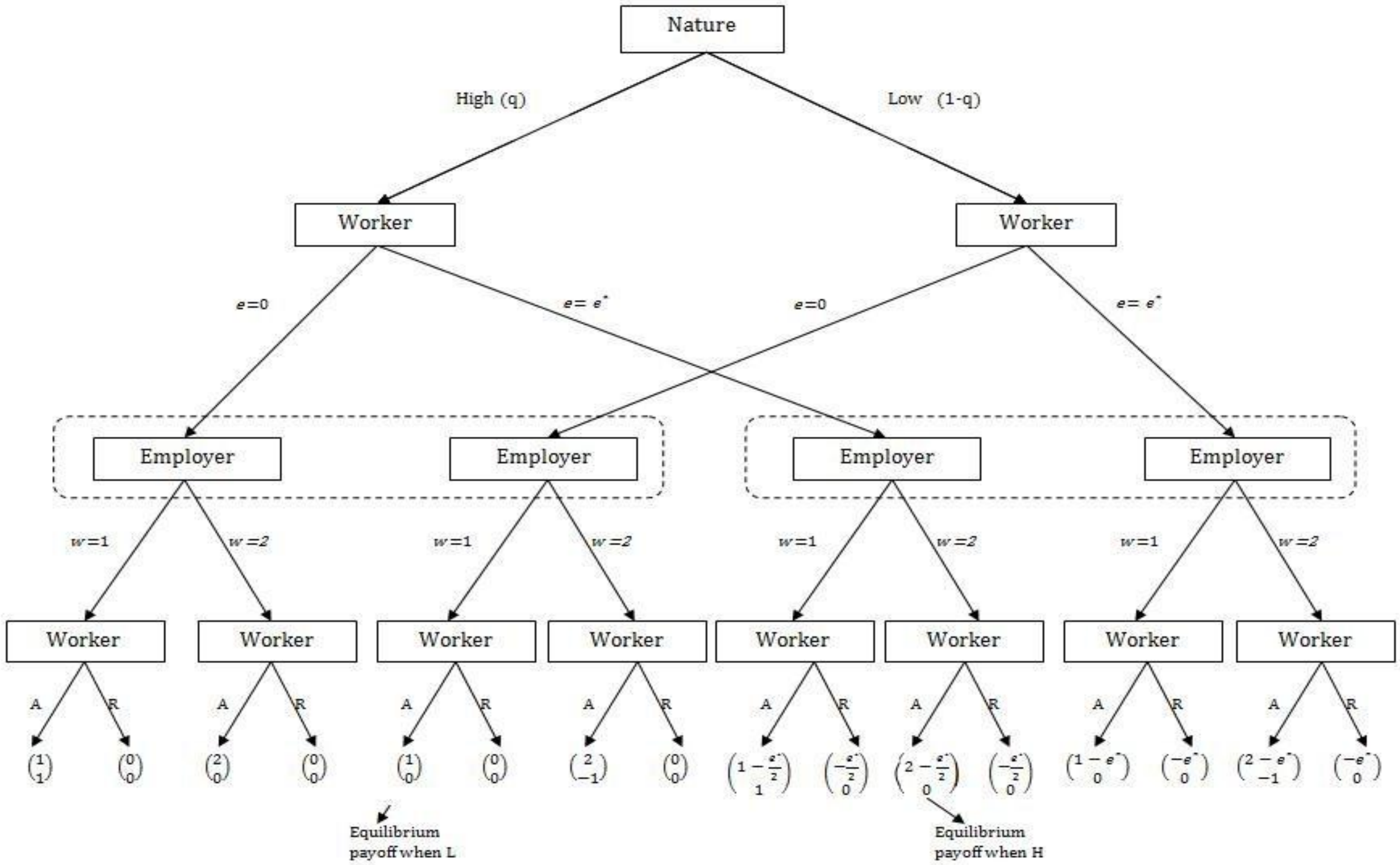


Figure 3.5 Game tree for the Signaling game

4 Models of Job Information Networks

In this chapter we restrict our attention to the way how information about job openings is transmitted through a network. We provide an overview of the models of job information networks described in the literature, before drawing up our own model, which combines strategic network formation models with signaling games theory.

4.1 Random Models of Job Information Networks

Several models that attempt to describe the transmission of information about job openings in a network of contacts have been presented in the literature. They can be split into two categories: *random* and *strategic* models.

In *random* models the network is created by some random process. A worker can then randomly lose his job or hear about a job opening. Ioannides and Datcher Loury (2004) prefer to call this class of models *exogenous* job information networks, because the structure of the networks does not depend on the strategies of the players and thus can be thought of as given.

On the contrary, in *strategic* models the structure of the network is given by individuals' uncoordinated actions. Ioannides and Datcher Loury (2004) call them models of *endogenous* job information networks. The topology of such networks depends on costs of link formation relative to benefits from the connection.

4.1.1 The Model of Montgomery

A random model of network formation where both Workers and Employers choose between formal and informal hiring channels was proposed by Montgomery (1991).

He develops a two-period model of the labor market where each Worker lives for one period, so a Worker lives either in period 1 or in period 2. Workers may be of two *types*, either *high* or *low* ability. For simplicity half of the Workers has high productivity and produces one unit of output while the other half produces zero units. Employers cannot observe the type of the Worker before offering him a wage and each Employer can hire at most one Worker in any period. Workers are unable to signal their productivity.

The profit of an Employer is equal to the output produced by his Worker minus the wage paid. Employers are free to enter the market in either period so the expected profit (for entering Employers) is driven to zero. Thus, offered wages will be equal to the expected productivity of the Workers in the market.

Each period-1 Worker knows with probability τ precisely one period-2 Worker and with probability $1 - \tau$ he does not know any period-2 Worker. If a period-1 Worker has a link with a period-2 Worker, then the latter is a Worker of the same type with probability $\alpha > \frac{1}{2}$. Note that some period-2 Workers may have ties with several period-1 Workers while others have none.

