UNIVERSITY OF ECONOMICS, PRAGUE

Bayesian Estimation of DSGE Models

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Declaration of Authorship

I, Milan Bouda, declare that this thesis titled, 'Bayesian Estimation of DSGE Models' and the work presented in it are my own. Literature and supporting materials are mentioned in the attached bibliography.

Prague, 31 July, 2014

Signature

"It is naive to try to predict the effects of a change in economic policy entirely on the basis of relationships observed in historical data, especially highly aggregated historical data."

Robert Lucas

"Essentially, all models are wrong, but some are useful."

George E. P. Box

"When the AIDS crisis hit, we did not turn over medical research to acupuncturists."

V.V. Chari

Abstract

Thesis is dedicated to Bayesian Estimation of DSGE Models. Firstly, the history of DSGE modeling is outlined as well as development of this macroeconometric field in the Czech Republic and in the rest of the world. Secondly, the comprehensive DSGE framework is described in detail. It means that everyone is able to specify or estimate arbitrary DSGE model according to this framework. Thesis contains two empirical studies. The first study describes derivation of the New Keynesian DSGE Model and its estimation using Bayesian techniques. This model is estimated with three different Taylor rules and the best performing Taylor rule is identified using the technique called Bayesian comparison. The second study deals with development of the Small Open Economy Model with housing sector. This model is based on previous study which specifies this model as a closed economy model. I extended this model by open economy features and government sector. Czech Republic is generally considered as a small open economy and these extensions make this model more applicable to this economy. Model contains two types of households. The first type of consumers is able to access the capital markets and they can smooth consumption across time by buying or selling financial assets. These households follow the permanent income hypothesis (PIH). The other type of household uses rule of thumb (ROT) consumption, spending all their income to consumption. Other agents in this economy are specified in standard way. Outcomes of this study are mainly focused on behavior of house prices. More precisely, it means that all main outputs as Bayesian impulse response functions, Bayesian prediction and shock decomposition are focused mainly on this variable. At the end of this study one macro-prudential experiment is performed. This experiment comes up with answer on the following question: is the higher/lower Loan to Value (LTV) ratio better for the Czech Republic? This experiment is very conclusive and shows that level of LTV does not affect GDP. On the other hand, house prices are very sensitive to this LTV ratio. The recommendation for the Czech National Bank could be summarized as follows. In order to keep house prices less volatile implement rather lower LTV ratio than higher.

Keywords: DSGE, Bayesian Estimation, New Keynesian, Alternative Monetary Policy, Small Open Economy, Housing.

JEL Classification: E12, E17, E62.

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Chapter 1

Introduction

1.1 Motivation

During the 1960s and 1970s, large-scale macroeconometric models in the tradition of the "Cowles Commission" were the main tool available for applied macroeconomic analysis[1]. These models were composed by dozens or even hundreds of equations linking variables of interest to explanatory factors, such as economic policy variables, and while the choice of which variables to include in each equation was guided by economic theory, the coefficients assigned to each variable were mostly determined on purely empirical grounds, based on historical data.

In the late 1970s and early 1980s, these models came under sharp criticism. On the empirical side, they were confronted with the appearance of stagflation, i.e. the combination of high unemployment with high inflation, which was incompatible with the traditional Phillips curve included in these models, according to which unemployment and inflation were negatively correlated. It was necessary to accept that this was not a stable relation, something that traditional macroeconometric models were poorly equipped to deal with. Strong criticism on the empirical side came from Sims (1980) who questioned the usual practice of making some variables exogenous, i.e. determined "outside" the model. This was an ad-hoc assumption, which excluded meaningful feedback mechanisms between the variables included in the models. But the main critique was on the theoretical side and came from Lucas (1976) when he developed an argument that became known as the "Lucas critique" which is described in subsection 1.1.1.

[1] The "Cowles Commission for Research in Economics" is an economic research institute founded by the businessman and economist Alfred Cowles in 1932. As its motto "science is measurement" indicates, the Cowles Commission was dedicated to the pursuit of linking economic theory to mathematics and statistics, leading to an intense study and development of econometrics. Their work in this field became famous for its concentration on the estimation of large simultaneous equations models, founded in a priori economic theory.

The Dynamic Stochastic General Equilibrium (DSGE) methodology attempts to explain aggregate economic phenomena, such as economic growth, business cycles, and the effects of monetary and fiscal policy, on the basis of macroeconomic models derived from microeconomic principles. One of the main reasons macro-economists seek to build micro founded models is that, unlike more traditional macroeconometric forecasting models, micro founded models should not, in principle, be vulnerable to the Lucas critique. DSGE models were developed as a reaction to Lucas critique.

The main purpose of this thesis is to present four related topics. First, it is necessary to introduce DSGE models and reasons of their origin. Second, to provide a description of DSGE framework which contains set of econometric methods and algorithms. Compliance with this framework guarantees well estimated DSGE model. Third, to quantify the efficiency of different monetary policy rules in the Czech economy. The New Keynesian DSGE model which is a workhorse in monetary policy analysis is used as a benchmark. Fourth, Small Open Economy (SOE) DSGE model with housing sector is derived and estimated as a reaction to critique of DSGE models. This critique says that well-developed housing sector is very often missing in DSGE models. This SOE model with housing sector and government represents the most valuable output of this thesis.

1.1.1 Lucas critique

The Lucas critique, named for Robert Emerson Lucas, Jr. His work on macroeconomic policymaking, argues that it is naive to try to predict the effects of a change in economic policy entirely on the basis of relationships observed in historical data, especially highly aggregated historical data. For details see Lucas (1976). The basic idea predates Lucas' contribution (related ideas are expressed as Campbell's Law and Goodhart's Law), but in a 1976 paper Lucas drove home the point that this simple notion invalidated policy advice based on conclusions drawn from large-scale macroeconometric models. Because the parameters of those models were not structural, i.e. not policy-invariant, they would necessarily change whenever policy (the rules of the game) was changed. Policy conclusions based on those models would therefore potentially be misleading. This argument called into question the prevailing large-scale econometric models that lacked foundations in dynamic economic theory. Lucas summarized his critique:

"Given that the structure of an econometric model consists of optimal decision rules of economic agents, and that optimal decision rules vary systematically with changes in the structure of series relevant to the decision maker, it follows that any change in policy will systematically alter the structure of econometric models."

The Lucas critique is, in essence, a negative result. It tells economists, primarily, how not to do economic analysis. The Lucas critique suggests that if we want to predict the effect of a policy experiment, we should model the deep parameters (relating to preferences, technology and resource constraints) that are assumed to govern individual behavior; so called micro-foundations. If these models can account for observed empirical regularities, we can then predict what individuals will do, taking into account the change in policy, and then aggregate the individual decisions to calculate the macroeconomic effects of the policy change. Shortly after the publication of Lucas' article, Kydland and Prescott (1977) published the article, where they not only described general structures where short-term benefits that are negated in the future through changes in expectations, but also how time consistency might overcome such instances. This article and subsequent research lead to a positive research program for how to do dynamic, quantitative economics.

1.2 Overview

The first chapter describes the motivation why DSGE models are necessary and useful as a tool for monetary or fiscal policy analysis. Next, two schools of DSGE modeling are described as well as their main differences. Next section is dedicated to DSGE modeling in the Czech Republic. DSGE models of the Czech National Bank and Ministry of Finance of the Czech Republic are described. There are also many studies which come from academic sphere which are described as well. In order to be in line with the most up to date research the DSGE modeling in the world is described. There are many institutions and countries which use DSGE model as a main tool for monetary policy analysis. I decided to describe two most advanced institutions and their DSGE research (the European Central Bank and Federal Reserve System). Moreover, to be consistent with current economic research I provide the most up to date criticism of DSGE models. Finally, description of two useful software packages which are very often used for simulation or estimation of DSGE models is presented.

The second chapter contains two main sections (economic model and Bayesian estimation). Section economic model describes how to derive the first order conditions and the log-linearization of these conditions. Section Bayesian estimation describes theoretical background behind the Bayesian analysis. Next, the estimation procedure is described as well as prior distributions used in DSGE modeling. Furthermore, the computation of the data likelihood is described as well as the approximation of the posterior distribution. There are also a few words about model evaluation and identification. The book summarizes this econometric tools (in the sense of DSGE model estimation) has not yet been written. The main contribution of this chapter lies in the creation of one comprehensive econometric framework which enables to simulate or estimate DSGE models. This framework is created as a synthesis of many scientific papers.

The third chapter deals with the impact of alternative monetary policy rules on the economy of Czech Republic. This chapter shows how to implement existing model and perform experiment with this model. The most favorite New Keynesian DSGE model of all times is taken from Galí (2008). Model is derived and log-linearized equations are described. Model specified by Galí (2008) is used as a benchmark which is later compared to models containing alternative versions of monetary policy rules. DSGE model is fully calibrated to the Czech economy and contains two observed variables. Parameters of the model are estimated using the Bayesian techniques. Bayesian estimation is in accordance with the framework described in the second chapter. Next, this benchmark model is estimated several times but with different version of monetary policy rule. Three monetary policy rules are taken from previous studies. The fourth forward looking monetary policy rule is introduced by me. Bayesian comparison technique is applied and the best performing Taylor rule is identified the forward looking version of the monetary policy rule. The main contribution of this chapter is both didactic and experimental. The most favorite DSGE model of all times is taken from Galí (2008) and precisely derived, calibrated and estimated. Furthermore, the four different versions of monetary policy rules are specified and their impact is quantified using the technique called the Bayesian comparison.

The fourth chapter describes the specification, derivation and estimation of the Small Open Economy Model (SOE) of the Czech Republic with Housing sector. Firstly, all agents which are present in our model are described. DSGE model consists of households, firms, central bank and government. Following the framework used by Aoki, Proudman, and Vlieghe (2002), there are two types of households. The first type of consumers is able to access the capital markets and they can smooth consumption across time by buying or selling financial assets. These households follow the permanent income hypothesis (PIH). The other type of household uses rule of thumb (ROT) consumption, spending all their income on consumption. ROT consumers are effectively completely credit constrained as they do not have any access to the credit markets. This dual differentiation of consumers is based on Campbell and Mankiw (1989). As well as in the previous chapter the data sources and calibration of all parameters are performed. Next, DSGE model is estimated using the framework described in the second chapter. Finally, the macroprudential experiment is performed. This experiment investigates impact of the level of Loan to Value ratio to house prices. This chapter is the main contribution of this thesis. The SOE model of the Czech Republic with special focus on housing is derived from micro foundations. This DSGE model is estimated with several departures[2] from DSGE framework. Finally, the macroprudential experiment is performed. This means that DSGE model contains parameter which drives the level of Loan to Value (LTV) ratio. Central bank may control this LTV ratio using macroprudential regulation. Sensitivity analysis is performed and the best level of LTV ratio is recommended.

1.3 Schools of DSGE modeling

Two competing schools of thought form the bulk of DSGE modeling.

First, the Real Business Cycle (RBC) theory builds on the neoclassical growth model, under the assumption of flexible prices, to study how real shocks to the economy might cause business cycle fluctuations. The paper of Kydland and Prescott (1982) is often considered as the starting point of RBC theory and of DSGE modeling in general. The RBC point of view is surveyed in Cooley (1995).

Second, New Keynesian DSGE models are built on a structure similar to RBC models, but instead assume that prices are set by monopolistically competitive firms, and cannot be instantaneously and costlessly adjusted.

1.3.1 Real Business Cycle (RBC) models

The grounds of RBC theory lie in the papers of Kydland and Prescott (1982) and Prescott (1986). This theory provide main reference framework for the analysis of economic fluctuations and this theory is still the core of macroeconomic theory. There are two impacts of RBC theory: methodological and a conceptual dimension. From the methodological point of view, RBC theory established the use of DSGE models as a central tool for macroeconomic analysis. Behavioral equations describing aggregate variables were thus replaced by first-order conditions of intertemporal problems facing consumers and firms. Assumptions on the formation of expectations gave way to rational expectations. In addition, RBC economists stressed the importance of the quantitative aspects of the modeling, as reflected in the central role given to the calibration, simulation, and evaluation of their models. The most striking dimension of the RBC revolution was, however, conceptual.

- *The efficiency of business cycles.* The bulk of economic fluctuations observed in industrialized countries could be interpreted as an equilibrium outcome resulting

[2]These departures are described in the fourth chapter.

from the economys response to exogenous variations in real forces (most importantly, technology), in an environment characterized by perfect competition and frictionless markets. According to that view, cyclical fluctuations did not necessarily signal an inefficient allocation of resources (in fact, the fluctuations generated by the standard RBC model were fully optimal). That view had an important corollary: Stabilization policies may not be necessary or desirable, and they could even be counterproductive. This was in contrast with the conventional interpretation, tracing back to Keynes (1936), of recessions as periods with an inefficiently low utilization of resources that could be brought to an end by means of economic policies aimed at expanding aggregate demand.

- *The importance of technology shocks as a source of economic fluctuations.* That claim derived from the ability of the basic RBC model to generate "realistic" fluctuations in output and other macroeconomic variables, even when variations in total factor productivity calibrated to match the properties of the Solow residual[3] are assumed to be the only exogenous driving force. Such an interpretation of economic fluctuations was in stark contrast with the traditional view of technological change as a source of long term growth, unrelated to business cycles.

- *The limited role of monetary factors.* Most important, given the subject of the present monograph, RBC theory sought to explain economic fluctuations with no reference to monetary factors, even abstracting from the existence of a monetary sector.

Its strong influence among academic researchers notwithstanding, the RBC approach had a very limited impact (if any) on central banks and other policy institutions. The latter continued to rely on large-scale macroeconometric models despite the challenges to their usefulness for policy evaluation with respect to Lucas (1976) or the largely arbitrary identifying restrictions underlying the estimates of those models, see Sims (1980). The attempts by Cooley and Hansen (1989) and others to introduce a monetary sector in an otherwise conventional RBC model, while sticking to the assumptions of perfect competition and fully flexible prices and wages, were not perceived as yielding a framework that was relevant for policy analysis.

The resulting framework, which is referred to as the classical monetary model, generally predicts neutrality (or near neutrality) of monetary policy with respect to real variables. That finding is at odds with the widely held belief (certainly among central bankers)

[3]The Solow residual is a number describing empirical productivity growth in an economy from year to year and decade to decade. Robert Solow defined rising productivity as rising output with constant capital and labor input. It is a "residual" because it is the part of growth that cannot be explained through capital accumulation or the accumulation of other traditional factors, such as land or labor. The Solow Residual is procyclical and is sometimes called the rate of growth of total factor productivity.

in the power of that policy to influence output and employment developments, at least in the short run. That belief is underpinned by a large body of empirical work, tracing back to the narrative evidence of Friedman and Schwartz (1963), up to the more recent work using time series techniques, as described in Christiano, Eichenbaum, Evans, Taylor, and Woodford (1999). In addition to the empirical challenges mentioned above, the normative implications of classical monetary models have also led many economists to call into question their relevance as a framework for policy evaluation. Thus, those models generally yield as a normative implication the optimality of the Friedman rule a policy that requires central banks to keep the short term nominal rate constant at a zero level even though that policy seems to bear no connection whatsoever with the monetary policies pursued (and viewed as desirable) by the vast majority of central banks. Instead, the latter are characterized by (often large) adjustments of interest rates in response to deviations of inflation and indicators of economic activity from their target levels. The conflict between theoretical predictions and evidence, and between normative implications and policy practice, can be viewed as a symptom that some elements that are important in actual economies may be missing in classical monetary models. Those shortcomings are the main motivation behind the introduction of some Keynesian assumptions, while maintaining the RBC apparatus as an underlying structure.

1.3.2 New Keynesian models (NKM)

Paper that first introduced this framework was Rotemberg and Woodford (1998). Introductory and advanced textbook presentations are given by Galí (2008) and Woodford (2003). Monetary policy implications are surveyed by Clarida, Gali, and Gertler (1999). Despite their different policy implications, there are important similarities between the RBC model and the New Keynesian monetary model. For details see Galí and Gertler (2007).

The New Keynesian monetary model, whether in the canonical form presented below or in its more complex extensions, has at its core some version of the RBC model. This is reflected in the assumption of (i) an infinitely-lived representative household that seeks to maximize the utility from consumption and leisure, subject to an intertemporal budget constraint, and (ii) a large number of firms with access to an identical technology, subject to exogenous random shifts. Though endogenous capital accumulation, a key element of RBC theory, is absent in canonical versions of the New Keynesian model, it is easy to incorporate and is a common feature of medium-scale versions.

Also, as in RBC theory, an equilibrium takes the form of a stochastic process for all the economy's endogenous variables consistent with optimal intertemporal decisions by households and firms, given their objectives and constraints and with the clearing of all markets. The New Keynesian modeling approach, however, combines the DSGE structure characteristic of RBC models with assumptions that depart from those found in classical monetary models. Here is a list of some of the key elements and properties of the resulting models:

- *Monopolistic competition.* The prices of goods and inputs are set by private economic agents in order to maximize their objectives, as opposed to being determined by an anonymous Walrasian auctioneer[4] seeking to clear all (competitive) markets at once.
- *Nominal rigidities.* Firms are subject to some constraints on the frequency with which they can adjust the prices of the goods and services they sell. Alternatively, firms may face some costs of adjusting those prices. The same kind of friction applies to workers in the presence of sticky wages.
- *Short run non-neutrality of monetary policy.* As a consequence of the presence of nominal rigidities, changes in short term nominal interest rates (whether chosen directly by the central bank or induced by changes in the money supply) are not matched by one-for-one changes in expected inflation, thus leading to variations in real interest rates. The latter bring about changes in consumption and investment and, as a result, on output and employment, because firms find it optimal to adjust the quantity of goods supplied to the new level of demand. In the long run, however, all prices and wages adjust, and the economy reverts back to its natural equilibrium.

It is important to note that the three aforementioned ingredients were already central to the New Keynesian literature that emerged in the late 1970s and 1980s, and which developed parallel to RBC theory. The models used in that literature, however, were often static or used reduced form equilibrium conditions that were not derived from explicit dynamic optimization problems facing firms and households. The emphasis of much of that work was instead on providing micro-foundations, based on the presence of small menu costs, for the stickiness of prices and the resulting monetary non-neutralities. For details, see Mankiw (1985), Blanchard and Kiyotaki (1987) and Ball and Romer (1990). Other papers emphasized the persistent effects of monetary policy on output, and the role that staggered contracts played in generating that persistence. For details see Fischer (1977) and Taylor (1980).

[4] A Walrasian auction, introduced by Léon Walras, is a type of simultaneous auction where each agent calculates its demand for the good at every possible price and submits this to an auctioneer. The price is then set so that the total demand across all agents equals the total amount of the good. Thus, a Walrasian auction perfectly matches the supply and the demand.

