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Value at Risk models for Energy Risk Management

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Declaration of authorship

I hereby declare and confirm that this thesis is entirely the result of my own work except where otherwise indicated. I gratefully acknowledge supervision and guidance I have received from Doc. Jiri Hnilica.

Prague, 25th August 2011 Martin Novak

Abstract

The main focus of this thesis lies on description of Risk Management in context of Energy Trading. The paper will predominantly discuss Value at Risk and its modifications as a main overall indicator of Energy Risk.

Keywords: Commodity, Risk, Value at Risk, Derivatives, Energy Trading, Portfolio Theo

Preface

It's not whether you're right or wrong that's important, but how much money you make when you're right and how much you lose when you're wrong.

George Soros

Risk is pervasive, represents important influence and circumstances of every decision-making of every human being, not only investors, banks or trading companies. Together with liquidity and return, risk forms "trinity" that determines character of every investment. Economic model of a rational investor assumes that every investor aims to maximize returns and minimize risks. Every investor's decisions are not made in vacuum. Adverse or favourable market conditions represent always certain form of risk that investors should mitigate.

Investors would never be able to successfully reduce or eliminate any risk in case they do not how big the risk is. Quantification of risk enables various methods and concepts that emerged mostly since the "volatility boom" during the oil crisis in 1970's. Among several risk measures, Value at Risk prevailed and became a standard at multiple areas.

Since its birth, Value at Risk has not been always flawless. In general, Value at Risk does not describe the worst loss, but it initially was not designed to do so. Therefore Value at Risk is often accompanied by similar but modified risk measures that reflect more accurately certain aspects of risk.

Motivation for the work presented in this thesis stemmed from my experience at E.ON Energy Trading SE where I spent four months as an intern and strived to get familiar with the overwhelming complexity of energy trading industry.

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Introduction

The prevailing motivation for selection of this topic of my thesis was the following problem: "Assuming that there are several commonly used models and concepts measuring risks of underlying assets, what is the optimal risk concept in case of energy commodities and what are the major conceptual differences pertaining to them?"

Structurally, the paper is divided into two major parts. Whereas the first part provides readers with an overview of an elementary theoretical foundation of the discussed topic, the part number two presents empirical applications and final conclusion.

The theoretical part comprises the first four chapters, each addressing various aspects of rather extensive thematic areas that introduce into the subject. In Chapter 1 the basic characteristic of Energy Trading is given, explaining basics of energy markets, energy commodities and giving a brief overview of trading and financial derivatives.

Risk Management in the context of Energy Trading is discussed in Chapter 2 together with Portfolio Theory and theory of Hedging, starting with a brief introduction and describing numerous classes of risks that traders at energy trading companies need to take into account.

Chapter 3 addresses the description of Value at Risk (VaR) measure with its history, fundamental parameters and three dominating methodologies how to estimate it most accurately, namely analytical (variance-covariance) approach, historical simulation and Monte Carlo simulation.

Multiple modifications of Value at Risk, discussed in Chapter 4, emerged from the widespread application of the original concept of Value at Risk in various business areas and brought to light miscellaneous adjustments and extensions. Especially in the Energy Trading industry models like Liquidity adjusted Value at Risk (LVaR), Expected Shortfall (or Conditional Value at Risk) or Modified Value at Risk (mVaR) are common. All these concepts were created to calm critics of the original VaR concept and overcome its significant deficiencies.

The empirical part of the thesis comprises Chapter 5 that attempts to apply the presented risk concepts (analytical VaR, historical simulation, Monte Carlo simulation, mVAR and CVaR) on the market portfolio consisting of various energy commodities and calculate its estimated values. Further, all the numerical values are compared, discussed and possible reasons for differences between the individual models are presented.

The last Chapter 6 concludes all the portfolio risk measurements and suggests further research. All definitions or models presented in this work are collected from available books, journals or articles, which are listed at the end of this thesis.

Appendix A represents a supplement to the Empirical part and provides readers with three charts displaying 10-year historical prices of underlying energy commodities from analysed hypothetical portfolio – natural gas, crude oil and gasoline.

Appendix B shows daily returns of the energy commodities mentioned above, displayed on three charts.

Finally, Appendix C presents three figures providing empirical distribution of 10-year historical prices of natural gas, crude oil and gasoline, overlaid by its best-fit distribution defined by modelling tool @Risk from Palisade Corp.

Chapter 1

Energy Trading

1.1. **Introduction**

The history of energy trading has encountered series of troubles in its progression. The energy markets have developed rapidly since the growth of oil spot markets in the 1970's. Until then prices of commodities were relatively stable. Nevertheless since the oil crisis in 1970's the price volatility became a market phenomenon. The recent market volatility is demonstrated in Figure 1. The liberalization of natural gas and electricity markets across Europe has led to the emergence of gas and power trading markets in many countries. Coal trading has also developed in recent years and the trading of carbon emissions has grown following the creation of the EU Emissions Trading Scheme[1] (EU ETS) in 2005 and the start of the implementation phase of the Kyoto protocol in 2008.

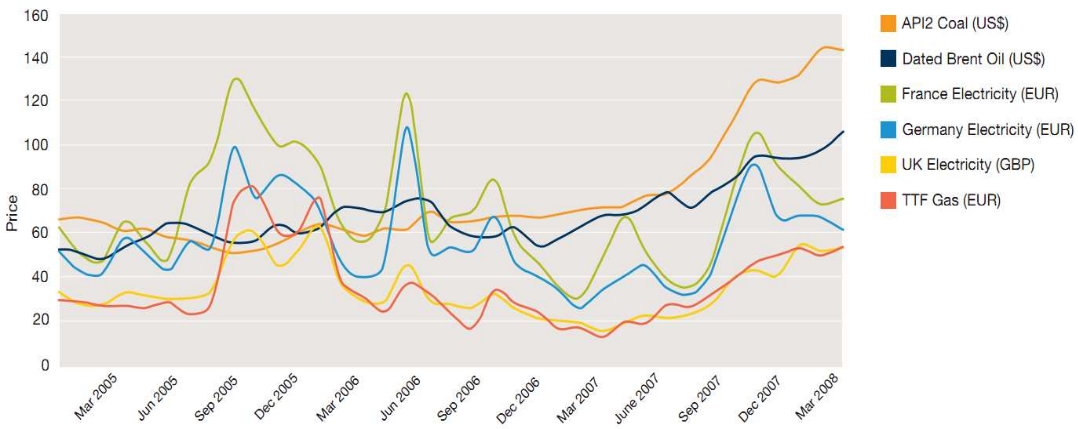

Figure 1 - Prices of a range of commodities (2005-2008) [PwC Commodity Risk in the oil & gas, power utility and mining sector]

The most important commodities in the energy market, which are described further in this chapter, are electricity, crude oil, natural gas, coal and emissions. All the products are usually utilized for heat or electricity generation. As both electricity and heat cannot be easily stored or transported, most of trades are arranged to future delivery at some location and time. Trading is usually performed either in the spot or forward market. Spot market lists commodities for immediate (on the day) delivery, whereas conditions of delivery in forward markets are given by details of underlying transaction. Spot markets can be very volatile due to the fact that no one can precisely predict actual energy demand. In order to cope with this

[1] http://ec.europa.eu/clima/policies/ets/index_en.htm

aspect, energy spot markets are relatively abundant and located in areas of the biggest energy consumption. High volatility sometime enables traders to take advantage of inverse changes in prices in nearby areas (e.g. cross-border trades). Another important feature of spot markets is that they may be subject to strict local regulation.
The high price volatility tends to disappear in forward markets, where trades are agreed between sellers and buyers prior to its delivery. Transactions are usually standardised and specified in matter of quantity, delivery, price etc.

Contracts have either physical or financial settlement. Physical delivery of actual commodity upon the specified delivery date and location is not common. More often traders close out their open positions with offsetting contracts prior to delivery.

All trading activities of energy commodities occur in two important forms. Trades are made either directly between counterparties or through an exchange. The direct form is called over the counter (OTC) trading, the latter exchange trading.
OTC trading lies in direct agreements between counterparties and involves counterparty risk of possible default if one party goes bankrupt. OTC trades are usually based on standard master agreements to facilitate negotiations. These standards are provided by associations such as EFET[2] or ISDA[3]. Companies usually measure the counterparty risk by credit scores or enter into credit default swaps (CDS) as a form of insurance against contractual default. More about counterparty risk discusses Chapter 2.

Trading on exchanges mitigates the counterparty risk and eliminates lot of paperwork. Trades are usually anonymous hence traders do not need to know every single detail of other traders. They can also use a broker who executes trades on behalf of its clients. On the other hand, exchange trading provides only standardized array of contracts so that any customized structured deals are carried out OTC. Exchanges require from every trading company to deposit margin to eliminate credit risk. Margin accounts of trading participants are settled every day by exchange's clearing house that handles credit and debits applied to the margin account. The system of exchange trading is illustrated in Figure 2.

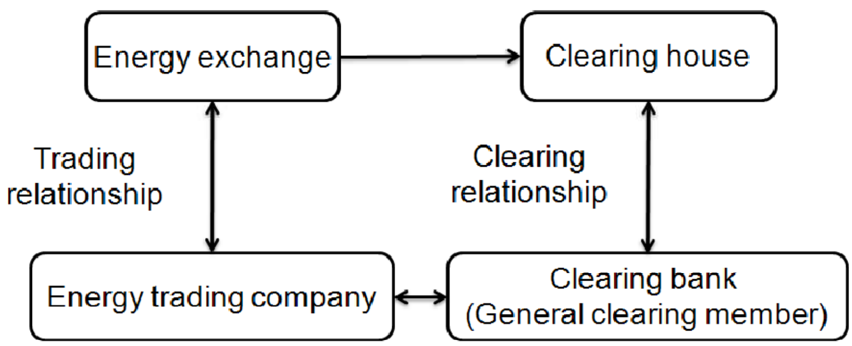

Figure 2 - Exchange trading ecosystem

[2] http://www.efet.org/
[3] http://www2.isda.org/

The most common derivative contracts for energy trading are forward, futures, option and swaps. Here I present only a short summary about the contracts, more information about principles of margining and fundamentals of derivative contracts could be found, for example, in Hull [26].

Futures and forwards are standardized agreements to buy or sell a commodity on a specified future delivery date at specified price. Futures contracts are traded on exchanges that allow choosing from limited number of product quality, delivery locations and periods. Futures contracts are freely liquid and transferable because an exchange always serves as counterparty. In contrast to futures, forwards are contracts defined directly by two counterparties.
Swaps are financial agreements between two counterparties that exchange a floating price for a fixed price. Swaps are usually settled in cash, rather than with physical delivery. Under a swap contract, buyers usually pay the fixed rate and receives floating rate. Sellers analogically pay the floating rate and receive the fixed rate. Swap rates are commonly based on a price reference (e.g. interest rates, OTC price report etc.).
Options are contracts that give a buyer the right (not the obligation) to buy or sell the underlying asset. Options are commonly used for risk mitigation of upside and downside price risk. Exchanges describe an option to sell as a put and to buy as a call. To distinguish between European and American options, European options are exercised only at maturity, whereas American any time up to maturity.

1.2. **Energy Markets**

In this section I would present basic features that make the energy market different from other markets. Afterwards I present fundamentals of the basic energy commodities.

In [21] author describes four basic attributes of energy markets – negative price, cyclicality, illiquidity and price transparency.
Prices of energy commodities might sometimes reach negative level. An example in [21] describes a situation when voltage of a power grid is overloaded as energy generators placed too much power in a grid. Grid operators might therefore make prices negative to encourage electricity producers to reduce power generation, because producers would rather give electricity away rather than restarting or shutting down their power plants. Due to higher operational costs, consumers get actually paid for increasing their consumption.
Forward prices tend to be cyclical due to several facts that define energy markets. Energy prices are determined mostly by short-term expectations of supply and demand, because energy and heat is not storable. Value of storing energy commodities could be very costly and decreases with the ease how easy energy could be produced in the future.
Energy markets are often illiquid. One of the reasons is represented by OTC contracts that could be customized, and that is why companies would rather use them to satisfy demand of its clients. OTC trades also bypasses limits to location and time to energy products. For example at distant locations there may be no possibility to buy an exchange traded contract for certain required time period.

OTC contracts between two counterparties are private agreements upon specified price without any obligation to make the prices public. In fact, energy trading companies usually publish estimates for their OTC contracts, nevertheless the real price information protect from any unauthorized eyes.

1.2.1. Electricity

Electricity is a secondary energy source. It is mainly produced by transforming heat energy into electrical energy in thermal power stations. Electricity supply is mostly generated from oil, coal, natural gas. Electricity cannot be stored and its transmission over long distances is rather costly. The process of liberalization of electricity markets started in the UK in 1990. Since 1996 the continental electricity trading follows the first EU Electricity Directive. Electricity markets in Europe are divided into regional markets with its own attributes. These markets are coordinated by own Transmission system operators (TSO), independent or governmental, that maintain the power grid. In deregulated markets all market participants have a guaranteed access to transmission lines.

In deregulated markets electricity prices are set in daily power auctions, where power generators submit price at which they are willing to supply. The price of power for producers and consumers is set to a single price called *wholesale price* or *clearing price*. Households and smaller consumers pay usually *retail price*, which is slightly higher. The clearing price of power is determined by the *merit order* of power plants. The power plants are activated according to price bids until the demand is satisfied. The electricity price is determined by the last activated power plant, which produces the most expensive electricity. The cost of bringing the last unit of electricity into the market is called the *marginal price of power*, and the recently activated power plant is the *marginal producer* [21].

The power auctions are either *day-ahead* or *real-time*. Day-ahead auctions set the electricity price for following day when the delivery will take place. Real-time auctions are run continuously during the actual delivery day. Market producers can buy additional electricity and thus balance actual demand. Real-time auction system includes only those power plants that can be quickly turned on and off.

While daily auctions are open to power producers that can place their produced power into the transmission grid, the forward power market is open to larger scale of participants who would like to enter either financial (with cash settlement) or physical (with delivery) contracts. There may be a situation, when there is a strong demand in a location the transmission system is overloaded, power providers have to activate power plants in area where electricity is required, regardless the merit order. These moments increase the *locational marginal price* in that area as the higher cost generators were activated *out-of-merit* order.

Usually, the locational marginal price is a combination of three parts: clearing price, congestion price and line loss charge. The clearing price is constant for everyone around the grid, only congestion price and line charge are market specific.

Power trading is highly dependent on accurate predictions of the future load on the power grid. The demand is driven by domestic, commercial, industrial or public sector that need to turn on the light, or heat up rooms. Power plant operators and schedulers create plans for maintenance, fuel supply and decide when a plant operates or not. The demand for electricity is cyclical and changes by seasons (weather and temperature), days a week or time of a day. The minimum load required at a given time is called *base load*, the maximum is the *peak load*. Electricity suppliers use a range of load profiles for each type of consumer to forecast demand and estimate bills. Time block starting usually from 7 a.m. until 11 p.m. is called *peak* hours, night time hours are *off-peak* hours.

Electricity generation normally involves the conversion of motion when the conducting coil rotates in a magnetic field. The most common are steam, gas turbines and combined cycles driven by heat emerged from burning fossil fuels, nuclear power or renewables.
Fossil fuel power plants burn oil, natural gas or coal, in order to produce steam. Burning fossil fuels produces as by-products pollutants and greenhouse gases, e.g. carbon dioxide (CO_2). The most polluting power plants are coal-fired plants with lower efficiency. Nuclear generators produce steam by nuclear fission of enriched uranium. Hydroelectricity is produced by turbines that are directly driven by water.
In energy trading the most important generators are fossil fuel power plants that have the strongest impact on determination of the clearing price. Fossil fuel power plants are usually described by the *heat rate*, which expresses its efficiency of transforming fuel into electricity:

$$Heat\ rate = \frac{Quantity\ of\ fuel\ used}{Quantity\ of\ power\ produced}. \quad (1.1)$$

Economics of power generation might be also estimated by *spark spreads*, expressed as a difference between the price of electricity and the cost of fuel input adjusted for thermal efficiency of the power plant:

$$Spark\ Spread = Price\ of\ electricity - (Price\ of\ gas\ \times Heat\ Rate). \quad (1.2)$$

While the spark spread is applied to gas-fired power plants, the dark to coal-fired power plants.
European power trading is either OTC (power forwards) or occurs on organized exchanges (power futures). Most European countries have national or regional markets. Nordic countries collaborate in Nordpool[4] exchange, where traders can buy or sell base load or peak load futures contracts for the Nordic, German or Dutch market. ICE Futures Europe[5] lists electricity contracts for the UK market. EEX[6] (European Energy Exchange) offers futures contracts for Phelix[7], German, Austrian and French Markets. Powernext[8] is a French market listing futures contracts for base load and peak load electricity. EPEX[9] (European Power

4 http://www.nordpoolspot.com/
5 https://www.theice.com/futures_europe.jhtml
6 http://www.eex.com/de/
7 http://www.eex.com/en/Market%20Data/Trading%20Data/Power/Phelix%20Futures%20|%20Derivatives
8 http://www.powernext.com/
9 http://www.epexspot.com/en/

Exchange), a joint venture of EEX and Powernext, provides spot trading for French, German, Austrian or Swiss spot contracts.

1.2.2. Coal

Use of coal is growing worldwide. Coal is a composition of solid hydrocarbons and other components that has a form of black or dark brown sedimentary rock used mainly to generate electricity, space heating and iron or steel production. Coal mines are either on the surface or underground. Coal mining is a sensitive political issue because of its environmental dimension.

Electricity prices are in dependence on coal prices, however because of the relatively cheaper price of coal, the effect on electricity is lower than has natural gas. Coal is a difficult commodity to handle with several drawbacks when compared with oil or natural gas. As a solid fuel, coal is harder to use in an engine, with more difficult manipulation. Coal combustion also takes more time to be turned on or off. Coal also creates dust, suffers degradation or contains impurities or pollutants.
Price of coal is relatively cheaper due to its limitations, mining characteristics or usage. If we look at the technology of extracting coal from the ground, it is not so complicated compared to oil or natural drilling. Another factor having negative influence on price is amount of pollutants emerging during combustion. Coal is supposed to be the largest source of air pollution worldwide.
Among advantages of coal we could count the ease to store it and transport. Coal is transported mostly by trains, shipped on barges in dry bulk carriers with iron ore. Coal is typically transported directly from mines to area of consumption (usually power plants). Due to the fact that transportation costs might account large percentage of the final price, power plants are commonly located near coal reserves in a mining area. Sometimes there is also a system of pipelines conveying crushed or compressed coal on continuous basis to power plants. Coal supply is in general based on long term contracts that eliminate the risk of price volatility.

Chemical structure of coal could be very variable, determining the quality of coal. Coal is classified according to carbon content and how much heat energy it can release. *Lignite*, *sub-bituminous*, *bituminous*, and *anthracite* are the most important fuel representatives of coal.

Type	*Fixed carbon*	*Volatile Matter*	*Moisture conten*	*Ash conten*
Peat[10]	10 %	20 %	65 %	5 %
Lignite	30 %	30 %	35 %	5 %
Bituminous	60 %	25 %	10 %	5 %
Anthracite	75 %	10 %	5 %	10 %

Table 1 - Coal composition [own material]

[10] Peat is decayed vegatation with lower energy content used mainly for power generation in Finland or Ireland.

Lignite (brown coal) has the lowest energy content, with around 30 % proportion of carbon. Lignite is seldom traded internationally because of its lower price due to the lower energy content. Its higher moisture content and tendency to spontaneous combustion limits the storage and transportation. Lignite is commonly used as cheaper fuel for power stations in nearby areas.

Sub-bituminous coal lies between lignite and bituminous as it has carbon content around 40 % and releases more heat energy. Lower moisture content enables transportation and storage. However the international trade with this grade of coal is not very often. In analogy with lignite, sub-bituminous coal is used commercially mostly as a fuel for power plants.
Bituminous coal contains around 60 – 70 % of carbon and two to three more heat energy than lignite coal. It is the most abundant coal in the USA. Bituminous coal contains also tar-like compounds (bitumen) and impurities. International markets with bituminous are very active as it is highly demanded by power producers and industrial (steel) companies. Colour of bituminous coal is black (dark black).
Anthracite has the highest carbon content with fewest impurities and produces relatively little pollution. It has a hard black surface. Due to its relative scarcity, anthracite has higher price than other types of coal.

When coal has any sulphur content, it may appear in combination with iron (pyrite crystals) or carbon (organic sulphur). Pyrite crystals can be relatively easily removed by coal washing process. Organic sulphur might be possibly removed by installation of scrubbers on the exhaust stacks of coal-fired power plants [21]. This process is called flue gas desulphurisation.
Coal is traded under long-term contracts OTC or on exchanges. The liberalization of the power generation markets in the USA and Europe has encouraged development of physical spot and financial derivative markets for coal.
Trading activity turns mostly around the high quality coal as it is favourable to transport it. Variable quality is a major challenge for coal traders. Trading uses standardized contracts as a price references, traders then negotiate price differential for different types of coal. The basic contract is SCoTA – the Standard Coal Trading Agreement, which is used mostly for international seaborne trading.
Coal prices in Europe are usually based on API indices (average pricing indices) that capture the average price of imported seaborne coal (particularly API 2).
Trading is mostly OTC with brokers. European coal trading on exchanges takes place EEX and ICE, where coal futures contracts are listed.
The major market players are electricity producers and large mining companies. The coal market has own specifics emerging from the transportation conditions and coal classification; trading activities are determined by impossibility of speculation among various grades of coal, furthermore there are limitations to execute transportation trades. As mentioned at the beginning of the section, there is almost no correlation between power and coal prices. Producers of electricity in coal-fired power plants can only speculate on the spread between coal and power prices.
Coal prices are quoted in currency units per weight, commonly in US Dollars/EUR per metric tonne.

1.2.3. Natural Gas

Natural gas is an emerging energy commodity market. For power generation, natural gas is cleaner than burning a coal, efficient, flexible and has lower carbon emissions. As its infrastructure has developed rapidly, natural gas became a primary residential fuel. The industry is still mostly dominated by national monopolies, only in the UK or USA the market is liberalized with active trading markets and independent transmission system operators.
Physically, natural gas is a non-renewable fossil fuel providing heat and used for power generation. It is a mixture of gaseous, colourless and odourless hydrocarbons, mostly methane (CH_4), ethane (C_2H_6), propane (C_3H_8) and butane (C_4H_{10}). The structure of natural gas is very various; therefore traders quote natural gas in units of energy, usually in *British Thermal Units* (Btu) or *Joules*. Heat content is usually expressed as *Gross or Net Calorific Value*. For consumers is natural gas usually offered in *therms* (1 Therm=100.10^3 Btu=29,3kWh).
Transmission systems operators (TSO) and industry regulators determine standards and oversee quality of natural gas entering their infrastructure. The most common quality standard in Europe is so-called Wobbe index, which is a measurement of burning characteristics. The Wobbe index (WI) is calculated as

$$WI = \frac{Gross\ Calorific\ Value}{\sqrt{Specific\ Gravitiy}}. \quad (1.3)$$

Natural gas is transported from sites of production to consumers through pipelines. The reason [21] is, that methane contains lower amount of energy per volume and requires a much larger storage container to hold the same quantity of heat energy. Rather than transport natural gas in large pressurized containers, it is more practical to use pipelines.
Another way of transportation enables process called *liquefaction* that, after cooling to approx. -162,2 °C, turns natural gas into liquid state. Natural gas in liquid form has higher density and therefore contains more heat energy per volume. LNG carriers transport liquid gas over oceans for longer distances.
Natural gas is associated with crude oil, coal beds or gas condensate fields. Through decomposition of various organic materials, methane arises as by-product. In case methane is surrounded by impermeable materials, the gas is trapped and gas reservoir is created. Natural gas usually extracted by drilled wells which use pressure of the gas in the reservoir. Extracted natural gas does not contain only methane but also numerous particles and pollutants that have to be removed. The basic way how to separate individual constituents from methane is cooling up to the point when large molecules turn into liquid form and smaller ones remain a gas.
Pipelines are often connected to a *gas hub*, where the most natural gas trading takes place. The most important European natural gas hubs are NBP (National Balancing Point) in the UK, TTF (Title Transfer Facility) in the Netherlands, Zeebrugge in Belgium, virtual trading hub NCG in Germany, PEG (Point d'Exchange de Gas) at French Powernext energy exchange or CEGH (Central European Gas Hub) in Austria (see Figure 3). In the Northern America, it is Henry Hub located on the Gulf Coast in Louisiana.