The timing of the game is as follows. First, Employers hire period-1 Workers through the market at an (equilibrium) wage w_1. After observing the productivity of his Worker, an Employer can make a *referral wage offer* to his Worker's social tie (if he has any). The referral wage offer made by Employer i is denoted w_{Ri}. Second, period-2 Workers compare all wage offers received and accept the highest or wait to find employment through the market. Third, period-2 Workers who have refused all offers or have not received any are hired through the market at an (equilibrium) wage w_2.

The equilibrium of the model involves aspects of a Nash equilibrium, since when the Employer offers a referral wage he is entering an auction against other potential Employers who might also make an offer to the same Worker. Montgomery proves the following result.

Proposition (Montgomery, 1991):

An Employer attempts to hire through referral if and only if he employs a high-productivity Worker in period 1, referral offers are dispersed over the interval $[w_2, \overline{w_R}]$.

The exact proof of this proposition is given in Montgomery (1991), but the idea behind it is that a Worker that enters the period 2 market is conditionally more likely to be connected to a low-productivity Worker or to lack connections. The expected value of such Worker is therefore less then ½ and so the wage w_2 is also less then ½. The referral offer has to be at least w_2 or it would never be accepted. Given that the Worker is connected to a period-1 high-productivity Worker and that w_2 is less then ½, the expected value of this Worker is more than w_2.

The maximum referral wage offer $\overline{w_R}$ is a wage at which the Employer attracts a referred Worker with probability 1. A referral wage offer generates a constant positive profit c over the interval $[w_2, \overline{w_R}]$, where c is given by $c(\alpha, \tau) = (2\alpha - 1)\tau / (e^{\alpha\tau} + e^{(1-\alpha)\tau})$. An offer lower than w_2 will never be accepted, while an offer higher than $\overline{w_R}$ increases the wage without increasing the probability of attracting a Worker.

An important aspect of this equilibrium is that Employers hiring through the market earn zero expected profit, while Employers making referral offers earn a positive expected profit c. They have a higher chance of finding high-productivity Workers through referrals than through the market. This is why Employers prefer to hire new Workers through referrals of their current employees. For the period-2 Workers having more social ties is advantageous, because it leads to higher expected wages.

4.1.2 The Model of Calvó-Armengol and Jackson

Calvó-Armengol and Jackson (2004; 2007) examine a model in which the information about jobs is transmitted through a network of Workers. This model does not take into account the role of the Employer.

In the simplest version of the model Workers are connected by an undirected network, represented by the matrix g with entries $g_{ij} \in \{0, 1\}$, such that $g_{ij} = g_{ji}$ for every i and j. Time evolves in discrete periods indexed by $t \in \{1, 2, \ldots\}$. The variable $s_{it} = 1$ indicates that agent i is employed at time t, $s_{it} = 0$ that he is unemployed. All jobs are considered to be identical and there is just one wage level.

Information about new job openings arrives randomly to the Workers in the networks at the beginning of each period. Each Worker directly hears about a job opening with a probability $a \in [0, 1]$. This job arrival process is independent across agents. If the Worker is unemployed, he takes the job and becomes employed. If we let $a = 1$, so that all Workers are sure to hear about a job in any period, every Worker that has been unemployed at the beginning of the period becomes employed. If the Worker is employed and hears about a job, he picks uniformly at random one of his unemployed neighbors and passes him the information. If he does not have any unemployed neighbors, the job information is lost as it cannot be passed to a second degree connection. At the end of each period, a Worker can lose his job with a probability $b \in [0, 1]$.

This model explains the phenomenon of *duration dependence* known from the empirical labor economics literature. A Worker that has been unemployed for a long time has a lower probability to find work in the next period than a Worker who is just recently unemployed. The explanation is that the probability of the Worker's neighbors being employed is decreasing with the time the Worker has been unemployed. This is due to the fact that the unemployed Worker has not been able to pass them any job information and that it is less likely that the neighbors are employed if the Worker has not heard about a job from them for a long period. So the longer the Worker is unemployed, the greater is the probability that his neighbors are also unemployed. Consequently, it is less likely that he will hear about a job opening from them. On the other hand, if a Worker is unemployed just for a short period, it is still likely that many of his neighbors are employed and that they will pass him a job information shortly.

The probability that the Worker will find work is also affected by the structure of the network. Figure 4.1 illustrates an example in which the employment rate of a Worker depends on his position in the network. Even though every Worker in the network has the same number of neighbors, the network is non-symmetric and the position of Workers 1 and 6 is advantageous. Calvó-Armengol and Jackson (2004) have used simulation to show that agents 1 and 6 have higher employment rates than other agents in the network. The link between agents 1 and 6 forms a bridge connecting the two groups and so none of their respective neighbors are linked to each other. This diversification in their social contacts allows them to have a higher probability of hearing about (at least) one job. In contrast, if the agent's neighbors are linked to one another, they are more likely to be either both employed (then the agent is more likely to hear about multiple jobs at once) or both unemployed (and the agent is less likely to hear about a job).

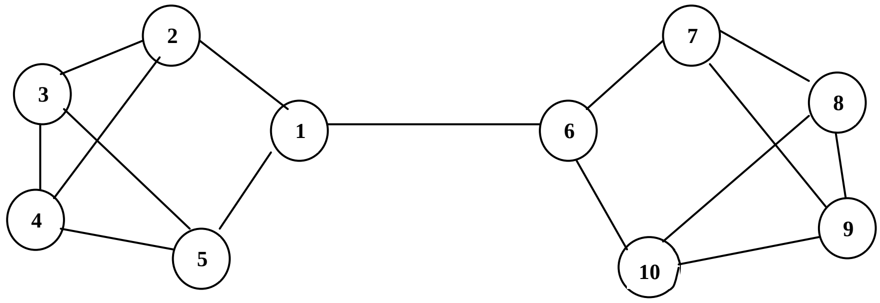


Figure 4.1 A network with a bridge (Calvó-Armengol and Jackson, 2004)

An important feature of this model is that a Worker can only benefit from direct connections with his neighbors. The benefit in this case is the possibility of hearing about a job from them. However, his second degree connections (or "friends of friends") do not bring him any benefits and moreover represent competition. If the Worker's neighbor hears about a job he decides randomly if he passes the information to the Worker or to some other neighbor. So it is advantageous for the Worker to have many neighbors, but it is not advantageous to have neighbors of neighbors. However, this is not a strategic model of network formation so the Worker cannot decide which links to form, the network structure is given.