The novelty of the new generation of monetary models has been to embed those features in a fully specified DSGE framework, thus adopting the formal modeling approach that has been the hallmark of RBC theory. Not surprisingly, important differences with respect to RBC models emerge in the new framework. First, the economys response to shocks is generally inefficient. Second, the non-neutrality of monetary policy resulting from the presence of nominal rigidities makes room for potentially welfare-enhancing interventions by the monetary authority in order to minimize the existing distortions. Furthermore, those models are arguably suited for the analysis and comparison of alternative monetary regimes without being subject to the Lucas critique.

1.4 DSGE modeling in the Czech republic

1.4.1 Academic sphere

Academic sphere is the driving force of progress in the field of DSGE modeling. DSGE models used by central banks and government are mainly inspired by partial results of scientists. Czech Republic has three universities which are focused, inter alia, on the development of DSGE models. The largest amount of theses and scientific papers comes from the Masaryk University in Brno followed by Charles University in Prague. University of Economics in Prague also started to publish doctoral theses and scientific papers in this economic field.

Masaryk University in Brno
Many papers about DSGE models come from this university. One may find e.g. Vašíček, Tonner, and Ryšánek (2011) or Vašíček, Tonner, and Polanský (2011). Many doctoral and master students are focused on DSGE topics. One can find e.g. master thesis of Herber (2009). This thesis deals with estimating potential output and output gap of the Czech economy. Next, the thesis introduces and estimates a simple DSGE model, where the potential output is defined as the output of the economy in case of non-existence of nominal rigidities.

The doctoral thesis written by Hloušek (2010) is focused on examination of importance of nominal rigidities - especially then wages and prices in the Czech economy. As a tool is used model of open economy with nominal rigidities. The conclusion is that nominal rigidities are such important feature of model. The model specification without nominal rigidities has the worst fit to data. Next conclusion, that wages are more rigid than prices, comes from two results. Firstly, estimated Calvo parameters show that wage contracts last much longer than price contracts. Secondly, the assumption of flexible nominal wages does not correspond to data well while model with flexible domestic prices has the best fit to data.

Next, the interesting thesis was published by Němec (2010). His thesis deals with on the verification of existence of hysteresis hypothesis in the Czech economy since the second half of the 1990s. Němec (2010) found positive and negative hysteresis in the Czech unemployment as a result of both tight and expansive economic policy. Rejecting the hysteresis hypothesis, the decreasing unemployment not accompanied by accelerating inflation is probably the result of structural changes on the labor market. Macroeconomic hysteresis effects are linked with economic growth in an interesting way. Comparing his empirical results for the Czech and New Zealand economies, one can conclude that hysteresis phenomenon is accompanied by a weak relationship between unemployment dynamic and economic growth dynamic. The empirical validity of Okun's law[5] is thus questioned in the case of the Czech Republic.

Doctoral student Musil (2009) tries to explore dynamic behavior of the Czech economy and monetary policy implications with the use of a NK DSGE model with a different approach to the foreign sector. Three types of micro-economically founded models with rigidities are introduced. These are the model with endogenous foreign sector, with exogenous foreign economy, and without foreign sector. The baseline model is the adjusted twocountry model and then it is reduced into a form of a small open economy and a closed economy model by additional assumptions. According to a parameter analysis it is evident that some parts of the adjusted two-country and small open economy model are similar. Some differences in estimated parameters can be identified for the closed economy model. Although the changes in the estimated parameters in various model structures are small, the effect on behavior can be important.

Tonner (2011) aimed his thesis on investigation the possible time-varying structure of DSGE models currently used for monetary policy analysis and forecasting. In order to replicate the observed data, models are often equipped with additional exogenous processes (technologies). These sector technologies are aimed to capture some sector specific (and often time-varying) part of an economy's behavior. Tonner (2011) extends a relatively rich small open economy model with a set of technologies which are tailored directly to the Czech data. Author find that the movement of technologies is a reflection of variability of structural parameters and thus technologies' incorporation enables to keep structural parameters relatively stable in time. Hence, such models can be regularly

[5] Okun's law is an empirically observed relationship relating unemployment to losses in a country's production. The "gap version" states that for every 1% increase in the unemployment rate, a country's GDP will be roughly an additional 2% lower than its potential GDP. The "difference version" describes the relationship between quarterly changes in unemployment and quarterly changes in real GDP. The stability and usefulness of the law has been disputed.

used for policy analysis and forecasting without having to work explicitly with time-varying structural parameters and nonlinear filtering.

Finally, Čapek (2012) examined whether there were any structural changes in the Czech economy approximated by a DSGE model estimated with Bayesian methods. The thesis identified several structural changes. One major structural change happened in the context of incorporation of inflation targeting regime by the Czech National Bank, the other major was a result of contemporary recession.

Charles University in Prague

Many researchers working for Charles University are also very tight connected with the Institute of Information Theory and Automation (UTIA), e.g. PhDr. Jaromír Baxa, Ph.D. has published many papers oriented on the New Keynesian theory. However, the number of theses and scientific papers published by Charles University is significantly lower than in case of Masaryk University in Brno. Master thesis written by Dudík (2009) focuses on the development of a smaller-scale non-linear DSGE model with typical New Keynesian features, which is subsequently applied for modeling the Czech economy business cycle. To this end, the model is estimated using maximum likelihood Bayesian method with the Kalman filter and the Metropolis-Hastings algorithm. Special care is paid to the issues of derivation and approximation of the model, in order to retain its non-linear nature. Although some of the properties of the estimated model are not fully satisfactory, the estimated model can be considered an useful approximation of the Czech economic reality.

Next, thesis written by Adam (2011) investigates the effects of government spending on aggregate economic variables in the Czech Republic. The standard RBC and New Keynesian models assume only forward-looking households despite the evidence of a significant fraction of non-optimizing households. These models do not provide reasonable predictions for the response of consumption: both models predict its fall following a government spending shock. Therefore, a variant of the New Keynesian model, where rule-of-thumb households coexist with optimizing households, is used for the analysis. We have found that fiscal policy has a positive impact on output, although government spending multiplier does not exceed one. Also, the impact on consumption is positive for several periods following a fiscal spending shock, which is consistent with the evidence.

Moreover, master thesis Zelený (2012) deals with the topic of fiscal policy. This policy has been long neglected in terms of fiscal policy's interdependence with other main macroeconomic variables. Author analyses the validity of different fiscal policy models for the case of Czech Republic. Different fiscal policy rules are put into otherwise identical benchmark and the models are compared. Zelený (2012) finds that the most plausible fiscal policy rule is of pro-cyclical type. The model assumes that interest groups can steal part of government income through corruption and voters cannot observe it, so they demand maximum fiscal spending in the good times. The logic of this model is in accordance with the current state of fiscal and economic behavior in Czech Republic.

Finally, master thesis Paulus (2012) is aimed to create a RBC model incorporating corrupting sector. The thesis contributes to the few existing DSGE models with corruption by introducing the corrupting sector into the sector of firms and political parties which is regarded as a sector of public procurements where firms bribe politicians for gaining public tenders. This setting is new and is supposed to catch better the phenomenon of political corruption. The model predicts that all shocks that positively affect the economy motivate firms to invest more into the bribes and vice versa. The increase of the overall level of corruption stimulates economy but is leading an economy to the instability. The model also examines the effect of various forms of fiscal spending in the households' utility function. The model exhibits several non-intuitive results (too high portion of stolen money by firms, stimulation of the economic performance caused by higher corruption and negative holding of government bonds) that should be solved in next research.

University of Economics in Prague

There are a few theses which come from the University of Economics in Prague. First, Průchová (2012) in her master thesis introduces the reader into the background of DSGE models. Next, Průchová (2012) derives the New Keynesian model of the Czech Republic. This is a small-scale DSGE model for closed economy. Furthermore, if anyone wants to develop new DSGE model then author describes general procedure which is recommended to follow.

The doctoral thesis Štork (2012) is focused on derivation of macro-finance model for analysis of yield curve and its dynamics using macroeconomic factors. Underlying model is based on basic Dynamic Stochastic General Equilibrium DSGE approach that stems from Real Business Cycle theory and New Keynesian Macroeconomics. The model includes four main building blocks: households, firms, government and central bank. Log-linearized solution of the model serves as an input for derivation of yield curve and its main determinants are pricing kernel, price of risk and affine term structure of interest rates which is based on no-arbitrage assumption. The thesis shows a possible way of consistent derivation of structural macro-finance model, with reasonable computational burden that allows for time varying term premium. A simple VAR model, widely used in macro-finance literature, serves as a benchmark. The paper also presents a brief comparison and shows an ability of both models to fit an average yield curve observed from the data. Lastly, the importance of term structure analysis is demonstrated using case of central bank deciding about policy rate and government conducting debt management.

Next, doctoral thesis Čížek (2013) describes general principals of contemporary macroeconometric models as well as their alternatives. Author formulates macroeconomic model of a monetary policy in order to describe fundamental relationships between real and nominal economy. Some parameters of original model are made endogenous. Despite this nonlinearity, author specified model in state space form with time-varying coefficients, which are solved by Kalman filter. There are two unexpected conclusions which are made by author. First, the estimation shows that application of Taylor rule (in order to simulate behavior of Central Bank) is not adequate. Second, the results indicate that interest rate policy of the European Central Bank has only a very limited effect on real economic activity of the European Union.

Furthermore, I published paper Bouda (2013) which deals with the estimation of the New Keynesian Phillips curve (NKPC). First, the history of the Phillips curve and the NKPC is outlined. Next, similar research and papers regarding the NKPC are mentioned. The main goal of the paper is to estimate the parameters of the NKPC using the Bayesian techniques. These techniques are widely used for the DSGE model estimation and this paper contains links to the source foreign literature. The NKPC is estimated as part of a fully calibrated Small Open Economy (SOE) DSGE model. The SOE DSGE model consists of households, firms, the government and the central bank. The estimation is performed on the Czech data and the period is from 2001Q1 to 2012Q2. The first output of the paper is the parameter estimates of the NKPC. The main finding is that the future expected inflation plays a crucial role in setting the level of inflation. Moreover, a shock decomposition of domestic and imported inflation is performed and the main output is that the domestic monetary policy shock causes crucial changes in the level of both domestic and imported inflation.

Moreover, Bisová, Javorská, Vltavská, and Zouhar (2013) present a preliminary study of a multi-sector extension of a canonical RBC model that allows to use data on the input-output structure with multiple sectors. Authors formulate a simple baseline model that allows for an arbitrary number of sectors with an arbitrary input-output structure. The practical obstacle to using the model in practice lies in the need to find approximate steady-state values of the variables. In the general case, finding a solution to the steady-state problem is difficult; however, authors provide an analytic solution to a special symmetric case, which can sometimes provide a close enough approximation for non-symmetric problems as well.

1.4.2 The Czech National Bank

The head of the Department of Macroeconomic Prognosis is RNDr. Tibor Hlédik, MA. Economists who work in the Czech National Bank (CNB) very often publish results of their research in CNB Working paper series as well as in other international and domestic journals. For example see Hlédik (2003), Beneš, Hlédik, Kumhof, and Vávra (2005) and Andrle, Hlédik, Kameník, and Vlček (2009).

Quarterly Projection Model

CNB's modern model history started with the Quarterly Projection Model (QPM) which is described in Warren Coats and Rose (2003). The model has three basic roles. As a research tool, it provides instruments of learning about the functioning of the economy and studying policy options. Within the context of the forecast, the model has two roles. First, it provides an organizational framework for the exercise, and a consistent story about how the short-term conjuncture will evolve into the medium term. A crucial part of this is a description of what needs to be done, conditional on all the other assumptions in the forecast, to respect the inflation target. This is the primary contribution of the model to a baseline scenario. Second, the model plays a more central role in dealing with uncertainty. Given the judgmental baseline forecast, the model can be used to study the implications of the major risks to the forecast.

CNB uses a few minor models which help with projection to QPM model. The detailed description is in Warren Coats and Rose (2003).

Hermin CR

The model Hermin Czech Republic is a four-sector (Manufacturing, Market Services, Agriculture, and Government) supply-side macro model that allows for conventional Keynesian cyclical effects. It is based on neoclassical theory in that the investment and labor decisions of firms in the two main sectors (Manufacturing and Market Services) follow cost minimization of CES production functions. The direct incorporation of income-output-expenditure identities permits both demand and supply side experiments.

Fiscal Policy Stance Model

CNB uses Fiscal Policy Stance Model which provides a basis for assessing fiscal policy stance and evaluating its impact on the economy. It separates the structural and cyclical part of the fiscal balance using information on the business cycle position of the economy and estimated elasticities of fiscal receipts with respect to the aggregate output. At this stage, zero elasticity is assumed for expenditures, mainly because the sorts of expenditures that might exhibit cyclical behavior sum to a relatively small fraction of total amount and there is too little experience to seek estimates.

Trade Balance Models

Two simple quarterly models have been built to catch potential inconsistencies in the foreign trade's discussions regarding the contribution of foreign demand to the expected changes in the domestic current account balance and economic growth. Reduced-form equations are estimated for the current account and net exports (in terms of their ratio to the real output) so that there is no need to investigate direct links between exports and imports.

Model of Optimizing IS Curve and Monetary Policy Effects

Two channels enable real impacts of monetary policy in a sticky-price model of a small open economy: those related to the real interest rate and the real exchange rate. Both are captured in an IS (or aggregate demand) curve.

MMI (NEAR-TERM INFLATION FORECAST)

MMI is a time-series model of the ARMAX class operated on a monthly basis. Its structure incorporates both elementary microeconomic theory of consumer demand and influence of aggregate macroeconomic development. The model focuses on net inflation, that is, inflation excluding administered prices. Two elements are featured to deal with the scarcity of domestic time series observations. First, the estimate of the AR part relies on polynomial distributed lags, which allows us to identify a gradually diminishing impact of the explanatory variables (supported by empirical evidence) with no substantial loss in degrees of freedom. Second, pseudo error-correction terms are included to stabilize the model behavior on reasonable medium or long term paths: the rate of inflation is related to levels of the unemployment rate and the real interest rate. The model has been used for short-term forecasting for about three years and its forecasting accuracy is tested continuously. The structure has not changed and the coefficients do not show severe instability although a gradual change is observed. The model is a part of the formal tools used by the Real Economy Division in compilation of their Near Term Forecast. It has shown relatively robust forecasting properties.

Small Linear Error-Correction Model with Model-Consistent Expectations

The key behavioral relationships include an aggregate demand curve for private demand, a simple fiscal rule for government spending (capturing some counter-cyclical properties of government consumption that are expected to be more pronounced in the future than at present), a UIP condition, a Phillips curve, an approximation for long-term interest rates and an exponential function approximating the CNBs disinflation target. Long interest rates, the exchange rate and inflation are jump variables in the model. The model is closed by an optimal (and time-consistent) monetary policy rule, which is derived by dynamic programming. The code of the model has been written in GAUSS[6]. The behavioral relationships were determined as a combination of estimation (OLS, TSLS) and calibration. The equations are specified in error-correction form. The model provides a tool within which the implications of the use of various monetary policy rules are studied.

Generalized Total Return Parity For The Exchange Rate

A model of the uncovered interest rate parity condition using secondary market yields (total returns) of default free instruments, instead of money market rates is being developed to asses market expectations for both CZK/EUR and EUR/USD exchange rates. This model, based on mainstream stochastic general equilibrium micro-foundations, shows very promising empirical performance.

G3 Model

The new structural model (g3) has been used as the core forecasting tool since July 2008 and replaced former QPM model. The model is a general equilibrium small open economy (SOE) model of the Czech Republic. It is a business cycle model to be used in forecasting and policy analysis, therefore forward-looking expectations are an important part of it. The model is structural and has consistent stock-flow national accounting. Its structure aims to capture the main characteristics of the Czech economy. The economic dynamics in the model result from the interactions of households, firms in individual sectors, the central fiscal and monetary authorities, and their counterparts in the rest of the world. The monetary policy authority in the model operates an inflation targeting regime and both households and firms are aware of the monetary policy regime operated. Hence, there are no credibility or communication uncertainty issues of monetary policy conduct in the model.

From the theoretical point of view the model follows the New-Keynesian tradition, implying important nominal frictions in the economy enriching the real business cycle dynamics. To capture important stylized facts of an emerging economy, the multi-sectoral nature of the model and a focus on permanently viewed economic shocks are important ingredients of the model. Trends in sectoral relative prices, real exchange rate appreciation, a high import intensity of exports, imperfect exchange rate pass-through, investment specific shocks, and an increase in trade openness are examples of the models features. The model relies on many standard modeling choices in the field of applied Dynamic General Equilibrium Models, employing a variety of nominal and real rigidities and frictions. The model is tailor-made for the Czech economy, yet many of its design features should be suitable for other small open emerging economies as well. The

[6]Matrix programming language for mathematics and statistics, developed and marketed by Aptech Systems. Its primary purpose is the solution of numerical problems in statistics, econometrics, time-series, optimization and visualization.

model is fully calibrated. The process of calibrations was very rigorous and lasted approx. two years. Calibration follow the minimal econometric approach recommended for DSGE models by Geweke (1999). For more details about g3 model see Andrle, Hlédik, Kameník, and Vlček (2009).

Research in the Czech National Bank

One of the most recent papers written by Ambriško, Babecký, Ryšánek, and Valenta (2012) describes satellite DSGE model which investigates the transmission of fiscal policy to the real economy in the Czech Republic. Model shares features of the Czech National Bank's current g3 forecasting model developed by Andrle, Hlédik, Kameník, and Vlček (2009), but contains a more comprehensive fiscal sector. Crucial fiscal parameters, related mainly to the specified fiscal rule, are estimated using Bayesian techniques. Authors calculate a set of fiscal multipliers for individual revenue and expenditure items of the government budget. Authors find that the largest real GDP fiscal multipliers in the first year are associated with government investment (0.4) and social security contributions paid by employers (0.3), followed by government consumption (0.2).

Next, paper Brázdik (2013) presents an extension of a small open economy DSGE model allowing the transition toward a monetary policy regime aimed at exchange rate stability to be described. The model is estimated using the Bayesian technique to fit the properties of the Czech economy. In the scenarios assessed, the monetary authority announces and changes its policy so that it is focused solely on stabilizing the nominal exchange rate after a specific transition period is over. Four representative forms of monetary policy are followed to evaluate their properties over the announced transition period. Welfare loss functions assessing macroeconomic stability are defined, allowing the implications of the transition period regime choice for macroeconomic stability to be assessed. As these experiments show, exchange rate stabilization over the transition period does not deliver the lowest welfare loss. Under the assumptions taken, the strict inflation-targeting regime is identified as the best-performing regime for short transition periods. However, it can be concluded that for longer transition periods the monetary policy regime should respond to changes in the exchange rate.

Papers published by CNB's economists can be found in CNB Working paper series which is available at http://www.cnb.cz/en/research/research_publications/cnb_wp.

1.4.3 Ministry of Finance of the Czech Republic

Ministry of Finance (MF) of the Czech Republic has a modeling unit which developed DSGE model called HUBERT, for details see Štork, Závacká, and Vávra (2009). The MF publishes papers mainly about fiscal policy, e.g. Štork and Závacká (2010). The model HUBERT describes the behavior of four basic agents in the economy: households, firms, government, and world. Although HUBERT is rather a simple version of standard DSGE models, it incorporates standard features of New Keynesian economics such as imperfect competition, habit formation of households, nominal and real rigidities. A current version of the model is intended both for policy analysis simulations and regular forecasts at the Ministry of Finance. Preliminary results show that the model produces reasonable outputs.