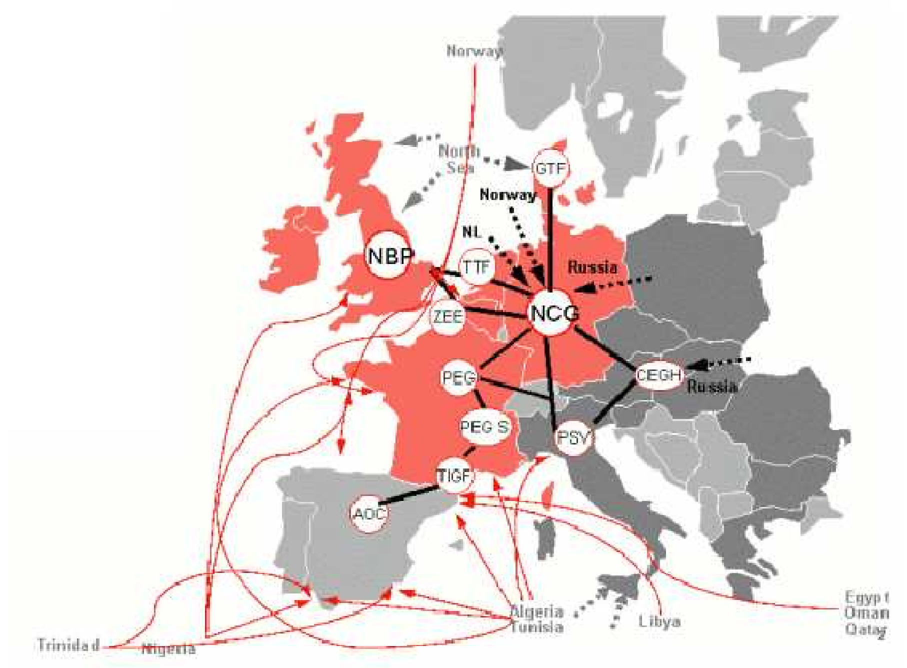

Figure 3 - European gas hubs (www.eon-energy-trading.com)

Natural gas trading takes place OTC and on organized exchanges. Natural gas prices are quoted in basis prices which are related to a basic index price. Basis price of a physical gas is a difference between the actual market price and the index price at a specific location (mostly natural gas hub). Natural gas is mostly traded with future contacts that are usually priced at Henry Hub price (in the USA) or NBP (in Europe). The basis price usually reflects additional costs, mostly transportation costs, therefore

$$all - in\ price = index\ price + basis\ price. \quad (1.4)$$

Prices of natural gas are cyclical, that is why traders usually speculate on its price by entering into spread trades. Spread trades are not dependent directly on movements of natural gas markets, but traders speculate on a difference between two securities by buying one and selling another [21]. Traders might enter into location spreads (speculation on the price between two locations), heat rates (speculation on price relationship between natural gas and electricity), time spreads (speculation on periods of higher and lower demand) or swing spreads (speculation on ability of a trader to store natural gas).

Spot and forward trading is usually separated. Spot prices are more volatile because they are based on gas supply with immediate availability. Forward prices reflect seasonal expectations of future demand and supply given by future macroeconomic issues. Forward prices follow regular pattern; they are higher in the winter and lower in the spring. Spot prices are not as seasonal as forward prices. In contrast to forward prices, spot prices take into account costs of transportation and storage.

Supply and demand of natural gas is driven mostly by temperatures. In winter periods the gas is used mostly for heating, in the summer the demand is lower as natural gas is used for power generation.

1.2.4. Oil

Oil is the world's biggest energy commodity market. The impact of oil on global economy is really high and crude oil industry is a subject of global attention and serves as a benchmark or the energy industry.
Oil is a convenient commodity with high energy content per volume therefore it is very popular fuel. In fact, oil (petroleum) is a liquid fossil fuel formed when decaying plant life becomes trapped in a layer of porous rock [21]. The decaying plants were then converted by heat and pressure into hydrocarbons. The first form of petroleum that is extracted by drills from the ground is called crude oil. Crude oil is usually separated into several components by distillation in a refinery.
The global crude oil demand is represented mostly by industrialized countries in the USA, Europe, China, India and the Middle East. The supply is concentrated and controlled by organizations such as OPEC. Transportation and storage costs are dependent on distances between extraction and consumption.
Crude oil markets could be described as a mutual interaction of four major participants: producers, refiners, marketers and consumers. Around one half of the global oil reservoirs are located in the Middle East, developing region with lower domestic oil consumption. Refineries are usually located near consumers. Trading companies are various firms that buy or sell oil products from miscellaneous intents (funds, investment banks, oil companies etc.). Consumption may be represented by industrial manufacturers or end consumers parking at a petrol station.

European oil trading is handled typically by weight, commonly set in *tonnes*. On the other hand, in the USA petroleum is traded by volume in *barrels*. A barrel equals 42 U.S. gallons or 159 litres. A relationship between barrel and a tonne is defined by conversion factor 7,33 barrels per tonne.

Oil is typically characterised by *density* and *sulphur content*. Density is measured by API gravity that classifies crude oil into five basic groups:

Classification	*API gravity*
Ultra-light	>50
Light	35-50
Medium	26-34
Heavy	10-25
Bitumen	<10

Table 2 - Crude oil classification [own materials]

Sulphur is an undesirable component in crude oil, causing acid rains. According to the sulphur content, crude oil is described either as *sweet* or *sour*. Sweet crude oil contains lower amount of sulphur.
Refined oil products are components separated from crude oil by process of distillation. Progressive boiling of crude oil at higher temperatures separates individual fractions and gases that are subsequently captured and cooled back into liquid state. With temperature rising, at the beginning of the process firstly the lighter fractions start to separate (LPG, gasoline), followed by medium distillates (kerosene, diesel). At the end the heavy and residual distillates remain (Heavy fuel oil, asphalt, waxes, lubricating oil etc.). Traditional distillation of a barrel is depicted in the Figure 4.
Post-distillation then liquidates sulphur content and increases the octane number of gasoline. The lightest oil products are usually more valuable, therefore refiners crack the heavier products into least dense. Petroleum converted mostly into lighter and high-end products is called *premium crude oil*.

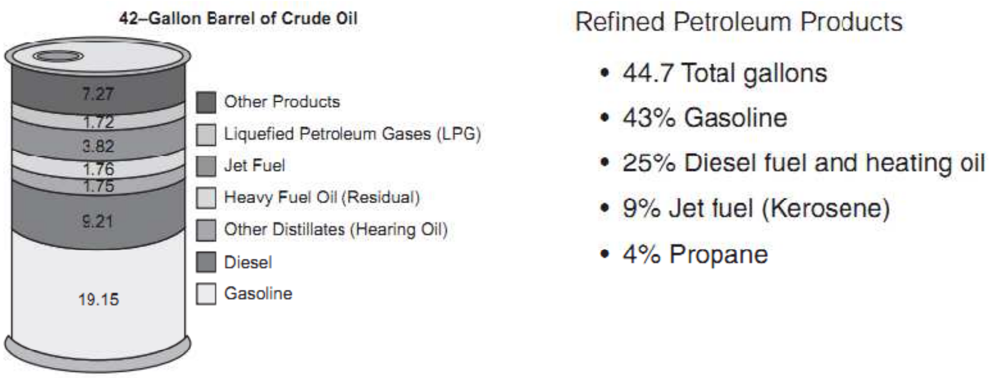

Figure 4 - Barrel breakdown by gallons [21]

Gasoline is used as a fuel for automobiles and various kinds of transportation. Gasoline is transported through pipelines that convey gasoline from refineries to terminals close to end consumers. At terminals, gasoline is adjusted to local conventions and regulatory directives (bio component added or ethanol). From terminals the fuel is finally transported by lorries to petrol stations. Price of gasoline depends mostly on prices of crude oil, consumer demand or characteristics of required additives.

Heating oil and diesel fuel have almost identical chemical structure and therefore used interchangeably. Price pattern for diesel fuel is similar to gasoline, whereas heating oil is determined by weather with peaks in the winter and lows in the summer.

Regions typically refer crude oil to a benchmark with defined attributes. In the USA the most common price reference is WTI crude oil (West Texas Intermediate), Europeans refer crude oil to the North Sea Brent Blend. To the rest belongs e.g. Arab Light or Bonny Light.

The link between price of a distilled oil product and price of crude oil is expressed by crack spread. Crack spreads reflect local supply and demand for oil products. Crack spread roughly represents the gross margin of a refiner. Refiners enter into crack spread to hedge the volatile prices crude oil. Refiners might go long the crack spread when they expect prices of crude oil

will decrease, so the spread will increase. They take short position in order to eliminate the risk when prices of crude oil would rise.
Refiners also assess quality of crude oil by comparing prices of its products and yields and calculating a weighted average value, so called *gross product worth* (GPW). GPW is calculated by multiplying the yield of each refinery product by the market price.

Oil spot trading in the Atlantic area is carried out usually at differential to WTI or Brent price references. Asia Pacific markets use often regional price markers such as Malaysian Tapis Oil, Dubai or Oman crude oil.
The price of oil is established in the liquid oil futures markets – Nymex[11] (WTI) and ICE[12] (Brent oil). Nymex lists monthly light sweet WTI crude oil futures and options, heating oil and gasoline options and futures contracts, all financial or with physical delivery. ICE provides futures, options and swaps contracts for crude oil and refined products. Other exchange handling oil futures contracts are Tocom[13] (Tokyo Commodities Exchange) or DME[14] (Dubai Mercantile Exchange).

1.2.5. Emissions

Emissions trading is relatively new energy commodity market created in order to limit carbon dioxide emissions. Global issues concerning climate change forced countries to adopt policies that would limit production of greenhouse gases and other pollutants. However reduction of carbon dioxide is very expensive as it increases price of electricity, which is produced mainly in coal-fired power plants. Complete elimination of coal as a fuel for steam generation would be unfeasible and not economical. Emissions markets endeavours to keep the prices at reasonable level.
The global issue concerning climate change arose in early 1990's. United Nations hosted numerous discussions and panels about global warming. Most of discussions failed over a question about responsibility and who should have paid the costs for reducing greenhouse emissions. There were several plans suggested by the world powers bringing in varied array of solutions. Finally due to lack of international compromise carbon emissions schemes were implemented unequally around the world.
The first global agreement about reduction of greenhouse gases was the *Kyoto protocol* adopted in 1997 by 37 industrialized states except the USA. These efforts in fact did not turn into a binding document. The Kyoto protocol establishes a *cap-and-trade* system imposing national and international ceilings for emissions. Under the Kyoto protocol the industrialized countries agreed to restrict their rights to emit the greenhouse gases by emissions trading, the clean development mechanism (CDM) and joint implementation (JI). Each country has certain assigned limit for emission, subsequently surpluses might be traded. CDM enables countries another production of emissions if they invest and assist developing countries with emission

[11] http://www.cmegroup.com/trading/energy/crude-oil/light-sweet-crude.html
[12] https://www.theice.com/productguide/ProductDetails.shtml?specId=219
[13] http://www.tocom.or.jp/
[14] http://www.dubaimerc.com/

reduction. JI is similar to CMD but takes place in other industrialized countries. However the Kyoto protocol is often disputed by the biggest carbon producers (USA and China).

In general, there are two main methods on which the emissions policies are based: a *carbon tax* and *cap-and-trade system*. Carbon tax is simply a punishment for every gram of any greenhouse gas that has to be paid. Carbon tax makes the polluting technologies more expensive and favours cleaner technologies. The collected tax revenue is subsequently reinvested as subsidies for greener technologies. On the other hand, cap-and-trade system creates a ceiling for total amount of emissions produced and provides tradable rights to emissions creation. Trading with right (allowances) is fully at discretion of each company. Whereas the cap-and-trade system creates variable fees based on current supply and demand, the carbon taxation adds fixed costs.

Both systems have certain drawbacks. Cap-and-trade system suffers from sound inability to monitor and track all emissions produced. Carbon dioxide is produced also by animals and human beings what the system cannot reflect. Major problem of carbon taxes is that energy companies can transfer them on consumers, so that there is no reason to improve their current technology.

Trading markets with emissions are based on cap-and-trade system. Rights to emissions give a license to pollute or use electricity produced by burning coal. The trading system is transparent and gives companies flexibility to meet the governmental requirements. Besides benefits for green producers, trading market also protect consumers as it keeps price of electricity at more affordable level.

Emissions rights are distributed either from the administrator of the cap-and-trade system or according to ability to reduce carbon dioxide. The latter enables countries to maintain their operations of coal-fired power plants until cleaner technologies are developed and implemented. Countries can also capture the carbon dioxide and store it in reservoirs similar to natural gas tanks. The storage of carbon dioxide is usually called *carbon sequestration*.

Carbon credits are traded OTC or on world exchanges. Besides the Carbon emissions allowances, market participants also trade with the project credits, such as CER certificates (Certified Emissions Reduction) issued under CMD or ERU (Emissions Reduction Units), which were generated under JI. Traders buy or sell electronic forms of certificates, there is no physical delivery. European exchanges offer various kinds of contracts and arrangements – spot, options or futures. The most significant exchanges are ECX[15] (European Climate Exchange), Nordpool, EEX, Bluenext[16], EXAA[17] (Energy Exchange Austria) or GME[18] (Gestore Mercato Elettrico) in Italy.

ECX lists EUA (European Union Allowances) futures contracts and options, also CER options and futures. Nordpool offers day-ahead spot and futures for EUA and futures for CER. EEX provides spot, futures contracts and options for EUA and finally futures for CER. Bluenext lists spot and futures contracts for EUA and CER. EXAA and GME both offer spot for EUA.

[15] https://www.theice.com/productguide/ProductGroupHierarchy.shtml?groupDetail=&group.groupId=19

[16] http://www.bluenext.eu/

[17] http://www.exaa.at/

[18] http://www.mercatoelettrico.org/En/Default.aspx

Chapter 2

Energy Risk Management

2.1. **Introduction**

Companies strive to achieve sustainability, efficiency and profitability despite uncertainty and various risks. Organizations need to obtain foresight to control and manage large portfolio of risks spread across complex business processes, relationships and markets. Main objective of corporate risk management is to manage uncertainty in company's business.
Risk and uncertainty is pervasive. Risk is a result of direct or indirect consequences of outcomes and events that were not accounted for, for which we are ill-prepared, and which affects individuals, firms, financial markets and society at large [42]. Risk management uses several financial tools to eliminate risk and its effects, for example insurance contracts, options or swaps. Financial tools might be rather costly; therefore risk managers should find a balance between costs and benefits of its application. There are several examples illustrating wrongdoings in risk management – Barings Bank, Metallgesellschaft or Enron.

Risk management has three common characteristics: *ex-ante risk*, *ex-post risk* and *robustness*. Ex-ante risk is usually mitigated by preventive policies and controls. Ex-post risk represents consequences of adverse events once they have occurred. Robustness is a characteristic of risk which is insensitive to randomness of its parameters [42].

A *risk measure* is a crucial term of the risk theory. In 1999 Artzner et al. [5] provided complex definition and properties of risk measures. If we denote Ω as the infinite set of states of nature and $\mathcal{G}$ the set of all risks, which are real-valued functions on Ω. Then we could think of a risk measure as a mapping $\mathcal{G} \rightarrow \mathbb{R}$. Risk measure ρ defined on $\mathcal{G}$ is called coherent is it satisfies these four axioms:

1. Monotonicity: $For\ all\ X\ and\ Y \in \mathcal{G}\ with\ X \leq Y, we\ have\ \rho(Y) \leq \rho(X).$
2. Subadditivity: $For\ all\ X_1\ and\ X_2 \in \mathcal{G}, \rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2).$
3. Positive homogeneity: $For\ all\ \lambda\ \geq 0\ and\ all\ X \in \mathcal{G}, \rho(\lambda X) = \lambda \rho(X).$
4. Translational invariance:
   $For\ all\ X \in \mathcal{G}\ and\ all\ real\ numbers\ \alpha, we\ have\ \ \rho(X + \alpha.r) = \rho(X) - \alpha.$

Monotonicity means that that higher value *Y* should be less risky. Dowd and Blake [13] add that it means that less has to be added to Y than to X to make it acceptable, and the amount is the risk measure. Subadditivity describes a portfolio comprising 2 positions has total risk not higher than the sum of the risks of these two positions. Positive homogeneity keeps proportion and size of the initial risky position. Translational invariance expresses that the certain amount added to the risky position reduces the risk by that amount.

2.2. **Decision under uncertainty and Expected utility**

A company may also pay an insurance to avoid possible future losses that might occur with certain probabilities. Insurance creates a marketplace for risk and enables risk exchange between decision makers of various risk preferences. Insurance represents *passive* risk management, whereas technological and innovations and, for example, loss prevention are *active* forms of risk management.

Decision making under uncertainty takes place among decision makers who have expectations about outcomes given by preferences described by probabilities. In theory of probability there are several principles that determine the decision making. The *Laplace criterion* says that, when the probabilities of the states of nature in a given problem are not known, we assume they are equally likely [42]. *Minimax criterion* selects the outcome that will have the least maximal loss no matter what conditions will occur in future. Decision maker with minimax attitude is loss-averse, satisfied with the best of all worst possible outcomes. In contrary, *maximax criterion* advises taking the best possible future outcome regardless of event probabilities.

Decision makers are driven by their utility emerging from their decision. Attitude towards risk is described by expected utility which is defined for $R \in \mathbb{R}$ as

$$E\big(u(R)\big) = \int u(R)p(R)dR, \quad (1.5)$$

where R are rewards with its probability $p(R)$ and utility $u(R)$. Every decision maker is supposed to be rational and therefore strives to maximize his utility function with probability p and outcomes π:

$$EU_i = \sum_{j=1}^{N} p_{ij}u\big(\pi_{ij}\big). \quad (1.6)$$

Final decision about the level of expected utility is influenced by decision maker's attitude towards risk. His attitude is described either as *risk aversion*, *risk neutrality* or *risk seeking*. Risk-averse individual would be willing to pay a certain premium in order to reduce the risk. as his preferences for risky outcomes are decreased, preferring a mean outcome [42]. For illustration, if we consider any uncertain reward $\tilde{R}$, its expected utility $E(u(\tilde{R}))$, then the *certainty equivalent* $\bar{R}$ is expressed as

$$\bar{R} = u^{-1}\{E[u(\tilde{R})]\}, \quad (1.7)$$

where the reward $\tilde{R}$ is

$$u(\bar{R}) = E\left(u(\tilde{R})\right). \quad (1.8)$$

Then the difference between the certainty equivalent and expected value $\hat{R} = E(\tilde{R})$ is the risk premium ρ, which is the amount the investor would rather pay to improve the uncertain prospects. The risk premium ρ is then

$$\rho = \hat{R} - \bar{R}. \quad (1.9)$$

2.3. **Various Risks**

2.3.1. Credit Risk

In energy industry credit risk appears mostly in a form of counterparty risk. Failure to deliver and fulfil contract obligations might happen even to highly-rated trading companies. The loss that company can suffer is determined by the amount at risk and its fraction that is recovered. The credit risk has three major components [29]: *Default risk*, *credit exposure risk* and *recovery risk*. Default risk is given by probability of default and expresses probability of counterparty's default. Credit exposure risk is determined by movements in the market value of claims on counterparty. When counterparty is in default, this risk is known as exposure at risk. Recovery risk is uncertainty that a company would receive some fraction of the claim after default of the counterparty.
Default of a counterparty is represented by a discrete model with variable b_i when default is defined as $b_i=1$ and no default $b_i=0$. Probability of default (PD) is often given by actuarial models (Altman's z-score) or credit rating agencies.

	Year									
Rating	1	2	3	4	5	6	7	8	9	10
AAA	0.00	0.00	0.04	0.07	0.12	0.21	0.31	0.48	0.54	0.62
AA	0.01	0.03	0.08	0.16	0.26	0.40	0.56	0.71	0.83	0.97
A	0.04	0.13	0.26	0.43	0.66	0.90	1.16	1.41	1.71	2.01
BBB	0.29	0.86	1.48	2.37	3.25	4.15	4.88	5.60	6.21	6.95
BB	1.28	3.96	7.32	10.51	13.36	16.32	18.84	21.11	23.22	24.84
B	6.24	14.33	21.57	27.47	31.87	35.47	38.71	41.69	43.92	46.27
CCC/C	32.35	42.35	48.66	53.65	59.49	62.19	63.37	64.10	67.78	70.80

Figure 5 - S&P Cumulative default rates [29]

Cumulative default rates c_n depicted in the Figure 5 represent the total probability of defaulting at any time between now and year n [29]. Probability of a company's survival up to year n is then given by:

$$(1 - c_n) = (1 - c_{n-1})(1 - d_n) = \prod_{i=1}^{n}(1 - d_i), \quad (1.10)$$

where d_i are annual default rates during year *i*. The cumulative probability of default is then summing of its annual probabilities for years *i*:

$$c_n = d_1 + k_2 + k_3 + \cdots + k_n, \quad (1.11)$$

where the annual probability for year *i* is equal to $k_i = (1 - c_{i-1})d_i$.

Recovery risk is represented by the loss given default (LGD) that expresses the part of the exposure lost in default. LGD is also defined as $1 - f$, where f is recovery rate. Among many factors, the recovery rate depends mostly on debt seniority, security or character of industry. In general, secured or senior debts have higher recovery rates than subordinated debts. Recovery rates are usually modelled as they are unstable and depending on several variables.

Companies often set a credit limit to each counterparty, global and market specific. Important methods to manage and control credit risk are netting arrangements, collaterals, bank guarantees or margin payments. Mutual netting agreement allows companies to offset obligations, resulting in one single net claim against counterparty [29]. Consider two counterparties with *N* derivative contracts between them. The potential loss without netting is the sum of all positively valued contracts [29]:

$$Gross\ loss = \sum_{i=1}^{N} \max(V_i, 0)\,. \quad (1.12)$$

With netting agreement the risk exposure is defined as positive sum of all market value of all contracts [29]:

$$Net\ loss = \max(V, 0) = \left(\sum_{i=1}^{N} V_i, 0\right). \quad (1.13)$$

Analogically, the gross (GRV) and net (NRV) replacement values, defined as the sum of worst-case loss over all counterparties *K*, can be calculated as:

$$GRV = \sum_{k=1}^{N} \text{gross loss}_k = \sum_{k=1}^{K}\left[\sum_{i=1}^{N_k} max(V_i, 0)\right]. \quad (1.14)$$

Under netting agreement the net replacement value is then

$$NRV = \sum_{k=1}^{N} \text{net loss}_k = \sum_{k=1}^{K} \left[max\left(\sum_{i=1}^{N_k} V_i, 0 \right) \right]. \quad (1.15)$$

2.3.2. Operational Risk

According to T. James [32] the operational risk is the risk of loss caused by failures in operational processes or the IT systems that support them, including those adversely affecting reputation, legal enforcement of contracts and claims. Many of the events causing the operational risk are common in many companies. There might be two ways how the risk could affect a company – directly or indirectly. A company could suffer direct losses from, for example, hardware failure, fraud, improper trades or scheduling errors. Indirect losses might result from, for instance, a failed client relationship or damages to a reputation.
In [32] the operational risk emerges mostly from four areas:

1. Process risk – marketing, trade execution, trade fraud etc.
2. **Human risks** – fraud, misuse of information, rogue traders, health and safety etc.
3. Technology risk - data corruption, programming errors, viruses, system failure etc.
4. External business environment – money laundering, compliance, legal risk etc.