Jackson (2008) examines a variation of this model where he allows agents to invest in education. A Worker is eligible for jobs only if he invests in education, otherwise he has a payoff of 0. The cost of education c_i varies across Workers. The variable x_i is equal to 1 if the Worker is educated, and $x_i = 0$ if he is not. A Worker can hear about a job from an employed and educated neighbor. The payoff of a Worker is his long-run employment rate minus the cost c_i. He will invest in education if the payoff is greater than 0. The model provides some explanation of poverty traps. If the whole group is uneducated, for just one member it is often not advantageous to become educated. On the other hand, if all his neighbors are educated it is more likely that it will be advantageous for the Worker to be educated as well.

4.2 Strategic Model of Calvó-Armengol

While in the model of Calvó-Armengol and Jackson discussed above the network of social interactions is given, Calvó-Armengol (2004) presents a model where the strategic consideration of the players affects the network structure. The basic structure of his model is similar to Calvó-Armengol and Jackson (2004; 2007): the network is undirected, a is the probability of hearing about a new job opportunity and b is the probability of losing a job. What is different is that the creation of a link is associated with some cost c shared by both players. The consent of both players is needed to create a link.

In the model, players create links in order to broaden their available information channels about potential jobs, meaning that they are engaged in *passive job search*. The effectiveness of this passive search is measured by the individual probabilities of getting a job through contacts.

Proposition (Calvó-Armengol, 2004):

The probability that player $i \in N$ gets a job through contacts is $P_i(g) = 1 - Q_i(g)$, where $Q_i(g) = \prod_{j \in N_i(g)} \left[1 - a(1-b)\frac{1-(1-b)^{n_j(g)}}{bn_j(g)}\right]$.

For all players $i \in N, Q_i(g)$ denotes the probability that player i does *not* find a job through contacts when the referral network is g. $N_i(g)$ denotes the neighborhood of node i and $n_j(g)$ the number of neighbors of node j.[6] It can be deduced from the expression of $P_i(g)$ given above that adding or removing a link only affects the two players concerned and their direct neighbors, because the job information cannot be passed to a second degree connection.

The probability $P_i(g)$ of player i getting a referred job increases with the size of his neighborhood $n_i(g)$ and decreases with the size of his direct neighbors' neighborhood $n_j(g), j \in N_i(g)$. Having more second degree contacts increases the competition for information.

In a *symmetric* network all players have the same number of neighbors μ and all positions in the network are equivalent. The probability of getting a referred job is then the same for all players and equal to $P = 1 - Q$, where $Q = \left[1 - a(1-b)\frac{1-(1-b)^{\mu}}{b\mu}\right]^{\mu}$. Calvó-Armengol (2004) shows that in a symmetric network P is strictly concave and has a global maximum at $\bar{\mu} > 1$. So the probability of getting a referred job increases with μ in sparse networks until some $\bar{\mu}$ is reached and then decreases with μ.

[6] Calvó-Armengol assumes that $N_i(g)$ does not contain i itself, $i \notin N_i(g)$.

In order to study the strategic network formation, we must define the individual payoffs of the players. Calvó-Armengol (2004) supposes that all jobs are associated with equal wage $w > 0$ and that creation of a link results in a cost $c > 0$ equal for both players who cooperate to form the link. The *expected net payoff* $Y_i(g)$ of some (initially employed) player $i \in N$ is given by $Y_i(g) = w\{(1-b) + b[a + (1-a)P_i(g)]\} - cn_i(g)$.

To characterize the topology of created networks, Calvó-Armengol (2004) uses the concept of pairwise stability introduced by Jackson and Wolinsky (1996). This concept supposes that two players must cooperate to form a link, while an individual can sever a link. A network is stable if no player benefits from adding or removing a link. Calvó-Armengol (2004) provides some characterizations of stable networks, but the model does not make it possible to describe the types of resulting networks as it was done for example in Bala and Goyal (2000). This situation where the topology of the network is much more difficult to study can be explained by the fact that in this case individual payoffs do not contain a component that is linear in the number of other players each player is connected with.

4.3 Developing a Model of Job Information Network with Signaling

Although there have been several models of job information networks presented in the literature, none of them allows education to be a signal of the productivity. We think that both the way the Employer is contacted and what he derives from the level of the Worker's education are important aspects of the job search process. Therefore we want to develop a model that combines the network theory with signaling games.

4.3.1 The Job Market Signaling Game

We first propose a simple model where the structure of the network is given. The network consists of an unemployed Worker, an Employer who is offering a job and another Worker who currently works for the Employer and who is a friend of the unemployed Worker. The unemployed Worker has two possibilities of contacting the Employer: a direct application or an indirect application through his friend as depicted in Figure 4.2.

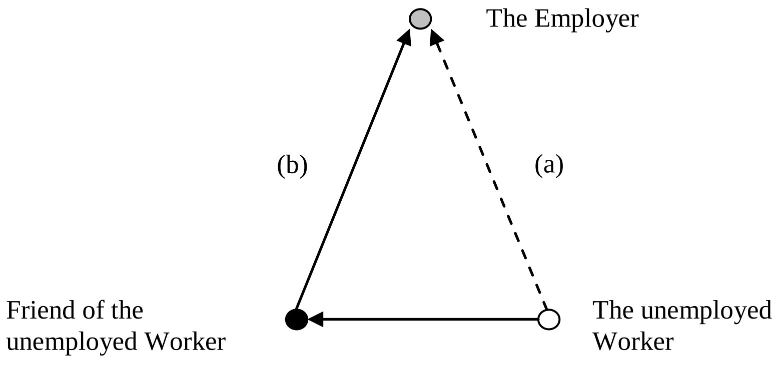


Figure 4.2 Possibilities of contacting the Employer: (a) direct application, (b) indirect application

The purpose of this model is to study which level of education the Worker will choose depending on his productivity and whether he will prefer a direct or an indirect application if he has both possibilities. It will serve as a basis for the model described in the next section.