1.5 DSGE modeling in the World

1.5.1 European Central Bank (ECB)

The European Central Bank (ECB) has developed a DSGE model, often called the Smets-Wouters model, which it uses to analyze the economy of the Eurozone as a whole (in other words, the model does not analyze individual European countries separately). For more details about Smets-Wouters model see Smets and Wouters (2003). The model is intended as an alternative to the Area-Wide Model (AWM), a more traditional empirical forecasting model which the ECB has been using for several years.

The equations in the Smets-Wouters model describe the choices of three types of decision makers: households, who made an optimal trade-off between consumption and worked hours, under a budget constraint; firms, who optimize their labor and capital to employ; and the central bank, which controls monetary policy. The parameters in the equations were estimated using Bayesian statistical techniques so that the model approximately describes the dynamics of GDP, consumption, investment, prices, wages, employment, and interest rates in the Eurozone economy. In order to accurately reproduce the sluggish behavior of some of these variables, the model incorporates several types of frictions that slow down adjustment to shocks, including sticky prices and wages, and adjustment costs in investment.

1.5.2 Federal Reserve System (FED)

Large-scale macroeconometric models have been used for forecasting and quantitative policy and macroeconomic analysis at the Federal Reserve Board for the past 40 years. Model design and development efforts at the Fed have been divided into two complementary research programs. One project, undertaken in the Division of Research and Statistics, focuses on the U.S. economy, and the other, residing in the Division of International Finance, is oriented toward the global economy. For some applications, the macro models maintained by the two divisions are combined to form a single world model.

The first generation FED models MPC and MCM were developed in the 1960's and 1970's and based on the reigning IS/LM/Phillips curve paradigm. During the 1970's and 1980's, the theoretical underpinnings of models of this type were seriously challenged. These criticisms, as well as improvements in econometric methodology and computational capabilities, led to a basic redesign of the FED macro models in the 1990's.

The second generation of these models is represented by significant improvement over their predecessors in the treatment of expectations, intertemporal budget constraints, and household and firm decision making, while at the same time holding to a high standard of goodness of fit. One can read more about these older models in Brayton, Levin, Tryon, and Williams (1997).

The FED currently uses multi-country model called SIGMA. It is DSGE model which was developed as a quantitative tool for policy analysis. More details about model SIGMA can be found in Erceg, Guerrieri, and Gust (2006). Another model which is currently used by FED is called FRB/Global. It is a large-scale macroeconomic model used in analyzing exogenous shocks and alternative policy responses in foreign economies and in examining the impact of these external shocks on the U.S. economy. FRB/Global imposes fiscal and national solvency constraints and utilizes error-correction mechanisms in the behavioral equations to ensure the long-run stability of the model. In FRB/Global, expectations play an important role in determining financial market variables and domestic expenditures. Simulations can be performed using either limited-information (adaptive) or model-consistent (rational) expectations. For more details about FRB/-Global see Levin, Rogers, and Tryon (1997).

1.6 Criticism of the DSGE approach

The introduction of Bayesian DSGE models in policy institutions has also been accompanied by increasing criticism of some of the elements and assumptions underlying this approach. In this section, the three of those criticisms are addressed. Kodera and Quang (2013) provide summary of reasons why DSGE models failed during financial crisis.

The main failures of DSGE models are explained by their emphasis on building macro models based on the neoclassical microeconomic assumptions of "rational behavior" of a representative agent that maximizes consumption under a budget constraint and maximizes profits in production with a resource constraint, within a very well behaved market clearing process and guided by rational expectations.

Rational behavior

North (1993), the 1993 Nobel prize winner for his contribution to institutional analysis demonstrated that under uncertainty it is not possible to assume the idea of a "rational behavior" defended by neoclassical economics and it more close to real life to accept that people learn and behaves by trial and error. Contemporary psychology dismisses the idea of a built in "homo economicus" that drives peoples behavior towards permanently maximizing their marginal utility or profits.

Kahneman and Tversky (1979) demonstrated that in decisions under uncertainty the empirical evidence did not point towards a "rational behavior of agents". Under uncertainty tends to emerge an asymmetric pattern that is quite different from the neoclassical rational behavior and does not follow the probability theory: the risk aversion drive dominates behavior. People prefer much more not to loose 100 USD than to win 100 USD even though probability theory does not subscribe such preference. This asymmetrical behavior summarized in the idea that when uncertainty increases the driving force is risk aversion is frontally opposed to the neoclassical assumption that the representative model agent is always maximizing profits or utility. In some periods agents may be maximizing profits but as soon uncertainty increases they rapidly shift towards risk aversion. Hence, human behavior and thus economic decisions can not be assumed as constant or permanent along time because they change following the increase or decrease in uncertainty, confidence and expectations. Uncertainty is not a particular case. It is the frequent environment in todays economies. The findings of Kahneman and Tversky (1979) are very much in line with Keynes (1936) animal spirits proposition that the human behavior is moved by deep forces that can not be explained by probability theory hence his emphasis on the variability of animal spirits and its consequence for investment decisions and macroeconomic performance.

We are left then with the evidence that risk aversion and the changing behavior of agents along time, following the ups and downs of uncertainty, is in fact a more reasonable assumption than the maximizing behavior of the representative agent. This is more than important because uncertainty rises with shocks affecting macroeconomic performance and these shocks are very frequent in todays globalized world.

Market clearing adjustment

The same happens with the idea of very well behave markets that always move towards equilibrium. It is useful to illustrate this subject with some contemporary Nobel Laureate contributions. Mirrlees (1996), the 1996 Nobel Prize winner demonstrated that under incomplete information, a quite common fact in the real world, market adjustment does not clear and does not ensure full employment of resources. The findings of 2001 Nobel Prize winner Akerlof (1970) showed that markets do not optimize and in many circumstances do not even clear or exist. Another 2001 Nobel Prize winner, Joseph E. Stiglitz, follows in his paper Stiglitz and Weiss (1983) a similar path to that of James Mirrlees but considering asymmetrical information reached a similar conclusion: there is no automatic market clear adjustment that warrants full employment. The 2001 Nobel Prize winner Michael Spence, working on the dynamics of information flows that allow for the development of labor markets, presents the same conclusion in the book Spence (1974). Krugman (1979), 2008 Nobel Prize winner for his contribution on the importance of international specialization and economic scales for international trade and growth concludes that larger scales and thus trends towards monopolistic markets do not point towards neoclassical optimal market adjustments. Oliver Williamson, 2009 Nobel Prize winner for his contribution Williamson (2002) underlines situations when it is not possible or too expensive to acquire information for each transaction - issue that generates frictions in market adjustments. Peter Diamond obtained the 2010 Nobel Prize for his search and friction analysis in labor markets. Thus, beside the previous contributions of a great number of top level economist to this subject, seven recent Nobel Laureates emphasized that the assumption of an automatic adjustment that clears the markets is far from been true due to quite a number of reasons.

Hence, there is plenty scientific evidence that a permanent trend towards market automatic adjustment in the sense of market clear with full employment does not exists. The fact that in 1985-2007 no trends towards recessions were verified in the USA can not be taken as hard evidence towards market efficient adjustment because in the same period: i) many episodes happened in USA showing that financial and other markets did not clear; ii) dozens of other market economies faced in 1985-2007 serious recessive trends and iii) before 1985-2007 hundreds of recessions took place in numerous market economies. For details see Reinhart and Rogoff (2009).

Rational expectations

George Lucas theory of rational expectation, which was deduced like a mathematical theorem, defines a type of expectation that for the average agent tends to be equal to the best guess of future events using all the information available today. It is assumed that future events do not significantly differ from the outcome of the future market equilibrium. Hence, rational expectations do not significantly differ from the future market equilibrium outcomes. In mathematical terms is equivalent to give a variable today the value that it will obtain in the market equilibrium tomorrow.

There is a branch of contemporary psychology that has dealt with human expectations. Professor Albert Bandura, former president of the American Psychologist Association and former Director of the Psychology Department of the University of Stanford, led a school of thought on this subject and made lasting theoretical and empirical contributions. Bandura (1985) developed what is today known as the Social Cognitive Theory. This approach emphasizes that human behavior is a result of the dynamic interaction between personal factors, behavioral patterns and environment. Behavior, for this approach, is regulated by a previous cognitive process. A key issue is the human capacity to understand and retain symbols: images, mental portraits, paintings, and above all words and language. Symbols are essential in the mechanism of human thinking because they allow human beings to store information in their memory.

Such information is going to be used afterwards by human beings in order to assess future behavior. Symbols are then the input to engage in actions assessing the future. It is through symbols that human beings can think about the future consequences of a certain behavior. Hence, through the stored symbols, the previous individual experiences create the expectation that a certain outcome will be the consequence of a certain behavior. The process of elaborating and storing symbols associated with previous experiences allows human beings to represent in their minds future events in the present. It is in the essence of contemporary psychology expectations analysis that symbols allow the storage of information in human memory and this information will be afterward used in order to assess, anticipate and guide future behavior.

Hence the Social Cognitive Theory emphasizes that in order to develop certain expectations is previously needed the storage of symbolic information about previous individual experiences. It is that stored symbolic information the necessary input that will allow us to form expectations towards the future. For this branch of Psychology, causality runs from previous experiences stored symbolically in the human brain towards the creation of expectations for the future. But for rational expectations theory causality runs in the opposite direction: the capacity to foresee the future market equilibrium is what determines todays expectations towards the future. For the rational expectation theory, the symbolic information storage process in the human brain is not relevant. The agents behave as having perfect foresight which allows them to form expectations towards the future which are equal to future market equilibrium values.

The fact is that the rational expectation theory has no connection at all with the branch of contemporary psychology that studies human expectations. Once it is analyzed at the light of contemporary psychology it sounds more as a convenient intellectual craft that pays little attention to real human behavior than a scientific economic approach.

Moreover, Buiter (2009) suggests that the rational expectation approach emphasizing that today prices depend of price expectations for tomorrow, can be extended in time towards the future. This extension process allows us to say that tomorrow prices will depend on price expectations for the day after tomorrow. Since this process is continuous and doesnt have an end we can conclude that today prices depend of price expectations for the infinite remote future. And once we get the infinite within our reasoning, the equation becomes undetermined. In the same way, rational expectations operate when there is only one future market equilibrium. Once we allow for multiple possible future equilibriums or the possibility of a future disequilibrium the hypothesis becomes inoperative.

Other critique
It has been argued that both econometrically and economically some of the shocks and frictions in common DSGE models are not well identified. The most forceful illustration of these identification problems in standard New Keynesian models has been provided by Canova and Sala (2009). These authors and Canova (this issue) show that the likelihood function often shows little curvature in key parameters of the model. Moreover, because of the highly non-linear nature, it is not always obvious where the identification problems lie, and it makes correct inference difficult. Clearly, acknowledging these identification problems must be an important element of any empirical analysis. However, as argued above, the Bayesian approach allows using prior information to address some of these identification problems. For example, Mackowiak and Smets (2008) discuss how the wealth of new micro information on price setting can be used in the specification and estimation of macro models. Clearly, there is never an unambiguous, one-to-one mapping between micro features such as the frequency of price changes and the simplified structural macroeconometric model.

However, confronting micro information with its macro-implications is a useful and informative exercise which can help reduce identification problems. It can also point to deficiencies in the specification of the model. Similarly, as shown above it is standard practice to calibrate some of the key parameters by using, for example, information on the great macroeconomic ratios. The analysis of Canova and Sala (2009) does highlight that it is important to check the informativeness of the data by comparing the prior distribution with the posterior distribution.

One of the criticisms of Chari, Kehoe, and McGrattan (2009) of the Smets and Wouters (2007) model is that the economic interpretation of some of the shocks is not clear. For example, the so-called wage mark-up shocks affecting the labor supply equation could be due to a variety of factors such as shifts in labor supply coming from changing preferences or participation rates, shifts in the bargaining power of unions or changing taxes. The welfare and policy implications of these different sources of wage mark-up variations can be quite different. Also in this case, using additional information may help solving this identification problem. For example, Galí (2009) shows that simply adding the unemployment rate to the observable variables may allow distinguishing between the first two sources of mark-up fluctuations. Second, the assumption of rational (or model-consistent) expectations and perfect information, which underlies most of the DSGE models, is obviously an extreme assumption. As argued above, it is a useful and consistent benchmark, in particular when analyzing the steady state effects of changes in the policy regime. By bringing out expectations explicitly, their impact can be discussed directly. At the same time, it is unreasonable to assume that in an uncertain world and taking into account that the model is an abstraction of reality agents use the model to form their expectations in a model-consistent way.

A number of avenues have been pursued to include learning and imperfect information in DSGE models. First, it is fair to say that addressing information problems at the micro level and analyzing its implications for the aggregate macro economy is still at an early stage and is only feasible in small highly stylized models. Second, a number of authors have introduced learning about shocks in the model. This will typically help explaining some of the persistence in the response of the economy to shocks. For example, Collard, Dellas, and Smets (2010) show that models with such signal extraction problems better fit the data. Third, an alternative is to endow the agents with forecasting regressions that are regularly updated. Milani (2005) and Wouters and Slobodyan (2009) find that DSGE models with learning mechanisms of this sort fit the macro-economic variables better than rational-expectations models and can also explain some of the time variation in the estimated persistence of inflation and variances. The third criticism has become loud since the outbreak of the financial crisis. Most DSGE models do not explicitly model a financial intermediation sector and rely on perfect arbitrage equations to model asset prices. As a result, there is only a limited role for financial phenomena such as agency problems arising from asymmetric information giving rise to debt constraints and the possibility of default. As discussed above, one of the models used at the ECB, the CMR model, does have an explicit banking sector and includes an agency problem with respect to the financing of investment by firms.

As in most other DSGE models, the banking sector itself is, however, not subject to asymmetric information problems and costly financing constraints. As one of the major propagation mechanisms of the current financial crisis has been tensions in the inter-bank market, it is not surprising that a lot of current research focuses on modeling a more explicit banking sector. Recent examples are Gertler and Karadi (2011), Dib and Christensen (2005) and Gerali, Neri, Sessa, and Signoretti (2010). It remains to be seen whether such extensions can capture the slow build-up of financial imbalances and associated credit and asset price booms that we have witnessed over the past decade and the sudden collapse in 2007 and 2008. Moving away from models that are linearized around a steady state is likely to be one condition for capturing such non-linear phenomena. Another feature that is often missing from DSGE models used in policy institutions is a well-developed housing market. Historical experience, as well as the current crisis, has highlighted the important role that overextended real estate markets play in many financial crises. The work published by Iacoviello (2005), which itself is based on Kiyotaki and Moore (1997) is one way of introducing financial frictions in real estate finance.

Despite these drawbacks DSGE models are the most common tool which is used by national banks and international financial institutions (e.g. International Monetary Fund, European Central Bank, etc.). The critique that the well-developed housing market is missing will be crushed in the 4. chapter of this thesis.

1.7 Software tools for simulation and estimation of DSGE models

There are no many software packages which allow simulation or estimation of DSGE models. Two packages are mentioned in this thesis: Dynare and IRIS. Applications in this thesis are estimated solely using Dynare.

1.7.1 Dynare

Dynare is a software platform for handling a wide class of economic models, in particular dynamic stochastic general equilibrium (DSGE) and overlapping generations (OLG) models[7]. The models solved by Dynare include those relying on the rational expectations hypothesis, wherein agents form their expectations about the future in a way consistent with the model. But Dynare is also able to handle models where expectations are formed differently: on one extreme, models where agents perfectly anticipate the future; on the other extreme, models where agents have limited rationality or imperfect knowledge of the state of the economy and, hence, form their expectations through a learning process. In terms of types of agents, models solved by Dynare can incorporate

[7]An overlapping generations model, abbreviated to OLG model, is a type of representative agent economic model in which agents live a finite length of time long enough to overlap with at least one period of another agent's life. All OLG models share several key elements. Individuals receive an endowment of goods at birth. Goods cannot endure for more than one period. Money endures for multiple periods. Individual's lifetime utility is a function of consumption in all periods.

consumers, productive firms, governments, monetary authorities, investors and financial intermediaries. Some degree of heterogeneity can be achieved by including several distinct classes of agents in each of the aforementioned agent categories. Dynare offers a user-friendly and intuitive way of describing these models. It is able to perform simulations of the model given a calibration of the model parameters and is also able to estimate these parameters given a dataset. In practice, the user will write a text file containing the list of model variables, the dynamic equations linking these variables together, the computing tasks to be performed and the desired graphical or numerical outputs.

A large panel of applied mathematics and computer science techniques is internally employed by Dynare: multivariate nonlinear solving and optimization, matrix factorizations, local functional approximation, Kalman filters and smoothers, MCMC techniques for Bayesian estimation, graph algorithms, optimal control, etc.

Various public bodies (central banks, ministries of economy and finance, international organizations) and some private financial institutions use Dynare for performing policy analysis exercises and as a support tool for forecasting exercises. In the academic world, Dynare is used for research and teaching purposes in postgraduate macroeconomics courses.

Download Dynare here: http://www.dynare.org

1.7.2 IRIS

IRIS is a free, open-source toolbox for macroeconomic modeling and forecasting in Matlab. IRIS integrates core modeling functions (including a versatile model file language, simulation, estimation, forecasting, or model diagnostics) with supporting infrastructure (including time series analysis, data management, or reporting) in a user-friendly command-oriented environment.

Download IRIS here: https://code.google.com/p/iris-toolbox-project/

Chapter 2

DSGE framework

2.1 Introduction

The main goal of this chapter is to summarize all methods and algorithms which are used in the estimation of DSGE models. The book summarizes this econometric tools (in the sense of DSGE model estimation) has not yet been written. Experienced readers may skip this chapter and continue reading of empirical studies in third and fourth chapter.

This chapter is divided into two sections. The first section describes the development of economic model. Before one proceeds to Bayesian estimation it is necessary to specify DSGE model (optimization problems of all agents) and to derive first order conditions and log-linearize equations which characterize the equilibrium. The second section deals with the Bayesian estimation of DSGE models. The large amount of foreign literature is cited in order to describe the most up to date algorithms and research in this field. The basic principles of Bayesian analysis are described. Next, the Bayesian estimation procedure is described. Moreover, the importance of prior distribution is highlighted. The computation of the data likelihood is described as well as the possibilities of approximation the posterior distribution. The last two subsections are dedicated to evaluation and identification of DSGE models.

2.2 Economic model

DSGE model describes behavior of agents in the economy. These agents optimize their utility functions subject to many constraints. Model can contain various scale of agents and their choice depends on specific purpose of each study. The incorporation of many different agents means that model is derived from microeconomic foundations.

DSGE model is a set of equations characterizing the equilibrium of examined system. This system may be specified in accordance with Real Business Cycle or New Keynesian theory. Nothing prevents to specify DSGE model according to some other economic theory.