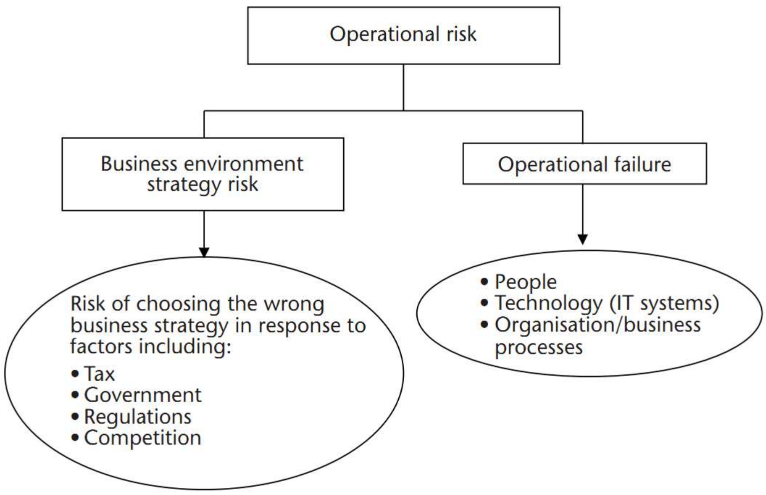

Figure 6 - Operational risk areas [32]

Operational risk is harder to quantify and manage than market of credit risk, but it is equally important for an energy trading company. Most operational risks are due to human errors or negligent processes. In this case, companies should double-check by the four-eyes principle. The process of mitigating operational risk has classical phases – data collection, risk reduction, control and containment or transfer. Collected data might serve as components of internal

KPIs and be benchmarked with industry survey such as ISDA Operations Benchmarking Survey[19].

2.3.3. Liquidity Risk

Liquidity in financial markets represents the possibility to transact larger amounts of securities quickly at low cost. Liquidity risk is characterized by the bid-ask spreads, turnover information or processing costs. Black [Black Towards] defines liquidity as a market attribute, when (a) there is an ask price and a bid price for an investor who wants to buy or sell immediately a minimal quantity imposed by the market; (b) the bid-ask spread is always tight; (c) in the absence of a "special" information, an investor who wants to buy or sell a large "block" immediately by paying a premium which is positively related to the volume.
Perfectly liquid markets hypothetically hold constantly a single bid-ask price, no matter how large the quantities are traded. In case of illiquid markets a trader is mostly unable to liquidate or hedge his positions quickly and under current market conditions.

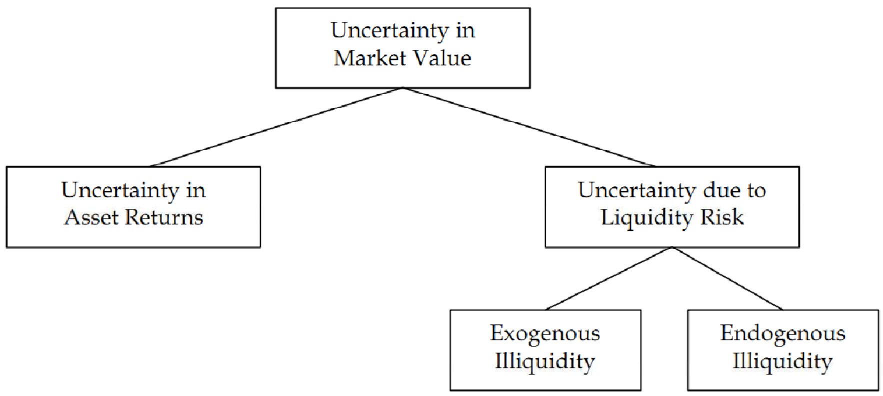

Figure 7 - Taxonomy of Market Risk [6]

There is a common recognition of two types of liquidity risk – *exogenous* and *endogenous*. According to A. Bangia et al. [6] is the *exogenous* illiquidity the result of market characteristics; it is common to all market players and unaffected by the actions of any one participant (although it can be affected by the joint action of all or almost all market participants). The market for liquid securities, such as G7 currencies, is typically characterized by heavy trading volumes, stable and small bid-ask spreads, stable and high levels of quote depth. Liquidity costs may be negligible for such positions when marking to market provides a proper liquidation value. In contrast, markets in emerging currencies or thinly traded junk bonds are illiquid and are characterized by high volatilities of spread, quote depth and trading volume.

Endogenous liquidity risk, in contrast, is specific to the position in the market and varies across market participants [6]. The exposure of any one participant is affected by the actions of that participant. It is mainly driven by the size of the position: the larger the size, the greater the

[19] http://www.isda.org/statistics/operbenchsurvey.html

endogenous illiquidity. If the market order to buy/sell is smaller than the volume available in the market at the quote, then the order transacts at the quote. In this case the market impact cost, defined as the cost of immediate execution, will be half of the bid-ask spread. If the size of the order exceeds the quote depth, the cost of market impact will be higher than the half-spread. The difference between the total market impact and half-spread is called the incremental market cost, and constitutes the endogenous liquidity component [9]. The link between the liquidation price and the total position held is depicted in Figure 8.

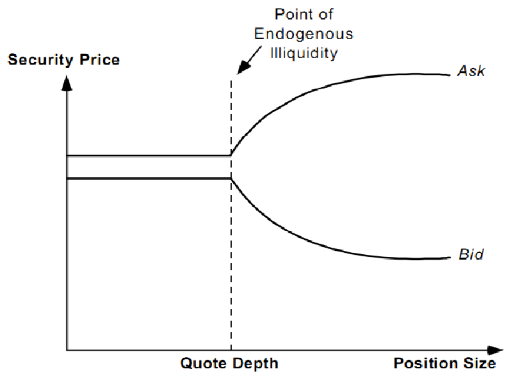

Figure 8 - Effect of position size on liquidation value [6]

Later in this paper I will show how the liquidity problem influenced quantification of portfolio risk. Rather than endogenous, I will briefly describe how the liquidity risk was incorporated into VaR calculations.

2.4. **Management Controls**

The Collapse of Barings in 1995, the UK's oldest bank at that time, rattled the then perception of management controls and surveillance. Later time revealed that the disaster was caused by complete lack of enforced management controls throughout the organisation.

The board of directors should approve all significant policies relating to the management of risks throughout the institution. These policies, which should include those related to derivatives activities, should be consistent with the organisation's broader business strategies, capital strength, management expertise and overall willingness to take risk. Senior management should be responsible for ensuring that there are adequate policies and procedures for conducting derivatives operations on both a long-range and day-to-day basis. Senior management should regularly evaluate the procedures in place to manage risk to ensure that those procedures are appropriate and sound.

This is only a short excerpt from the Risk Management Guidelines for Derivatives written by Bank for International Settlements. The material gives general recommendation to develop and maintain that defines purpose and handling with derivatives. Many control failures that result in substantial monetary losses might be hypothetically reduced or avoided by efficient management controls established by senior management and Board.
T. James [32] segments failures of the control mechanisms into five main categories:

1. **Management Control**

Any situation when senior management promotes and rewards managers who focus more on profit generation with lower interest in implementation of internal control policies indicates serious control breakdown. Such information would send a negative message that internal controls do not hold the highest priority.

2. **Risk Assessment**

Once a company suffered large losses in the past, it should not omit to assess risks of current derivatives and trading activities. There should always be an analytical review of procedures and risk management systems when senior managers decide to start using more complex derivative instruments.

3. **Segregation of duties**

As the Barings collapse showed, one of the main reasons for such troubles has been lack of segregation of duties. Typically, supervision of business areas with contradictory interests should not be held by a single person with high individual responsibility. Ordinarily most of trading companies have developed a rigid internal structure that Front Office (trading and commercial activities) is strictly separated from the Back Office (risk management, reporting, trade settlement and confirmation).

4. **Reporting**

Communication is undoubtedly one of the most important prerequisites of effective control procedures and policies. Senior management should ensure that the organisational structure helps the information flow and sharing of the internal risk guidelines, because every key staff member should be fully aware of risk policies for derivatives. In contrary to the senior managers reports and figures should be reported.

5. **Reviews/Audits**

Internal or external auditing should examine risk procedures and policies in order to ensure the company there are no weaknesses in company's controls or significant risk exposure. Senior management should pay the highest attention to such reports and prioritize its remedy.

2.5. Trading Policy

Risk management policies for derivative trading vary in every trading company. The most important documents are generally stored in a common workspace (intranet) that every staff member can access them. T. James in his book Energy Price Risk [32] mentioned six joint components which almost all risk policies contain.

1. **Board level approval**

The Board of Directors should ensure, develop and approve an effective policy that drives the usage of derivatives. Senior management implements an independent review of risks and rewards and also takes responsibility of underlying instruments defined by these policies being consistent with corporate business strategy, commercial objectives and risk appetite.

2. **Policies and Procedures**

Internal procedures and policies should further cover a definition of trading authorities and its roles, management reporting lines, market position limits, counterparty and documentation approvals or setting of valuation procedures.

3. **Control and Supervision**

Senior management should properly supervise the derivative activities and ensure that the trading activities with derivatives in in compliance with corporate policy and external regulation, internal system of controls is review regularly, computer systems are robust and secured from any intrusion of unauthorised personnel.

4. **Organization, roles and responsibility**

A sound risk management system provides an independent framework for reporting, monitoring and controlling of all possible risk aspects. Key staff members should are supposed to have clear responsibility and function within the risk management system. Risk managers should be equipped with appropriate valuation and market risk tools and techniques and serious limit excesses to the Board.

5. **Credit procedures**

Credit risk aspects represent important part of all risk procedures. Any exposure to credit risk should be analysed and mitigated through effective credit management tools (collateralisation, credit default derivatives, credit insurance, netting agreements, credit risk limits etc.).

6. **Legal and documentary**

Legal department of a company is responsible for appropriate use of a legal documentation and ensures compliance and necessary authority. Financial health is one of most important prerequisites for any successful trading activity. Therefore a company should conclude and maintain a list of authorised existing and potential brokers and counterparties and obtain warranties from them. OTC derivative deals are often concluded by standardised master trading agreements that enhance certainty and smoothness of a deal conclusion and keep up-front legal costs down as well.

2.6. Back Office

The Back Office (BO) has a crucial role in maintaining the operations of an energy trading business. Elements of BO operations vary across the energy trading industry but I will try to give a simple explanation of practicalities of common BO tasks.
The BO is an essential part of any trading organisation. All Efforts to implement a flawless risk management and reporting structure in a company would be wasted if an appropriate BO system is not in place. With growing volumes of OTC trading prompt and efficient confirmation and settlement of deals are necessary to mitigate counterparty risk and ensure profit realization. BO is also vital for protecting an organisation from fraud or unauthorised position of traders. The Collapse of Barings and the global crisis placed tighter scrutiny on BO processes and forced trading companies to accelerate and refine confirmations, for example, by using electronic trade matching systems such as eCM[20].

BO Principles

BO is necessarily an independent part of a company and strictly separated from Front Office (FO) so that traders have no chance to influence or even come across deal confirmation or settlement. These core principles are also regulatory requirements that could prevent fraud. This separation of BO and FO does not have to be necessarily physical (depends mostly on facilities), but anyhow access to own processes, systems and records must be prohibited. Personal relationships between FO and BO staff should be also monitored.
Independent inspection of deals at Confirmation and Settlement is based on so-called Cross-Checking principle. Every task is examined by "four eyes" and rotates among staff according to a timetable.

BO Tasks

In general, BO ensures that all deals entered into trading system by FO were entered and concluded correctly. Basic scheme of main BO tasks is depicted in the Figure 9.

[20] eCM = electronic confirmation matching (http://www.efet.org/Standardisation/eCM_Standards_5404.aspx)

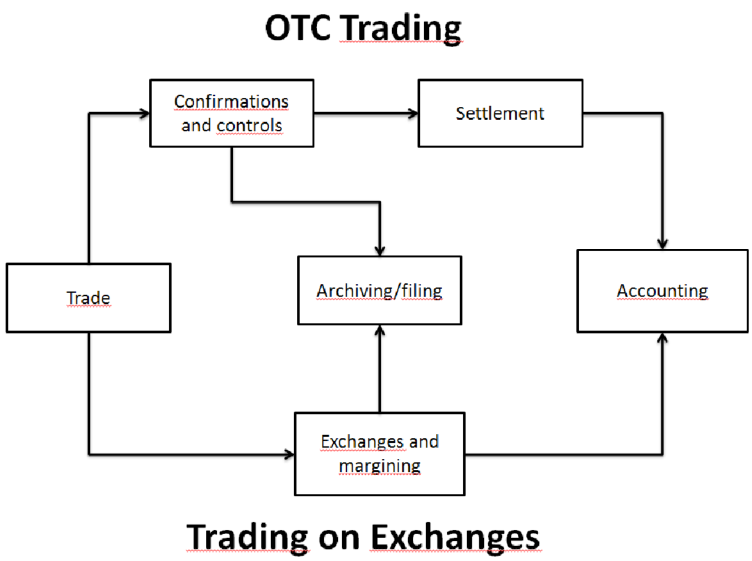

Figure 9 - Back Office Tasks

Confirmation of OTC deals ensures that both parties agree upon the conditions of the contract. When BO checks internally details a deal, sends seller an official confirmation document (usually by fax) to buyer for his approval. Any inconsistencies between internal record of a deal and a official confirmation received from a counterparty should be resolved promptly with FO or a counterparty itself. Strong market players use usually modern software tools based on mutual cooperation and data sharing such as eCM or Trayport[21].

On the other hand, confirmation of a deal is followed by settlement, which ensures that terms of delivery and payment resulting from the contract are fulfilled at expiry date. BO checks and calculates contract conditions under the term of the deal (e.g. taxes, quantity, prices, invoicing etc.). BO is also responsible for reconciliation of exchange accounts which includes margin payments, checking broker fees, data maintenance (addresses, contact details, tax numbers, expiry dates etc.). BO maintains and archives trade records of deals for a purpose of auditing and risk management.

Settlement process comprises mostly invoicing and netting. Invoices confirming the payment details are issued monthly, based on underlying master agreement for each deal. Financial deals are often settled by netting accompanied by netting statement, which means that all mutual payables and receivables are aggregated and netted ad therefore the account is settled by a net payment of its difference.
Energy trading companies can apply for OTC clearing services, so that its BO is no longer fully involved in confirmation and settlement of OTC deals. Agreement with clearing houses enables companies to eliminate counterparty risk and reduce workload by swapping the counterparty for a clearing house. OTC deals are then, similarly as exchange trading, marked to market and margined daily. OTC clearing services are in general provided by power exchanges – ICE Clear Europe[22], EEX[23], Nordpool[24], Powernext[25] etc.

21 http://www.trayport.com/en/splash/
22 https://www.theice.com/clear_europe.jhtml

2.7. **Portfolio Management**

The basic concept of a modern portfolio theory was established by Nobel Prize winner Harry Markowitz in 1952 when he published an article titled "Portfolio Selection" in the Journal of Finance. According to the article of Mark Rubinstein from the University of California in Berkley [1], Markowitz was not the first who considered the urge of diversification of investments; his paper was the first comprehensive mathematical formalization of the idea of diversification.

Prior to Markowitz's work, the oldest appreciation of benefits emerging from diversification was probably William Shakespeare in his play the Merchant of Venice where Antonio appeases himself that his wealth is securely diversified. Another significant contribution to the problem of utility and risk allocated on a set of goods made Daniel Bernoulli in his famous article presenting the mathematical solution of the St. Petersburg Paradox in 1738. Following Markowitz's steps, in 1952 James Tobin expanded on the portfolio theory by including a risk-free asset in the analysis which enriched the theory of efficient frontier and the capital market line. Through leveraging of deleveraging a portfolio on the efficient frontier by adding a riskless asset, portfolios on the capital market line are enabled to outperform those on the efficient frontier. In 1964 the famous article about Capital Asset Pricing Model (CAPM) was published by William Sharpe in the journal of Finance. According to Sharpe, all investors should possess the market portfolio, leveraged or deleveraged with a riskless asset. Sharpe also introduced β as the sensitivity of the expected excess asset returns to the market ones.

Investors might minimize their exposure to various risks by investing into a collection of unrelated assets rather than by holding a single one. This concept of diversification is often explained with the traditional saying "don't put all eggs in one basket."
Returns of any diversified portfolio are uncorrelated and consist of expected returns weighted according to proportions of individual components of the portfolio.

Based on Markowitz's article [35], let $\tilde{R}_i$ be the rate of return of an asset in a single period which is considered as random variable and μ_i the expected return of an asset i. Then the risk of a single asset *i* measured by variance and standard deviation is:

$$\tilde{R}_i = \frac{P_{t+1} - P_t}{P_t} \quad (1.16)$$

Then variance and standard deviation are defined as follows:

$$\sigma_i^2 = E(\tilde{R}_i - \mu_i)^2 \quad (1.17)$$

[23] http://www.eex.com/
[24] http://www.nordpoolspot.com/
[25] http://www.powernext.com/

$$\sigma_i = \sqrt{\sigma^2} \quad (1.18)$$

Risk on the portfolio level includes fundamental statistical measurements describing how the numbers are spread out from the mean. Variance of two-asset portfolio includes covariance coefficient. Let X an Y be two random variables, then

$$var[X] = E\big[X - E[X]\big]^2 = \sigma_X^2 \quad (1.19)$$

$$var[X \pm Y] = var[X] + var[Y] \pm 2cov(X, Y) \quad (1.20)$$

$$cov(X, Y) = E[XY] - E[X]E[Y] \quad (1.21)$$

For $var[X], var[Y] > 0$ the correlation coefficient is defined as:

$$\rho(X, Y) = \frac{cov(X, Y)}{\sqrt{var[X].var[Y]}} \quad (1.22)$$

Matrix Ψ of a type i×j (i,j=1..n) where on the main diagonal (i=j) there are var[X_i] and cov(X_i,X_j) otherwise, is called the covariance matrix:

$$\Psi = \begin{pmatrix} \sigma_{11}^2 & cov_{12} & \cdots & cov_{1n} \\ cov_{21} & \sigma_{22}^2 & \cdots & cov_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ cov_{n1} & cov_{n2} & \cdots & \sigma_{nn}^2 \end{pmatrix} \quad (1.23)$$

For a portfolio comprising several assets, Markowitz [35] showed that it is not a security's own risk that is important to an investor, but rather the contribution the security makes to the variance of his entire portfolio, that is determined by its covariance with all assets in the portfolio.

Let *N* be number of assets in a portfolio, θ_i weight of the ith asset in the portfolio, then

$$\boldsymbol{\theta} = (\theta_1 \quad \theta_2 \quad \cdots \quad \theta_N)^T \quad (1.24)$$

$$\boldsymbol{\mu} = (\tilde{R}_1 \quad \tilde{R}_2 \quad \cdots \quad \tilde{R}_N)^T. \quad (1.25)$$

Let $\tilde{R}_p$ be rate of return of a portfolio, μ_p the expected return of a portfolio, thus we arrive at

$$\mu_p = E\big[R_p\big] = \sum_{i=1}^{N} E\big[\theta_i \tilde{R}_i\big] = \sum_{i=1}^{N} \theta_i \mu_i \quad (1.27)$$

$$\tilde{R}_p = \sum_{i=1}^{N} \theta_i \tilde{R}_i \quad (1.26)$$

Covariance σ_{ip} between rates of return of *i-th* asset and the entire portfolio are given by

$$\sigma_{ip} = E\left[(\tilde{R}_i - \mu_i)\sum_{j=1}^{N}\theta_j(\tilde{R}_j - \mu_j)\right] = \sum_{j=1}^{N}\theta_j E[(\tilde{R}_i - \mu_i)(\tilde{R}_j - \mu_j)]$$
$$= \sum_{j=1}^{N}\theta_j\sigma_{ij} \qquad (1.29)$$

Analogically, covariance σ_{ij}between rates of return of *i-th* and *j-th* asset can be defined as

$$\sigma_{ij} = E[(\tilde{R}_i - \mu_i)(\tilde{R}_j - \mu_j)] = \sigma_i\sigma_j\rho_{ij} \quad (1.28)$$

The most important portfolio risk measure is variance which expresses riskiness of the entire portfolio. There are two possible ways how two calculate the variance. Risk of a portfolio measured by variance σ_p^2 is defined as:

$$\sigma_p^2 = E\left[(\tilde{R}_p - \mu_p)^2\right] = E\left[\sum_{i=1}^{N}\theta_i(\tilde{R}_i - \mu_i)\right]^2$$
$$= E\left[\sum_{i=1}^{N}\sum_{j=1}^{N}\theta_i\theta_j(\tilde{R}_i - \mu_i)(\tilde{R}_j - \mu_j)\right]^2$$
$$= \sum_{i=1}^{N}\theta_i^2\sigma_i^2 + 2\sum_{i=1}^{N}\sum_{j=i+1}^{N}\theta_i\theta_j\sigma_{ij} \qquad (1.30)$$
$$= \sum_{i=1}^{N}\theta_i^2\sigma_i^2 + 2\sum_{i=1}^{N}\sum_{j=i+1}^{N}\theta_i\theta_j\sigma_i\sigma_j\rho_{ij}$$

We could also define the portfolio variance as follows:

$$\sigma_p^2 = E\left[(\tilde{R}_p - \mu_p)^2\right] = E\left[(\tilde{R}_p - \mu_p)\sum_{i=1}^{N}\theta_i(\tilde{R}_i - \mu_i)\right]$$
$$= \sum_{i=1}^{N}\theta_i E[(\tilde{R}_p - \mu_p)(\tilde{R}_i - \mu_i)] = \sum_{i=1}^{N}\theta_i\sigma_{ip} \qquad (1.31)$$

$$\sigma_p^2 = var(R_p) = \sum_{i=1}^{N}\sum_{j=1}^{N}\theta_i\theta_j\, cov(R_i, R_j) = \sum_{i=1}^{N}\sum_{j=1}^{N}\theta_i\sigma_{ij}\theta_j \quad (1.32)$$

If we use the covariance matrix $\mathbf{\Psi}$ defined above, the portfolio risk is equal to

$$\sigma_p^2 = \boldsymbol{\theta^T \Psi \theta} = (\theta_1 \quad \cdots \quad \theta_N) \begin{pmatrix} \sigma_{11}^2 & cov_{12} & \dots & cov_{1n} \\ cov_{21} & \sigma_{22}^2 & \dots & cov_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ cov_{n1} & cov_{n2} & \cdots & \sigma_{nn}^2 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_N \end{pmatrix}. \quad (1.33)$$

Investors might choose from various portfolios of expected return (E) and variance (V). According to Markowitz [35], investors should select those attainable portfolios with minimum V for given E or more and maximum E for given V or less. Efficient portfolio [18] is then a portfolio that has either higher expected return (for a given risk) or has lower risk (for a given expected return). An efficient combination of assets lies on so-called efficient frontier, a concept introduced by Markowitz. According to [18], efficient frontier is an upper part of parabola that interconnects the minimum risky portfolio H and asset A with the highest expected return:

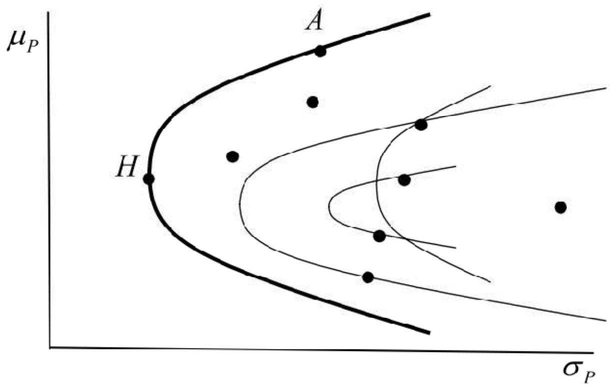

Figure 10 - Efficient Frontier [18]

2.8. **Portfolio sensitivity**

As the value of a portfolio moves in various directions die to movements in market price, quantity, volatility, correlation or time, companies necessarily need a certain precision when predicting and managing its value.
Following Pilipovic [36], we can define the value of a portfolio as the cumulative value of all its *N* assets at a time *t*:

$$\prod\nolimits_t \sum_{n=1}^{N} (y_n)_t (A_n)_t \quad (1.34)$$

where Π_t is the portfolio value at time *t*, A_n *n*-th portfolio asset and y_n number of *n*-th asset. Change in the value of a portfolio could be expressed as

$$d\Pi = f(dv_m, dt) \quad (1.35)$$

where dv_m is the change in the *m*-th market variable.