Suppose that the *cost of using an indirect application* is u. This cost is entirely carried by the unemployed Worker. He makes a strategic decision about the way he contacts the Employer and about the level of education he chooses.

As in the numerical example of a Signaling game presented in chapter 1, we suppose that the costs of obtaining education depend on the productivity of the Worker. In particular, the costs for a low-productivity Worker (L) are two times higher than for a high-productivity Worker (H) for each level of education e:

$$c(H, e) = \frac{1}{2}e$$

$$c(L, e) = e.$$

We further suppose that the output y of the Worker does not depend on his education but only on his productivity and that

$$y(H, e) = 2, \forall e$$

$$y(L, e) = 1, \forall e.$$

Suppose now that the Employer believes that the Worker has high productivity with probability 1 if he (a) applies for the job directly and has education at least e_2, or (b) applies for the job through a high-productivity friend and has education at least e_1, where $0 \leq e_1 < e_2$. The Employer believes that the Worker has low productivity with probability 1 in all other cases. The intuition behind it is that the referral of his current employee makes the Employer believe that the Worker has high productivity at a lower level of education than he would in case of a direct application.

Note that this is true only if the current employee has high productivity. The Employer is able to observe the productivity of his current employee, but not the productivity of a Worker before hiring him. If the current employee has low productivity, the Employer would require his friend to have the same level of education e_2 as a Worker applying directly to believe that he has high productivity. So there is no point in applying for a job through a low-productivity friends because indirect applications are more costly and the required level of education is the same as in case of a direct application. The Worker would always prefer a direct application if he has both possibilities.

The Employer will offer the Worker a wage w corresponding to his expected output. Given this fact and the beliefs of the Employer, the wage schedule is as follows:

- $w = 1$ in case of a direct application and $e < e_2$,
- $w = 2$ in case of a direct application and $e \geq e_2$,
- $w = 1$ in case of an indirect application and $e < e_1$,
- $w = 2$ in case of an indirect application and $e \geq e_1$,

where $0 \leq e_1 < e_2$.

The Worker chooses his optimal education and the way he contacts the Employer according to this wage schedule. If he chooses $e < e_1$, his optimal choice is $e = 0$, because education is costly and there are no benefits of increasing his education until he reaches e_1. If he chooses $e \geq e_1$ and an indirect application, he will in fact choose $e = e_1$, since increasing his education would not bring him any benefits. Similarly, if he chooses a direct application, he will in fact choose $e = e_2$.

The payoff of the Worker is the wage he receives minus the cost of education minus the cost of using an indirect application. We will now analyze for which levels of e_1 and e_2 the Employer's beliefs will be confirmed and we will have a separating signaling equilibrium. In case of a direct application, the optimal choice for L is $e = 0$ if:

$$w(0, d) - c(L, 0) > w(e_2, d) - c(L, e_2)$$

$$1 - 0 > 2 - e_2$$

$$e_2 > 1,$$

and the optimal choice for H is $e = e_2$ if:

$$w(e_2, d) - c(H, e_2) > w(0, d) - c(H, 0)$$

$$2 - \frac{1}{2}e_2 > 1 - 0$$

$$1 > \frac{1}{2}e_2$$

$$2 > e_2.$$

A low-productivity Worker (L) will prefer a direct application with $e = 0$ over an indirect application with $e = e_1$ if:

$$w(0, d) - c(L, 0) > w(e_1, i) - c(L, e_1) - u$$

$$1 - 0 > 2 - e_1 - u$$

$$e_1 > 1 - u,$$

and a high-productivity Worker (H) will prefer an indirect application with $e = e_1$ over a direct application with $e = 0$ if:

$$w(e_1, i) - c(H, e_1) - u > w(0, d) - c(H, 0)$$

$$2 - \tfrac{1}{2}e_1 - u > 1 - 0$$

$$1 - u > \tfrac{1}{2}e_1$$

$$2 - 2u > e_1.$$

So for each e_1, e_2 that satisfy the conditions

$$1 - u < e_1 < 2 - 2u,$$

$$1 < e_2 < 2,$$

we have a signaling equilibrium.

Assuming that $e_1 \geq 0$, we have $0 \leq 2 - 2u$, which gives $u \leq 1$. As $1 - u \neq 2 - 2u$, we have $u \neq 1$, which gives $u < 1$. So the cost of using an indirect application must be smaller than 1 for the equilibrium to hold. Putting together the conditions $1 - u < e_1$ and $u < 1$, we receive $0 < e_1$. So the level of education e_1 must be strictly larger than zero for the equilibrium to hold.

Let's now examine the relation between the cost of education and the cost of application. A high-productivity Worker (H) will prefer an indirect application with $e = e_1$ over a direct application with $e = e_2$ if:

$$w(e_1, i) - c(H, e_1) - u > w(e_2, d) - c(H, e_2)$$

$$2 - \tfrac{1}{2}e_1 - u > 2 - \tfrac{1}{2}e_2$$

$$\tfrac{1}{2}(e_2 - e_1) > u. \tag{4.1}$$

So for a Worker of type H the difference between the level of education e_2 and e_1 must be more than two times larger than the cost of using a link to prefer the indirect application. If $e_2 - e_1 = 2u$, he is indifferent between the two possibilities. If on the other hand $e_2 - e_1 < 2u$ the Worker will prefer a direct application.