2.2.1 Derivation of the first order conditions

After economic formulation of the DSGE model it is necessary to find equations characterizing the equilibrium, i.e. constraints, first-order conditions, etc. To solve optimization problems, one should use the techniques of dynamic programming described by Stokey and Lucas (1989). The calculation of first-order conditions of optimality is performed using the Lagrangian. The method of Lagrange multipliers is a strategy for finding the local maximum and minimum of a function subject to equality constraints. This technique is described in Uhlig (1995).

Subsequently, the non-linear system with rational expectations may be written in compact notation as

$$E_t \left[f_\theta \left(x_{t+1}, x_t, x_{t-1}, \varepsilon_t \right) \right] = 0, \tag{2.1}$$

where x is the vector collecting all the exogenous variables, ε is the vector collecting the exogenous stochastic shocks and θ is the vector collecting the "deep" parameters of the model.

2.2.2 Log-linearization

Log-linearization of the necessary equations (2.1) characterizing the equilibrium of the system makes the equations approximately linear in the log-deviations from the steady state. The basic principle of log-linearization is to use a Taylor approximation around the steady state to replace all equations by approximations, which are linear functions in the log-deviations of the variables. Formally, let X_t be a vector of variables, $\bar{X}$ their steady state and $x_t = \log X_t - \log \bar{X}$ the vector of log-deviations. The vector $x_t \cdot 100$ is interpreted as the difference of the variables from their steady state levels in period t in percent. The necessary equations which characterize the equilibrium may be written as

$$1 = f\left(x_t, x_{t-1} \right) \tag{2.2}$$

$$1 = E_t \left[g\left(x_{t+1}, x_t \right) \right], \tag{2.3}$$

where $f(0,0) = 1$ and $g(0,0) = 1$, i.e. the left-hand side of 2.2 and 2.3. Taking first-order approximations around $(x_t, x_{t-1}) = (0,0)$ yields

$$\begin{aligned} 0 &\approx f_1 \cdot x_t + f_2 \cdot x_{t-1} \\ 0 &\approx E_t \left[g_1 \cdot x_{t+1} + g_2 \cdot x_t \right]. \end{aligned}$$

One obtains a linear system in x_t and x_{t-1} in the deterministic equations and x_{t+1} and x_t in the expectational equations. This linear system can be solved with the method of undetermined coefficients. In the large majority of cases, there is no need to differentiate the functions f and g explicitly. Instead, the log-linearized system can usually be obtained as follows. Multiply out everything before log-linearizing. Replace a variable X_t with $X_t = \bar{X} e^{x_t}$, where x_t is a real number close to zero. Let likewise y_t be a real number close to zero. Take logarithms, where both sides of an equation only involve products, or use the following three building blocks, where a is some constant

$$\begin{aligned} e^{x_t + a y_t} &\approx 1 + x_t + a y_t \\ x_t y_t &\approx 0 \\ E_t \left[a e^{x_{t+1}} \right] &\approx E_t \left[a x_{t+1} \right] \text{ up to a constant.} \end{aligned}$$

For example, these examples yield

$$\begin{aligned} e^{x_t} &\approx 1 + x_t \\ a X_t &\approx a \bar{X} x_t \text{ up to a constant} \\ (X_t + a) Y_t &\approx \bar{X} \bar{Y} x_t + \left(\bar{X} + a \right) \bar{Y} y_t \text{ up to a constant.} \end{aligned}$$

If the equations satisfy steady state relationships then these constants are eliminated.

2.3 Bayesian estimation

2.3.1 Literature review

There are many papers on how to take DSGE models to the data and how to work with these models empirically. At the beginning of this macroeconometric research field classical estimation techniques prevailed. There has been a trend toward advanced econometric methods for the last several years due to better computational skills. Bayesian estimation is now the most common technique when working with DSGE models. The classical approach (non-Bayesian) has been elaborated extensively. Surveys of these methods can be found in papers of Kim and Pagan (1995) or Canova (2007) which also provides introduction to Bayesian estimation.

Another overview is provided by An and Schorfheide (2005). Ruge-Murcia (2007) introduces and compares following methods: GMM (Generalized Method of Moments), ML (Maximum Likelihood) with Bayesian priors, SMM (Simulated Method of Moments) and Indirect Inference. A very extensive and detailed discussion and overview of Bayesian estimation is provided by Fernandez-Villaverde (2009). The main difference among all methods is in amount of information each method is able to handle. Methodological discussion of various estimation and model evaluation techniques can be found in Sims (1996) or Kydland and Prescott (1996).

Following Ruge-Murcia (2007), several advantages of fully-fledged econometric estimation vs. calibration are noteworthy. First, parameter values are obtained using the model of interest. Parameter values taken from some other study may be inconsistent with the model's assumptions. Furthermore, one can estimate all relevant parameters, even those where there is no microeconomic study so far. Second, in order to take care of parameter uncertainty, confidence intervals for models's response to a shock can be constructed more easily. Finally, it is easier to evaluate these models.

The use of Bayesian estimation yields the following benefits. An and Schorfheide (2005) point out that Bayesian estimation takes advantage of the general equilibrium approach. In contract, GMM estimation is based on equilibrium relationships, so it fits the model to a vector of aggregate time series which are based on the likelihood function generated by the model. Last, prior distributions can be used to incorporate additional information into the parameter estimation. Bayesian estimation outperforms the techniques of GMM and ML in small samples. Moreover, in case of mis-specified models, Bayesian estimation and model comparison are according to Rabanal and Rubio-Ramirez (2005) consistent.

2.3.2 Bayesian analysis

The most frequently used statistical methods are known as frequentist (or classical) methods. These methods assume that unknown parameters are fixed constants, and they define probability by using relative frequencies. It follows from these assumptions that probabilities are objective and that you cannot make probability statements about parameters. Frequentist believes that a population mean is real, but unknown, and unknowable, and can only be estimated from the data. Knowing the distribution for the sample mean, one constructs a confidence interval, centered at the sample mean. And that's because to a frequentist the true mean, being a single fixed value, doesn't have a distribution. Either the true mean is in the interval or it is not. So the frequentist can't say there's a 95% probability that the true mean is in this interval, because it's either already in, or it's not. And that's because to a frequentist the true mean, being a single fixed value, doesn't have a distribution. The sample mean does. Thus the frequentist must use circumlocutions like "95% of similar intervals would contain the true mean, if each interval were constructed from a different random sample like this one."

Bayesian methods offer an alternative approach. They treat parameters as random variables and define probability as "degrees of belief" and that is the probability of an event is the degree to which you believe the event is true. It follows from these postulates that probabilities are subjective and that you can make probability statements about parameters. The term "Bayesian" comes from the prevalent usage of Bayes' theorem in this area. Bayes' theorem was developed by the Reverend Thomas Bayes (1702-1761). His paper Bayes and Price (1763) was published posthumously. Comprehensive treatments of Bayesian analysis offer Jeffreys (1961), Zellner (1971), Geweke (1999). Empirical application is presented by Canova (2007). The comprehensive description of Bayesian methods presents Koop (2003).

2.3.2.1 Bayesian inference

The following steps describe the essential elements of Bayesian inference. A probability distribution for θ is formulated as $\pi(\theta)$, which is known as the prior distribution, or just the prior. The prior distribution expresses your beliefs, for example, on the mean, the spread, the skewness, and so forth, about the parameter before you examine the data. Given the observed data y, you choose a statistical model $p(y|\theta)$ to describe the distribution of y given θ. Next, the beliefs are updated about θ by combining information from the prior distribution and the data through the calculation of the posterior distribution $p(\theta|y)$. This update is performed using the Bayes' theorem

$$p(\theta|y) = \frac{p(y|\theta)\,p(\theta)}{p(y)} = \frac{p(y|\theta)\,\pi(\theta)}{\int p(y|\theta)\,\pi(\theta)\,d\theta}.$$

The quantity

$$p(y) = \int p(y|\theta)\,\pi(\theta)\,d\theta \tag{2.4}$$

is the normalizing constant of the posterior distribution. This quantity $p(y)$ is also the marginal distribution of y, and it is sometimes called the marginal distribution of the data. The likelihood function of θ is any function proportional to $p(y|\theta)$ and that is $L(\theta) \propto p(y|\theta)$ (symbol $\propto$ is abbreviation for "proportional to" and is often used in the sense of "varies directly as"). Another way to write Bayes' theorem is

$$p(\theta|y) = \frac{L(\theta)\,\pi(\theta)}{\int L(\theta)\,\pi(\theta)\,d\theta}.$$

The marginal distribution $p(y|\theta)$ is an integral; therefore, as long as it is finite, the particular value of the integral does not provide any additional information about the posterior distribution. Hence, $p(\theta|y)$ can be written up to an arbitrary constant, presented here in proportional form as

$$p(\theta|y) \propto L(\theta)\,\pi(\theta)\,. \tag{2.5}$$

Generally, Bayes' theorem defines how to update existing knowledge with new information. At the beginning, one has a prior belief $\pi(\theta)$, and after learning information from data y, one changes or updates the belief on θ and obtain $p(\theta|y)$. These are the essential elements of the Bayesian approach.

In theory, Bayesian methods offer a very simple alternative to statistical inference. All inferences follow from the posterior distribution $p(\theta|y)$. However, in practice, only the most fundamental problems enable you to obtain the posterior distribution with straightforward analytical solutions. Bayesian analysis requires very often sophisticated computations, including the use of simulation methods. Samples are generated from the posterior distribution and then these samples are used for the estimation of the quantities of interest.

2.3.2.2 Prior distribution

A prior distribution of a parameter is the probability distribution that represents your uncertainty of the parameter before the current data are examined. Multiplying the prior distribution and the likelihood function together leads to the posterior distribution of the parameter. One uses the posterior distribution to carry out all inferences. One cannot carry out any Bayesian inference or perform any modeling without using a prior distribution.

Objective Priors versus Subjective Priors

Bayesian probability measures the degree of belief that you have in a random event. By this definition, probability is highly subjective. It follows that all priors are subjective priors. Not everyone agrees with this notion of subjectivity when it comes to specifying prior distributions. There has long been a desire to obtain results that are objectively valid. Within the Bayesian paradigm, this can be somewhat achieved by using prior distributions that are "objective" which have a minimal impact on the posterior distribution. Such distributions are called objective or non-informative priors. However, while non-informative priors are very popular in some applications, they are not always easy to construct. For more details about objective Bayesian versus subjective Bayesian analysis see Berger (2006) and Goldstein (2006).

Non-informative Priors

Roughly speaking, a prior distribution is non-informative if the prior is "flat" relative to the likelihood function. Thus, a prior $\pi(\theta)$ is non-informative if it has minimal impact on the posterior distribution of θ. Other names for the non-informative prior are vague and flat prior. Many statisticians favor non-informative priors because they appear to be more objective. However, it is unrealistic to expect that non-informative priors represent total ignorance about the parameter of interest. In some cases, non-informative priors can lead to improper posteriors (non-integrable posterior density). You cannot make inferences with improper posterior distributions. In addition, non-informative priors are often not invariant under transformation; that is, a prior might be non-informative in one parameterization but not necessarily non-informative if a transformation is applied. A common choice for a non-informative prior is the flat prior, which is a prior distribution that assigns equal likelihood on all possible values of the parameter. Intuitively this makes sense, and in some cases, such as linear regression, flat priors on the regression parameter are non-informative. However, this is not necessarily true in all cases. For details see Kass and Wasserman (1994).

Informative Priors

An informative prior is a prior that is not dominated by the likelihood and that has an impact on the posterior distribution. If a prior distribution dominates the likelihood, it is clearly an informative prior. These types of distributions must be specified with care in actual practice. On the other hand, the proper use of prior distributions illustrates the power of the Bayesian method; information gathered from the previous study, past experience or expert opinion can be combined with current information in a natural way.

Conjugate Priors

A prior is said to be a conjugate prior for a family of distributions if the prior and posterior distributions are from the same family, meaning that the form of the posterior has the same distributional form as the prior distribution. For example, if the likelihood is binomial, $y \sim Bin(n, \theta)$, a conjugate prior on θ is the beta distribution; it follows that the posterior distribution of θ is also a beta distribution. Other commonly used conjugate prior/likelihood combinations include the normal/ normal, gamma/Poisson, gamma/gamma, and gamma/beta cases. The development of conjugate priors was partially driven by a desire for computational convenience, conjugacy provides a practical way to obtain the posterior distributions. The Bayesian procedures do not use conjugacy in posterior sampling.

Jeffreys' Prior

A very useful prior is Jeffreys' prior. It satisfies the local uniformity property, a prior that does not change much over the region in which the likelihood is significant and does not assume large values outside that range. It is based on the Fisher information matrix. Jeffreys' prior is defined as follows

$$\pi(0) \propto |I(\theta)|^{1/2},$$

where $||$ denotes the determinant and $I(\theta)$ is the Fisher information matrix based on the likelihood function $p(y|\theta)$

$$I(\theta) = -E\left[\frac{\partial^2 \log p(y|\theta)}{\partial \theta^2}\right].$$

Jeffreys' prior is locally uniform and hence non-informative. It provides an automated scheme for finding a non-informative prior for any parametric model $p(y|\theta)$. It is important to recognize that Jeffreys' prior is not in violation of Bayesian philosophy, it is the form of the likelihood function that determines the prior but not the observed data, since the Fisher information is an expectation over all y and not just the observed y. Another appealing property of Jeffreys' prior is that it is invariant with respect to one-to-one transformations. The invariance property means that, if you have a locally uniform prior on θ and $\phi(\theta)$ is a one-to-one function of θ, then $p(\phi(\theta)) = \pi(\theta)$. $|\phi'(\theta)|^{-1}$ is locally uniform prior for $\phi(\theta)$. This invariance principle carries through to multidimensional parameters as well. While Jeffreys' prior provides a general recipe for obtaining non-informative priors, it has some shortcomings: the prior is improper for many models and it can lead to improper posterior in some cases, and the prior can be cumbersome to use in high dimensions. For details see Jeffreys (1946).

2.3.2.3 Advantages and Disadvantages

Generally speaking, when the sample size is large, Bayesian inference often provides results for parametric models that are very similar to the results produced by frequentist methods. There are general advantages and disadvantages to Bayesian inference.

The advantages of Bayesian approach are as follows. Bayesian analysis provides a natural and principled way of combining prior information with data, within a solid decision-theoretical framework. You can incorporate past information about a parameter and form a prior distribution for future analysis. When new observations become available, the previous posterior distribution can be used as a prior. All inferences logically follow from Bayes theorem. It provides inferences that are conditional on the data and are exact, without reliance on either asymptotic approximation or the "plug-in" principle. Small sample inference proceeds in the same manner as if one had a large sample. It obeys the likelihood principle: if two distinct sampling designs yield proportional likelihood functions for θ, then all inferences about θ should be identical from these two designs. Classical inference does not obey the likelihood principle, in general. It provides interpretable answers, such as "the true parameter θ has a probability of 0.95 of falling in a 95% credible interval". In addition, it provides a convenient setting for a wide range of models, such as hierarchical models and missing data problems. MCMC, along with other numerical methods, makes computations tractable for virtually all parametric models.

The disadvantages of the Bayesian approach are as follows. It does not tell the analyst how to select a prior. There is no correct way to choose a prior. Bayesian inferences require skills to translate subjective prior beliefs into a mathematically formulated prior. If you do not proceed with caution, you can generate misleading results. It can produce posterior distributions that are heavily influenced by the priors. From a practical point of view, it might sometimes be difficult to convince subject matter experts who do not agree with the validity of the chosen prior. It often comes with a high computational cost, especially in models with a large number of parameters. In addition, simulations provide slightly different answers unless the same random seed is used. Note that slight variations in simulation results do not contradict the early claim that Bayesian inferences are exact: the posterior distribution of a parameter is exact, given the likelihood function and the priors, while simulation-based estimates of posterior quantities can vary due to the random number generator used in the procedures.

2.3.3 Estimation procedure

The focus of the thesis is on the Bayesian estimation of DSGE models. An and Schorfheide (2005) list three main characteristics of this approach. The GMM estimation is based on equilibrium relationships and the Bayesian analysis is system based and fits the solved DSGE model to a vector of aggregate time series. For example Linde (2005) argues by means of Monte Carlo simulations, that the Full Information Maximum Likelihood approach improves the estimation results considerably in comparison with single-equation methods even if the model and the policy rule are misspecified. Simultaneous estimation of all equations allows the unambiguous interpretation of structural shocks and it is an important advantage for policy analysis. For details see Canova (2002) or Christiano, Eichenbaum, and Evans (2005).

Prior distributions can be used to incorporate additional information into the parameter estimation. Likelihood-based inference presents a series of issues. Specially the lack of identification which means multiple maximum, over-parametrization. The maximum is given by a complex multidimensional combination rather than by a single point in the parameter space. From the computational point of view, the Bayesian approach and the use of a prior makes the optimization algorithm more stable. The estimation in this thesis follows Schorfheide (2000). The two-stage estimation procedure is applied and it involves calibration and Bayesian Maximum Likelihood methods. The simplified algorithm consists of six steps. For details see algorithm 1.

Algorithm 1: Brief overview of the Bayesian estimation

1. Construct a log-linear representation of the DSGE model and solve it or transform it into the state space model.
2. Specify prior distributions for the structural parameters, fix the parameters which are not identifiable.
3. Compute the posterior density numerically, using draws from the prior distribution and the Kalman filter to evaluate the likelihood of the data.
4. Draw sequences from the joint posterior of the parameters using the Metropolis-Hastings algorithm. Check whether the simulated distribution converge to the posterior distribution.
5. Construct statistics of interest using the draws in 4.
6. Evaluate the model and examine sensitivity of the results to the choice of priors.

2.3.3.1 Model solutions

This section presents the transformation of a DSGE model to a rational expectations system. The DSGE model may be estimated in its nonlinear form, then the log-linearization (the 1. step of the algorithm 1) is unnecessary. However, the methods for estimation of non-linear models are computationally extremely demanding, which causes that at this point only the most basic RBC model has been estimated by nonlinear likelihoods methods. For details see Fernandez-Villaverde and Francisco Rubio-Ramirez (2004). The full Bayesian nonlinear approach is hardly feasible on currently available computers. The estimation of the seminal nonlinear DSGE model using its second order approximation around the steady state and the particle filter for evaluation of the likelihood is 200-1000 slower than estimation based on log-linearized version of the model with the likelihood evaluated by Kalman filter recursion.

The set of equilibrium conditions of a DSGE models take the form of a non-linear rational expectation system of variables vector s_t and innovations u_t and it may be written in the form

$$E_t\left[G_t\left(s_{t+1}, s_t, u_t\right)\right] = 0.$$

The rational expectations system has to be solver before the model can be estimated and the solution takes the form

$$s_t = A_t\left(s_{t-1}, u_t, \theta\right), \tag{2.6}$$

where s_t may be seen as a state vector and the equation 2.6 is a nonlinear state transition equation. The parameter vector θ in the equation 2.6 indicates the dependence of the solution on a parameter constellation. Currently, a lot of numerical techniques are available to solve rational expectations systems. The algorithms to construct a second-order accurate solution have been developed by Judd (1998), Kim, Kim, Schaumburg, and Sims (2005), Collard and Juillard (2001) and Schmitt-Grohe and Uribe (2004). The so called perturbation methods which includes higher order terms in the approximation, and therefore takes both curvature and risk into account are discussed in Juillard (1996) and Judd (1998).