Risk managers need to know answers to questions like, "Will the portfolio value increase or decrease if the price of oil continues to fall?" Sensitivity analysis can indicate relationships between value of a portfolio and value of its underlying assets. In general, a trader often estimates an impact of price changes on their P/L statement. Traders could obtain better clarity in their portfolio values by considering the five more common sensitivity measures usually called Greeks:

2.8.1. Delta Δ

Delta belongs to the unitless first-order Greeks. The portfolio delta expresses the sensitivity of the value Π_t to the changes in spot or forward prices.

$$\Delta_t^{spot} = \frac{\partial \Pi_t}{\partial \mathrm{S}_t} \quad (1.36)$$

$$\Delta_{t,T}^{forward} = \frac{\partial \Pi_t}{\partial \mathrm{F}_{t,T}} \quad (1.37)$$

Often there is a need to express the portfolio delta in currency terms. This could be achieved easily by

$$\Delta_{currency} = \Delta \times \text{tick amount} \quad (1.38)$$

The tick amount is the monetary value of a tick (a minimum price movement) depending on price quotation in the market.

2.8.2. Vega V

Vega is another first-order derivative which is interpreted as the portfolio value change caused by unit change in the volatilities. The portfolio vega is first derivative of the portfolio value divided by volatility (represented by the standard deviation σ):

$$\mathrm{V}_{t,T}^{product} = \frac{\partial \Pi_t}{\partial \sigma_{t,T}^{product}} \quad (1.39)$$

Vega is usually expressed as a one-volatility-point change. The volatility change is often estimated as a 1 % movement of the current volatility V, therefore the single volatility point is calculated as follows:

$$V_{0,01} = V \times 0{,}01. \quad (1.40)$$

In contrast to Δ, the portfolio vega is expressed in currency units reflecting changes in market variables caused by 1 % movement in the volatility.

In the commodity derivative markets the vega for option on spot price or future price is very handy. The formula is based on the foremost Black-Scholes model where the gamma measure Γ is implemented. Therefore the vega for an option on spot price S with volatility σ in a time to expiration $\tau = (T-t)$ is

$$V_S = \Gamma_S^2 \sigma_S \sqrt{\tau}. \quad (1.41)$$

and its variation on forward price F is

$$V_F = \Gamma_F^2 \sigma_F \sqrt{\tau}. \quad (1.42)$$

2.8.3. Theta Θ

Portfolio value changes due to time decay of underlying assets, which is function to the time to expiration. Value of out-the-of money options, which approach its expiry date, declines as the probability that the options will end up in the money is reduced. Time decay is measured by theta. Theta is also first order derivative expressing change in the portfolio value Π_t with respect to time t:

$$\Theta = \frac{\partial \Pi_t}{\partial \mathrm{t}} \quad (1.43)$$

Theta represents the annual portfolio value change expressed in currency units per time. In order to adjust the time horizon for some shorter period, we have to normalize it by

$$\Theta_{t,t+dt} = \frac{\partial \Pi_t}{\partial \mathrm{t}} \times dt \quad (1.44)$$

where dt might be selected as a calendar day (1/365), a trading day (1/252), a week (1/52) or a month (1/12).
Similarly as vega, the Black-Scholes model could be also adjusted for the theta when trading with options. The formulas for both spot and future price take the interest rate r into account as follows:

$$\Theta_{Call,S} = -\left\{\frac{1}{2} S^2 \sigma_S^2 \Gamma_S + \frac{\partial \mathrm{C}_S}{\partial \mathrm{r}} \left(\frac{r}{\tau}\right)\right\} \quad (1.45)$$

$$\Theta_{Call,F} = -\left\{\frac{1}{2} F^2 \sigma_F^2 \Gamma_F + \frac{\partial \mathrm{C}_F}{\partial \mathrm{r}} \left(\frac{r}{\tau}\right)\right\} \quad (1.46)$$

$$\mathrm{P}_S = \mathrm{C}_S - \mathrm{S} + \mathrm{Ke}^{-rt} \quad (1.47)$$

$$\mathrm{P}_F = \mathrm{C}_F + (\mathrm{K} - \mathrm{F})\mathrm{e}^{-rt} \quad (1.48)$$

$$\Theta_{Put} = \Theta_{Call} + \mathrm{rKe}^{-rt} \quad (1.49)$$

2.8.4. Rho P

Rho is a seldom tool nowadays. It is the first-order derivative that measures an influence of the interest rate *r* on the overall value of a portfolio:

$$\mathrm{P} = \frac{\partial \Pi_t}{\partial \mathrm{r}_{t,T}^{product}}. \quad (1.50)$$

2.8.5. Gamma Γ

The portfolio gamma is the second-order derivative of portfolio value measuring sensitivity to the change in delta. Gamma is expressed for options either for spot *S* or forward price *F* by

$$\Gamma_t^{spot} = \frac{\partial \Delta_t^{spot}}{\partial \mathrm{S}_t} = \frac{\partial^2 \Pi_t}{\partial \mathrm{S}_t^2} \quad (1.51)$$

$$\Gamma_{t,T}^{fwd} = \frac{\partial \Delta_{t,T}^{fwd}}{\partial \mathrm{F}_{t,T}} = \frac{\partial^2 \Pi_t}{\partial \mathrm{F}_{t,T}^2}. \quad (1.52)$$

Financial derivatives with a certain level of optionality carry gamma risk, therefore forwards do not. In case a trading company uses options on averages of forward prices, the option prices then simultaneously carry risk due to the forward prices of various expiry dates. This risk is called the cross-gamma risk. Cross-gamma measures the change in portfolio delta with respect to a particular forward price with expiration time T_1, as the forward price with the expiration at T_2 moves by one currency unit [36]:

$$\gamma_{t,T_1,T_2}^{fwd} = \frac{\partial \Delta_{t,T_1}^{fwd}}{\partial \mathrm{F}_{t,T_2}} = \frac{\partial^2 \Pi_t}{\partial \mathrm{F}_{t,T_1} \partial \mathrm{F}_{t,T_2}}. \quad (1.53)$$

2.9. Hedging

Hedging is the process of entering into contracts to reduce portfolio risk [36]. Portfolio managers open hedge transactions to offset the price risk. Aim of hedging is to neutralize an exposure that remains in a portfolio.

Financial derivatives as forwards or futures are often used for hedging purpose. If an investor holds a short position in a forward contract against a long position in the commodity, the investor entered into a *short hedge* position. In contrary, *long hedge* is a situation if an investor holds short commodity and enters into a long forward position. Price movements of the underlying assets cause also changes in prices of its forwards or futures. In case the investor making short hedge transaction expects the commodity price to fall, the forward contract price would fall in the same proportion to make an offsetting profit on the derivative. Success of hedging strategies thus depends strongly on interactions between spot and forward prices. Relationship between spot/forward prices and time horizon describes *basis risk*. Hull [26] defines the general basis risk in time t and maturity T as spot price of asset to be hedged minus futures price of contract used:

$$B_{t,T} = S_t - F_{t,T}. \quad (1.54)$$

Figure 11 shows various levels of basis risk depending on time horizon. Development of the basis is always determined by two rules: First, the initial basis $B_{0,T}$ ($t=0$) could be easily observed as an investor knows the spot price S_0 and the forward $F_{0,T}$. Second, the basis at the time of maturity T is always equal to zero as the spot and forward price converges in T ($S_T=F_{T,T}$).

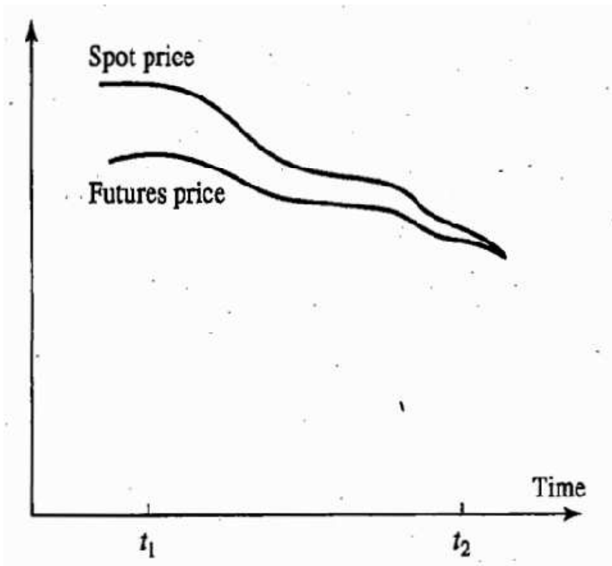

Figure 11 - Basis over time [26]

Reilly and Brown [Investment Analysis] gives an example of an investor who entered into a short hedge to secure his long commodity position and agreed to sell it at T through a short forward contract. The initial Basis at $t=0$ is $B_{0,T}=F_{0,T} - S_0$. When the investor liquidates his position in $t<T$ (therefore he will not be able to deliver the underlying asset), he will have to sell the commodity position for S_t and buy back the forward contract for $F_{t,T}$. Finally after the short hedge liquidation at t he makes a profit

$$B_{t,T} - B_{0,T} = (S_t - F_{t,T}) - (S_0 - F_{0,T}). \quad (1.55)$$

Once an investor enters into a hedge contract, he faces only the basis risk instead of the absolute price movements of underlying assets. The development of the *cover basis* $B_{t,T}$ depends on the correlation coefficient between the spot and forward prices. Mutually correlated spot and forward contract prices creates negligible basis.

Prerequisite of an effective hedging is the optimal hedge ratio that reflects the mutual correlations and thus minimizes the variance of the value of the hedged positions. Let's assume a short hedger who is long a commodity unit and short N forward contracts, then the profit at t equals [Reilly Brown Investment analysis]

$$\Pi_t = (S_t - S_0) - \left(F_{t,T} - F_{0,T}\right)(N) = (\Delta S) - (\Delta F)(N) \quad (1.56)$$

and its variance is given by

$$\sigma_{\Pi}^2 = \sigma_{\Delta \mathrm{S}}^2 + (N^2)\sigma_{\Delta \mathrm{F}}^2 - 2(N)COV_{\Delta \mathrm{S},\Delta \mathrm{F}}. \quad (1.57)$$

The optimal hedge ratio N^* is then expressed by equation

$$N^* = \frac{COV_{\Delta \mathrm{S},\Delta \mathrm{F}}}{\sigma_{\Delta \mathrm{F}}^2} = \left(\frac{\sigma_{\Delta \mathrm{S}}}{\sigma_{\Delta \mathrm{F}}}\right)\rho, \quad (1.58)$$

where ρ is the correlation between the changes in spot and forward price.

Chapter 3

Value at Risk

3.1. **Introduction**

Value at Risk (VaR) is a statistical measure of possible portfolio losses due to an adverse movement in prices of each of the portfolio components, simply gives you an answer to a question [8]: how much money might the firm lose due to normal market movements? Its biggest advantage is that it summarizes the total market risk into a single number which is easy to understand to senior managers, directors or shareholders. Its importance has further increased after VaR was adopted as an international regulatory standard for measuring market risk under the Basel II[26].

The concept was first used by large investment banks in the late 1980's but the underlying idea originated a decade before. Following Linsmeier & Pearson [34], in the 1970's after the fall of the Bretton Wood system of fixed exchange rates, financial markets have witnessed both enormous volatility and proliferation with development of financial derivatives. Due to enlargement of derivative instruments in portfolios, its flexibility and growing linkages among financial institutions, companies searched for a portfolio level quantitative measure of market risks. Increasing market has led to the demand for management of the volatility of market rates and prices. In 1973 Fischer Black and Myron Scholes published their renowned article "The Pricing of Options and Corporate Liabilities" that contained their ground-breaking option pricing model. With these tools financial firms could have better quantified and measured volatility and market risks.

G. Holton [25] traced origins of Value at Risk back to 1922 when first non-mathematical discussions about portfolio constructions took place. The very first VaR measure was published by Leavens in 1945 in Markowitz's article Mean-variance analysis in portfolio choice and capital markets, where he researched risk of a bond portfolio. Modern VaR model has its roots in the modern portfolio theory (MPT) developed mainly by Markowitz, Roy, Sharpe or Tobin. In the 1980's, Kenneth Garbade constructed VaR model for bond portfolio based on price sensitivity to changes in bond's yields. In 1993 Garbade's work was followed by Thomas Wilson who published VaR measure tailored for a trading environment. Probably the most significant creadit is claimed to JP Morgan Chase that developed in the late 1980's comprehensive firm-wide Value at Risk system. The investment bank then implemented VaR into a new reporting scheme for its daily Treasury meetings with bank's Chairman Dennis Weatherstone. The main architect of the modern VaR model was Till Guldimann from JP

[26] Basel II: International Convergence of Capital Measurement and Capital Standards: A Revised Framework - Comprehensive Version : The First Pillar – Minimum Capital Requirement [accessed from bis.org]

Morgan Chase who later popularized and promoted VaR internally. Guldimann participated also in a research group that developed a service named RiskMetrics provided by JP Morgan Chase that comprised software tools with technical documents.
Modern Value at Risk measure has been widely adopted by regulators of banking institutions all around the world. Basel Committee on Banking Supervision suggested banks had a possibility to calculate their capital requirements for market risk with their in-house Value at Risk model (with some parameters determined by the Basel Committee). This step was followed the same year by US Federal Reserve allowing banks to use their internal models of VaR. The general proliferation of this risk measure was completed by US Security and Exchange Commission that proposed VaR as a possible market risk disclosure measure.

3.2. **Definitions**

The question placed at the beginning of this chapter should be rephrased into more specific form: how much money (or more) might the firm lose over time period T with probability X. Having this simple definition in mind, let's now move forward to formal definitions.

VaR is defined as the predicted worst-case loss at a specific confidence level (e.g. 95%) over a certain period of time (e.g. 1 day). [30]

Value at Risk is a measure of the maximum potential change in value of a portfolio of financial instruments with a given probability over a pre-set horizon. [30]

Value at Risk is the worst loss over a target horizon such that there is a low, prespecified probability that the actual loss will be larger. [29]

All the definitions involve two important quantitative factors, the time horizon (holding period) and the confidence level.

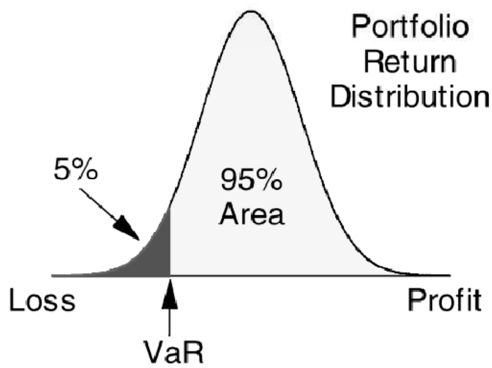

Figure 12 – Value at Risk (PDF) [30]

For higher detail, let's have a look at the mathematics of Value at Risk.

3.3. **Parameters**

Skewness in accordance with [24] expresses asymmetry in a probability distribution of a random variable. Skewness of a random variable X is defined as

$$\eta_1 = skew(X) = \frac{E[(X-\mu)^3]}{\sigma^3} \quad (3.1)$$

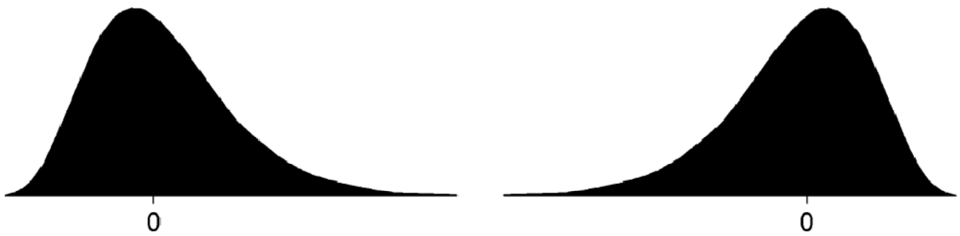

Figure 13 – Two probabilistic Density functions having the same mean and variance. Left one is positively skewed, the one on the right is skewed negatively [24]

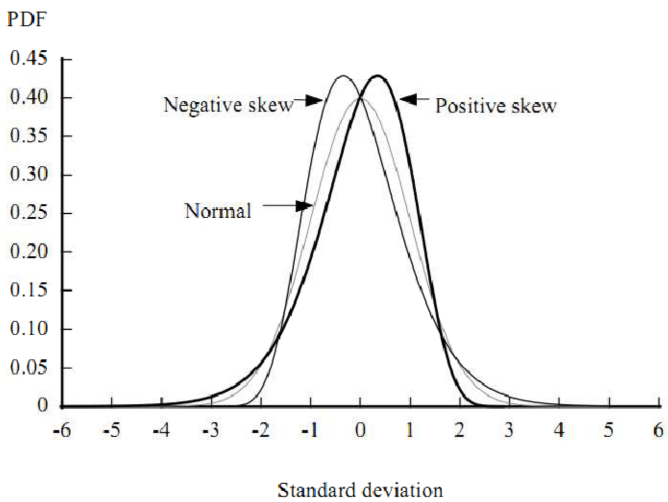

Figure 14 – Skewness [30]

Kurtosis [24] describes the shape of a probability distribution of a random variable. Kurtosis of a random variable X is defined as

$$\eta_2 = kurt(X) = \frac{E[(X-\mu)^4]}{\sigma^4} \quad (3.2)$$

If the kurtosis of any distribution is greater than 3, it is described as leptokurtic. On the other hand, if less than 3, than the distribution is said to be platykurtic. Leptokurtic distributions are simultaneously 'peaked' with fat tails. Platykurtosis describes 'less peaked' distributions with thinner tails.

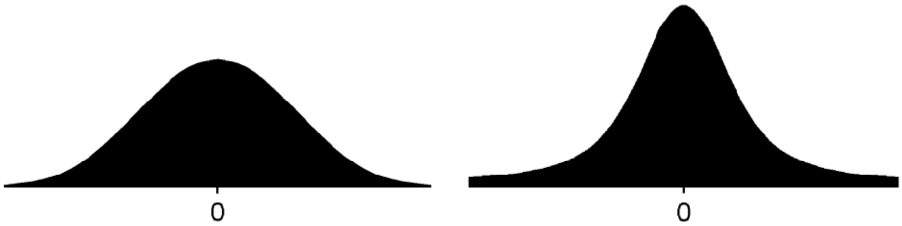

Figure 15 - Platykurtic distribution (on the left), Leptokurtic distribution (on the right)

3.4. Fundamentals of distributions

One of the most important parameter of Value at Risk models is probability distribution. Standard set of distributions is very numerous and basic functions are either continuous or discrete. I would mention only the most common continuous ones with summarized overview providing elementary information. Manual for @RISK 5 gives detailed overview about all distributions provided by the @RISK 5 modelling software. There also several joint distributions consisting mixture of basic distributions

3.4.1. Normal $\mathcal{N}(\mu, \sigma^2)$

The normal distribution is characterized by two main parameters: a mean μ and a variance σ^2. Let X be normal random variable, then its density function is

$$\phi(x) = \frac{\exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)}{\sigma\sqrt{2\pi}}. \quad (3.3)$$

Random variable X having normal distribution is often transformed into standardised normal variable Z, by this formula:

$$Z = \frac{X-\mu}{\sigma}. \quad (3.4)$$

It implies $dz = {dx}/{\sigma}$, thus the normal distribution function is:

$$\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-x^2/2}\, dx \quad (3.5)$$

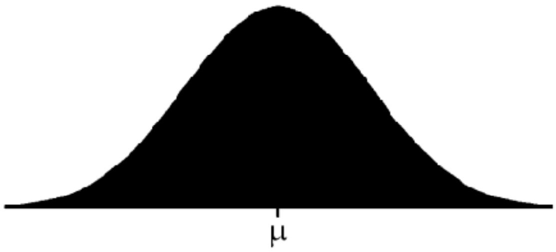

Figure 16 - Density function of a normal distribution

Distribution of normal density function is depicted in Figure 16. With the distribution function $\Phi(z)$ relates quantile function, inverse to the function of the standard normal distribution function. The quantile function is an important component of the Value at Risk measure. The quantile function is defined as

$$\Phi^{-1}(\alpha) = \inf\{z \in \mathbb{R} : \Phi(z) \geq \alpha\}. \quad (3.6)$$

Regardless its mean and variance, the normal distribution has always:

$$\eta_1 = skew(X) = 0 \quad (3.7)$$

$$\eta_2 = kurt(X) = 3 \quad (3.8)$$

Usually a normal distribution defined as $\mathcal{N}(0,1)$ is called the standard normal distribution. Therefore, let Z be random variable $Z \backsim \mathcal{N}(0,1)$, then a random variable X might be then expressed as a polynomial:

$$X = \sigma Z + \mu \quad (3.9)$$

3.4.2. Lognormal ln$\boldsymbol{\mathcal{N}(\mu, \sigma^2)}$

By definition [24], random variable X is lognormally distributed if the natural logarithm of X is normally distributed. Lognormal distribution has 2 parameters: a mean μ and a variance σ^2. Probability density function of lognormal distribution is given by

$$\Phi(x) = \begin{cases} \dfrac{\exp\left(-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right)}{x\sigma\sqrt{2\pi}} & for\ x > 0. \\ 0 & \end{cases} \quad (3.10)$$

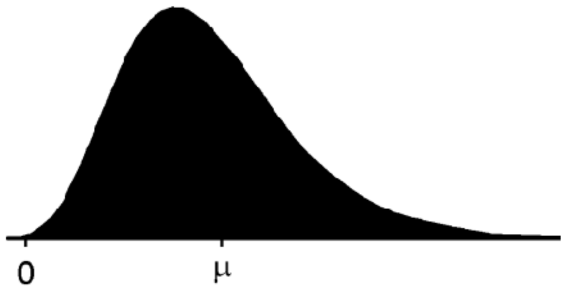

Figure 17 - Density function of a lognormal distribution

Skewness and kurtosis are calculated according to the following formulas:

$$\eta_1 = skew(X) = [\exp(s^2) + 2]\sqrt{\exp(s^2) - 1} \quad (3.11)$$

$$\eta_2 = kurt(X) = \exp(4s^2) + 2\exp(3s^2) + 3\exp(2s^2) - 3 \quad (3.12)$$

Mean and standard deviation are expressed by

$$\mu = \exp\left(\frac{2m + s^2}{2}\right) \quad (3.13)$$

$$\sigma = \sqrt{exp(2m + 2s^2) - exp(2m + s^2)}, \quad (3.14)$$

where

$$m = \ln(\frac{\mu^2}{\sqrt{\sigma^2 + \mu^2}}) \quad (3.15)$$

$$s = \sqrt{\ln\left[\left(\frac{\sigma}{\mu}\right)^2 + 1\right]}. \quad (3.16)$$

3.4.3. Chi-squared χ_k^2

Standard random variable X is of a chi-squared distribution with n degrees of freedom and noncentrality parameter

$$\delta^2 = \sum_{i=1}^{n} \delta_i^2 \quad (3.17)$$

if for X applies:

$$X = (Z_1 + \delta_1)^2 + (Z_2 + \delta_2)^2 + (Z_3 + \delta_3)^2 + \cdots + (Z_n + \delta_n)^2 \quad (3.18)$$

where $Z_1, Z_2, \ldots, Z_n$ are *n* independent standard normal variables and $\delta_1, \delta_2, \ldots, \delta_n$ are *n* constants.
According to standards, the distribution is ussualy denoted as $\chi^2(n, \delta^2)$. In case $\delta^2 = 0$ the distribution is said to be centrally chi-squared, otherwise $(\delta^2 \neq 0)$ it is non-centrally chi-squared.