The inequality (4.1) can be rewritten as

$$e_1 < e_2 - 2u.$$

Using the equilibrium condition $e_2 < 2$, we receive

$$e_1 < e_2 - 2u < 2 - 2u,$$

which is the first equilibrium condition. So the first equilibrium condition always holds if (4.1) holds and $e_2 < 2$.

We have shown that if $1 - u < e_1 < 2 - 2u$, $0 < e_1$ and if $1 < e_2 < 2$, the optimal level of education for L is $e = 0$. When choosing the way of contacting the Employer, L will always prefer a direct application because it has the lowest cost.

To summarize we will state the strategies and beliefs that form a perfect Bayesian equilibrium for this game:

Worker's strategy: Not acquire education when he has low productivity. When he has high productivity choose either $e = e_1$ and apply for the job through a high-productivity friend, or choose $e = e_2$ and apply directly, depending on the relation between the cost of using an indirect application and the difference in the required levels of education.

Employer's strategy: Offer the Worker a wage $w = 2$ only if he (a) applies for the job directly and has education at least e_2, or (b) applies for the job through a high-productivity friend a has education at least e_1. In all other cases offer him a wage $w = 1$.

Employer's beliefs: The Worker certainly has high productivity if he (a) applies for the job directly and has education at least e_2, or (b) applies for the job through a high-productivity friend a has education at least e_1. In all other cases he certainly has low productivity.

4.3.2 A Strategic Model with Signaling

In the previous model the structure of the network was given and the Worker considered only the costs of contacting the Employer directly or through friends he already had. But forming links representing friendship with other Workers can be also subject of strategic decision. In this section we outline a possible structure of a model of job information networks which allows for strategic network formation and signaling.

We consider a network where all Workers are initially employed and none of them are linked. The timing of the game is as follows. They first choose with how many other Workers they wish to form links. In our model, we do not consider links between Workers and Employers and among Employers themselves. In the next step, each Worker may lose his job with probability $b \in]0,1[$. Then each Worker hears about a job opening with probability $a \in]0,1[$. If he is employed he does not use the information and the information is lost. If he is unemployed, he can apply for the job directly or through an employed high-productivity friend, if he has any. A Worker has the possibility to invest in education when applying for a job.[7]

[7] Perhaps it would seem more logical if the Worker chose his education when he forms links with other Workers. But to be able to apply the principles described in the previous section, we must allow him to choose his education at the same time as he chooses the way he contacts the Employer.

We assume that there is no competition for the jobs between Workers. If a Worker hears about a job and applies for it, he gets the job. A Worker remains unemployed if he does not hear about a job. Having more friends does not increase the probability of hearing about a job. The advantage of having friends is that it opens the possibility of applying for the job through a high-productivity friend, which requires lower education. The disadvantage is that forming links is associated with some cost l and using an indirect application is associated with cost u. So the Worker must compare those costs of forming and using links with the benefits of having them.

We suppose that both Workers must agree to form a link, both of them bear the cost of link formation l and both get the benefit from having the link (the possibility of using the link for an indirect application). We have shown in the previous section that a low-productivity Worker will always prefer a direct application. So if the Worker has low productivity, he will refuse to form any links. He would have to pay the cost of link formation l and it would not bring him any benefits. So only high-productivity Workers will create links with each other.

This finding brings more motivation to our assumption that when the Worker applies through a high-productivity friend the Employer believes that he has high productivity at a lower level of education than when he applies directly. If the Employer knows that only high-productivity Workers form links with each other, he will believe that the friend of his high-productivity employee will have high productivity as well. However, the conditions for the signaling equilibrium given in the previous section must hold. The lower bound for e_1 is given by the inequalities $1-u<e_1$ and $0<e_1$. So when the cost of using an indirect application u approaches 1, the level of education e_1 can approach 0. But e_1 can never be equal to zero, because the equilibrium would fail to hold.

We already know that a low-productivity Worker will not form any links. We will now study how many links a high-productivity Worker will wish to form depending on the parameters $a, b, l\,, u\,, e_1$ and e_2. To start, we summarize what we know about those parameters:

- a ... probability of hearing about a job opening

$$0<a<1$$

- b ... probability of losing a job

$$0<b<1$$

- l ... cost of link formation

$$0<l$$

- u ... cost of using an indirect application

$$0<u<1$$

- e_1 ... level of education needed to receive $w = 2$ in case of an indirect application

$$0 < e_1$$

$$1 - u < e_1 < 2 - 2u$$

- e_2 ... level of education needed to receive $w = 2$ in case of a direct application

$$e_1 < e_2$$

$$1 < e_2 < 2$$

We further know that a high-productivity Worker will prefer an indirect application over a direct application if

$$\tfrac{1}{2}(e_2 - e_1) > u.$$

From now on we assume that this condition holds, because otherwise even high-productivity Workers would not form any links. If they preferred a direct application or were indifferent between the two possibilities, they would have no reason to form costly links.

We will compare the *expected net payoff* Y_i of some (initially employed) Worker i when having 0, 1 and 2 friends. The expected net payoff Y_i is given by:

$$\begin{aligned} &Y_i(number\ of\ friends) \\ &\qquad = Pr(i\ keeps\ job).(wage - cost\ of\ link\ formation) \\ &\qquad + Pr(i\ fired\ and\ reemployed).(wage \\ &\qquad - costs\ of\ forming\ and\ using\ links\ and\ costs\ of\ education). \end{aligned}$$

In our case, the probabilities are:

$$\Pr(i\ keeps\ job) = 1 - b$$

- **1 friend:**

$$\Pr(i\ fired\ and\ friend\ fired) = b^2$$

$$\Pr(i\ fired\ and\ friend\ not\ fired) = b(1 - b)$$

- **2 friends:**

$$\Pr(i\ fired\ and\ both\ friends\ fired) = b^3$$

$$\begin{aligned} &\Pr(i\ fired\ and\ at\ least\ one\ of\ two\ friends\ not\ fired) \\ &\qquad = \Pr(i\ fired).\{1 - \Pr\,(both\ friends\ fired)\} = b(1 - b^2) \end{aligned}$$