In the context of system-based DSGE model estimation linear approximation methods are very popular because they lead to state-space representation of the DSGE model that can be analyzed with the Kalman filter (see algorithm 4). Application of linear approximation makes feasible even the large scale DSGE models. Several solution algorithms have been put forward by Blanchard and Kahn (1980), Uhlig (1995), Anderson (2010), Klein (2000) and Sims (2002). The solution of the DSGE model may be: unstable, stable (determinacy), or there are multiple stable solutions (indeterminacy). In this thesis estimation follows the assumption of determinacy. Since the linearized model cannot be solved analytically because of singularity problem, it is decided to apply the method by Sims (2002).

The method involves using the QZ decomposition[1] (which is described later) to solve the generalized eigenvalue problem. It produces the solution quickly and enables us to solve the model for many different values of the underlying parameters in a reasonable amount of time.

The transformation of DSGE model into a state space model (the 1. step of the algorithm 1) might be detailed as follows. The model's equation are log-linearized around the non-stochastic steady state vector $\overline{s}$, where $\overline{s}$ is the solution of $(G_t\left(\overline{s}, \overline{s}, 0\right) = 0)$. The log-linearized equations yield a first order linear difference equation system of the following

[1]often called Generalized Schur decomposition

form also known as a Blanchard and Kahn's formulation

$$G_0 E_t s_{t+1} = G_1 s_t + F u_t.$$

Furthermore, the vector s_t is extended by replacing terms of the form $E_t s_{t+1}$ with $\widetilde{s_t} = E_t s_{t+1}$. The restriction linking the newly defined elements of the s_t to its old elements is then added in a form of equation $s_t = \widetilde{s}_{t+1} + \eta_t$. Models with more lags, or with lagged expectations, or with expectations of more distant future values, can be accommodated by in this framework by expanding the vector s_t. Finally, the system may be rewritten in the following form

$$\Gamma_0 s_t = \Gamma_1 s_{t-1} + \Psi u_t + \Pi \eta_t, \tag{2.7}$$

where η_t is a rational expectations error and $E_t \eta_{t+1} = 0$ for all t. This notation is suggested by Sims (2002) and implies that all variables dated t are observable at t, thus no separate list of what is predetermined is needed to augment the information that can be read off from the linearized equations themselves. Before the description of the solution method, it is important to understand what is meant by the solution of 2.7. The main objective is to express the sequence of $\{s_{t+1}\}_{i=1}^{\infty}$ as a function of realizations of the exogenous random process $\{u_{t+i}\}_{i=1}^{\infty}$ and some initial conditions for the state vector. This requires solving for the endogenously determined η_t. This procedure consists of three steps. For details see algorithm 2.

Algorithm 2: Algorithm for solving the rational expectations system

1. Triangularize the system 2.7 using the QZ decomposition.
2. Find the set of solutions to the transformed system and determine the η_t such that the stable solution is unique.
3. Reverse-transform this solution into the format of the original system.

In the 1. step of the algorithm 2, the Γ_0 and Γ_1 are using the QZ factorization decomposed into a unitary and upper triangular matrices such that

$$\Gamma_0 = Q' \Lambda Z',$$
$$\Gamma_1 = Q' \Omega Z',$$

where Q and Z are both unitary and possibly complex. It means that $Q'Q = Z'Z = I$. The $'$ symbol indicates here both transposition and complex conjugation. The matrices Λ and Ω are possibly complex and are upper triangular. Although the QZ decomposition is not unique, the collection of values for the ratios of diagonal elements of Λ and Ω, denoted by $\left\{\psi_i = \frac{\omega_{ii}}{\lambda_{ii}}\right\}$, is unique. Furthermore, we can always choose the matrices Λ, Ω and Z in a way that the generalized eigenvalues are organized in exceeding order. Defining $r_t \equiv Z' s_t$ and pre-multiplying 2.7 by Q we obtain the transformed system of the form

$$\Lambda r_t = \Omega r_{t-1} + Q\Pi\eta_t + Q\Psi u_t. \tag{2.8}$$

Let $\overline{\xi}$ denote the maximum growth rate allowed for any component of s. $\overline{\xi}$ may be available from the transversality condition of the economic problem. For growth rate larger than $\overline{\xi}$ the system become explosive. In particular we can partition the system 2.8 so that $|\psi_i| \geq \overline{\xi}$ for all $i > k$ and $|\psi_i| < \overline{\xi}$ for all $i \leq k$. Hence the system 2.8 may be expanded as

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ 0 & \Lambda_{22} \end{bmatrix} \begin{bmatrix} r_t^1 \\ r_t^2 \end{bmatrix} = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ 0 & \Omega_{22} \end{bmatrix} \begin{bmatrix} r_{t-1}^1 \\ r_{t-1}^2 \end{bmatrix} + \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \left[\Pi\eta_t + \Psi u_t\right]. \tag{2.9}$$

Because of the way the generalized eigenvalues are grouped, the lower block of equations 2.9 is purely explosive. It has a solution that does not explode as long as we solve it forward to make r^2 a function of future $u's$ and $\eta's$, such that the latter offset the exogenous process in a way that put r^2 on a stationary path

$$\begin{aligned} Z'_{\cdot 2} s_t &= r_t^2 \Omega_2 2^{-1} \Lambda_2 2 r_{t+1}^2 - \Omega_2 2^- 1 Q_{2\cdot} \left[\Pi\eta_{t+1} + \Psi u_{t+1}\right] \\ &= -\sum_{i=1}^{\infty} \left(\Omega_{22}^{-1}\Lambda_{22}\right)^{i-1} \Omega_{22}^{-1} Q_{2\cdot} \left[\Pi\eta_{t+1} + \Psi u_{t+i}\right] \end{aligned} \tag{2.10}$$

Taking expectations conditional on information available at time t leaves the left hand side of 2.10 unchanged, i.e. $E_t r_t^2 = r_t^2$. The right hand side becomes then

$$\begin{aligned} & -\sum_{i=1}^{\infty} \left(\Omega_{22}^{-1}\Lambda_{22}\right)^{i-1} \Omega_{22}^{-1} Q_{2\cdot} \left[\Pi\eta_{t+1} + \Psi u_{t+i}\right] \\ = & -E_t \left[-\sum_{i=1}^{\infty} \left(\Omega_{22}^{-1}\Lambda_{22}\right)^{i-1} \Omega_{22}^{-1} Q_{2\cdot} \left[\Pi\eta_{t+1} + \Psi u_{t+i}\right] \right] \end{aligned} \tag{2.11}$$

Since $E_t \eta_{t+1} = 0$ and $E_t u_{t+i} = 0$ for $i \geq 1$, we get $Z'_{\cdot 2} s_t = r_t^2 = 0$ as a solution for the explosive block. The equality in 2.10 follows on the assumption that $\left(\Omega_{22}^{-1}\Lambda_{22}\right)^t \to 0$ as $t \to \infty$. Note that if some of diagonal elements of $\Lambda = 0$, there are equations in 2.9 containing no current values of r and this corresponds to the singularity in matrix Γ_0. While these cases does not imply explosive paths, the corresponding components of 2.10 are still valid. For instance we have $0 r_t^i = \psi_i i r_{t-1}^i + F\left(\eta_t, u_t\right)$. It can be still solved for r_{t-1}^i producing corresponding component of 2.10. For further details see Sims (2002). In the absence of any additional constraints, this implies that the upper block of equation

2.9 can support any solution of the form

$$[\Lambda_{11}]\left[r_t^1\right] = [\Omega_{11}]\left[r_{t-1}^1\right] + [Q_{1.}]\left[\Pi\eta_t + \Psi u_t\right],$$

which still depends on the endogenous η_t. The equality 2.11 imposes, however, certain constraints on the left-hand side and on η_t. Knowing that $E_t\eta_{t+i} = 0$ and $E_t u_{t+i} = 0$ we obtain

$$-\sum_{i=1}^{\infty}\left(\Omega_{22}^{-1}\Lambda_{22}\right)^{i-1}\Omega_{22}^{-1}Q_{2\cdot}\left[\Pi\eta_{t+1} + \Psi u_{t+i}\right].$$

We further notice that all future shocks can be eliminated by taking expectations conditional on information available at time $t+1$. After doing som and shifting the equation one period backward we obtain

$$Q_{2.}\Pi\eta_t = -Q_{2.}\Psi u_t. \tag{2.12}$$

Sims (2002) concludes from 2.12 that a necessary and sufficient condition for existence of a solution is that the space of $Q_{2.}\Psi$ is to be contained in that of $Q_{2.}\Pi$. Assuming a solution exists, we can combine 2.12 with some linear combination of equations in 2.9 to obtain a new complete system in r that is stable. What remains to be done is to free the new equation from references to the endogenous form η. Form 2.12 we see that $Q_{2.}\Pi\eta_t$ depends on exogenous shock at time t. The 2.9 involves, however, other linear combination of $\eta, Q_{1.}\Pi\eta_t$. In general it is possible that knowing $Q_{2.}\Pi\eta_t$ is not sufficient to tell the value of $\eta, Q_{1.}\Pi\eta_t$, in which case the solution to the model is not unique. To assure that solution is unique it is necessary and sufficient that the row space of $Q_{1.}$ be contained in that of $Q_{2.}$. Which is equivalent to

$$Q_{1.}\Pi = \Phi Q_{2.}\Pi$$

for some matrix Φ. Pre-multiplying 2.9 by matrix $[I - \Phi]$ we obtain the system free of reference to η

$$\begin{bmatrix}\Lambda_{11} & \Lambda_{12} & \Phi\Lambda_{22}\end{bmatrix}\begin{bmatrix}r_t^1\\ r_t^2\end{bmatrix} = \begin{bmatrix}\Omega_{11} & \Omega_{12} & \Phi\Omega_{22}\end{bmatrix}\begin{bmatrix}r_{t-1}^1\\ r_{t-1}^2\end{bmatrix} + [0]\,\eta + \left[Q_{1.}\Psi - \Phi Q_{2.}\Psi\right]u_t. \tag{2.13}$$

Combining 2.13 with 2.10 we obtain

$$\begin{bmatrix}\Lambda_{11} & \Lambda_{12} - \Phi\Lambda_{22}\\ 0 & I\end{bmatrix}\begin{bmatrix}r_t^1\\ r_t^2\end{bmatrix} = \begin{bmatrix}\Omega_{11} & \Omega_{12} - \Phi\Omega_{22}\\ 0 & 0\end{bmatrix}\begin{bmatrix}r_{t-1}^1\\ r_{t-1}^2\end{bmatrix}$$

$$+E_t\begin{bmatrix}0\\ -\sum\limits_{i=1}^{\infty}\left(\Omega_{22}^{-1}\Lambda_{22}\right)^{i-1}\Omega_{22}^{-1}Q_{2\cdot}\Psi u_{t+i}\end{bmatrix} + \begin{bmatrix}Q_{1\cdot}\Psi - \Phi Q_{2\cdot}\Psi\\ 0\end{bmatrix}u_t.$$

Since exogenous shocks are serially uncorrelated $E_t\left[-\sum_{i=1}^{\infty}\left(\Omega_{22}^{-1}\Lambda_{22}\right)^{i-1}\Omega_{22}^{-1}Q_{2\cdot}\Psi u_{t+i}\right]=$ 0. The solution of the system in s can be recovered using that $Z's_t = r_t$

$$s_t = As_{t-1} + Ru_t \text{ and } u_t \sim N\left(0, \Sigma_u\right),$$

where

$$A = \left(\begin{bmatrix} \Lambda_{11} & \Lambda_{12} - \Phi\Lambda_{22} \\ 0 & I \end{bmatrix} Z'\right)^{-1} \begin{bmatrix} \Omega_{11} & \Omega_{12} - \Phi\Omega_{22} \\ 0 & 0 \end{bmatrix} Z'$$

and

$$R = \left(\begin{bmatrix} \Lambda_{11} & \Lambda_{12} - \Phi\Lambda_{22} \\ 0 & I \end{bmatrix} Z'\right)^{-1} \begin{bmatrix} Q_{1\cdot}\Psi - \Phi Q_{2\cdot}\Psi \\ 0 \end{bmatrix}$$

The matrices A, R and Σ_u are functions of structural parameters stored in the vector θ.

2.3.3.2 Setting up a state space framework

Estimation of the DSGE model requires the transformation into a state space form which represents the joint dynamic evolution of an observable random vector y_t and a generally unobserved state vector s_t. Precisely, y_t stacks the time t observations that are used to estimate the DSGE model. The measurement equation completes the model by specifying how the state interacts with the vector of observations. The evolution of the state vector s_t is governed by a dynamic process of the form

$$s_t = A_t\left(s_{t-1} + u_t\right), \tag{2.14}$$

where A_t is a function which may depend on time. The innovation vector u_t is a serially independent process with mean zero and finite covariance matrix $\Sigma_{t,u}$, which may also depend on time. The measurement equation determines the vector of observations y_t as possibly time dependent function of the state and of the error term u_t^m.

$$y_t = B_t\left(s_t\right) + u_t^m. \tag{2.15}$$

The vector u_t^m as also a serially independent process with mean zero and finite covariance matrix $\Sigma_{t,m}$. Some preliminary components about the functioning of the model are in order. Having explicitly specified an initial condition, i.e. a distribution for the state vector s_t at time $t = 0$, or simply s_0, the process s_t is started by a draw from this distribution and evolves according to 2.14. The process has the Markov property, see Lemke (2005). That is, the distribution of s_t at time t given the entire past realizations of the process, is equal to the distribution of s_t given s_{t-1} only. The evolution of the observation vector y_t is determined by the state vector. In addition, the error u_t^m

measures the deviations between the systematic component $B_t(s_t)$ and the observed vector y_t.

The general set-up above may be restricted by making the following assumptions. First, the functions $A_t(\cdot)$ and $B_t(\cdot)$ define linear transformations. Second, the distributions of s_0, u_t and u_t^m are normal. Models satisfying these assumptions are referred to as a linear Gaussian state space models and are the common structure used in the estimation of DSGE models. The transition equation is given by

$$s_t = As_{t-1} + Ru_t, \tag{2.16}$$

which in the DSGE context coincides with the model solution 2.14. For the measurement equation we have

$$y_t = Gx_t + Bs_t + Hu_t^m, \tag{2.17}$$

where x_t is the vector of predetermined variables. The joint evolution of state innovation and measurement error are assumed to satisfy

$$\begin{bmatrix} u_t \\ u_t^m \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_u & 0 \\ 0 & \Sigma_m \end{bmatrix}\right),$$

where $\begin{bmatrix} u_t \\ u_t^m \end{bmatrix}$ and $\begin{bmatrix} u_{t-i} \\ u_{t-i}^m \end{bmatrix}$ are independent for all t and i. The initial conditions write $s_0 \sim N\left(\bar{s}_0, \bar{P}_0\right)$ and finally $E_t\left(u_t s_0'\right) = 0$ and $E_t\left(u_t^m s_0'\right) = 0$ for all t. It is possible that exogenous or predetermined variables enter both the transition equation and the measurement equation. For example, the vector of predetermined variables x_t stores the long-run rates of technology growth, steady state inflation and steady state nominal interest rate which all are functions of structural parameters. The naming u_t^m as a measurement error stems from the use of the state space framework in the engineering or natural sciences. In the context of the DSGE model estimation it might account for the discrepancies between the variables of theoretical model and definitions of aggregates used by statistical offices. However, in many applications the issue of incorporating the measurement errors into the state space model is dedicated by the need to obtain a non-singular forecast error covariance matrix resulting while predicting the vector y_t. This singularity is an obstacle to likelihood estimation. In general, the DSGE model generates a rank-deficient covariance matrix for y_t if the number of shocks stacked in the vector u_t is lower than the number of time series to be matched. Adding measurement u_t^m reflecting the uncertainty regarding the quality of the data to equation 2.15 or augmenting the stochastic nature of the theoretical DSGE model solves the singularity problem. The former procedure was applied in Ireland (2004). In this thesis is used the latter approach, applied in Smets and Wouters (2003), by considering models in which the number of structural shocks is at least as high as the number of observable variables (time series).

2.3.3.3 Impulse Response Functions and second moments

In this section, some complementary issues related to the model solution are discussed. In particular it is shown how the solution may be used to study the dynamic properties of the model from a quantitative point of view. Basically, two issues are discussed here: Impulse Response Functions (IRF) and computation of second moments. The IRF and the second moments are computed for an estimated model. Subsequently, they may be used for validation of the model. The impulse response function of a variable to a shock gives the expected response of the variable to a shock at different horizons. In other words this corresponds to the best linear predictor of the variable if the economic environment remains the same in the future. Provided the solution of the system is known, the immediate response to one of the fundamental shocks. Shock k is given by

$$s_t = Au_{k,t},$$

where $u_{k,t}$ is a vector with all entries equal to zero except one, which stands for the shock k. The response at horizon j is then given by

$$s_{t+j} = As_{t+j-1},$$

Let us focus on the moments for the system 2.7. Since the system is linear the theoretical moments can be computed directly. In what follows we consider the stationary representations of the system such that the covariance of the state vector is $\Sigma_{ss} = E\left(s_{t+j}s'_{t+j}\right)$ whatever j. Hence, we have

$$\begin{aligned} \Sigma_{ss} &= A\Sigma_{ss}A' + AE\left(s_{t-1}u'_t\right)R + RE\left(u_t s'_{t-1}\right)A' + R\Sigma_u R' \\ &= A\Sigma_{ss}A' + R\Sigma_u R' \end{aligned}$$

Solving this equation for Σ_{ss} can be achieved remembering that $vec\,(ABC) = (A \otimes C')\,vec\,(B)$, hence

$$vec\,(\Sigma_{ss}) = (I - A \otimes A)^{-1}\,vec\left(R\Sigma_u R'\right).$$

The computation of covariances at leads and lags proceeds in a very similar way. From the model solution 2.14 we know that

$$s_t = A^j s_{t-j} + \sum_{i=0}^{j-1} A^i R u_{t-i}.$$

Hence,

$$E\left(s_t {s'}_{t-j}\right) = A^j E\left(s_{t-j} {s'}_{t-j}\right) + \sum_{i=0}^{j-1} A^i R E\left(u_{t-i} {s'}_{t-j}\right)$$

Since u is the vector of innovations orthogonal to any past values such that $E\left(u_{t-i} {s'}_{t-j}\right) = 0$ if $i < j$ and for $i = j$, $E\left(u_{t-i} {s'}_{t-j}\right) = E_t\left(u_{t-i}\left(A s_{t-j-1} + R u_{t-i}\right)'\right) = \Sigma_u R'$. Then the previous equation reduces to

$$E\left(s_t {s'}_{t-j}\right) = A^j \Sigma_{ss} + A^j R \Sigma_u R'.$$

2.3.4 Prior distributions

The 2. step of Algorithm 1 requires specification of the prior densities. This can bring additional information which is not in the estimation sample $\{y\}_{t=1}^T$ as mentioned in sub-subsection 2.3.2.2. As is evident from 2.5 the prior re-weights the likelihood. Therefore, the Bayesian approach helps to tackle problems which are common in the Maximum Likelihood (ML) estimation. There are three following problems.