The probability density function for central chi-squared distribution is:

$$\Phi(x) = \begin{cases} \dfrac{x^{\frac{(n-2)}{2}} exp\left(-\frac{x}{2}\right)}{2^{\frac{n}{2}}\Gamma(\frac{n}{2})} & for\ x > 0 \\ 0 & \end{cases} \quad (3.19)$$

For noncentral:

$$\Phi(x) = \begin{cases} \dfrac{exp[-(x+\delta^2)/2]}{2^{\frac{n}{2}}} \displaystyle\sum_{j=0}^{\infty} \frac{x^{j-1+n/2}\delta^{2j}}{\Gamma\left(j+\frac{n}{2}\right)2^{2j}j!} & for\ x > 0 \\ 0 & \end{cases} \quad (3.20)$$

where $\Gamma()$ is gamma function:

$$\Gamma(y) = \int_0^{\infty} e^{-z} z^{y-1} dz \quad (3.21)$$

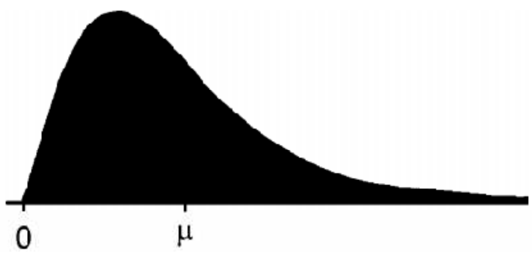

Figure 18 - Density function of a chi-squared distribution

The standard deviation, skewness and kurtosis are defined as follows:

$$\sigma = \sqrt{2(n + 2\delta^2)}, \quad (3.22)$$

$$\eta_1 = skew(X) = \frac{2^{3/2}(n + 3\delta^2)}{(n + 2\delta^2)^{3/2}}, \quad (3.23)$$

$$\eta_2 = kurt(X) = 3 + \frac{12(n + 4\delta^2)}{(n + 2\delta^2)^2}. \quad (3.24)$$

3.5. **Value at Risk**

3.5.1. Absolute Value at Risk

Value at Risk has been already defined at the beginning of Chapter 3 but now I have to introduce two kinds of VaR as seen from two different point of view – absolute and relative VaR.
The Absolut Value at Risk $VaR_{t,a}$ is defined for a portfolio, for a time horizon *t* with probability *a* as the highest potential loss L towards initial portfolio value, which may an investor occur on time horizon *t* with probability *1-a.* Definition of the absolute VaR is proceeds from the quantile function, inverse to the function of the standard normal distribution function.

Let $\alpha \in (0,1)$ represents the confidence level and *L* is the loss of portfolio, then the absolute VaR is:

$$VaR_{t,\alpha}^{abs} = -inf\{L \in \mathbb{R}: \Phi(L) \geq 1 - \alpha\}. \quad (3.25)$$

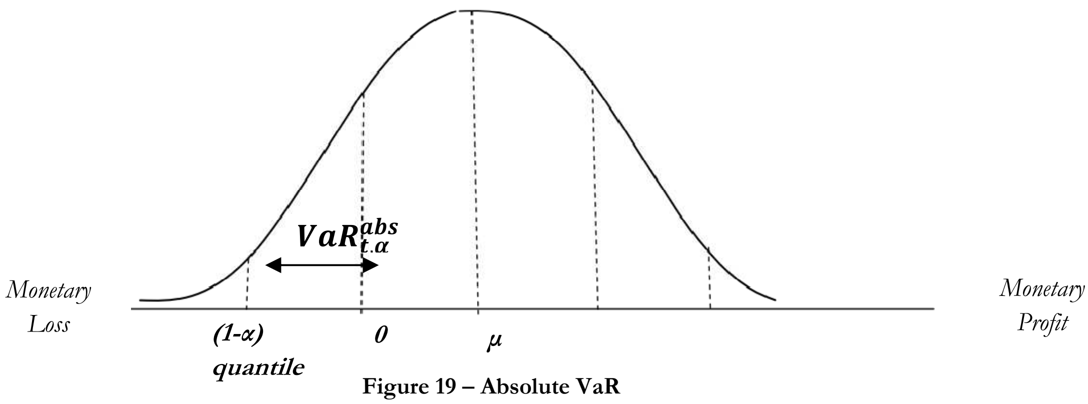

Figure 19 – Absolute VaR

3.5.2. Relative Value at Risk

Contrary to absolute VaR which defined simply as the absolute monetary loss of a portfolio, the relative VaR is for a given confidence level described as the loss relative to the mean. According to [19] is the relative VaR a difference between the expected change in the value of a portfolio and the change that corresponds to the cut-off rate of return determined by selected confidence level.

Let P_0 be the initial portfolio value, t be the time horizon, μP_0 be the expected change of portfolio's value, $\alpha_c P_0$ be the portfolio loss corresponding to the cut-off point, then the relative Value at Risk is defined as:

$$VaR_{t,\alpha}^{rel} = \mu P_0 - \alpha_c P_0 = (\mu - \alpha_c) P_0. \quad (3.26)$$

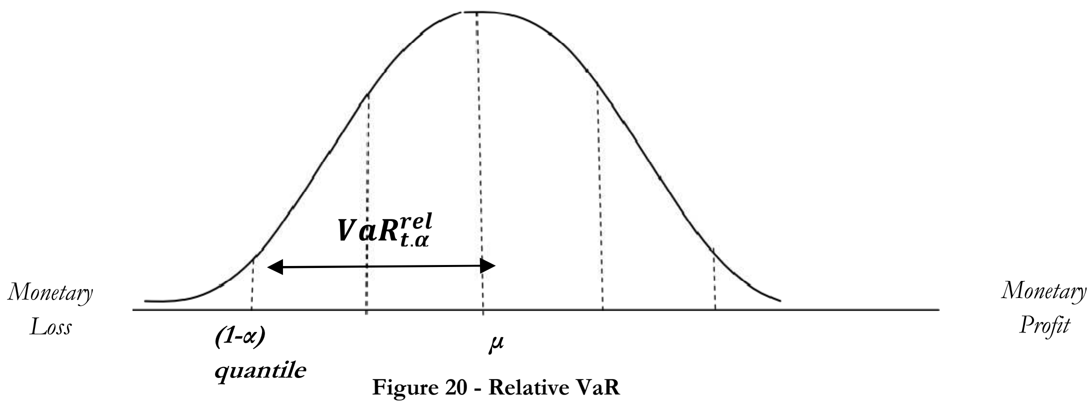

Figure 20 - Relative VaR

3.5.3. Nonparametric Value at Risk

Definition of Value at Risk is based on an appropriate distribution function, that describes characterizes the value at risk. In case we possess only limited amount of historical data, from which we cannot sufficiently derive the actual distribution function, we need to make an assumption according to the available historical data and estimate the shape of the distribution of returns.

According to [29], let W_0 be the initial investment and R the random rate of return. Value of the portfolio is thus $W = W_0 (1+R)$, where expected return and volatility of R are defined statistically as μ and σ. Then the lowest portfolio value at the confidence level c as $W^* = W_0(1 + R^*)$.
The relative Value at Risk is defined as the loss relative to the mean:

$$VAR(mean) = E(W) - W^* = -W_0(R^* - \mu) \quad (3.27)$$

The absolute Value at Risk is described as the loss relative to zero or without reference to the expected value:

$$VAR(zero) = W_0 - W^* = -W_0 R^* \quad (3.28)$$

According to [29], relative VAR is conceptually more appropriate, because it views risk in terms of deviations from the mean. Value at Risk is derived from the distribution of the future portfolio value *f(w)*. To measure VAR, we try to find the worst possible realization of *W** such that probability of exceeding this value is *c* :

$$c = \int_{W^*}^{\infty} f(w)dw \quad (3.29)$$

Naturally, probability of a portfolio value lower than the minimal W^*, $p = P(w \leq W^*)$ is $1 - c$, then:

$$1 - c = \int_{W^*}^{\infty} f(w)dw = P(w \leq W^*) = p \quad (3.30)$$

W^* is the quantile of the distribution, that represents a threshold value with fixed probability of being exceeded.

3.5.4. Parametric Value at Risk

Parametric models of VAR presuppose that the distribution belongs to the group of parametric distributions (e.g. normal distribution). Value at Risk in this case, is computed as a standard deviation of the portfolio multiplied by a factor depending on a confidence level. If we select, for example, a normal distribution as a description of the portfolio values, we should fit the data by translating the general distribution *f(w)* into the desired normal distribution function *Φ(ε)*, *ε* has mean equal 0 and standard deviation 1. If $W^* = W_0(1+R^*)$, where R* is negative ($R^* = -|R^*|$), then the standard normal deviate α>0 is

$$-\alpha = \frac{-|R^*| - \mu}{\sigma} \quad (3.31)$$

Probability is then equivalent to

$$1 - c = \int_{\infty}^{W^*} f(w)dw = \int_{-\infty}^{-|R^*|} f(r)dr = \int_{-\infty}^{-\alpha} \Phi(\varepsilon)\, d\varepsilon \quad (3.32)$$

Value at Risk is then represented by the deviate *a* and the probability *p* is

$$p = N(x) = \int_{-\infty}^{x} \Phi(\varepsilon)d\varepsilon \quad (3.33)$$

According to the equation (3.31) we can express the cut-off return R^* is

$$R^* = -\alpha\sigma + \mu \quad (3.34)$$

If we consider the parameters σ and μ as annual, the time interval is Δt years, we can expand the equation (3.34) as follows:

$$VAR(mean) = -W_0(R^* - \mu) = W_0 \alpha\sigma\sqrt{\Delta t} \quad (3.35)$$

In terms of this equation, VAR equals a multiple of a portfolio value, standard deviation of the distribution, the time horizon and the statistical factor depending on the confidence level.

Absolute loss that investors might achieve is then:

$$VAR(zero) = -W_0 R^* = W_0(\alpha\sigma\sqrt{\Delta t} - \mu\Delta t) \quad (3.36)$$

3.6. **Methods**

3.6.1. Analytical (Variance-Covariance) method

The variance-covariance approach assumes that returns of the underlying market variables – profits and losses – have multivariate normal distribution. The portfolio returns are supposed to be a linear weighted combination of the returns on each of *N* asset:

$$R_{p,t+1} = \sum_{i=1}^{N} w_i R_{i,t+1}, \quad (3.37)$$

$$\theta_i = \frac{w_i}{w_p}, \quad (3.38)$$

$$w_p = \sum_{i=1}^{N} w_i. \quad (3.39)$$

That means that the portfolio could be decomposed on individual linear cash flows. Analytical VaR is then simply a multiple of standard deviation of a portfolio. For the calculation of VaR we necessarily need to determine a covariance matrix and weights of all portfolio components. Similarly we can assume that the portfolio return at the (1-α) quantile is also normally distributed, thus $R \sim N(\mu, \sigma^2)$. The cut-off value u_α of the standard normal distribution:

$$\begin{aligned} 1 - \alpha = P(\Phi \leq -u_\alpha) = P\left(\frac{N-\mu}{\sigma} \leq -u_\alpha\right) = P(N \leq \mu - \sigma u_\alpha) \\ = P(N \leq u_\alpha) \end{aligned} \quad (3.40)$$

Now we can define the rate of portfolio return as:

$$R^* = \mu - \sigma . u_\alpha \quad (3.41)$$

while

$$-u_{1-\alpha} = u_\alpha \quad (3.42)$$

The absolute VaR is then described by:

$$VaR_{t,\alpha} = -P_0 . (\mu - \sigma\mu_\alpha) \quad (3.43)$$

And relative VaR is defined as:

$$VaR_{t,\alpha} = P_0 . \big(\mu - (\mu - \sigma\mu_\alpha)\big) = P_0 \sigma u_\alpha . \quad (3.44)$$

In case the time horizon for which the VaR is calculated is not unitary (1 day period) over the horizon in which we monitor the volatility, we can modify the volatilities:

$$\sigma_t = \sqrt{t}\sigma_1 \quad (3.45)$$

Where we use one-day volatility σ_1. For determination of volatilities in an interval between t_1 and t_2 we can adjust the volatilities as follows:

$$\sigma_{t_1} = \sqrt{t_1}\sigma_1, \quad (3.46)$$

$$\sigma_{t_2} = \sqrt{t_2}\sigma_2, \quad (3.47)$$

so we finally arrive at

$$\sigma_{t_2} = \sqrt{\frac{t_2}{t_1}}\sigma_{t_1} . \quad (3.48)$$

The initial VaR equation is then adjusted by taking the time factor into account:

$$VaR_{t_2,\alpha} = \sqrt{\Delta \mathrm{t}} VaR_{t_1,\alpha} = \sqrt{\frac{t_2}{t_1}} P_0 \sigma_{t_1} u_\alpha . \quad (3.49)$$

The VaR measure might be flexibly adjusted according to the desired holding period. Holding period depends mostly on the business circumstances and character of the industry. The prerequisite of the normal distribution makes the VaR determination rather flexible as VaR modifications between several confidence levels are not very difficult. The most common quantiles are:

Confidence level	u_α
90%	1,282
95%	1,645
99%	2,326

Table 3 - Normal distribution (source: http://statistika.vse.cz/download/materialy/tabulky.pdf)

Analytical Value at Risk for a linear portfolio necessarily takes correlations among numerous risk factors and assets into consideration. In case a portfolio is composed of $i=1,\dots,N$ assets in which we invest amount of P_i we can calculate the portfolio's volatility as standard deviation:

$$\sigma_p = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} P_i P_j \sigma_i \sigma_j \rho_{ij}} \quad (3.50)$$

As variance of the portfolio:

$$\sigma_p^2 = \sum_{i=1}^{N} P_i^2 \sigma_i^2 + 2\sum_{i=1}^{N}\sum_{j<i} P_i P_j \sigma_i \sigma_j \rho_{ij} =$$

$$= \begin{pmatrix} P_1 & \cdots & P_N \end{pmatrix} \begin{pmatrix} \sigma_{11}^2 & \sigma_{12} & \sigma_{13} & \cdots & \sigma_{1N} \\ \vdots & & & & \vdots \\ \sigma_{N1} & \sigma_{N2} & \sigma_{N3} & \cdots & \sigma_{NN}^2 \end{pmatrix} \begin{pmatrix} P_1 \\ \vdots \\ P_N \end{pmatrix} \quad (3.51)$$

The portfolio Value at Risk is then multiple of the initial portfolio value P_0, portfolio volatility and desired confidence level:

$$VaR_{t,\alpha} = P_0 \sigma_p u_\alpha = P_0 \sqrt{\sum_{i=1}^{N} \theta_i^2 \sigma_i^2 u_\alpha^2 + 2\sum_{i=1}^{N}\sum_{j=i+i}^{N} u_\alpha^2 \theta_i \theta_j \sigma_i \sigma_j \rho_{ij}} =$$

$$= \sqrt{\sum_{i=1}^{N} (VaR_i)^2 + 2\sum_{i=1}^{N}\sum_{j=i+i}^{N} (VaR_i)(VaR_j)\rho_{ij}} = \sqrt{\boldsymbol{P} \times \boldsymbol{V} \times \boldsymbol{P^T}} \quad (3.52)$$

where $\boldsymbol{P} = (u_{1-\alpha}.P_1 \quad \cdots \quad u_{1-\alpha}.P_N)$ and $\boldsymbol{V}$is the covariance matrix capturing connections between the assets. This approach that takes into account diversification between individual portfolio components is called the Diversified VaR. On the other hand, the Undiversified VaR is simply sum of individual isolated components. Additional and more detailed description of incremental or marginal VaR could be found in Jorion [29].

The Variance-Covariance approach is widely used and often adopted as the basic way of VaR calculation. The biggest advantage is that this approach is rather simple, intuitive and and its calculation is efficient. Investors or risk managers could use Riskmetrics dataset or collect their own historical price series. In contrast, the analytical VaR presents static method that is mostly effective for rather short time horizons. Furthermore the assumption of normally distributed returns is not realistic for energy markets which are characterized by fat-tailed return distributions.

Linear positions with Options

So far I have only covered theory of linear model of a portfolio without any financial derivatives. It means that all positions in a portfolio were linear – there is a linear relationship between movements in the underlying risk factor and the profit and loss profile of the dependent variable [19].
Following Hull [26], I consider linear model of a portfolio consisting of an option and a stock of a price *S*. Then

$$\Delta = \frac{\delta P}{\delta S} \quad (3.53)$$

is the rate of change in the value of the portfolio, where δS is 1-day change in the price of the stock and δP is 1-day change of the portfolio's value. Further, we can define

$$\delta x = \frac{\delta S}{S} \quad (3.54)$$

as the 1-day percentage change in the price of the stock. Now the relationship between the change in the portfolio δP and δx might approximated by

$$\delta P = \Delta . S . \delta x \quad (3.55)$$

If an investor holds *k* positions in the underlying market assets, analogically we can define this relationship between δP and δx_i as follows:

$$\delta P = \sum_{i=1}^{k} \Delta_i . S_i . \delta x_i \quad (3.56)$$

where S_i is the value of *i*-th market variable and Δ_i is the rate of change in the value of the portfolio with respect to the *i*-th market variable.

Nonlinear positions

The above described linear model for a portfolio with options is an approximation. To achieve higher accuracy in VaR calculations, I will introduce quadratic model, often called as the Delta-Gamma model. The linear model with options is also called the Delta-Normal method. Delta Δ is defined as the rate of change in the value of a portfolio with respect to an underlying asset. On the other hand, Gamma Γ is defined as the rate of change of Delta with respect to an underlying asset. Gamma and Delta both belong to the Greeks, the vital sensitivity risk measures. Positive Gamma causes that the probability distribution has a tendency to be positively skewed. When Gamma is negative, it tends to be negatively skewed.

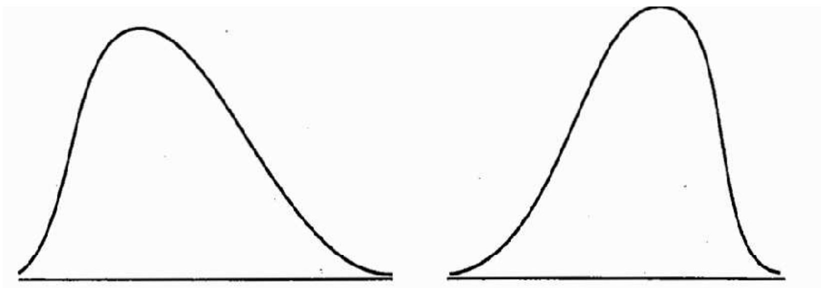

Figure 21 – Positive and negative Gamma (Skewness) [26]

The Figure 21 illustrates that the Value at Risk of a portfolio depends strongly on the left tails of the probability distribution. When we compare the portfolio's distribution function to normal probability density function, then the positive Γ portfolio tends to less heavy left fat tail than the normal distribution [26]. The opposite situation for the negative Γ portfolio (heavy left fat tail) is depicted in the Figure 22. As indicated in both Figure 21 and 22, the VaR measure would be in case of the negative (positive) Γ portfolio rather low (higher).

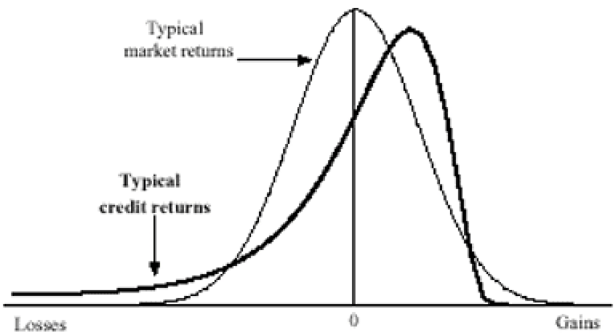

Figure 22 - fat tails to the left (Negative Gamma)

If we suppose a portfolio comprised of an asset with its price S, δ and γ are delta and gamma of the portfolio. According to the Taylor series expansion we can get

$$\delta P = \Delta \delta S + \frac{1}{2}\Gamma(\delta S)^2. \quad (3.57)$$

As we already know that

$$\delta x = \frac{\delta S}{S}, \quad (3.58)$$

Now we can adjust the equation (3.57) like this:

$$\delta P = S\Delta\delta x + \frac{1}{2}\Gamma S^2(\delta x)^2. \quad (3.59)$$

For a portfolio comprising of *k* assets which are dependent on only one of the market variable, we can generalize the equation (3.59) into

$$\delta P = \sum_{i=1}^{k} S_i\Delta_i\delta x_i + \frac{1}{2}\sum_{i=1}^{k}\Gamma_i S_i(\delta x_i)^2, \quad (3.60)$$

where S_i is the value of i-th asset in the portfolio, Δ_i and Γ_i are the delta and gamma of the portfolio with respect to the *i*-th portfolio's asset.

3.6.2. Historical Simulation

Historical simulation uses obtained historical time series of market variables. The simulation consists in revaluing the portfolio for numerous historical scenarios and builds a 'hypothetical' distribution of profit and losses based on how the portfolio would have behaved in the past [8]. Historical simulation does not make any assumptions about the distribution of the portfolio returns; we only assume that risk in the past influences the future risk. The fact that energy markets are in general very dynamic with a certain every day development makes this method not very applicable.
Calculation of Value at Risk is based on historical prices of the risk factors and price developments to revaluate the portfolio value within each scenario.
Advantage of this method is primarily the simplicity that makes this approach quite easy to understand, intuitive and straightforward. Historical simulation is also relatively easy to implement with a high performance of computation. This method is also relatively easy to explain to senior managers when they ask. If you obtain a dataset of historical prices, this method can be applied to all instruments.
On the other hand, this method can produce misleading values in case the past data were atypical and corresponding with the recent situation. Last but not least, historical simulation does not enable a risk manager to perform any 'what-if' analyses to discover impacts of each part of the model [34].

Practical steps to calculate VAR starts with the gathered data of the past market development (typically daily market prices). Into the obtained time series, we will include also the value of the current portfolio and estimate VAR as the quantile from density of the distribution.

Absolute Value at Risk was earlier expressed by this definition:

Let $\alpha \in (0,1)$ be the confidence level and X the loss, then:

$$VaR_{t,\alpha}^{abs} = -\inf\{x \in \mathbb{R}: F(x) \geq 1 - \alpha\}. \quad (3.61)$$

Following [20], risk of the current portfolio loss is measured by the price changes in the past. Let P_t is the daily price of a portfolio at time *t*. Absolute yield achieved in period between t and t-1 is

$$D_t = P_t - P_{t-1} \quad (3.62)$$

relative yield for the same period is computed as follows:

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} \quad (3.63)$$

and finally logarithmic yield is

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) = \ln(1 + R_t). \quad (3.64)$$

If we consider prices captured in a *k-day* period, the formulas above could be easily adjusted. The k-day portfolio yield is

$$R_t = \frac{P_t - P_{t-k}}{P_{t-k}} \quad (3.65)$$

The yield $1 + R_t$ for k-day period could be expressed as a multiplication of 1 day yields:

$$\begin{aligned} 1 + R_t(k) &= (1 + R_t)(1 + R_{t-1})(1 + R_{t-2}) \dots (1 + R_{t-k-1}) \\ &= \frac{P_t}{P_{t-1}} . \frac{P_{t-1}}{P_{t-2}} \dots \frac{P_{t-k-1}}{P_{t-k}} = \frac{P_t}{P_{t-k}} \end{aligned} \quad (3.66)$$

Yield of entire portfolio during period $t = 1 \dots T$ consisting of N individual assets reflects yield and weight of each asset. Let $R_{j,t}$ be yield of *j*-asset in period *t* and $w_{j,T}$ be the weight of the *j*-asset in the portfolio. Then yield of the portfolio $R_{p,t}$ in a moment *t* equals

$$R_{p,t} = \sum_{j=1}^{N} w_{j,T} R_{j,t} \quad (3.67)$$

All results achieved from the formula (3.67) are then sorted in ascending order as follows:

$$R_{p,(1)} < R_{p,(2)} < \dots < R_{p,(T-1)} < R_{p,(T)} \quad (3.68)$$

Depending on the desired confidence level, we empirically select the threshold value (cut-off point) as (1-α)-quantile, 0<α<1:

$$\tilde{u}_{(1-\alpha)} = \begin{cases} R_{(\lfloor T(1-\alpha)\rfloor+1)}, & if\ T(1-\alpha) \notin \mathbb{Z} \\ \frac{1}{2}\left(R_{(T(1-\alpha))} + R_{(T(1-\alpha)+1)}\right), & if\ T(1-\alpha) \in \mathbb{Z} \end{cases} \quad (3.69)$$

Value at Risk is then the value that should not be exceeded at the selected confidence level. Absolute Value at Risk is thus:

$$VaR_{t,\alpha} = -\tilde{u}_{(1-\alpha)}.P_0 \quad (3.70)$$

Nonlinear portfolio requires slightly different approach. All factors influencing the Value at Risk measure of a portfolio need to be mapped and identified, all changes in their price development during period *t* for *t=0,…,T* are gathered in a time series. Nonlinear portfolio in a *j*-day has then *T* scenarios of its development in *j+1* moment.