We assume that if a high-productivity Worker is employed and keeps the job, he receives a wage equal to two. The *expected net payoffs* then are:

- **0 friends:**

$$Y_i(0) = (1-b)2 + ba(2 - \tfrac{1}{2}e_2)$$

- **1 friend:**

$$Y_i(1) = \underbrace{(1-b)(2-l)}_{\text{Keeps job}} + \underbrace{b^2a\big(2 - l - \tfrac{1}{2}e_2\big)}_{\text{Fired and friend fired, uses direct}} + \underbrace{b(1-b)a(2 - l - u - \tfrac{1}{2}e_1)}_{\text{Fired and friend not fired, uses indirect}}$$

- **2 friends:**

$$Y_i(2) = \underbrace{(1-b)(2-2l)}_{\text{Keeps job}} + \underbrace{b^3a\big(2 - 2l - \tfrac{1}{2}e_2\big)}_{\text{Fired and both friends fired, uses direct}} + \underbrace{b(1-b^2)a(2 - 2l - u - \tfrac{1}{2}e_1)}_{\text{Fired and at least one of his 2 friends not fired, uses indirect}}$$

The Worker will compare those expected net payoffs to decide how many links he wishes to form. He will prefer to form one link over forming zero links if:

$$Y_i(1) > Y_i(0).$$

This is true when $l < t_1$, where

$$t_1 = \frac{b(1-b)a\{u - \frac{1}{2}(e_2 - e_1)\}}{b - 1 - ba}.$$

The calculation of t_1, the threshold value for forming one link, is given at the end of this chapter. If the cost of link formation l does not exceed this threshold value, the high-productivity Worker prefers forming one link over forming zero links. As $b - 1 - ba < 0$ and $u - \frac{1}{2}(e_2 - e_1) < 0$, the threshold value t_1 is larger than zero ($t_1 > 0$) for all values of the parameters a, b, u , e_1 and e_2, which satisfy the conditions summarized above.

The Worker will prefer to form two links over forming one link if:

$$Y_i(2) > Y_i(1),$$

which is true when $l < t_2$, where

$$t_2 = \frac{b^2(1-b)a\{u - \frac{1}{2}(e_2 - e_1)\}}{b - 1 - ba}.$$

The calculation of t_2, the threshold value for forming two links, is also given at the end of the chapter. As $b - 1 - ba < 0$ and $u - \frac{1}{2}(e_2 - e_1) < 0$, t_2 is larger than zero ($t_2 > 0$) for all values of the parameters a, b, u , e_1 and e_2, which satisfy the conditions summarized above. It can be easily seen that $t_2 = bt_1$. As $0 < b < 1$, it follows that $t_2 < t_1$.

So if the cost of forming a link l is smaller than some positive threshold value t_1 given by the formula above, a high-productivity Worker will prefer to form a link over not forming a link. If moreover l is smaller than t_2, which is a positive threshold value smaller than t_1, the high-productivity Worker prefers forming two links over forming one link. So the lower is the cost of link formation l, the more links the high-productivity Worker will wish to form, when other parameters remain the same.

We will now continue the calculations in a similar way for higher numbers of links in order to find a general formula specifying for which level of l the Worker will wish to form one more link when having n friends. We first need to calculate the *expected net payoff* Y_i of some (initially employed) Worker when having $3, n$ and $n+1$ friends.

The corresponding probabilities are:

- **3 friends:**

$$\Pr(i\ fired\ and\ all\ 3\ friends\ fired) = b^4$$

$$\begin{aligned}&\Pr(i\ fired\ and\ at\ least\ one\ of\ 3\ friends\ not\ fired)\\ &\qquad = \Pr(i\ fired).\{1 - \Pr\ (all\ 3\ friends\ fired)\} = b(1 - b^3)\end{aligned}$$

- ***n* friends:**

$$\Pr(i\ fired\ and\ all\ n\ friends\ fired) = b^{\mathrm{n}+1}$$

$$\begin{aligned}&\Pr(i\ fired\ and\ at\ least\ one\ of\ n\ friends\ not\ fired)\\ &\qquad = \Pr(i\ fired).\{1 - \Pr\ (all\ n\ friends\ fired)\} = b(1 - b^n)\end{aligned}$$

And the *expected net payoffs* are:

- **3 friends:**

$$Y_i(3) = \underbrace{(1-b)(2-3l)}_{\text{Keeps job}} + \underbrace{b^4 a\left(2 - 3l - \tfrac{1}{2}e_2\right)}_{\substack{\text{Fired and all 3 friends}\\ \text{fired, uses direct}}} + \underbrace{b(1-b^3)a(2 - 3l - u - \tfrac{1}{2}e_1)}_{\substack{\text{Fired and at least one of his 3 friends not}\\ \text{fired, uses indirect}}}$$

- ***n* friends:**

$$Y_i(n) = \underbrace{(1-b)(2-nl)}_{\text{Keeps job}} + \underbrace{b^{n+1} a\left(2 - nl - \tfrac{1}{2}e_2\right)}_{\substack{\text{Fired and all } n \text{ friends}\\ \text{fired, uses direct}}} + \underbrace{b(1-b^n)a(2 - nl - u - \tfrac{1}{2}e_1)}_{\substack{\text{Fired and at least one of his } n \text{ friends}\\ \text{not fired, uses indirect}}}$$

- **$\boldsymbol{n+1}$ friends:**

$$Y_i(n+1) =$$

$$\underbrace{(1-b)(2-nl-l)}_{\text{Keeps job}} + \underbrace{b^{n+2}a\left(2-nl-l-\tfrac{1}{2}e_2\right)}_{\substack{\text{Fired and all } n+1 \\ \text{friends fired, uses direct}}} + \underbrace{b(1-b^{n+1})a(2-nl-l-u-\tfrac{1}{2}e_1)}_{\substack{\text{Fired and at least one of his } n+1 \text{ friends} \\ \text{not fired, uses indirect}}}$$

The Worker will prefer to form three links over forming two links if:

$$Y_i(3) > Y_i(2).$$

This is true when $l < t_3$, where

$$t_3 = \frac{b^3(1-b)a\left\{u - \frac{1}{2}(e_2 - e_1)\right\}}{b - 1 - ba}.$$

The calculation of t_3, the threshold value for forming three links, is given at the end of the chapter. Again it can be easily seen that $t_3 = bt_2$ and that $t_3 < t_2$ as $0 < b < 1$.