First, the estimates of structural parameters obtained by ML procedures based on a set of observations $\{y\}_{t=1}^T$ are often at odds with out of sample information.

Second, due to the nature of DSGE models the likelihood function often peaks in the parameter region which is at odds with the micro evidence.

Finally, priors adding curvature to a likelihood function that may be flat in some dimensions of the parameter space and influence the shape of of the posterior distribution and allow its numerical optimization.

In the DSGE context non-degenerate priors are typically selected to be centered around standard calibrated values of the structural parameters and are often motivated by the microeconomic evidence. Since the macro theory hardly ever gives us a guidance regarding the volatility of structural shocks, in the applications to be presented in this thesis, the priors are centered so that the model roughly replicates the volatility of the data. The standard errors of the prior distributions generally reflect subjective prior uncertainty faced by an analyst. One could also specify standard errors so as to cover the range of existing estimates. For details see Onatski and Williams (2010).

In some applications, it may be convenient to select diffuse priors over a fixed range to avoid imposing too much structure on the data. However, in the majority of applications, the form of the prior reflects computational convenience. Canova (2007) suggests to assume gamma or inverse gamma distribution for parameters bounded to be positive (e.g. standard deviation of structural shocks). Beta distributions for parameters bounded between zero and one (e.g. parameters of the shocks persistence, Calvo stickiness parameters, indexation parameters or habit persistence parameters). Normal distribution is recommended for the other parameters.

2.3.5 Computation of the data likelihood

This section is focused on the issues related to the computation of the likelihood of the state space model. The general concepts of filtering distribution and prediction distribution are introduced. Subsequently, the algorithms applicable for linear Gaussian models are presented.

2.3.5.1 Problem of filtering and prediction

As mentioned before the computation of the posterior requires beforehand the evaluation of the data likelihood. This, however, in a state space framework, is associated with the more general problem of estimation of the unobservable sequence of state variables $\{s_t\}_{t=1}^{T}$ using a set of observations $Y_T = \{y_t\}_{t=1}^{T}$. For fixed t we consider the problem of estimating s_t in terms of $Y_i = \{y_j\}_{j=1}^{i}$. If $i = t$ the problem is called filtering problem, $i < t$ defines a prediction problem and the case $i > t$ is refer to as a smoothing problem. Besides predicting the unobservable state, we consider here the problem of forecasting the observation vector y_t. Applying the squared error as optimality criterion, the best estimators of the state vector are functions $\hat{s}_t$ that satisfy

$$E\left[\left(s_t - s_t\left(Y_i\right)\right)\left(s_t - s_t\left(Y_i\right)\right)'\right] \geq E\left[\left(s_t - \hat{s}_t\left(Y_i\right)\right)\left(s_t - \hat{s}_t\left(Y_i\right)\right)'\right] \tag{2.18}$$

for every function $s_t\left(Y_i\right)$. The inequality sign denotes that the difference of the right hand side and the left hand side is a semidefinite matrix.

The optimal estimator in terms of 2.18 for s_t conditional on Y_i is given by the conditional expectation

$$\hat{s}_t = \left(Y_i\right) = E\left(s_t|Y_i\right) = \int s_t p\left(s_t|Y_i\right) ds_t \tag{2.19}$$

This means that for finding the optimal estimators $s_t\left(Y_t\right)$ (filtered state), $s_t\left(Y_{t-1}\right)$ (predicted state) and $s_t\left(Y_T\right)$ (smoothed state), one has to find the respective conditional densities. We will refer to them as filtering, prediction and smoothing densities $p\left(s_t|Y_t\right)$, $p\left(s_t|Y_{t-1}\right)$ and $p\left(s_t|Y_T\right)$, and then compute the conditional expectation given by 2.19. Similarly, for obtaining $\hat{y}_t\left(Y_{t-1}\right)$, the optimal one-step predictor for the observation vector, one has to find the conditional density $p\left(y_t|Y_{t-1}\right)$ in order to compute $E\left(y_t|Y_{t-1}\right)$.

Required conditional densities may be constructed iteratively. This procedure is described in algorithm 3. For derivation see Lemke (2005).

Algorithm 3: A generic algorithm for computation of conditional densities

1. Initialize the predictive density with $p\left(s_0|Y_0\right)=p\left(s_0\right)$.
2. Given the density $p\left(s_{t-1}|Y_{t-1}\right)$ compute the predictive density of the state vector

$$p\left(s_{t-1}|Y_{t-1}\right)=\int p\left(s_t|s_{t-1}\right)p\left(s_{t-1}|Y_{t-1}\right)ds_{t-1}. \tag{2.20}$$

3. Compute the predictive density of the observables

$$p\left(y_t|Y_{t-1}\right)=\int p\left(s_t|s_t\right)p\left(s_t|Y_{t-1}\right)ds_t. \tag{2.21}$$

4. Compute the filtering density of the state

$$p\left(s_t|Y_t\right)=\frac{p\left(y_t|s_t\right)p\left(s_t|Y_{t-1}\right)}{p\left(y_t|Y_{t-1}\right)}. \tag{2.22}$$

5. Repeat steps 2-4 until $t=T$.

The conditional density $p\left(s_t|Y_t\right)$ required for computing the smoothing estimates $E\left(s_t|Y_T\right)$ (for $t=1,\ldots,T-1$) is obtained by backward integration

$$p\left(s_t|Y_t\right)=p\left(s_t|Y_t\right)\int\frac{p\left(s_{t-1}|Y_T\right)p\left(s_{t-1}|s_t\right)}{p\left(s_{t+1}|Y_t\right)}ds_{t+1}. \tag{2.23}$$

Computation of $p\left(s_{t-1}|Y_T\right)$ is straightforward given the recursive formula 2.23. Since the state space model contains unknown parameters stored in the vector θ, both in transition and measurement equation, we are interested in the evaluation of the joint data density or the data likelihood $p\left(y_1,\ldots,y_T\right)$. This density may be written as a product of conditional densities using the so-called prediction error decomposition

$$p\left(y_1,\ldots,y_T\right)=\prod_{t=1}^{T}p\left(y_t|Y_{t-1}\right). \tag{2.24}$$

The conditional densities $p\left(y_t|Y_{t-1}\right)$ are obtained within the iterative procedure 2.20-2.22. Thus for a given parameter constellation θ the iterations above can be used to compute the log-likelihood

$$\ln L\left(Y_t|\theta\right)=\sum_{t=1}^{T}\ln p\left(y_t|Y_{t-1},\theta\right),$$

which is subsequently used to construct the log-posterior. Note that here we have explicitly added the argument θ, but also densities 2.20-2.24 are conditional on θ.

2.3.5.2 Prediction, filtering and likelihood of linear Gaussian models

The subsection above gives a general exposition of filtering and prediction problems. The integration steps required in algorithm 3 for functions $A_t(\cdot)$ and $B_t(\cdot)$ can be performed only under two very special circumstances. First, when the support of the state variables is discrete and finite then the integrals are just summations. Second, when the state and the measurement equations are both linear and the disturbances are Gaussian. In former case Sequential Monte Carlo (SMC) methods (e.g. Particle filter) can be applied.

Fernandez-Villaverde and Rubio-Ramirez (2006) is one of the first studies in which these techniques are used for DSGE models. The idea of this method is straightforward. The filtering density is obtained in two steps. First, draw a large number of realizations from the distribution s_{t+1} conditioned on y_t. Second, assign them a weight which is determined by their distance, computed using a measurement equation, from y_{t+1}. The advantage of SMC methods is that they are also applicable to non-linear approximations. The disadvantage is much more computational time, as opposed to Kalman filter, is required. SMC methods are very sensitive to outliers and degeneracies frequently arise. The researcher has to monitor carefully the numerical efficiency indicators of the SMC. The computations simplify if transition and measurement equations are linear, For the model given by 2.16 and 2.17 the transition density $p(s_{t+1}|s_t)$ and measurement density $p(y_t|s_t)$ are normal. It implies that also filtering and prediction densities are normal.

$$\begin{aligned} s_t|Y_{t-1} &\sim N\left(s_{t|t-1}, P_{t|t-1}\right) && (2.25)\\ s_t|Y_t &\sim N\left(s_{t|t}, P_{t|t}\right) && (2.26)\\ y_t|Y_{t-1} &\sim N\left(y_{t|t-1}, F_{t|t-1}\right) && (2.27)\end{aligned}$$

Since the multivariate normal densities are completely described by their means and covariance matrix, it is sufficient to find the sequences of conditional means, s_{t+1}, s_t, y_{t-1} and the sequences of conditional covariance matrices, $P_{t|t-1}$, $P_{t|t}$ and $F_{t|t-1}$ to evaluate the likelihood of the data. These quantities can be iteratively obtained from the Kalman filter. For the observer system the 2.16 and 2.17, the Kalman filter is described in algorithm 4.

Under normality assumptions 2.25-2.27, the distribution of y_t conditional on Y_{t-1} is the n-dimensional normal distribution with mean $y_{t|t-1}$ and variance-covariance matrix $F_{t|t-1}$. Thus, the conditional density of y_t can be according to Hamilton (1994) written

Algorithm 4: Kalman filter

1. Select initial conditions. If all eigenvalues of A are less than one in absolute value, set $s_{1|0} = E(s_1)$ and $P_{1|0} = AP_{1|0}A' + R\Sigma_u R'$ or $vec\left(P_{1|0}\right) = \left(I - (A \otimes A')^{-1}\right) vec\left(R\Sigma_u R'\right)$, in which case the initial conditions are the unconditional mean and variance of the process. When some of the eigenvalues of A are greater than one, initial conditions cannot be drawn from the unconditional distribution and one needs a guess (say, $s_{1|0} = 0$, $P_{1|0} = \kappa I$, κ is very large) to start the iterations. For more information about diffuse Kalman filter see Koopman and Durbin (2003).

2. Predict y_t and construct the mean square of the forecasts using the information from $t-1$.

$$E\left(y_{t|t-1}\right) = Bs_{t|t-1} \tag{2.28}$$

$$\begin{aligned} E\left(y_{t|t-1}\right)\left(y_{t|t-1}\right)' &= EB'\left(s_t - s_{t|t-1}\right)\left(s_t - s_{t|t-1}\right)' B + H\Sigma_m H' \\ &= B'P_{t|t-1}B + H\Sigma_m H' = F_{t|t-1} \end{aligned} \tag{2.29}$$

3. Update state equation estimates (after observing y_t)

$$s_{t|t} = s_{t|t-1} + P_{t|t-1}BF_{t|t-1}^{-1}\left(y_t - Bs_{t|t-1}\right) \tag{2.30}$$

$$P_{t|t} = P_{t|t-1} - P_{t|t-1}BF_{t|t-1}^{-1}BP_{t|t-1} \tag{2.31}$$

where $F_{t|t-1}$ is defined in 2.29.

4. Predict the state equation random variables next period

$$s_{t+1|t} = As_{t|t} = As_{t|t+1} + K_t v_t \tag{2.32}$$

$$P_{t+1|t} = AP_{t|t}A' + R\Sigma_u R' \tag{2.33}$$

where $v_t = y_t - \hat{y}_{t|t-1} = y_t - Bs_{t|t-1}$ is the one step ahead forecast error in predicting the observed variables vector and

$$K_t = AP_{t|t-1}BF_{t|t-1}^{-1} \tag{2.34}$$

is the Kalman gain.

5. Repeat steps 2-4 until $t = T$. The equations 2.30 and 2.31 provide the input for the next step of the recursion.

as

$$p\left(y_t|Y_t,\theta\right)=\left[(2\pi)^{n/2}\sqrt{\left|F_{t|t-1}\right|}\right]^{-1}\exp\left\{-1/2\,\left(y_t-\hat{y}_{t|t-1}\right)'F_{t|t-1}^{-1}\left(y_t-\hat{y}_{t|t-1}\right)\right\}.$$

The log-likelihood function becomes then

$$\begin{aligned}\ln L\left(Y_T|\theta\right) &= \ln p\left(y_1,\ldots,y_T|\theta\right)\\ &= -\frac{T_n}{2}\ln\left(2\pi\right)-\frac{1}{2}\sum_{t=1}^{T}\ln\left|F_{t|t-1}\right|-\frac{1}{2}\sum_{t=1}^{T}v'_tF_{t|t-1}^{-1}v_t.\end{aligned}\tag{2.35}$$

The function 2.35 only depends on the prediction errors $v_t = y_t - \hat{y}_{t|t-1}$ and their covariance matrices $F_{t|t-1}$ which are both the outputs of the Kalman filter.

Let's touch on the problems related to with applying the Kalman filter to a model with linear measurement and transition equation for which the errors are not Gaussian. This paragraph summarizes the discussion provided by Lemke (2005). If we drop the assumption of Gaussian errors, the Kalman filter outputs $s_{t|t-1}$, $y_{t|t-1}$ and $s_{t|t}$ still preserve an optimality property. They are the linear projections of s_t and y_t on Y_{t-1} and Y_t, respectively. Hence, they are estimators which have smallest mean square errors in the restricted class of all linear estimators. However, they are not conditional expectations any more, since in the non-Gaussian case, the conditional expectations function is generally nonlinear in the conditioning variables. If in the linear model the state innovations and measurement error are not Gaussian, one can still obtain estimates of the model parameters by falsely assuming normality, computing the log-likelihood by means of the Kalman filter, and maximizing it with respect to θ. This approach is known as quasi-maximum likelihood estimation. Under certain conditions it will still lead to consistent estimators which are asymptotically normally distributed.

2.3.6 Approximations of the posterior distribution

In this section the methods for approximation of the posterior distribution are presented. First, the simulation methods including Markov Chain Monte Carlo (MCMC) are presented. Secondly, the numerical optimization methods, used to locally approximate the posterior are described.

2.3.6.1 Posterior simulations

Having specified the likelihood and the prior, we proceed to analyze the posterior distribution. Knowledge of the posterior is required for implementation of the Bayesian inference, the objective of which is

$$E\left(h\left(\theta\right)|Y_T,M\right) = \int h\left(\theta\right)p\left(\theta|Y_T,M\right)d\theta \tag{2.36}$$
$$= \frac{\int h\left(\theta\right)L\left(Y_T|\theta,M\right)p\left(0|M\right)d\theta}{p\left(Y_T|M\right)}.$$

Since only the kernel of the posterior

$$p^*\left(\theta|Y_T,M\right) = L\left(Y_T|\theta,M\right)p\left(0|M\right)$$

is available but the marginal density $p\left(Y_T|M\right)$ is unknown, the above expression cannot be evaluated analytically. Only in a very special situations, the integral 2.36 can be approximated using the method of Monte Carlo integration. Then, producing a random sequence $\{\theta_k\}_{k=1}^{n_{sim}}$ using the kernel $p^*\left(\theta|Y_T,M\right)$ one can guarantee that

$$\frac{1}{n_{sim}}\sum_{k=1}^{n_{sim}} h\left(\theta_k\right) \to E\left(h\left(\theta\right)|Y_T,M\right)$$

"almost surely" as $n_{sim} \to \infty$. The almost surely means that convergence is subject to some regularity conditions of the function $h\left(\theta\right)$ specifically absolute convergence of the integral 2.36 must be satisfied. For details see Geweke (1996). However, as the posterior kernel is usually analytically intractable, it is likewise impossible to generate the random numbers from it directly. What may be done instead is to generate the random numbers from different analytically tractable distributions and correct these draws to better approximate the posterior distribution.

Formally, a sequence of $\{\theta_k\}_{k=1}^{n_{sim}}$ together with a generic weighting function $w\left(\theta_k\right)$ with the property that

$$\frac{\sum\limits_{k=1}^{n_{sim}} w\left(\theta_k\right)h\left(\theta_k\right)}{\sum\limits_{k=1}^{n_{sim}} w\left(\theta_k\right)} \to E\left(h\left(\theta\right)|Y_T,M\right) \text{ "almost surely" as } n_{sim} \to \infty$$

is the subject of interest. There is a huge amount of literature dealing with this issue. From acceptance sampling, importance sampling, to MCMC approaches (Gibbs sampler and the class of Metropolis-Hastings algorithms). The latter approach is the one which is applied in the recent literature on Bayesian analysis of DSGE models.

The MCMC methods have become very popular because there has been a dramatic decrease in the cost of computing in the last few years. In order to present the posterior simulations algorithms used in this thesis, we first familiarize the reader with the basic concepts of Markov chains and subsequently give a general idea of the MCMC method.

Introduction to Markov chains

Before introducing the MCMC methods, a few general and introductory comments on Markov chains are in order. Let X_t denote the value of a random variable at time t, and let the state space refer to the range of possible X values. The random variable is a Markov process if the transition probabilities between different values in the state space depend only on the random variables current state

$$P\left(X_{t+1} = z_j | X_0 = z_k, \ldots, X_t = z_i\right) = P\left(X_{t+1} = z_j | X_t = z_i\right)$$

Thus for a Markov random variable the only information about the past needed to predict the future is the current state of the random variable. A Markov chain refers to a sequence of random variables $(X_0, \ldots, X_n)$ generated by a Markov process. A particular chain is defined by its transition probabilities, $p(i,j) = P(i \to j)$, which is the probability that a process at state z_i moves to z_j in a single step.

$$p(i,j) = P(i \to j) = P\left(X_{t+1} = z_j | X_t = z_i\right)$$

Let

$$\pi_j(t) = P\left(X_t = z_j\right)$$

denote the probability that the chain is in state j at time t, and let $\pi(t)$ denote the vector pf the state space probabilities at step t. We start the chain by specifying a starting vector $\pi(0)$.

The probability that the chain has state value z_i at step $t+1$ is given by ChapmanKolmogorov equation[2], which sums over the probabilities of being in a particular state at the current step and the transition probability from that state into state z_i,

$$\begin{aligned}
\pi_i(t+1) &= P\left(X_{t+1} = z_i\right) \\
&= \sum_k P\left(X_{t+1} = z_i | X_t = z_k\right) P\left(X_t = z_k\right) \\
&= \sum_k P(k \to i)\, \pi_k(t) = \sum_k P(k,i)\, \pi_k(t)
\end{aligned}$$

Iterations of Chapman-Kolmogorov equation describe the evolution of the Markov chain. More compactly the Chapman-Kolmogorov equation may be written in a matrix form as follows. Define the transition matrix P as the matrix whose $P_{i,j}$ element is equivalent

[2] In mathematics, specifically in probability theory and in particular the theory of Markovian stochastic processes, the ChapmanKolmogorov equation is an identity relating the joint probability distributions of different sets of coordinates on a stochastic process. The equation was arrived at independently by both the British mathematician Sydney Chapman and the Russian mathematician Andrey Kolmogorov.

to the probability $P(i,j)$. This implies that $p(i,j) = P(i \to j) = 1$. The Chapman-Kolmogorov equation becomes

$$\pi(t+1) = \pi(t)P.$$

Iterating the above equation yields

$$\pi(t) = \pi(0)P^t.$$

Defining the n-step transition probability $\pi_{i,j}^{(n)}$ as the probability that the process is in the state j given that it started in state i n periods ago. For example,

$$\pi_{i,j}^{(n)} = P(X_{t+n} = z_j | X_t = z_i)$$

this probability is also an i,j element of P^n.