Definition: Let V_i^k be the value of *k*-th market factor (k=1,…,n) during *i*-th day (i=1,…,m). Let's suppose that the portfolio is now in *j*-th day. Then we can calculate tomorrow's *i*-th scenarios for the portfolio value (*j+1*-th day) as follows:

$$V_{j+1,i}^k = V_j^k \frac{V_i^k}{V_{i-1}^k} \quad (3.71)$$

Than the overall portfolio value at the time (*j+1*-th day) is simply a function of all market factors:

$$P_{j+1,i} = f(V_{j+1,i}^1, \dots, V_{j+1,i}^n) \quad (3.72)$$

Relative change of the portfolio value is

$$R_{p,i} = \frac{P_{j+1,i} - P_j}{P_j} \quad (3.73)$$

where P_j is the portfolio value during the *j*-th day. Values of the nonlinear portfolio are than sorted the same way as described above in the case of linear portfolio. Value at risk (the cut-off point) is than selected from these values as the desired quantile based on the confidence level.

3.6.3. Monte Carlo Simulation

The Monte Carlo simulation method has some common features with historical simulation. The main contrast is that the simulation is not carried out using N observed changes of the

market factors during last N time periods and then generates N hypothetical portfolio values, Monte Carlo simulation uses statistical distributions that fits and approximate expected changes of the portfolio value. Within the selected distribution, generator of random value generates sufficiently enough hypothetical values of the portfolio. Monte Carlo simulation is based on the statistical simulation of the joint behaviour of all relevant market variables and uses this simulation to generate future possible values of the portfolio.
Process of calculating the Monte Carlo Value at Risk starts with a specification of a parametric stochastic process for its risk factors. The second step entails simulation of fictitious price paths for all risk factors.
Behaviour of portfolio components is described by a stochastic process - e.g. Brownian motion (Wiener process) or Random walk. Geometric Brownian motion assumes that innovations in the asset prices are uncorrelated over time and that small movements in prices can be described by stochastic differential equation

$$dS_t = \mu_t S_t dt + \sigma_t S_t dz \quad (3.74)$$

where μ is the expected return, dz is normally distributed random variable with a variance dt [29]. The random variable creates unpredictable shocks in the price development as it does not follow any information from the past price history of the asset. By the definition [20] a Brownian motion has independent increments and can be modified to a continuous process. Parameter μ_t stands for drift and σ_t represent volatility at the time t. In practice, the increments of dt can be approximated by small changes of Δt. Let t be the present time, T the target time and τ=T-t be the time horizon of VAR. for the generation of random price variables S_{t+1} over time τ, we divide τ by n steps of the process as follows: $\Delta t = \tau/n$. Now for discrete changes we can adjust the stochastic equation accordingly:

$$\Delta S_t = S_{t-1}(\mu \Delta t + \sigma \epsilon \sqrt{\Delta t}) \quad (3.75)$$

Where ϵ is a standard normal random variable; $\epsilon \sim N(0,1)$. This process then generates a mean $E(\Delta S/S) = \mu \Delta t$ and variance $V(\Delta S/S) = \sigma^2 \Delta t$. Monte Carlo simulation of the future prices in every of the n steps of simulation continues for ϵ for $i = 1, \dots, n$. In the next step the price is modelled as $S_{t+1} = S_t + S_t(\mu \Delta t + \sigma \epsilon_1 \sqrt{\Delta t})$ and $S_{t+2} = S_{t+1} + S_{t+1}(\mu \Delta t + \sigma \epsilon_2 \sqrt{\Delta t})$ etc. The simulation continues until the point until $S_{t+n} = S_T$.

Let's assume that the initial price is S_0 and then I can express the simulated future prices in increments of time as

$$S_{t+\Delta t} = S_t\left(1 + \mu \Delta t + \sigma \epsilon \sqrt{\Delta t}\right) \Rightarrow S_{\Delta t} = S_0\left(1 + \mu \Delta t + \sigma \epsilon_{\Delta t} \sqrt{\Delta t}\right) \quad (3.76)$$

After two incremental time periods:

$$S_{2\Delta t} = S_0\left(1 + \mu \Delta t + \sigma \epsilon_{\Delta t} \sqrt{\Delta t}\right)\left(1 + \mu \Delta t + \sigma \epsilon_{2\Delta t} \sqrt{\Delta t}\right) \quad (3.77)$$

In general, in the moment when the simulation reaches the target time horizon T:

$$S_T = S_0 \prod_{i=1}^{n} \left(1 + \mu \Delta t + \sigma \epsilon_{i\Delta t} \sqrt{\Delta t}\right) \quad (3.78)$$

The random numbers ϵ are based on the probability distribution that we estimate. These numbers are actually 'pseudo' random because they are generated from an algorithm using a predefined rule [29]. These numbers, generated mostly by computer programmes, use uniform distribution in the interval [0,1]. In [19], a random drawing from uniform distribution must be transformed into a random drawing from standard normal distribution (using the inverse distribution function of the standard normal distribution). The overall number of iteration depends on the desired accuracy and technology. Too short simulation cycles might make the range of possible portfolio values incomplete and thus incorrect.

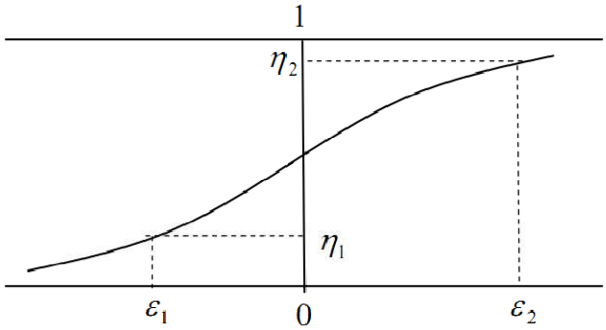

Figure 23 - VAR random numbers generation [19]

Monte Carlo simulation applied on a portfolio composed of several assets need to reflect also possible correlation among each of its assets. Correlations might be captured in a correlation matrix that is an adjustment of the covariance matrix (1.23):

$$C = \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & 1 & \cdots & \rho_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n1} & \rho_{n2} & \cdots & 1 \end{pmatrix} \quad (3.79)$$

Cholesky factorization enables us to add a correlation structure into the simulated process. Uncorrelated variables are generated independently according to the equation, where $\epsilon_{j,t}$values are independent across time period and series *j=1,…,N* [29]:

$$\Delta S_{j,t} = S_{j,t-1}(\mu_j \Delta t + \sigma_j \epsilon_{j,t} \sqrt{\Delta t}) \quad (3.80)$$

Set of transformed random numbers ϵ are simulated by set of original independent variables η that are generated by computer software.
By Cholesky factorization of the correlation matrix $\boldsymbol{C = AA^T}$ we can obtain the vector ϵ:

$$\epsilon_1 = \eta_1 \quad (3.81)$$

$$\epsilon_2 = \rho\eta_1 + \sqrt{(1-\rho^2)}\eta_2 \quad (3.82)$$

with ρ as the correlation coefficient of variables ϵ. Correlated variables are generated by the Cholesky factorization, where $\boldsymbol{A}$ is a lower triangular matrix (zeros in the upper right corners). Vector of *N* independent values η, which all have variance equal to 1, than generates the variables $\epsilon = \boldsymbol{A}\eta$, where $\boldsymbol{A} = a_{ij}$. As a consequence, the vector of generated variables is:

$$\begin{gathered} \epsilon_1 = a_{11}\eta_1, \\ \epsilon_2 = a_{21}\eta_1 + a_{22}\eta_2, \\ \cdots \\ \epsilon_n = a_{n1}\eta_1 + a_{n2}\eta_2 + \cdots + a_{nn}\eta_n. \end{gathered} \quad (3.83)$$

If we consider two variable situation, the correlation matrix is by Cholesky factorization decomposed as follows [29]:

$$\begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \rho & \sqrt{1-\rho^2} \end{pmatrix} \begin{pmatrix} 1 & \rho \\ 0 & \sqrt{1-\rho^2} \end{pmatrix} \quad (3.84)$$

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \rho & \sqrt{1-\rho^2} \end{pmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} \quad (3.85)$$

Usage of appropriate random number generator in a simulation model estimates the portfolio values. Number of computer software (the Crystal Ball from Oracle or @Risk from Palisade Software) display the simulation output as a histogram that makes all the process of determination of VaR easier.

The major problems of the Monte Carlo simulation approach are that it is relatively complex and thus difficult to understand and implement. The intensity of computer skills and hardware requirements are rather high. On the other hand, this method is intuitive and user can use own datasets from historical time series. The flexibility is higher as the user can apply several hypothetical scenarios and study each of its cases.

Chapter 4

VaR modifications

4.1. Introduction

Value at Risk measure has quickly reached recognition and popularity. The reasons for its rapid proliferation into whole financial sector are presented in [13]:

- VaR measures risk across different positions and risk factors and can be applied to various kinds of portfolios, unlike traditional methods (Greeks, portfolio theory measures etc.).
- VaR aggregates the risk resulting from positions with respect to correlations among them.
- VaR is a holistic portfolio risk measure taking all influential risk factors into account.
- VaR gives probabilities of loss amounts that could be suffered, whereas many traditional measures answer only "what if?" questions.

However, the Value at Risk evinces some drawbacks and inconsistencies. The most significant weaknesses are primarily the fact that VaR does not inform about tail losses and the problem with the sub-additivity axiom.

Acerbi et al. [3] introduces a paradox that illustrates shortcomings of VaR. Accerbi considers two portfolios A and B, whereas portfolio A (made for instance of long option positions) is of value 1000 € with a maximum downside level of 100 € and the worst 5 % cases on a fixed time horizon T are all of maximum downside. VaR at 5 % on this time horizon would then be 100 €. Another portfolio B is also of 1000 € which on the other hand invests also in strong short futures positions that allow for a potential unbounded maximum loss. We could easily choose B in such a way that its VaR is still 100 € on the time horizon T. However, in portfolio A the 5 % worst case losses are all of 100 €. In portfolio B the 5 % worst case losses range from 100 € to some arbitrarily high value. Acerbi then places a question: Which portfolio is more risky? According to VaR 5 % they bear the same risk.

According to Dowd and Blake [13] VaR can only tell us the most we can lose in good states where a tail event does not occur; it tells nothing about how much we can lose in bad states where a tail event does occur. To illustrate this problem, we can assume there is an investment that has very high expected rate of return but also higher possible loss. In case the possible higher loss would not affect the VaR measurement of the investment, an investor would

proceed with the investment regardless of the size of the higher expected rate of return or the higher loss. This could leave the investor exposed to possibility of very high losses.
In many companies traders have to fulfil their VaR-defined targets that determine also their remuneration. Hypothetically a trader might have several positions in out-of-the-money options that might make a profit in positive states of the world (when things go well) and make a loss when the trader is unlucky. The fact that VaR does not inform about outcomes that would happen in bad states can motivate the trader to "optimize" the VaR target in order to maximize his personal remuneration.

4.2. **Sub-additivity axiom**

Loosely said, the sub-additivity means that aggregation of individual risks does not increase the total risk. The problem is that sub-additive risks might inflate an estimate of the total risk.

The subadditivity axiom is part of so-called coherent measures of risk which are properties of risk that help to predict and manage risks efficiently if these axioms are fulfilled. Although these four axioms have been already presented in Chapter 2, I would mention the subadditivity axiom again. The axiom is defined as [5]:

$$for\ all\ X_1\ and\ X_2\ \in \mathcal{G}, \rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2) \quad (4.1)$$

where $\mathcal{G}$ is set of all risks and $\rho(X)$ is measure to the risk X. This formula says that the combination of two returns on financial instruments never reaches higher risk than sum of its individual parts. The risk $\rho(X)$ might be also interpreted as the minimum extra cash an investor has to add to the risky position X to be allowed to proceed with his plan [5]. If we put $\rho(X) = VaR_\alpha(X)$ we see that VaR is not the coherent risk measure, as it does not fulfil the axiom of subadditivity. According to [5], any coherent risk measure arises as the supremum of the expected negative of final net worth for some collection of "generalized scenarios" or probability measure on states of world:

$$\rho(X) = sup\left\{E_{\mathbb{P}}\left[\frac{-X}{r}\right]\middle|\mathbb{P} \in \mathcal{P}\right\}. \quad (4.2)$$

where r is total return on a reference investment and ρ is coherent risk measure, $\mathcal{P}$ represents family of probability measures on the set of states of nature [5]. To construct a coherent risk measures we should focus more on expected loss (CVaR) than or minimum loss from set of worst losses (VaR). We should be concerned about the shape of the tail of the underlying distribution of returns. To overcome this drawback of VaR Artzner [5] introduces an alternative that is called "Tail Conditional Expectation" or "TailVaR" which consider not only he excess but the whole of the variable X. The Tail Conditional Expectation is defined as

$$TCE^{(\alpha)}(X) = -E_{\mathbb{P}}\left(\frac{-X}{r}\middle|\frac{-X}{r} \leq -VaR_\alpha(X)\right) \quad (4.3)$$

and is usually known as Expected Shortfall or Conditional VaR.

4.3. **Expected Shortfall (Conditional Value at Risk)**

Expected Shortfall has many names. Actuaries call it Conditional Tail Expectation or Tail VaR. in financial risk circles it is labelled as Expected Tail Loss, Tail Conditional Expectation, Conditional VaR, Tail Conditional VaR or Worst Conditional Expectation [13].

Expected Shortfall satisfies all four axioms of coherent risk measure, subadditivity included.

$$ES^{(\alpha)}(X) + ES^{(\alpha)}(Y) = ES^{(\alpha)}(X + Y) \quad (4.4)$$

CVaR is the expected loss in case a tail event occurs. Several authors consider CVaR as more conservative than VaR and agree that CVaR is in general more suitable risk measure that produces better incentives for traders than VaR. Jaschke [27] recommends CVaR for quantification of minimal required capital and banking supervision.

Rockafellar and Uryasev [37] interpret CVaR for continuous loss distribution at a given confidence level as the expected loss given that the loss is greater than the VaR at that level, or for that matter, the expected loss given that the loss is greater than or equal to the VaR.

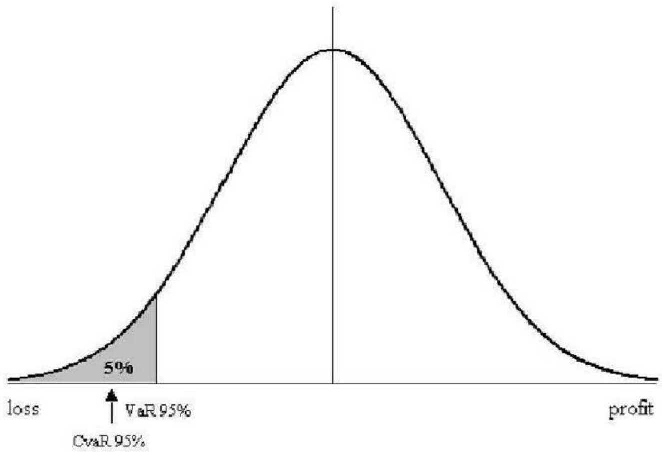

Figure 24 – Robustness of CVaR [47]

Mathematically, CVaR is defined for continuous loss distribution, for given confidence level *a* and time horizon *t* as the conditional expectation of the losses exceeding VaR [47]:

$$\mathrm{CVaR}_{\alpha} = \frac{1}{1-\alpha}\int_{-\infty}^{VaR_{\alpha}} rp(r)dr \quad (4.5)$$

or

$$\mathrm{CVaR}_{\alpha} = E(x | x \leq VaR_{\alpha}). \quad (4.6)$$

where *p(r)* is the probability density, *r(t)* the expected rates of return with respect to the holding period *t* with confidence level $\alpha \in (0,1)$.
Equivalent definition might be found in Acerbi et al. [3]. Let *X* be the profit-loss of a portfolio on a specified time horizon *T* and let $\alpha = A\% \in (0,1)$ some specified probability level. The Expected Shortfall of the portfolio is then defined as[27]

$$ES^{(\alpha)}(X) = -\frac{1}{\alpha}\left(E\left[X\,\mathbb{1}_{\{X \le x^{(\alpha)}\}}\right] - x^{(\alpha)}\left(P\left[X \le x^{(\alpha)}\right] - \alpha\right)\right) \quad (4.7)$$

where $x^{(\alpha)}\left(P\left[X \le x^{(\alpha)}\right] - \alpha\right)$ is interpreted as the exceeding part to be subtracted from the expected value $E\left[X\,\mathbb{1}_{\{X \le x^{(\alpha)}\}}\right]$ when $\{X \le x^{(\alpha)}\}$ has probability larger than $\alpha = A\%$. On the other hand, if the probability distribution of loss is continuous, then $P\left[X \le x^{(\alpha)}\right] = \alpha$ and $ES^{(\alpha)} = TCE^{(\alpha)}$.

CVaR is a convex function with respect to portfolio positions. Within the concept of CVaR in case of discontinuous loss probability, Rockfellar and Uryasev [37] further define CVaR$^-$ (lower CVaR) and CVaR$^+$ (upper CVaR). In general, we can express inequality $CVaR^- \le CVaR \le CVaR^+$. Whereas CVaR is a coherent risk measure, CVaR$^-$ and CVaR$^+$ are not. CVaR can be viewed as a weighted average of VaR and CVaR$^+$.
To define upper and lower CVaR we have to start with definition of the distribution function for the loss $z = f(x, y)$:

$$\Psi(x, \xi) = P\{y | f(x, y) \le \xi\} \quad (4.8)$$

where $f(x, y)$is continuous in x and measurable in y.
VaR can be equivalently defined as α-VaR of the loss associated with a decision *x*:

$$\xi_\alpha(x) = min\{\xi | \Psi(x, \xi) \ge \alpha\}. \quad (4.9)$$

Now we can finally arrive at the definition of upper and lower CVaR. Also, α-CVaR$^+$ of the loss associated with a decision x is the value

$$\phi_\alpha^+(x) = E\{f(x, y) | f(x, y) > \xi_\alpha(x)\}, \quad (4.10)$$

whereas α-CVaR$^-$ of the loss represents the value

$$\phi_\alpha^-(x) = E\{f(x, y) | f(x, y) \ge \xi_\alpha(x)\}. \quad (4.11)$$

[27] Notation $\mathbb{1}_{\{Relation\}} = \begin{cases} 1, if\ Relation\ is\ true \\ 0, if\ Relation\ is\ false \end{cases}$

Comparison of behaviour and convexity of the risk measures is depicted in the Figure 25. CVaR might be easily estimated on (1-α) % level of confidence as weighted average of calculated VaR values of our loss distribution [13].

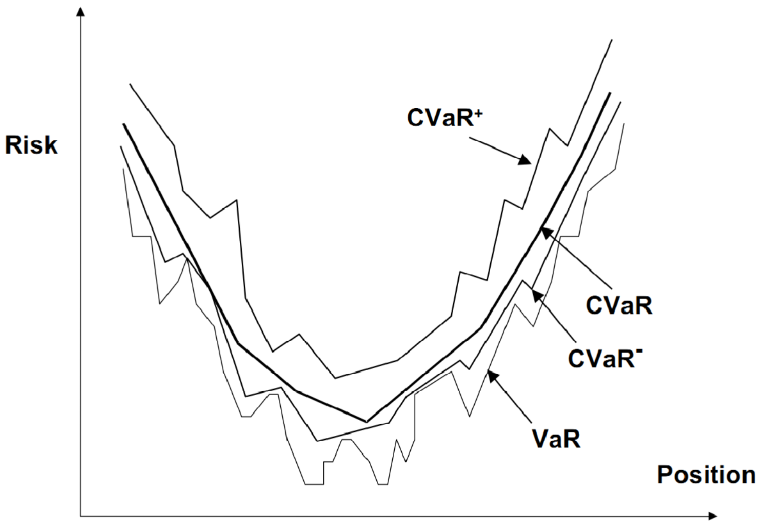

Figure 25 – Convexity of CVaR [37]

4.4. **Liquidity adjusted VaR**

Liquidity risk has been already described in Chapter 2. Liquidity risk has become a key issue especially in the aftermath of the "credit crunch".
In the contemporary literature there are several approaches to the Liquidity-adjusted-VaR (LVaR). Sunando Roy [39] categorizes 6 main groups of methods that could be found in literature – Ad-hoc approach (Lengthening Time Horizon), Optimal Liquidation Approach/Transaction Costs, Liquidation Discount Approach, Exogenous Liquidation Approach, Market Price Response Approach or Intraday Liquidity Risk.
In this paper I follow the LVaR model proposed by A. Bangia et al. [6] that quantifies exogenous liquidity risk. Volatility risk is characterized by the volatility of the observed spread with no reference to the relationship of the realized spread to trade size [6].
In [6] the model of LVaR consists of parametric VaR model, which is enriched by incorporation of the exogenous liquidity risk. Firstly, for the time *t* the authors define one-day asset returns r_t as the logarithmical difference of mid-prices:

$$r_t = \ln(P_t) - \ln(P_{t-1}) = \ln\left(\frac{P_t}{P_{t-1}}\right). \quad (4.12)$$

If we assume one-day time horizon over which the one-day returns are normally distributed, than the 99% worst value is defined

$$P_{0,99} = P_t e^{(E[r_t] - 2{,}33\sigma_t^2)}, \quad (4.13)$$

where 2,33 is the multiple for the standard deviation arising from the assumption of normal distribution. With another assumption, that the daily returns $E[r_t]$ are equal zero, the standard parametric VaR is given by

$$VaR_{0,99} = P_t\left(1 - e^{(-2{,}33\sigma_t^2)}\right). \quad (4.14)$$

Unfortunately the equation above only considers the volatility of the mid-price. Therefore we have to take into account the exogenous liquidity risk (market conditions) under tail-event circumstances. The exogenous cost of liquidity (COL) relies on an average spread $\bar{S}$

$$\bar{S} = \frac{[\bar{P}(ask) - \bar{P}(bid)]}{\bar{P}(mid)} \quad (4.15)$$

and volatility of a spread $\alpha\tilde{\sigma}$

$$COL = \frac{1}{2}[P_t(\bar{S} + \alpha\tilde{\sigma})] \quad (4.16)$$

where P_t is today's mid-price and $\bar{S}$ the average relative spread. The distribution of the $\bar{S}$ spread is then non-normal. Finally the LVaR model assumed for the 99th percentile is given by

$$LVaR = VaR + COL = P_t\left(1 - e^{(-2{,}33\sigma_t^2)}\right) + \frac{1}{2}[P_t(\bar{S} + \alpha\tilde{\sigma})] \quad (4.17)$$

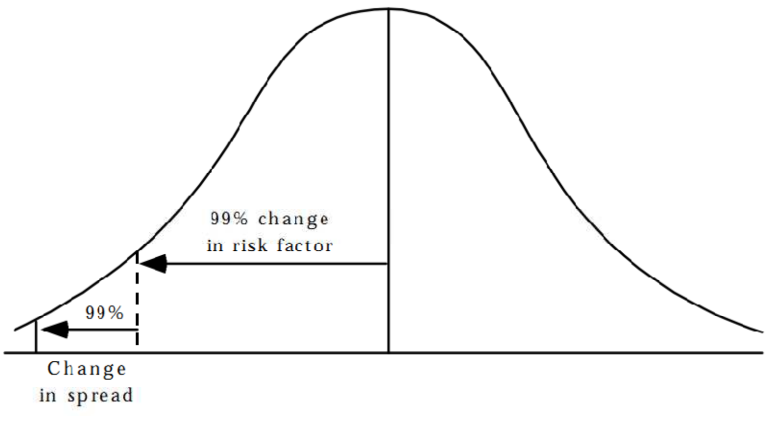

Figure 26 - Liquidty Value at Risk (LVAR)

In case the assumption of normally distributed daily returns is violated, the LVaR formula above underestimates the overall risk. Therefore the authors designed a correction factor θ as

$$VaR_{0,99} = P_t\theta\left(1 - e^{(-2,33\sigma_t^2)}\right), \quad (4.18)$$

Where

$$\theta = 1{,}0 + \phi \ln\left(\frac{\kappa}{3}\right), \quad (4.19)$$

κ is kurtosis and ϕ is a constant depending on the tail probability (for normal distribution $\kappa = 3$ and $\theta = 1$). Value of the constant ϕ is usually calculated by regression.