In case the Worker has n friends, he will wish to form one more link if it increases his *expected net payoff*, that is if:

$$Y_i(n+1) > Y_i(n).$$

This is true when $l < t_{n+1}$, where

$$t_{n+1} = \frac{b^{n+1}(1-b)a\left\{u - \frac{1}{2}(e_2 - e_1)\right\}}{b - 1 - ba}.$$

So if the cost of link formation l is smaller than some threshold value t_{n+1}, a high-productivity Worker who has n friends will wish to form one more link. The calculation of this threshold value t_{n+1} is given at the end of this chapter. This threshold value is always positive ($t_{n+1} > 0$ for all values of the parameters n, a, b, u , e_1 and e_2, which satisfy the given conditions). It can be easily shown that $t_{n+1} = bt_n$, which implies that $t_{n+1} < t_n$ for all values of n.

We have found a general formula for the threshold value for each number of friends the Worker may have. If the cost of link formation l is below this value, the high-productivity Worker will wish to form one more link. This value depends on the parameters a, b, u , e_1 and e_2 of the model.

If we know all the parameters of the model including the cost of link formation l, we can determine how many links high-productivity Workers will form. They will form exactly n links if $t_{n+1} < l < t_n$. The resulting structure of the network will be such that high-productivity Workers will form the same number of links with each other depending on the parameters of the model and low-productivity Workers will not form any links.

Calculation of t_1

$$Y_i(1) > Y_i(0)$$

$$(1-b)(2-l) + b^2a(2-l-\tfrac{1}{2}e_2) + b(1-b)a(2-l-u-\tfrac{1}{2}e_1) > (1-b)2 + ba(2-\tfrac{1}{2}e_2)$$

$$2-l-2b+bl+2b^2a-b^2al-\tfrac{1}{2}b^2ae_2+2ba-bal-bau-\tfrac{1}{2}bae_1-2b^2a+b^2al+b^2au+\tfrac{1}{2}b^2ae_1 > 2-2b+2ba-\tfrac{1}{2}bae_2$$

$$-l+bl-bal > -\tfrac{1}{2}bae_2+\tfrac{1}{2}b^2ae_2+bau+\tfrac{1}{2}bae_1-b^2au-\tfrac{1}{2}b^2ae_1$$

$$l(b-1-ba) > bau(1-b) - \tfrac{1}{2}ba(e_2-e_1)(1-b)$$

$$l(b-1-ba) > b(1-b)a\left\{u-\tfrac{1}{2}(e_2-e_1)\right\}$$

$$l < \frac{b(1-b)a\{u-\frac{1}{2}(e_2-e_1)\}}{b-1-ba}$$

As $(b-1-ba) < 0$.

Calculation of t_2

$$Y_i(2) > Y_i(1)$$

$$(1-b)(2-2l) + b^3a(2-2l-\tfrac{1}{2}e_2) + b(1-b^2)a(2-2l-u-\tfrac{1}{2}e_1) > (1-b)(2-l) + b^2a(2-l-\tfrac{1}{2}e_2) + b(1-b)a(2-l-u-\tfrac{1}{2}e_1)$$

$$2-2l-2b+2bl+2b^3a-2b^3al-\tfrac{1}{2}b^3ae_2+2ba-2bal-bau-\tfrac{1}{2}bae_1-2b^3a+2b^3al+b^3au+\tfrac{1}{2}b^3ae_1$$
$$> 2-l-2b+bl+2b^2a-b^2al-\tfrac{1}{2}b^2ae_2+2ba-bal-bau-\tfrac{1}{2}bae_1-2b^2a+b^2al+b^2au+\tfrac{1}{2}b^2ae_1$$

$$-l+bl-bal > -\tfrac{1}{2}b^2ae_2+\tfrac{1}{2}b^3ae_2+b^2au+\tfrac{1}{2}b^2ae_1-b^3au-\tfrac{1}{2}b^3ae_1$$

$$l(b-1-ba) > b^2au(1-b) - \tfrac{1}{2}b^2a(e_2-e_1)(1-b)$$

$$l(b-1-ba) > b^2(1-b)a\left\{u-\tfrac{1}{2}(e_2-e_1)\right\}$$

$$l < \frac{b^2(1-b)a\{u-\tfrac{1}{2}(e_2-e_1)\}}{b-1-ba}$$

As $(b-1-ba) < 0$.

Calculation of t_3

$$Y_i(3) > Y_i(2)$$

$$(1-b)(2-3l) + b^4a(2-3l-\tfrac{1}{2}e_2) + b(1-b^3)a(2-3l-u-\tfrac{1}{2}e_1)$$
$$> (1-b)(2-2l) + b^3a(2-2l-\tfrac{1}{2}e_2) + b(1-b^2)a(2-2l-u-\tfrac{1}{2}e_1)$$

$$2-3l-2b+3bl+2b^4a-3b^4al-\tfrac{1}{2}b^4ae_2+2ba-3bal-bau-\tfrac{1}{2}bae_1-2b^4a+3b^4al+b^4au+\tfrac{1}{2}b^4ae_1$$
$$> 2-2l-2b+2bl+2b^3a-2b^3al-\tfrac{1}{2}b^3ae_2+2ba-2bal-bau-\tfrac{1}{2}bae_1-2b^3a+2b^3al+b^3au+\tfrac{1}{2}b^3ae_1$$

$$-l+bl-bal > -\tfrac{1}{2}b^3ae_2+\tfrac{1}{2}b^4ae_2+b^3au+\tfrac{1}{2}b^3ae_1-b^4au-\tfrac{1}{2}b^4ae_1$$

$$l(b-1-ba) > b^3au(1-b) - \tfrac{1}{2}b^3a(e_2-e_1)(1-b)$$

$$l(b-1-ba) > b^3(1-b)a\left\{u-\tfrac{1}{2}(e_2-e_1)\right\}$$

$$l < \frac{b^3(1-b)a\{u-\frac{1}{2}(e_2-e_1)\}}{b-1-ba}$$

As $(b-1-ba) < 0$.