Finally, a Markov chain is said to be a irreducible if for all i,j and $n\pi_{i,j}^{(n)} > 0$. That is, all states communicate with each other. A chain is aperiodic when the number of steps required to move between two states is not required to be multiple of some integers. A Markov chain may also reach a stationary distribution π^*, where the vector of probabilities of being in any particular state is independent on the initial condition. This distribution satisfies

$$\pi^* = \pi^* P.$$

The conditions for existence of a stationary distribution π^* are that the chain is irreducible and aperiodic. Sufficient conditions for a unique stationary distribution are detailed as follows

$$P(j \to k)\pi_j^* = P(k \to j)\pi_k^*. \tag{2.37}$$

If equation 2.37 holds for all i,k the Markov chain is said to be reversible. This reversibility condition implies that

$$(\pi P)_j = \sum_i \pi_i P(i \to j) = \sum_i \pi_j P(j \to i) = \sum_i P(j \to i) = \pi_j.$$

The basic idea of discrete-state Markov-chain can be generalized to a continuous state Markov process by having a probability kernel $P(x,y)$ that satisfies

$$\int P(x,y)\,dy = 1$$

and the continuous extension of Chapman-Kolmogorov equation is

$$\pi_t(y) = \int \pi_{t-1}(x)P(x,y)\,dy.$$

Finally, the stationary distribution satisfies

$$\pi^*(y) = \int \pi^*(x) P(x, y)\, dy.$$

Markov Chain Monte Carlo methods

The main problem with applying Monte Carlo is in obtaining samples from complex probability distribution, $p(\theta|Y_T, M)$, in our case. The ability to solve this problem is a root of MCMC methods. In particular, they trace to attempts by mathematical physicists to integrate very complex functions by random sampling and the resulting Metropolis-Hastings (M-H) algorithm. For details see Metropolis, Rosenbluth, Rosenbluth, Teller, and Teller (1953) and Hastings (1970). A detailed review of of this method is given by Neal (1993) and Geweke (1998).

The main goal is to draw samples from the distribution $p(\theta|Y_T, M) = \frac{p^*(\theta|Y_T,M)}{p(Y_T|M)}$, where $p(Y_T|M)$ may be treated as an unknown normalizing constant, which is in fact very difficult to compute. The Metropolis algorithm according to Metropolis, Rosenbluth, Rosenbluth, Teller, and Teller (1953) generates a sequence of draws from this distribution. The Metropolis algorithm is described in algorithm 5.

Algorithm 5: The Metropolis algorithm

---

1. Start with any initial value θ_0 satisfying $p^*(\theta|Y_T, M) > 0$. Set $k = 0$.
2. Using the current value of θ, sample a candidate point $\theta^{candidate}$ from some jumping distribution $q(\theta_1, \theta_2)$, which is the probability of returning a value θ_2 given a previous value of θ_1. This distribution is also referred to as the proposal or candidate-generating distribution. The only restriction on the jump density is that it is symmetric, it means that $q(\theta_1, \theta_2) = q(\theta_2, \theta_1)$.
3. Given the candidate point $\theta^{candidate}$, calculate the ratio of the density at the candidate point $\theta^{candidate}$ and the current point θ_{k-1}

$$r = \frac{p\left(\theta^{candidate}|Y_T, M\right)}{p(\theta_{k-1}|Y_T, M)} = \frac{p^*\left(\theta^{candidate}|Y_T, M\right)}{p^*(\theta_{k-1}|Y_T, M)}.$$

   Notice that one consider the ratio $p(\theta|Y_T, M)$ under two different values of θ the constant $p(Y_T|M)$ cancels out.
4. If the jump increases the density $(r > 1)$, accept the candidate point $\theta_k = \theta^{candidate}$ and return to step 2. If the jump decreases the density $(r < 1)$ then with probability r accept the candidate point, else reject it, set $\theta_k = \theta_{k-1}$ and return to step 2. Do until $k = n_{sim}$.

We can summarize the Metropolis sampling as first computing

$$r = \frac{p\left(\theta^{candidate}|Y_T, M\right)}{p\left(\theta_{k-1}|Y_T, M\right)} = \frac{p^*\left(\theta^{candidate}|Y_T, M\right)}{p^*\left(\theta_{k-1}|Y_T, M\right)}$$

and then accepting a candidate point with probability r which represents the probability of move.[3] This generates a markov chain $(\theta_0, \theta_1, \ldots, \theta_{n_{sim}})$ as the probability from θ_k to θ_{k+1} depends only on θ_k and not on the history of the chain. Following a sufficient burn-in period, the chain approaches its stationary distribution and the samples from the vector $\left(\theta_{n_{burn-in}}, \ldots, \theta_{n_{sim}}\right)$ are samples from the distribution of interest $p\left(Y_T|M\right)$. Hastings (1970) generalized the Metropolis algorithm by using an arbitrary probability function $q\left(\theta_1, \theta_2\right) = P\left(\theta_1 \to \theta_2\right)$ and setting the acceptance probability for a candidate point as

$$r = \min\left(\frac{p^*\left(\theta_k|Y_T, M\right) q\left(\theta^{candidate}, \theta_{k-1}\right)}{p^*\left(\theta_{k-1}|Y_T, M\right) q\left(\theta_{k-1}, \theta^{candidate}\right)}, 1\right).$$

Assuming that the proposal distribution is symmetric. For example, $q\left(x, y\right) = q\left(y, x\right)$, the original Metropolis algorithm may be according to Chib and Greenberg (1995) recovered.

Metropolis-Hastings algorithm

Next, the Metropolis-Hastings (M-H) algorithm is demonstrated. M-H sampling generates a Markov chain whose equilibrium density is the candidate density $p\left(x\right)$ (here $p\left(x\right)$ is a shortcut for our density of interest $p\left(\theta|Y_T, M\right)$). To shows this, it is sufficient that the M-H transition kernel satisfies equation 2.37. This demonstration is based on Chib and Greenberg (1995). Using M-H algorithm it is sampled from $q\left(x, y\right) = P\left(x \to y|q\right)$ and then accept the move probability $r\left(x, y\right)$, and the transition probability kernel is given as follows

$$P\left(x \to y\right) = q\left(x, y\right) r\left(x, y\right) = q\left(x, y\right) \min\left(\frac{p\left(y\right) q\left(y, x\right)}{p\left(x\right) q\left(x, y\right)}, 1\right).$$

If the M-H kernel satisfies $P\left(x \to y\right) p\left(x\right) = P\left(y \to x\right) p\left(y\right)$ or $q\left(x, y\right) r\left(x, y\right) p\left(x\right) = q\left(y, x\right) r\left(y, x\right) p\left(y\right)$ for all x, y then that stationary distribution from this kernel corresponds to draws from the target distribution. Below the three possible cases are analyzed.

1. Let $q\left(x, y\right) p\left(x\right) = q\left(y, x\right) p\left(y\right)$. Hence $r\left(x, y\right) = r\left(y, x\right) = 1$ implying $P\left(x, y\right) p\left(x\right) = q\left(y, x\right) p\left(x\right)$ and $P\left(y, x\right) p\left(y\right) = q\left(y, x\right) p\left(y\right)$ and hence $P\left(x, y\right) p\left(x\right) = P\left(y, x\right) p\left(y\right)$,

[3] The Metropolis algorithm uses the mechanism of acceptance-rejection sampling. The basic idea of this mechanism is to generate a random vector from a distribution that is similar to the approximated distribution and then to accept that draws with probability that depends on the drawn value of the vector. If this acceptance probability function is chosen correctly then the accepted values will have the desired distribution.

fulfilling the reversibility condition 2.37.

2. Let $q(x,y)\,p(x) > q(y,x)\,p(y)$, in which case

$$r(x,y) = \frac{p(y)\,q(y,x)}{p(x)\,q(x,y)} \text{ and } r(x,y) = 1.$$

Hence

$$\begin{aligned} P(x,y) &= q(x,y)\,r(x,y)\,p(x) \\ &= q(x,y)\,\frac{p(y)\,q(y,x)}{p(x)\,q(x,y)}p(x) \\ &= q(y,x)\,p(y) \\ &= q(y,x)\,r(y,x)\,p(y) \\ &= P(y,x)\,p(y)\,. \end{aligned}$$

3. Let $q(x,y)\,p(x) < q(y,x)\,p(y)$. Then

$$r(x,y) = 1 \text{ and } r(y,x) = \frac{p(x)\,q(x,y)}{p(y)\,q(y,x)}.$$

Hence

$$\begin{aligned} P(y,x) &= q(y,x)\,r(y,x)\,p(y) \\ &= q(x,y)\,\frac{p(x)\,q(x,y)}{p(y)\,q(y,x)}p(y) \\ &= q(x,y)\,p(x) \\ &= q(x,y)\,r(x,y)\,p(x) \\ &= P(x,y)\,p(x)\,. \end{aligned}$$

Choosing a Jumping distribution

There are existences of two general approaches for choosing the jumping distribution. One may decide either for random walks or independent chain sampling. While using the proposal distribution based on a random walk chain, the new value of y equals to current value x plus a random variable z. In this case $q(x,y) = g(y-x) = g(z)$, the density associated with the random variable z. If $g(z) = g(-z)$, the density for the random variable z is symmetric, then the Metropolis sampling can be used as $q(x,y)/q(y,x) = g(z)/g(-z) = 1$. The variance of the proposal distribution is selected to get better mixing. Under a proposal distribution using an independent chain, the probability of jumping to point y is independent of the current position x of the chain. Thus, the candidate value is simply drawn from a distribution of interest, independent of the current value. Any number of standard distributions can be used for $g(y)$. In this case, the proposal distribution is generally not symmetric, as $g(x)$ is generally not equal to $g(y)$, and M-H sampling must be used. For details see Walsh (2004). In this thesis the posterior simulations are performed using the Random Walk Metropolis (RWM) algorithm. For details see Schorfheide (2000) and description in algorithm 6.

Algorithm 6: Random Walk Metropolis algorithm

1. Use a numerical optimization routine to maximize the logarithm of the posterior kernel $\ln L(Y_T|\theta) + \ln p(\theta)$. Denote the posterior mode by $\tilde{\theta}$. See algorithm 8 below. Or algorithm 9 may be applied in case that the algorithm 8 does not provide solution due to absence of positive definite matrix.
2. Let $\Sigma_{\tilde{\theta}}^{-1}$ be the inverse of the numerically computed Hessian at the posterior mode $\tilde{\theta}$.
3. Draw θ_0 from $N\left(\tilde{\theta}, \Sigma_{\tilde{\theta}}^{-1}\right)$. Draw from the multivariate normal distribution centered at the posterior mode.
4. For $k = 1, \ldots, n_{sim}$ draw from the proposal distribution $N\left(\theta_{k-1}, c\Sigma_{\tilde{\theta}}^{-1}\right)$ centered at the last accepted draw. The distribution $N\left(\theta_{k-1}, c\Sigma_{\tilde{\theta}}^{-1}\right)$ corresponds to the transition distribution q defined above. c is the scaling factor set to improve the efficiency of the algorithm. For details see Gelman, Carlin, Stern, and Rubin (2003). The jump from θ_{k-1} is accepted ($\theta_k = \theta^{candidate}$) with probability $\min\left[1, r\left(\theta_{k-1}, \theta^{candidate}|Y_T\right)\right]$ and reject ($\theta_k = \theta_{k-1}$) otherwise (see the acceptance-rejection sampling in footnote 3). Here

$$r\left(\theta_{k-1}, \theta^{candidate}|Y\right) = \frac{\exp\left(\ln L\left(Y_T|\theta^{candidate}\right) + \sum_{i=1}^{N} \ln p\left(\theta_i^{candidate}\right)\right)}{\exp\left(\ln L\left(Y_T|\theta_{k-1}\right) + \sum_{i=1}^{N} \ln p\left(\theta_{i,k-1}\right)\right)}.$$

Parameter constellations not yielding the unique stable solution are rejected.

The series of accepted draws $\{\theta_{k-1}\}_{k=1}^{n_{sim}}$ is serially correlated, therefore the number of draws n_{sim} and scaling factor c should be chosen to assure that the sequence $\{\theta_{k-1}\}_{k=n_{burn\_in}}^{n_{sim}}$ converges to the posterior distribution. In its simplistic form the convergence check may be performed using the CUMSUM statistics for each element θ^i of the vector θ

$$CUMSUM_{\theta^i}(j) = \frac{1}{j}\sum\nolimits_j \frac{\theta_j^i - \bar{\theta}_j^i}{\sqrt{\operatorname{var}\left(\theta_j^i\right)}}, \text{ where} j = 1, 2, \ldots, J.$$

In order to avoid the so-called local optima problem, it is reasonable to start the optimization from different points in the parameter space to increase the likelihood that the global optimum is found. Similarly, in Bayesian computation it is helpful to start MCMC from different regions of the parameter space or simply run the parallel posterior simulations and check whether the results in all blocks converge. The convergence can be assessed by comparing variation between and within simulated sequences until within variation approximates between variation, as suggested by Gelman, Carlin, Stern, and Rubin (2003).

Only when the distribution of each sequence is close to that of all the sequences mixed together, they can all be used to approximate the posterior distribution. The between-chain variance and pooled within-chain variance are defined by

$$B = \frac{n_{sim}}{m-1}\sum_{j=1}^{m}\left(\hat{\theta}_{\cdot j} - \hat{\theta}_{\cdot\cdot}\right)^2,$$

where $\hat{\theta}_{\cdot j} = \frac{1}{n_{sim}}\sum\limits_{i=1}^{n_{sim}}\theta_{ij}$ and $\hat{\theta}_{\cdot\cdot} = \frac{1}{m}\sum\limits_{j=1}^{m}\hat{\theta}_{\cdot j}$,

$$W = \frac{1}{m}\sum_{j=1}^{m}s_j^2,$$

where $s_j^2 = \frac{1}{n_{sim}-1}\sum\limits_{i=1}^{n_{sim}}\left(\theta_{ij} - \hat{\theta}_{\cdot j}\right)^2$ and m is the number of sequence and n_{sim} the number of draws in each sequence. The marginal posterior variance of each parameter will be a weighted average of W and B

$$\widehat{\mathrm{var}\left(\theta|Y\right)} = \frac{n_{sim}-1}{n_{sim}}W + \frac{1}{n_{sim}}B.$$

To check the convergence we calculate the Potential Scale Reduction Factor (PSRF) for each parameter

$$\hat{R} = \sqrt{\frac{\widehat{\mathrm{var}\left(\theta|Y\right)}}{W}},$$

which declines to 1 as $n \to \infty$. If the PSRF is high, one should proceed with further simulations to improve the inference. It is common practice to complement the convergence measures by visualization of the MCMC chains. These visualizations are useful especially when analyzing reasons of convergence problems. For details see Brooks and Gelman (1998).

If convergence is satisfactory, the posterior expected value of a parameter function $h(\theta)$ might be approximated by $\frac{1}{n_{sim}}\sum\limits_{k=1}^{n_{sim}}h(\theta_k)$, which is the step 5 of Algorithm 1. Throughout the study the approximations of the posterior distribution of the following parameter functions are used: the posterior mean of the parameters, parameter 90% confidence intervals as well as confidence intervals for the impulse response functions and replicated moments of the variables.

2.3.6.2 Numerical optimization of the posterior

In order to increase efficiency, the RWM algorithm starts at the posterior mode. The computation of the posterior mode and the matrix of second derivatives at the posterior mode may also be useful for local approximations of the posterior and subsequently for evaluating the marginal density. The maximization of the log-posterior may be performed by extending the Kalman filter algorithm. For details see algorithm 7.

Algorithm 7: Procedure for numerical optimization of the posterior

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1. Choose some initial $\theta = \theta_0$.
2. Do steps 1-4 of algorithm 4 (Kalman filter).
3. After each step save $v_t = y_t - y_{t|t-1}$ and $F_{t|t-1}$. Construct the log-likelihood using prediction error decomposition 2.35. Assign the prior and compute the log-posterior 2.5.
4. Update initial estimates of θ using the unconstrained optimization routine which is represented by algorithm 8.
5. Repeat steps 2-4 until a convergence criterion is met.

Algorithm 7 is accomplished by Newton's type optimization routine. Consider the following maximization problem $\max \ln p\left(\theta|Y_T, M\right)$. Suppose that $\ln p\left(\theta|Y_T, M\right)$ is twice continuously differentiable with respect to θ. The first order necessary condition means that if $\ln p\left(\theta|Y_T, M\right)$ achieve its minimum at a point $\tilde{\theta}$, then

$$\nabla \ln p\left(\tilde{\theta}|Y_T, M\right) = 0, \tag{2.38}$$

where $\tilde{\theta}$ is a stationary point. Since function $\ln p\left(\theta|Y_T, M\right)$ is complicated and it is impossible to solve 2.38 analytically, numerical methods are required.

The basic idea of Newton's method is to generate a sequence of points approximating a solution of 2.38. In particular, the Taylor approximation of

$$\nabla \ln p\left(\theta|Y_T, M\right) \approx g_0 + \Sigma_0\left(\theta, \theta_0\right)$$

is considered, where $g_0 = \ln p\left(\theta|Y_T, M\right)$ and $\Sigma_0 = \nabla^2 \ln p\left(\theta_0|Y_T, M\right)$. The fundamental idea of this method is to solve the linear system of equations given by

$$g_0 + \Sigma_0\left(\theta - \theta_0\right) = 0 \tag{2.39}$$

instead of 2.38 and take the solution of 2.39 as a new solution to 2.38. In general one can write the Newton's method as $\theta_{k+1} = \theta_k - g_k \Sigma_k^{-1}$ for $k = 0, 1, 2, \ldots$.

Newton's method is in its original form ineffective for the optimization of the posterior distributions of DSGE models. This is because the method requires the evaluation of the Hessian matrix at each step, which is computationally extremely expensive. The method also does not guarantee that the sequence of $\ln p(\theta|Y_T, M)$ at each step is monotonically decreasing. For this reason is applied the quasi-Newton method which is described in algorithm 8. It is assumed that one is able to calculate the sequence of estimates of Σ_k^{-1}.