4.5. **Modified VaR (Cornish-Fisher expansion)**

In case the distribution of loss is not normal, but there are only "small" deviations from the normal distribution function, we could approximate the non-normal distribution using the Cornish-Fisher expansion. This method analytically approximates the quantile α_c to accommodate non-normal skewness and kurtosis. Just like VaR, mVaR might fail to be subadditive.
The Cornish-Fisher expansion is power series approximation of the quantile function that is approximated by calculating the first moments of the series as long as the moments of distribution are known.

The approximation lies in a replacement of the standard α_c in the normal VaR formula [16]:

$$\begin{aligned} a_c^{CF} = \alpha_c + \frac{1}{6}(\alpha_c^2 - 1)E(X^3) + \frac{1}{24}(\alpha_c^3 - 3\alpha_c)E(X^4) \\ - \frac{1}{36}(2\alpha_c^3 - 5\alpha_c)E(X^3)^2. \end{aligned} \quad (4.20)$$

In other words, the Cornish-Fischer expansion corrects the previously defined Gaussian VaR (GVaR) for the portfolio skewness s_p and kurtosis k_p. When s_p and k_p are equal to zero (that is the case of the normal distribution), mVaR equals Gaussian VaR [10]. The relationship between Gaussian VaR and modified VaR is given by

$$\begin{aligned} mVaR(\alpha) = GVaR(\alpha) \\ + \sqrt{m_2}\left[-\frac{1}{6}(\alpha_c^2 - 1)s_p - \frac{1}{24}(\alpha_c^3 - 3\alpha_c)k_p + \frac{1}{36}(2\alpha_c^3 - 5\alpha_c){s_p}^2\right], \end{aligned} \quad (4.21)$$

where m_2 is the second central moment. The sample moments corrects the normal VaR and lead to a VaR value closer to the true VaR. There are of course several deficiencies of this method. Users must consider that the approximation gives worse results for probabilities closer to 0 or 1, left and right corner of the tail. Further, mVaR works well only for non-normal distribution closer to the normal one. Approximation of distribution shows fat tails or high degree of skewness will not work appropriately [23].

Calculation of the modified VaR for a portfolio comprising *N* assets with weights *w* is described in Peterson and Boudt [11]. If we denote *r* the (possibly non-normal) returns on portfolio assets with mean μ and covariance matrix $\sum$, firstly we have to calculate the $N \times N^2$ co-skewness matrix

$$M_3 = E[(r-\mu)(r-\mu)'\otimes(r-\mu)'] \quad (4.22)$$

and the $N \times N^3$ co-kurtosis matrix

$$M_4 = E[(r-\mu)(r-\mu)'\otimes(r-\mu)'\otimes(r-\mu)'], \quad (4.23)$$

where $\otimes$ is the Kronecker product[28]. Using the *q*-th portfolio moment $m_q = E\left[\left(r_p - w'\mu\right)^q\right]$ we can define prerequisites m_2, m_3 and m_4 for the portfolio skewness and kurtosis:

$$\begin{aligned} m_2 &= w'\Sigma w \\ m_3 &= w'M_3(w\otimes w) \qquad (4.23) \\ m_4 &= w'M_4(w\otimes w\otimes w). \end{aligned}$$

Then the portfolio skewness s_p and excess kurtosis k_p is calculated by

$$s_p = {}^{m_3}\big/_{\sqrt{m_2^3}} \quad (4.24)$$

$$k_p = {}^{m_4}\big/_{m_2^2} - 3. \quad (4.25)$$

Under the assumption of normally distributed returns with confidence level *a* is the portfolio (Gaussian) VaR given by the equation

$$GVaR(\alpha) = -w'\mu - a_c\sqrt{w'\Sigma w} \quad (4.26)$$

---

[28] $\boldsymbol{A}_{m,n} \otimes \boldsymbol{B}_{p,q} = \boldsymbol{C}_{mp,nq} = \begin{bmatrix} a_{11}b_{11} & a_{11}b_{12} & \cdots & a_{11}b_{1q} & \cdots & \cdots & a_{1n}b_{11} & a_{1n}b_{12} & \cdots & a_{1n}b_{1q} \\ a_{11}b_{21} & a_{11}b_{22} & \cdots & a_{11}b_{2q} & \cdots & \cdots & a_{11}b_{1q} & a_{11}b_{1q} & \cdots & a_{1n}b_{2q} \\ \vdots & \vdots & \ddots & \vdots & & & \vdots & \vdots & \ddots & \vdots \\ a_{11}b_{p1} & a_{11}b_{p2} & \cdots & a_{11}b_{pq} & \cdots & \cdots & a_{1n}b_{p1} & a_{1n}b_{p2} & \cdots & a_{1n}b_{pq} \\ \vdots & \vdots & & \vdots & \ddots & & \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots & & \ddots & \vdots & \vdots & & \vdots \\ a_{m1}b_{11} & a_{m1}b_{12} & \cdots & a_{m1}b_{1q} & \cdots & \cdots & a_{mn}b_{11} & a_{mn}b_{12} & \cdots & a_{mn}b_{1q} \\ a_{m1}b_{21} & a_{m1}b_{22} & \cdots & a_{m1}b_{2q} & \cdots & \cdots & a_{mn}b_{21} & a_{mn}b_{22} & \cdots & a_{mn}b_{2q} \\ \vdots & \vdots & \ddots & \vdots & & & \vdots & \vdots & \ddots & \vdots \\ a_{m1}b_{p1} & a_{m1}b_{p2} & \cdots & a_{m1}b_{pq} & \cdots & \cdots & a_{mn}b_{p1} & a_{mn}b_{p2} & \cdots & a_{mn}b_{pq} \end{bmatrix}$

Since the returns of the assets could be skewed or heavily tailed, we could reach better estimate for the portfolio riskiness by using the Cornish-Fisher expansion, replace the a_c by a_c^{CF} that the portfolio modified VaR is given by

$$mVaR(\alpha) = -w'\mu - a_c^{CF}\sqrt{w'\Sigma w} \quad (4.27)$$

where the quantile a_c^{CF} is set as

$$a_c^{CF} = \alpha_c + \frac{(\alpha_c^2 - 1)s_p}{6} + \frac{(\alpha_c^3 - 3\alpha_c)k_p}{24} - \frac{(2\alpha_c^3 - 5\alpha_c){s_p}^2}{36}. \quad (4.28)$$

Chapter 5

Empirical Part

Introduction

This chapter aims to illustrate theoretical concepts of Value at Risk measures applied on an example of an US-based energy trading company that holds a large portfolio of gasoline, crude oil and natural gas reserves. We assume that all trading activities considered are denominated in USD therefore no FX risk is taken into account.
Based on theoretical portfolio of this energy trading company comprising energy commodities, we analyse and comment the differences between diverse risk concepts such as Value at Risk (VaR), Conditional VaR (CVaR) and modified VaR (mVaR). The Value at Risk measure is calculated using three methods – analytical (variance-covariance) method, historical and Monte Carlo simulation. Calculations of CVaR and mVAR are based on the variance-covariance VaR.

At the beginning I describe composition of the portfolio, providing you with information about statistical and financial features. Afterwards I carry out several normality tests to ensure what attributes have the underlying distribution of the portfolio, whether its returns are normally distributed or not. After the data testing, I turn to the calculation of the Value at Risk measures applied on the portfolio. I use the methods presented earlier in Chapter 3 – variance-covariance approach, historical and Monte Carlo simulation. All the reached results are then compared and commented. For all calculations, simulations and graphing is carried out using Microsoft Excel with Palisade's @Risk[29] and StatTools[30].

Assumptions

In all calculations I consider simplifying assumptions of average daily volatilities received from the analysis of historical time series.
Furthermore, I have to consider composition of the portfolio being static during the time period I calculate VaR. Energy trading companies usually perform higher trading activities during the day and estimate VaR continuously or at the end of the day. This would be the case of a dynamic portfolio approach which would reflect all influences emerging from changes in market positions.

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[29] http://www.palisade.com/risk/
[30] http://www.palisade.com/stattools/

Data

The company decided to undertake a large investment of 50 million USD to hold a portfolio comprising three energy commodities US Gulf Coast conventional gasoline, WTI crude oil futures contracts and Henry Hub Gulf Coast natural gas. Each of the three assets appears in certain proportions (weights) that split the total invested amount as depicted in Table 4.

Component	w_i	*Amount in USD*
Gasoline	0,1895	9 475 000
WTI Crude Oil	0,5525	27 625 000
Natural Gas	0,258	12 900 000
Total	**1**	**50 000 000**

Table 4 - Structure of the portfolio

For all calculations I used 10-year time series of daily closing spot prices published by the US Energy Information Administration (*http://www.eia.gov/*). All observations started on 4/1/2000 and ended on 4/1/2010, what totally represents 2752 observations. All daily spot prices of the WTI crude oil and the US Gulf Coast Conventional Gasoline were in USD per barrel, whereas the daily spot prices of the Henry Hub Gulf Coast Natural Gas were in USD/MMBtu.

Value at Risk of a portfolio is determined by price movements of its underlying assets for a certain time period. We consider daily changes in prices. Development of daily return on the portfolio of energy commodities is depicted in Figure 27. The portfolio exhibits significant fluctuations of its daily value. The biggest changes occurred around 786th observation, which corresponds with enormous jumps in natural gas prices in February 2003. Graphs for each of the underlying asset are located in Appendix B. Development of historical prices of the underlying assets is displayed in Figures in Appendix A.

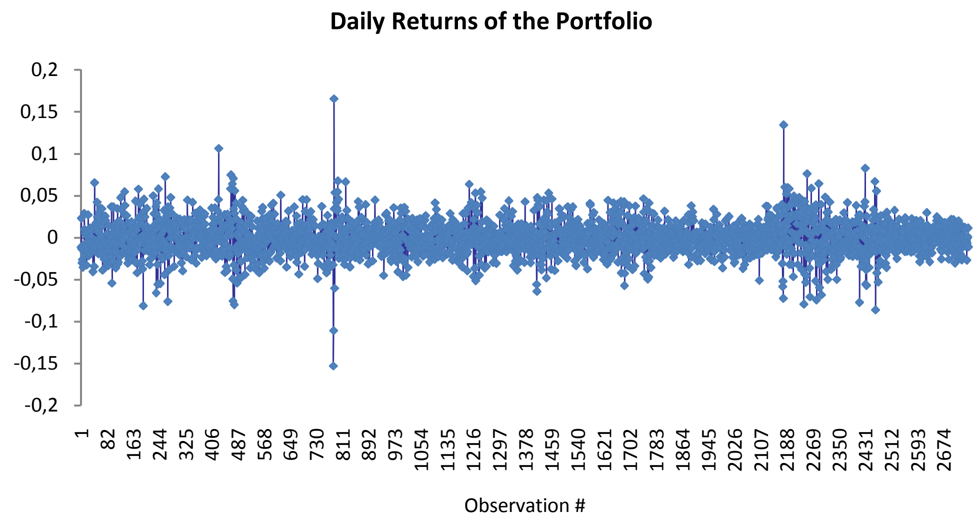

Figure 27 - Daily Returns on the Portfolio

Portfolio was constructed according to the modern portfolio theory presented in Chapter 2. Average daily returns, daily and annual volatilities for the underlying assets and the portfolio are depicted in Table 5. For calculation of annual volatility according to the formula [VaR kapitola], I supposed 252 trading days a year.

	Gasoline	WTI Crude Oil	Natural Gas	Portfolio
Average daily return	-0,048%	-0,046%	-0,027%	-0,041%
Daily volatility	3,445%	2,571%	4,882%	2,057%
Annual volatility	54,695%	40,819%	77,496%	32,649%

Table 5 - Characteristics of the portfolio and the time series

Following Table 6 depicts descriptive statistics of the underlying data. All values were calculated in @Risk[31] modelling tool by estimating the distribution of the inputs. Table 6 proves the assumption that the mean value μ of the expected return gets close or is almost equal to zero.

	Gasoline	WTI Crude Oil	Natural Gas	Portfolio
Minimum	-0,3717	-0,1641	-57,670%	-0,1528
Maximum	0,3868	0,1654	0,5682	0,1656
Mean	-0,000476	-0,000463	-0,00027	-0,00042
Mode	0,0000 [est]	-0,00732 [est]	0,0000 [est]	-0,00918 [est]
Median	-0,000519	-0,00131	0	-0,00127
Std. Deviation	0,0345	0,0257	0,0488	0,0206
Skewness	-0,1612	0,2064	-0,5144	0,2178
Kurtosis	17,9051	7,0228	22,9042	7,5977
Number of observations	2752	2752	2752	2752

Table 6 - Descriptive statistics

For further calculations of VaR we assume for the time series of daily returns to be mutually independent. The correlation coefficients are given in Table 7. Independence is normally tested with e.g. Pearson's test in programme R. For simplification, we do not prove the independence.
The correlation structure (given in Table 7) shows slightly bigger correlation between gasoline and the crude oil. This is understandable as gasoline is a derivative of crude oil.

	Gasoline	WTI Crude Oil	Natural Gas
Gasoline	1	0,053353687	0,05238297
WTI Crude Oil	0,05335369	1	0,0038845
Natural Gas	0,05238297	0,003884503	1

Table 7 - Correlation coefficients

31 http://www.palisade.com/risk/

Distribution and Normality

Calculated values in Table 6 indicate deviations of the portfolio from the normal distribution (Figure 28). Coefficients of skewness and kurtosis show that the portfolio is not normally distributed.

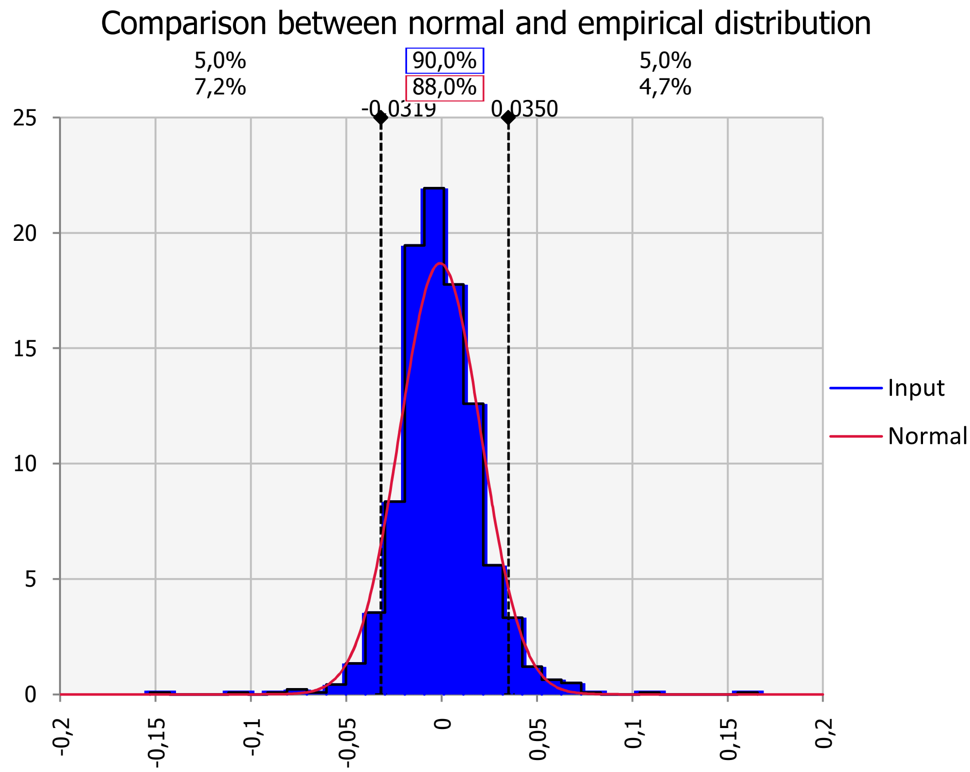

Figure 28 - Empirical and normal distribution (PDF)

There are several normality tests that can be used, e.g. Jarque-Bera test, Anderson-Darling test, Kolmogorov-Smirnov test, Shapiro-Wilk or Pearson's Chi-square test. For the purpose of this paper, I carried out three tests of the normality in the modelling tool @Risk using the functionality Distribution Fitting. The daily returns on the portfolio were tested in Chi-square test, Kolmogorov-Smirnov test and Anderson-Darling test. Results are showed in Table 8. The @Risk software selected the LogLogistic distribution as the best-fit. All the mentioned tests its strengths and weaknesses (e.g. K-S statistic does not detect tail discrepancies very well, A-D statistic highlights differences between the tails of the fitted distribution and input data etc.) but all of them unanimously selected LogLogistic distribution (Figure 29).

	Chi-square	K-S	A-D
LogLogistic	28,9462	0,0219	0,9776
Logistic	41,1773	0,025	1,5378
Lognorm	63,6730	0,0482	6,8091
Normal	65,8794	0,0480	7,3154
InvGauss	66,5988	0,0463	7,041

Table 8 - Fitting Distributions tests (@Risk)

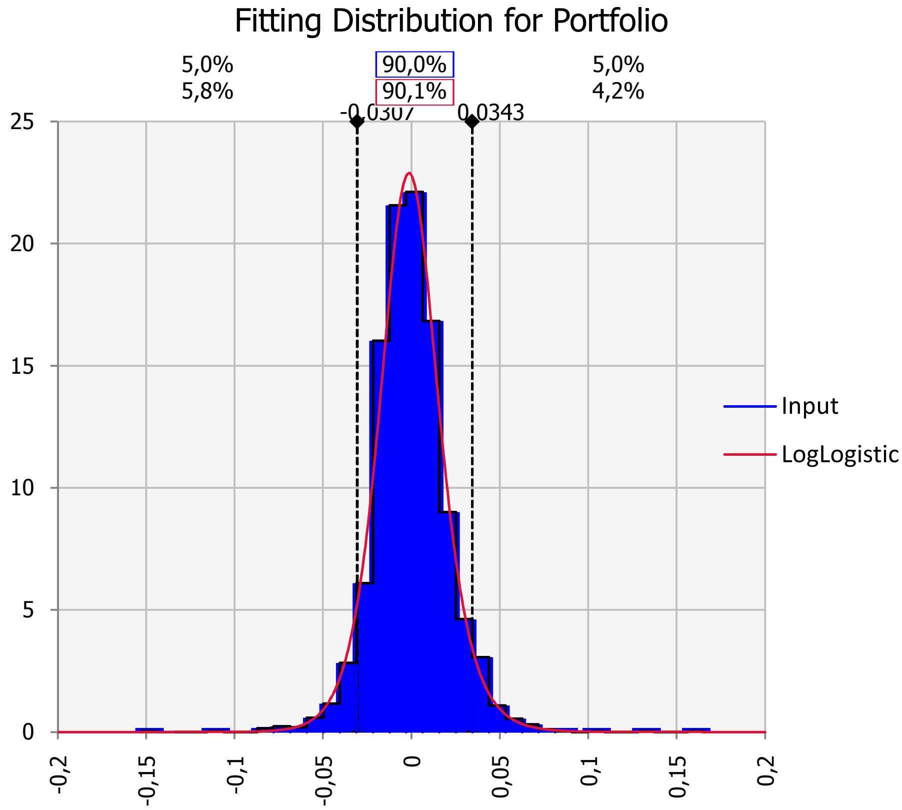

Figure 29 - Best-fit distribution for Portfolio (@Risk)

Historical Value at Risk

For the calculation of historical VaR I followed all steps described in Chapter 3. According to the selected confidence level, the cut-off points were found from sorted time series. VaR for each portfolio component can be found in Table 9.

	Gasoline		WTI crude oil		Natural Gas	
Conf. level	*Cut-off p.*	*VaR*	*Cut-off p.*	*VaR*	*Cut-off p.*	*VaR*
90,00%	-3,774%	357 590	-2,932%	809 903	-4,652%	600 108
91,00%	-3,983%	377 433	-3,076%	849 727	-5,021%	647 747
92,00%	-4,114%	389 820	-3,184%	879 718	-5,362%	691 712
93,00%	-4,463%	422 890	-3,393%	937 356	-5,814%	750 013
94,00%	-4,743%	449 373	-3,637%	1 004 656	-6,311%	814 150
95,00%	-5,075%	480 841	-3,824%	1 056 345	-7,048%	909 219
96,00%	-5,466%	517 938	-4,212%	1 163 596	-7,611%	981 773
97,00%	-5,935%	562 381	-4,563%	1 260 423	-8,649%	1 115 747
98,00%	-6,685%	633 389	-5,227%	1 444 016	-10,481%	1 352 022
99,00%	-7,910%	749 465	-6,300%	1 740 498	-13,623%	1 757 373

Table 9 – Value at Risk of underlying portfolio assets

VaR of the entire portfolio is given by the sorted time series of its returns. Here we could distinguish between NvaR and DvaR. Whereas NvaR is given by plain sum of VaR values of its underlying assets, DvaR is a Value at Risk measure that takes portfolio diversification into account. Value of the diversification affect is thus given simply by differential between the two measures.

	Portfolio			
Conf. level	*Cut-off p.*	*DvaR*	*NvaR*	*Div. Effect*
90,00%	-2,247%	1 123 388	1 767 601	644 213
91,00%	-2,362%	1 180 978	1 874 907	693 929
92,00%	-2,525%	1 262 353	1 961 250	698 897
93,00%	-2,670%	1 335 224	2 110 259	775 034
94,00%	-2,871%	1 435 291	2 268 179	832 889
95,00%	-3,073%	1 536 634	2 446 405	909 771
96,00%	-3,343%	1 671 659	2 663 307	991 649
97,00%	-3,647%	1 823 293	2 938 551	1 115 259
98,00%	-4,254%	2 126 856	3 429 426	1 302 570
99,00%	-5,301%	2 650 302	4 247 336	1 597 034

Table 10 - Value at Risk of the Portfolio

Analytical Value at Risk (Variance-Covariance)

Relative 1-day Value at Risk, defined by (3.26), takes daily volatilities into account. With daily volatilities for gasoline, crude oil and natural gas captured in Table 5, the results for single portfolio commodities are in Table 11.

		Gasoline		WTI Crude Oil		Natural Gas	
Conf. level	*St. normal*	*Cut-off p.*	*VaR*	*Cut-off p.*	*VaR*	*Cut-off p.*	*VaR*
90,00%	-1,282	-4,416%	418 370	-3,295%	910 330	-6,256%	807 053
91,00%	-1,341	-4,619%	437 697	-3,448%	952 384	-6,545%	844 336
92,00%	-1,405	-4,841%	458 694	-3,613%	998 070	-6,859%	884 839
93,00%	-1,476	-5,085%	481 781	-3,795%	1 048 305	-7,204%	929 375
94,00%	-1,555	-5,357%	507 565	-3,998%	1 104 409	-7,590%	979 114
95,00%	-1,645	-5,667%	536 972	-4,229%	1 168 396	-8,030%	1 035 841
96,00%	-1,751	-6,032%	571 522	-4,502%	1 243 572	-8,546%	1 102 489
97,00%	-1,881	-6,480%	613 996	-4,836%	1 335 992	-9,182%	1 184 424
98,00%	-2,054	-7,076%	670 459	-5,281%	1 458 848	-10,026%	1 293 342
99,00%	-2,326	-8,015%	759 450	-5,982%	1 652 485	-11,357%	1 465 010

Table 11 - Analytical VaR of the portfolio components

Monte Carlo Value at Risk

In the Monte Carlo simulation I assumed that the random returns followed the continuous geometric Brownian motion (3.75). Considering the 252 trading a year on average, the daily returns following the Brownian motion are given by

$$\Delta S_i = S_{t-1}\left(\mu.0{,}004 + \sigma_i \varepsilon \sqrt{0{,}004}\right). \quad (5.1)$$

To reflect the mutual correlations among the assets, I used the Cholesky factorization and decomposed the correlation matrix into a lower triangular matrix as follows

	Gasoline	WTI Crude Oil	Natural Gas
Gasoline	100,00%	0,00%	0,00%
WTI Crude Oil	5,34%	99,86%	0,00%
Natural Gas	5,24%	0,11%	99,86%

Table 12 - Cholesky factorization of the Correlation matrix

The Monte Carlo simulation was carried out using the @Risk modelling tool. The simulation itself comprises of generated 5000 random values for each of the portfolio component. These values were afterwards correlation-adjusted by multiplying with the lower triangular Cholesky matrix. The final simulated daily returns of the entire portfolio are depicted in the Figure 30.