Calculation of t_{n+1}

$$Y_i(n+1) > Y_i(n)$$

$$(1-b)(2-nl-l) + b^{n+2}a(2-nl-l-\tfrac{1}{2}e_2) + b(1-b^{n+1})a(2-nl-l-u-\tfrac{1}{2}e_1)$$
$$> (1-b)(2-nl) + b^{n+1}a(2-nl-\tfrac{1}{2}e_2) + b(1-b^n)a(2-nl-u-\tfrac{1}{2}e_1)$$

$$2-nl-l-2b+nbl+bl+2b^{n+2}a-nb^{n+2}al-b^{n+2}al-\tfrac{1}{2}b^{n+2}ae_2+2ba-nbal-bal-bau-\tfrac{1}{2}bae_1-2b^{n+2}a+nb^{n+2}al+b^{n+2}al$$
$$+b^{n+2}au+\tfrac{1}{2}b^{n+2}ae_1$$
$$> 2-nl-2b+nbl+2b^{n+1}a-nb^{n+1}al-\tfrac{1}{2}b^{n+1}ae_2+2ba-nbal-bau-\tfrac{1}{2}bae_1-2b^{n+1}a+nb^{n+1}al+b^{n+1}au$$
$$+\tfrac{1}{2}b^{n+1}ae_1$$

$$-l+bl-bal > -\tfrac{1}{2}b^{n+1}ae_2+\tfrac{1}{2}b^{n+2}ae_2+b^{n+1}au+\tfrac{1}{2}b^{n+1}ae_1-b^{n+2}au-\tfrac{1}{2}b^{n+2}ae_1$$

$$l(b-1-ba) > b^{n+1}au(1-b)-\tfrac{1}{2}b^{n+1}a(e_2-e_1)(1-b)$$

$$l(b-1-ba) > b^{n+1}(1-b)a\left\{u-\tfrac{1}{2}(e_2-e_1)\right\}$$

$$l < \frac{b^{n+1}(1-b)a\{u-\frac{1}{2}(e_2-e_1)\}}{b-1-ba}$$

As $(b-1-ba) < 0$.

Conclusion

The aim of this thesis was to develop a model of job information network which allows for signaling. In order to do so, we have studied the theory of network formation and the signaling games theory, before focusing on the models of job information networks. There exist several models of job information networks in the literature which we have used as a source of inspiration, but none of them makes signaling of the Worker's productivity possible.

In the model of Montgomery, Workers may be of two types, either high or low productivity. But they are unable to signal their productivity by the level of their education. Similarly to our model, the Workers have the possibility to find a job through referral of a high-productivity friend. The difference is that in Montgomery's model the Employer makes a referral wage offer only to a friend of his high-productivity employee. In our model it is the Worker who chooses the way how to contact the Employer. He will apply through a friend only if both of them have high productivity and the condition $\frac{1}{2}(e_2 - e_1) > u$ holds, in all other cases he will apply directly.

In both models it is advantageous for the Employer to hire through referrals. In Montgomery's model he has a higher chance to find a high-productivity Worker through referrals than through the market. In our model he knows that only high-productivity Workers form links with each other, so when a Worker applies through a high-productivity friend and has education at least e_1, he certainly has high productivity. Another difference between the models is that in the model of Montgomery the links between period-1 and period-2 Workers are created randomly, while in our model the link formation depends on the strategic decision of the Workers and there is just one period.

The model of Calvó-Armengol and Jackson shares two main characteristics with our model: the probability of hearing about a job opening a and the probability of losing a job b. However, other aspects of the model are different. In the model of Calvó-Armengol and Jackson, Workers do not differ by their productivity and there is just one wage level. They are able to pass information about a job opening to an unemployed neighbor if they do not need it. In our model there are two types of Workers depending on their productivity and two wage levels. Information about a job opening received by an employed Worker is lost. The main difference is that in the model of Calvó-Armengol and Jackson the network structure is given and the Worker cannot decide which links he wishes to form, while in our model he makes a strategic decision about the number of links formed.

The strategic network formation is a feature that our model has in common with the model of Calvó-Armengol. His model is based on the model of Calvó-Armengol and Jackson, but the network structure is affected by strategic consideration of the players. In his model the probability of getting a job increases with the number of friends the Worker has, but decreases with the number of friends of those friends. On the contrary, in our model the probability of getting a job does not depend on the number of friends.

This is the main characteristic of our model: having more friends does not increase the probability of getting a job, but it increases the probability of being able to apply through an indirect application. The indirect application is advantageous for high-productivity Workers because to get the wage level corresponding to their output they have to reach a lower level of education than in case of a direct application. Low-productivity Workers always prefer direct applications, so they do not form any links. The number of links formed by high-productivity Workers is given by the cost of link formation l and other parameters of the model.

A possible variation of our model could allow passing of the job information to unemployed neighbors. Then the probability of getting a job would increase with the number of friends and thus the Workers would be more motivated to form links. If the cost of link formation l is low enough, also low-productivity Workers would form links, even though they would not use them for indirect applications but only to increase the probability of hearing about a job. The resulting structure of the network would be much more complicated.

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