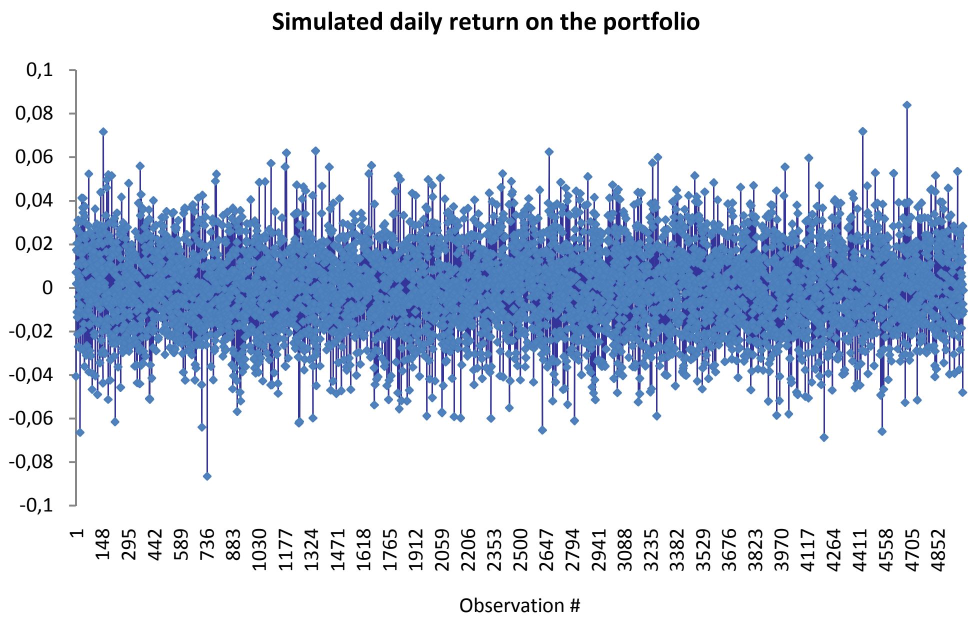

Figure 30 - Monte Carlo simulation of daily returns on the portfolio

From the sorted series of 5000 simulated daily portfolio returns, results of the portfolio Value at Risk on respective confidence are showed in Table 13.

Conf. level	*Cut-off p.*	*VaR*
90,00%	-2,629%	1314612
91,00%	-2,761%	1380318
92,00%	-2,853%	1426560
93,00%	-2,991%	1495285
94,00%	-3,154%	1577007
95,00%	-3,329%	1664385
96,00%	-3,593%	1796489
97,00%	-3,881%	1940691
95,00%	-3,329%	1664385
98,00%	-4,246%	2123018
99,00%	-4,833%	2416377

Table 13 - Simulated Value at Risk

Value at Risk modifications

Both considered variations of the Value at Risk measure proceeds from the analytical VaR. CVaR should inform us about the maximal expected loss occurring with respective probability at the left tail of the distribution. Firstly I adjust the quantile of the distribution according to the formula

$$\mathrm{a}_{\mathrm{c}}^{\mathrm{CVaR}} = \frac{PDF_{SN}(\mathrm{a}_{\mathrm{c}})}{CDF_{SN}(-1 \times \mathrm{a}_{\mathrm{c}})}. \quad (5.2)$$

Then the relative CVaR for t=1 is given by

$$CVaR_{1,\alpha}^{rel} = \mathrm{a}_{\mathrm{c}}^{\mathrm{CVaR}} \times P_0 \times \sigma_P. \quad (5.3)$$

where P_0 is the initial value of the portfolio and σ_P is the standard deviation. Table 14 shows the adjusted quantiles.

	90,00%	*95,00%*	*99,00%*
a_{c}	-1,28155	-1,64485	-2,32635
$\mathrm{a}_{\mathrm{c}}^{\mathrm{CVaR}}$	-1,75498	-2,06271	-2,66521

Table 14 - Quantiles adjusted for CVaR

Now I turn to Cornish-Fisher approximation to estimate portfolio's modified Value at Risk. I can straightforwardly use the formulas (4.28) to adjust the quantile a_{c}. The differences are presented in Table 15.

	90,00%	*95,00%*	*99,00%*
a_c	-1,28155	-1,64485	-2,32635
a_c^{CF}	-1,43088	-1,7405	-2,37876

Table 15 - Normal and modified quantiles used in mVaR

Based on the adjusted quantiles a_c^{CF} applied on the variance-covariance approach to Value at Risk, we arrive with values of mVAR and CVaR depicted in Table 16.

Confidence level	*90,00%*	*95,00%*	*99,00%*
CVaR	1 804 723	2 121 175	2 740 752
mVaR	1 471 434	1 789 871	2 446 183

Table 16 - mVaR and CVaR of the portfolio

Discussion

The achieved results of historical and analytical VaR are subject to the simplifying assumptions of constant daily (average) volatilities. In addition, I did not consider any foreign exchange risk which is usually present in trading activities.
All the results are based on analysis of 10-year time series of daily spot prices. We usually assume that with sufficient number of historical daily prices, distribution of returns will exhibit similar patterns as it did recently. In fact, commodity markets exhibit continuous development. Hence, any unexpected adverse or favourable market event would surely influence the results of VaR. In such cases, applicability of the historical time series is dubious. Risk managers usually try to avoid these drawbacks by stress-testing or sensitivity analyses.
The final outcomes are summarized in Table 17 and inform us about the value of the worst 1-day loss we could expect with 90 %, 95 % respectively 99 % probability.

Confidence level	*90,00%*	*95,00%*	*99,00%*
Historical VaR	1 123 388	1 536 634	2 650 302
Var-Cov VaR	1 317 874	1 691 472	2 392 282
Monte Carlo VaR	1 314 612	1 664 385	2 416 377
CVaR	1 804 723	2 121 175	2 740 752
mVaR	1 471 434	1 789 871	2 446 183

Table 17 - Comparison of 1-day VaR measures evaluated by alternative approaches

From the results of the hypothetical portfolio, which evince larger differences in dependence on the used method, we conclude that the parametric methods give lower risk estimates than the non-parametric historical simulation or Monte Carlo simulation. The demonstrated mVaR seem to be a suitable supplement to the parametric VaR as it accounts non-normality and adjust the earnings both ends of the distribution.
By definition CVaR focuses on the worst losses below the quantile. The numbers shows that VaR measures underestimate expected losses for higher quantiles. We can observe that the ratio $\frac{CVaR}{VaR}$ (depicted in Table 18) approaches to 1 with decreasing a_c.

Confidence level	*90,00%*	*95,00%*	*99,00%*
CVaR / Var-Cov VaR	1,36942076	1,25404	1,145665

Table 18 - CVaR/VaR ratio

Hence, it is tempting to say that for purposes of the hypothetical portfolio the most universal method is CVaR. However, all the results should be stress-tested or back-tested first.

Chapter 6

Conclusions

Purpose of this thesis was to give a basic overview and information about the field of energy trading and commodities and summarize methods and approaches used in Risk Management departments of modern energy trading companies. Energy companies mostly use Value at Risk (VaR) and its related measures, namely Liquidity adjusted VaR, Conditional VaR and modified VaR. Albeit contemporary literature might provide curious readers with other issues and similar at-Risk based measures in contemporary literature, these presented in this thesis are the most common.

In Chapter 1 energy commodity markets were described with stress on specifics that differentiate them from other financial markets. Each commodity has unique chemical structure that determines how it is stored or how traders handle with it in energy markets. Energy commodity markets contend with higher volatility and seasonality and these market specifics influence also risk management at energy trading companies which is mostly dominated by issues related to market and credit (counterparty) risk.

Principles of risk management with focus on energy trading are described in Chapter 2. Energy companies also plentifully use hedging and maintain large portfolios comprising various market positions. On that account, fundamentals of hedging and modern portfolio management are covered at the end of Chapter 2.

Contemporary literature for Value at Risk is abundant, covering almost all aspects of this risk measure. Issues related to methodology of VaR are plentifully examined by various authors who always try to develop and improve current approaches. Aim of this thesis was to present three basic methods used for VaR calculation, namely analytical (parametric) approach, historical simulation and Monte Carlo simulation. All the methods were also applied on hypothetical portfolio consisting of three commodities – natural gas, crude oil and gasoline. Historical prices were observed back from January 2000 to January 2010.

Historical simulation is sometimes considered as a simple method nevertheless users can face several difficulties with accuracy and demanding data analysis. This method is not praised mostly for its disadvantages connected with historical price extremes. In the empirical part the historical simulation showed the lowest level of the VaR estimations. Therefore I would not recommend this method as a stand-alone indicator of the worst possible losses. On the other hand, historical simulation might serve as a beneficial tool completing some missing pictures of market risk.

Analytical and Monte Carlo Value at Risk exhibited more accurate and similar values. Monte Carlo simulation comprised 5000 scenarios following pattern of the geometric Brownian motion. Monte Carlo simulation seems to me as the most efficient and sophisticated method because it does not require too many approximations and assumptions (e.g. normality of distribution). Of course, success of this approach depends mostly on correctness of the simulation model and number of iterations performed, but in contrary to analytical and historical VaR, this method is not so biased to the underlying historical prices.

Theory of VaR presented in Chapter 3 discovers also obvious deficiencies in underestimation of losses in fat tails or subadditivity problem. Therefore the Chapter 3 is followed by a section providing readers with modifications of the standard VaR, namely VaR modified by Cornish-Fisher expansion, Expected Shortfall (CVaR) or Liquidity adjusted VaR. Expected Shortfall is often recommended as one of the most suif VaR-based concept as it is coherent risk measure which eliminates the drawback of VaR related to subadditivity.

Chapter 4 covers modification of VaR which correct the analysed drawbacks of VaR. Liquidity adjusted VaR is very useful risk measurement incorporating exogenous market volatility. The concept requires reliable data to be correctly performed. Due to lack of free market data I did not include this approach in the Empirical part. Most of data are not freely provided by energy exchanges and any estimations or approximations would not be worthy.
Modified VaR and CVaR also adjust quantiles in order to reflect expected losses more accurately. The Empirical part proves that analytical VaR generally underestimates expected losses for higher quantiles and provides readers with results of CVaR and mVAR which give more realistic values.

I am fully aware that this thesis does not represent any comprehensive source of information neither about energy commodities, risk management issues nor Value at Risk matters. This would not be possible; it was not even my intent. Despite my simplifications, assumptions and approximations that I had to accept in the Empirical part, I would be glad if this paper could help anyone and serve as a source of fundamental information about Value at Risk models for Energy Risk Management.

Apendix A

A.1 Historical Prices of underlying assets

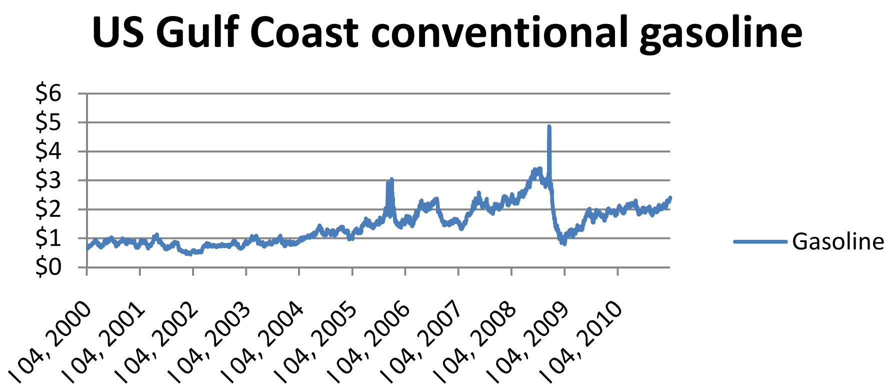

Figure 31 - Historical prices of US gasoline [own calculations]

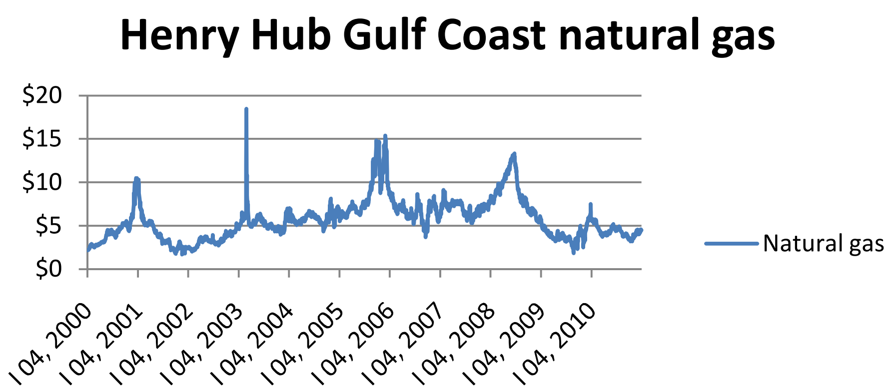

Figure 32 - Historical prices of Henry Hub natural gas

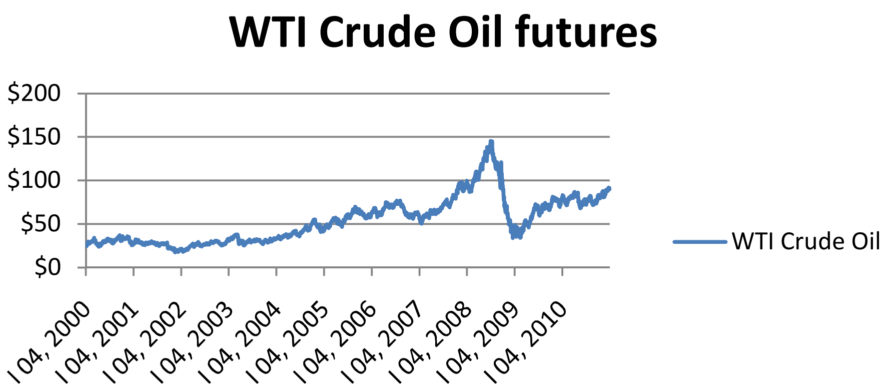

Figure 33 - Historical prices of WTI crude oil [own calculations]

Apendix B

B.1 Daily returns of underlying assets

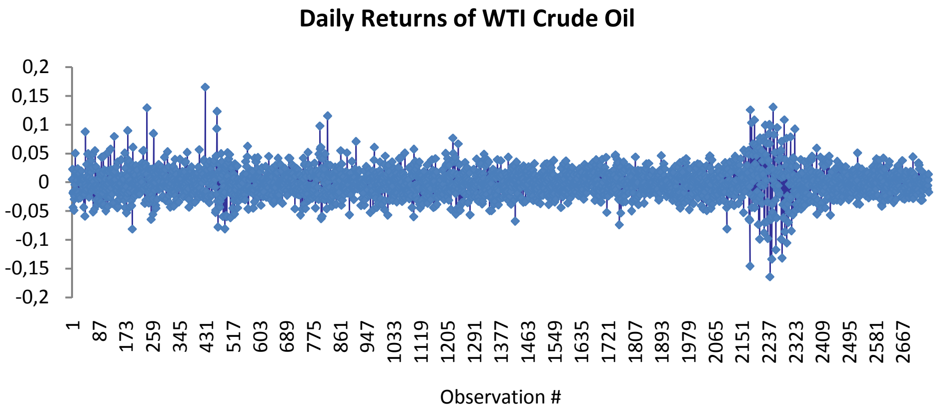

Figure 34 - Daily returns of WTI crude oil [own calculations]

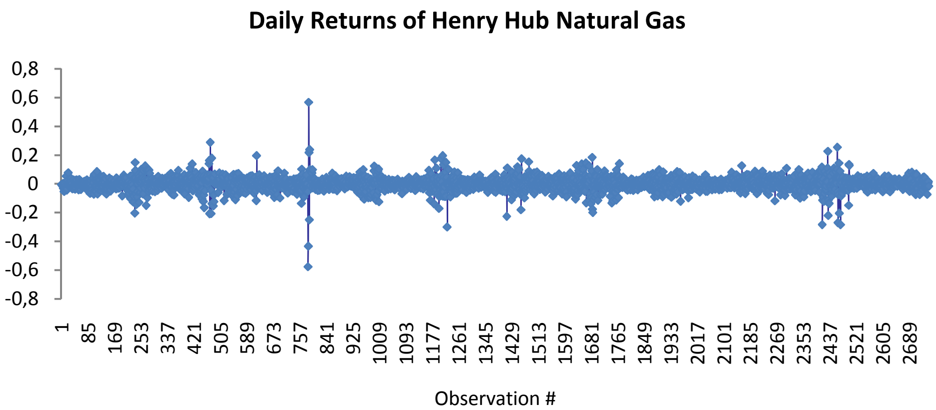

Figure 35 - Daily returns of Henry Hub natural gas [own calculations]

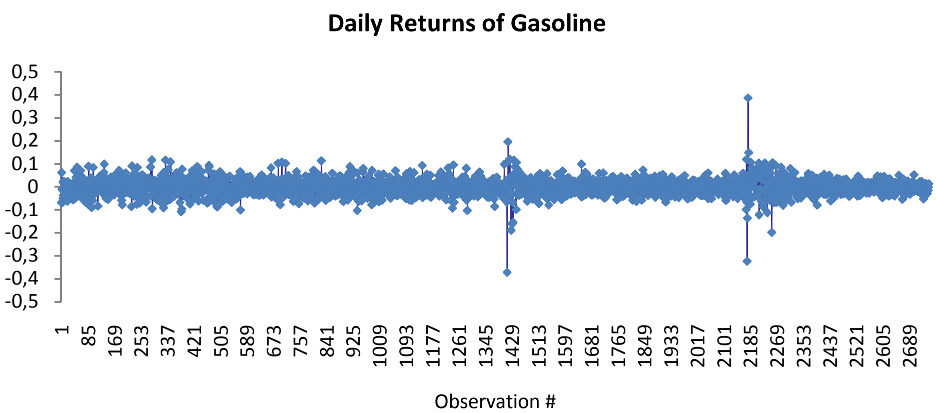

Figure 36 - Daily returns of US gasoline [own calculations]

Apendix C

C.1 Fitted distributions

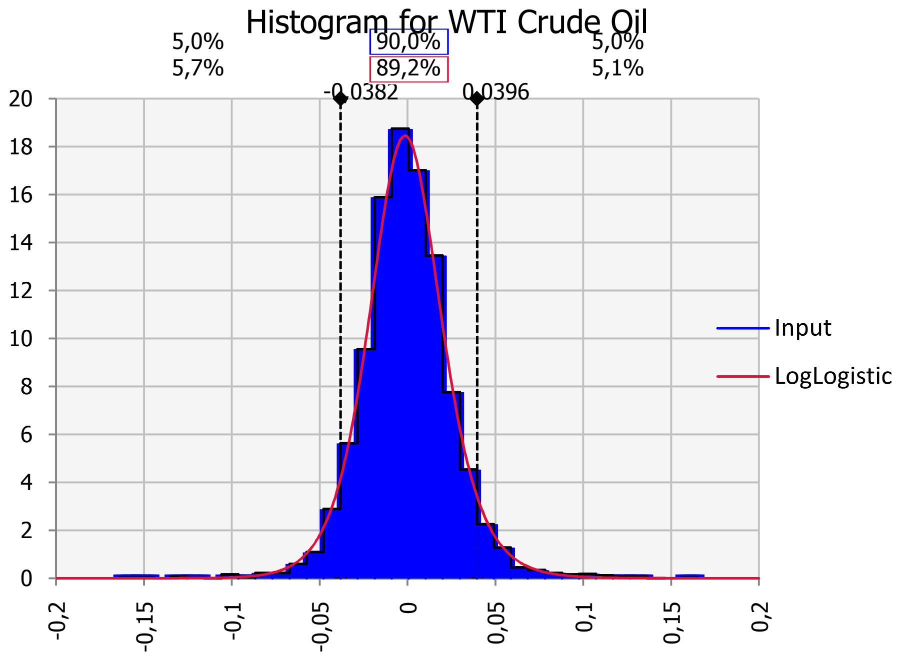

Figure 37 - Fitted Distribution for WTI crude oil

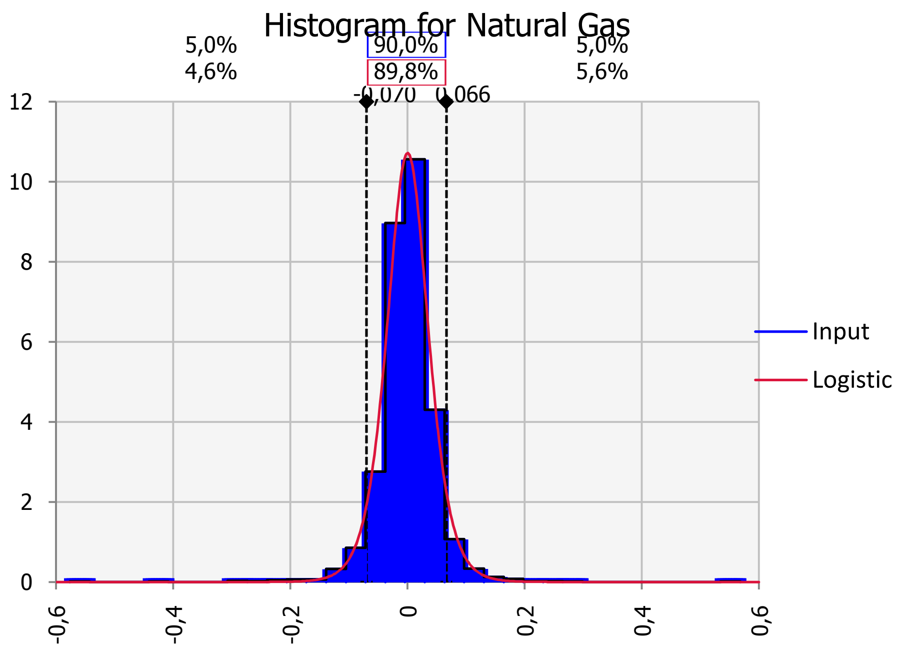

Figure 38 - Fitted Distribution for natural gas

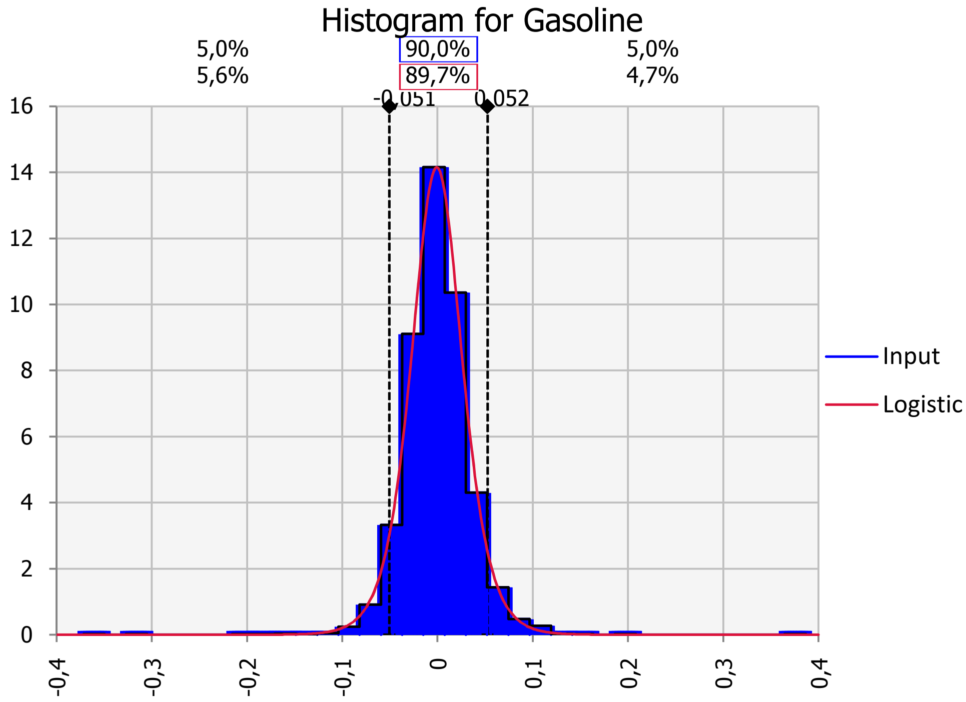

Figure 39 - Fitted Distribution for gasoline

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