Algorithm 8: The Quasi-Newton method with line search

1. Choose some initial $\theta = \theta_0$, set $k = 0$.
2. Calculate gradient $g_k = \nabla \ln p(\theta_k|Y_T, M)$ and estimate (using the BFGS method which is defined below) the inverse Hessian Σ_k^{-1}. When $g_k = 0$ then stop. In this thesis is used the $'csminwel'$ algorithm which is developed by Chris Sims. This algorithm is also robust against certain pathologies common on likelihood functions, e.g. 'cliffs', i.e. hyperplane discontinuities.
3. Find the maximum of quadratic approximation of the posterior $\ln p(\theta_k|Y_T, M)$. Since the posterior is in fact not quadratic solve for the optimum iteratively setting $\theta_{k+1} = \theta_k + d_k$, where $d_k = -\Sigma_k^{-1} g_k$ is called the direction of search. The direction is a vector describing a segment of a path from the starting point to the solution, where the inverse of the Hessian Σ_k^{-1} determines the angle of the direction and the gradient, g_k determines its size extreme. Check if under parametrization θ_{k+1} the DSGE model yields the unique stable solution and if $\theta_{k+1} \in \Theta$. If any of these conditions are not met then set $\ln p(\theta_{k+1}|Y_T, M) = -\infty$.
4. When the quadratic approximation of the posterior $\ln p(\theta_k|Y_T, M)$ is good, the Hessian is well conditioned and the convergence quadratic. In the case of DSGE models, the posterior, i.e. the function being optimized, can be not well behaved in the region of θ_k. To deal with this, the Newton is redefined as $\theta_{k+1} = \theta_k + \alpha_k d_k$, where α_k is called the step length and is determined by a local optimization of the function, called a line search, that is given the direction and the starting point $\alpha_k = \arg\min \ln p(\theta_k - \alpha_k d_k|Y_T, M)$.
5. Set $k = k + 1$ go to step 2. Repeat steps 2-4 until the convergence criterion is met. The convergence criterion is represented by a relative gradient, a gradient adjusted for scaling and may be stated as $\max \left| \ln p(\theta_{k+1}|Y_T, M) \frac{g_{k+1}}{\theta_{k+1}} \right| < \varepsilon$, where ε is very small number.

In the implementations of the quasi-Newton algorithm, one normally requires that the length of the step α_k satisfies the Wolfe conditions

$$\begin{aligned} \ln p(\theta_k + \alpha_k d_k|Y_T, M) - \ln p(\theta_k|Y_T, M) &\leq \delta_1 \alpha_k d'_k g_k, \\ d'_k \nabla \ln p(\theta_k + \alpha_k d_k|Y_T, M) &\geq \delta_2 d'_k g_k, \end{aligned}$$

where $\delta_1 \leq \delta_2$ are constant in $(0, 1)$. Now, it is explained how is the Σ_k^{-1} calculated. It is the key point of quasi-Newton method. Suppose, that $g_0 = \nabla \ln p(\theta_0|Y_T, M)$, $\Sigma_0 = \nabla^2 \ln p(\theta_0|Y_T, M)$ and θ_1 are calculated by Newton's method. Instead of calculating $\nabla^2 \ln p(\theta_1|Y_T, M)$, it is important to find the matrix Σ_1 to replace the $\nabla^2 \ln p(\theta_1|Y_T, M)$. Note that,

$$\nabla \ln p(\theta_0|Y_T, M) - \nabla \ln p(\theta_1|Y_T, M) \approx \nabla^2 \ln p(\theta_0|Y_T, M)(\theta_0 - \theta_1)$$

and the Σ_1 has to satisfy the following condition

$$\nabla \ln p(\theta_0|Y_T, M) - \nabla \ln p(\theta_1|Y_T, M) = \Sigma_1(\theta_0 - \theta_1)$$

or equivalently to find Σ_1^{-1} such that

$$\Sigma_1^{-1}(\nabla \ln p(\theta_0|Y_T, M) - \nabla \ln p(\theta_1|Y_T, M)) = (\theta_0 - \theta_1)$$

This condition is called quasi-Newton condition. In general it may be written as follows

$$\Sigma_{k+1}^{-1}\gamma_k = \delta_k, \tag{2.40}$$

where $\delta_k = \nabla \ln p(\theta_{k+1}|Y_T, M) - \nabla \ln p(\theta_k|Y_T, M)$ and $\delta_k = \theta_{k+1} - \theta_k$. If the matrix Σ_{k+1}^{-1} can be found then one is able to compute the search direction $d_{k+1} = -\Sigma_{k+1}^{-1}g_{k+1}$. However, the matrix satisfying 2.40 is not unique. The general idea to construct the Σ_{k+1}^{-1} is to update it from Σ_k^{-1} using the gradient information at both θ_k and θ_{k+1}. The most important methods for estimation of the inverse Hessian matrix are the Broyden's method, see Broyden (1965) and BFGS method, see Broyden (1970), Fletcher (1970), Goldfarb (1970) and Shanno (1970). The best performing method is BFGS, see Dai (2002). The BFGS method uses rank two correction form Σ_k^{-1}, i.e.

$$\Sigma_{k+1}^{-1} = \Sigma_k^{-1} + auu' + bvv',$$

where Σ_{k+1}^{-1} must also satisfy the quasi-Newton condition

$$\Sigma_k^{-1}\gamma_k + auu'\gamma_k + bvv'\gamma_k = \delta_k,$$

where u and v are not unique in this case. An obvious solution is $u = \delta_k$, $v = \Sigma_k^{-1}\gamma_k$, $a = 1/(u'\gamma_k)$ and $b = -1/(v'\gamma_k)$. Hence

$$\Sigma_{k+1}^{-1} = \Sigma_k^{-1} + \frac{\delta_k \delta'_k}{\delta'_k \gamma_k} - \frac{\Sigma_k^{-1}\gamma_k \gamma'_k \Sigma_k^{-1}}{\left(\Sigma_k^{-1}\gamma_k\right)'\gamma_k}. \tag{2.41}$$

Using the 2.41 can be transformed into

$$\Sigma_{k+1} = \Sigma_k + \frac{\delta_k {\delta'}_k}{{\delta'}_k \gamma_k} - \frac{\Sigma_k \gamma_k {\gamma'}_k \Sigma_k}{(\Sigma_k \gamma_k)' \gamma_k}.$$

Algorithm 9: Monte Carlo Optimization

1. In some situations the posterior mode (that will be used to initialize the Metropolis Hastings (MH) and to define the jumping distribution) is hard to obtain with standard (newton like) optimization routines.

2. For the MH algorithm we don't need to start from the posterior mode. We only need to start from a point (in parameters space) with a high posterior density value and to use a good covariance matrix for the jumping distribution.

3. The idea is to use a MH algorithm with a diagonal covariance matrix (prior variances or a covariance matrix proportional to unity) and to continuously update the posterior covariance matrix and the posterior mode estimates through the MH draws.

4. After each MH-draw θ_t in the posterior distribution we update the posterior mean, the posterior mode and the posterior covariance as follows

$$\mu_t = \mu_{t-1} + \frac{1}{t}\left(\theta_t - \mu_{t-1}\right),$$

$$\Sigma_t = \Sigma_{t-1} + \mu_{t-1}{\mu'}_{t-1} - \mu_t {\mu'}_t + \frac{1}{t}\left(\theta_t {\theta'}_t - \Sigma_{t-1} - \mu_{t-1}{\mu'}_{t-1}\right),$$

$$\text{mode}_t = \begin{cases} \text{if } p\left(\theta_t|Y\right) \mathrm{p}\left(\text{mode}_{t-1,}|Y\right) \text{ then } \theta_t \\ \qquad \text{otherwise mode}_{t-1}. \end{cases}.$$

The inverse Hessian is calculated as

$$\Sigma_{k+1}^{-1} = \Sigma_k^{-1} + \left(1 + \frac{{\gamma'}_k \Sigma_k^{-1} \gamma_k}{\delta_k \gamma_k}\right) \frac{\delta_k {\delta'}_k}{{\delta'}_k \gamma_k} - \left(\frac{\delta_k \gamma' \Sigma_k^{-1} + \Sigma_k^{-1} \gamma_k {\delta'}_k}{{\delta'}_k \gamma_k}\right).$$

Since Σ_{k+1}^{-1} is the unique solution of the following problem

$$\left|\Sigma^{-1} - \Sigma_k^{-1}\right| \to \min_{\Sigma^{-1}},$$

subject to

$$\begin{aligned} \Sigma^{-1} &= \left(\Sigma^{-1}\right)', \\ \Sigma^{-1}\gamma_k &= \delta_k, \end{aligned}$$

this means that, among all symmetric matrices satisfying the quasi-Newton condition, Σ_{k+1}^{-1} is the closest to the current matrix Σ_{k+1}^{-1}.

2.3.7 Model evaluation

The last step in the Bayesian estimation procedure is the evaluation of the model. This section is focused on the assessment of model's relative fit which is typically conducted by applying Bayesian inference and decision theory to the extended model space. The assessment of absolute fit of the model can be implemented by a sampling based model check. The model is considered as inaccurate if it is very unlikely to reproduce with the particular feature of the data. Such model checks though they provide valuable insights about the overall quality of the estimated model.

2.3.7.1 Assessment of the DSGE models

A natural method to assess the empirical validity of the DSGE model is to compare its predictive performance (measured by integrated or marginal likelihood) with other available models including DSGE models or perhaps an even larger class of non-structural linear reduced form models. Marginal likelihood $p\left(Y_T|M_i\right)$ measures how well model M_i predicts the observed data Y_T. First, consider the distribution of the sequence $y_{u+1},\ldots,y_t$ conditional on the data Y_u and model M_i, for details see Geweke (1998).

$$p\left(y_{u+1},\ldots,y_t|Y_u,M_i\right)=\int p\left(\theta|Y_u,M_i\right)\prod_{s=u+1}^{t}p\left(y_s|Y_{s-1},\theta,M_i\right)d\theta. \tag{2.42}$$

2.42 may be interpreted as the predictive density of $y_{u+1},\ldots,y_t$ conditional on Y_u and model M_i, because the judgment on $y_{u+1},\ldots,y_t$ is done based on Y_u and before observing $y_{u+1},\ldots,y_t$. Following the observation of $y_{u+1},\ldots,y_t$ expression 2.42 is the known number the so-called predictive likelihood of $y_{u+1},\ldots,y_t$ conditional on Y_u and the model M_i. Furthermore, $p\left(y_1,\ldots,y_t|Y_0,M_i\right)=P\left(Y_t|M_i\right)$ if $Y_0=\{\varnothing\}$. Substituting for the posterior density in 2.42 is obtained

$$\begin{aligned}
p\left(y_1,\ldots,y_t|Y_0,M_i\right) &= \int\left\{\begin{array}{c}\frac{p(\theta|M_i)\prod_{s=1}^{u}p(y_s|Y_{s-1},\theta,M_i)}{\int p(\theta|M_i)\prod_{s=1}^{u}p(y_s|Y_{s-1},\theta,M_i)d\theta}\\ \times\prod_{s=u+1}^{t}p\left(y_s|Y_{s-1},\theta,M_i\right)\end{array}\right\}d\theta\\
&= \frac{p\left(\theta|M_i\right)\prod_{s=1}^{t}p\left(y_s|Y_{s-1},\theta,M_i\right)d\theta}{p\left(\theta|M_i\right)\prod_{s=1}^{u}p\left(y_s|Y_{s-1},\theta,M_i\right)d\theta}\\
&= \frac{p\left(Y_t|M_i\right)}{p\left(Y_u|M_i\right)}.
\end{aligned}$$

Hence for any $0 \leq u = s_0 < s_q = t$, is obtained

$$\begin{aligned} p\left(y_1, \ldots, y_t | Y_0, M_i\right) &= \frac{p\left(Y_{s_1}|M_i\right)}{p\left(Y_{s_0}|M_i\right)} \frac{p\left(Y_{s_2}|M_i\right)}{p\left(Y_{s_1}|M_i\right)} \cdots \frac{p\left(Y_{s_q}|M_i\right)}{p\left(Y_{s_{q-1}}|M_i\right)} \\ &= \prod_{l=1}^{q} p\left(y_{s_{l-1}+1}, \ldots, y_{s_l}|Y_{s_{l-1}}, M_i\right). \end{aligned}$$

This decomposition shows that the marginal likelihood, if $u = 0$ and $t = T$, summarizes the out of sample model performance as expressed in predictive likelihoods $p\left(Y_T|M_i\right) = \prod_{l=1}^{q} p\left(y_{s_{l-1}+1}, \ldots, y_{s_1}|Y_{s_{l-1}}, M_i\right)$. The computation of the marginal likelihood $P\left(Y_t|M_i\right)$ and more precisely the computation the integral 2.4 is unfeasible analytically in most cases. There have been proposed methods for estimation of the marginal likelihood using a sample from the posterior distribution. The most popular are the estimators by Geweke (1998). Alternatively, the calculation of integral 2.4 may be based on the local approximations of the posterior. The marginal data density of the DSGE model is approximated by Geweke's modified harmonic mean estimator. Harmonic mean estimator is based on the following identity

$$\frac{1}{p\left(Y_T|M_i\right)} = \int \frac{f\left(\theta\right)}{L\left(Y_T|\theta, M_i\right) p\left(\theta\right)} p\left(\theta|Y_T, M_i\right) d\theta,$$

where $f\left(\theta\right)$ has the property that $\int f\left(\theta\right) f\theta = 1$. Conditional on the choice of θ the estimator of $p\left(Y\right)$ is

$$\hat{p}\left(Y_T|M_i\right) = \left[\frac{1}{n_{sim}} \sum_{k=1}^{n_{sim}} \frac{f\left(\theta_k\right)}{L\left(Y_T|\theta_k, M_i\right) p\left(\theta_k\right)}\right]^{-1},$$

where θ_k is drawn from the posterior distribution $\max \ln p\left(\theta|Y_T, M\right)$ using Random Walk Metropolis algorithm 6. To make the numerical approximation efficient, $f\left(\theta\right)$ is chosen so that the summands are of equal magnitude. Geweke (1998) proposed to use the density of a truncated multivariate normal distribution

$$\begin{aligned} f\left(\theta\right) = \tau^{-1} (2\pi)^{-1d/2} \left|V_\theta\right|^{-1/2} \exp\left(-0.5\left(\theta - \bar{\theta}\right)' V_\theta^{-1} \left(\theta - \bar{\theta}\right)\right) \\ \times p\left\{\left(\theta - \bar{\theta}\right)' V_\theta^{-1} \left(\theta - \bar{\theta}\right) \leq F_{\chi_N^2}^{-1}\left(\tau\right)\right\}, \end{aligned}$$

where $\bar{\theta}$ and V_θ are the posterior mean and posterior covariance matrix. N is the dimension of parameter vector θ, $F_{\chi_N^2}$ is the cumulative density of a χ^2 random variable with N degrees of freedom and $\tau \in (0, 1)$. When the likelihood is highly peaked around the mode and close to symmetric, the posterior kernel density can be locally approximated

Value of Bayes factor	**Intepretation**
$B_{ij} < 1$	support M_j
$1 \leq B_{ij} < 3$	very slight support for M_j
$3 \leq B_{ij} < 10$	slight evidence against M_j
$10 \leq B_{ij} < 100$	strong evidence against M_j
$B_{ij} \geq 100$	decisive evidence against M_j

TABLE 2.1: Interpratation of Bayes factor

by the multivariate normal density. The Laplace approximation looks as follows

$$\ln p\left(Y_T|\theta, M_i\right) + \ln p\left(\theta|M_i\right) \approx \ln p\left(Y_T|\tilde{\theta}, M_i\right) + \ln p\left(\tilde{\theta}|M_i\right) + \frac{1}{2}\left(\theta - \tilde{\theta}\right)' \Sigma_{\tilde{\theta}} \left(\theta - \tilde{\theta}\right),$$

where $\tilde{\theta}$ denotes the posterior mode and $\Sigma_{\tilde{\theta}}$ is the Hessian computed at the posterior mode. Integrating with respect to θ is obtained the following estimator of the marginal likelihood

$$\hat{p}\left(Y_T|M_i\right) = \left(2\pi\right)^{\frac{N}{2}} \left|\Sigma_{\tilde{\theta}}\right|^{-\frac{1}{2}} \hat{p}\left(\tilde{\theta}|Y_T, M_i\right) p\left(\tilde{\theta}, M_i\right),$$

where N is the number of estimated parameters. Having computed the approximation of 2.4 Bayesian model selection is done pairwise comparing the models through posterior odds ratio

$$PO_{i,j} = \frac{p\left(Y_T|M_i\right) p\left(M_i\right)}{p\left(Y_T|M_j\right) p\left(M_j\right)}, \tag{2.43}$$

where the prior odds $p\left(M_i\right)/p\left(M_j\right)$ are updated by the Bayes factor $p\left(Y_T|M_i\right)/p\left(Y_T|M_j\right)$. The interpretation of Bayes factor is suggested by Jeffreys (1961) in Table 2.1.

Adolfson, Laseen, Linde, and Villani (2007) and Smets and Wouters (2004) suggest to evaluate Bayesian estimated DSGE models by point estimates and apply standard statistical tools. The residuals can be tested for serial correlation. Next, neglected autoregressive conditional heteroskedasticity can be tested. Root Mean Square Errors (RMSE) of both DSGE models can be compared. Vector autoregression models can test the stability of estimated parameters. These statistical tools help to construct more realistic DSGE models.

2.3.8 Identification

This section is aimed on issue of parameter identification and issues related to Bayesian sensitivity analysis. Identification problems have been studied in econometric theory since 1950's. For details see Koopmans and Reiersol (1950) and for more recent contribution see Pesaran (1981). Identification issues related to DSGE models are studied by Lubik and Schorfheide (2006) and Canova and Sala (2009). The problem of parameter identification can be defined as the ability to draw inference about the parameters of theoretical model from an observed sample. There are several reasons which cause that data do not deliver the sufficient information for an unambiguous identification of the parameters.

First, the data might not distinguish between different structural forms of the model. It means that the loss function upon which the models are estimated does not account for the distinct features of alternative models.

$$\min_{\theta} L(\theta, M_1) = \min_{\xi} L(\xi, M_2),$$

where $L(\cdot)$ is the loss function and θ and ξ are parameter vectors of models M_1 and M_2 respectively.

Second, some of the estimated parameters might enter the loss function proportionally. Then, partitioning the parameter vector θ to θ_1 and θ_2 and the parameter space to $\Theta = \left[\Theta^1, \Theta^2\right]$ is obtained

$$\min_{\theta_1,\theta_2} L(\theta_1, \theta_2, M_1) = \min_{\theta_1} L(\theta_1, \theta_2, M_1), \forall \theta_2 \in \Theta_2 \subset \Theta^2.$$

This problem is referred to as a partial identification. In practical application, the easiest way to handle this problem is to estimate only one of the parameters entering the loss function proportionally and to fix the rest.

Third, even though all parameters enter the loss function independently and the population objective function is globally concave, its curvature may be insufficient.

$$L\left(\tilde{\theta}, M_1\right) - L(\theta, M_1) \leq \varepsilon, \forall \theta \in \Theta^* \subset \Theta,$$

where $\tilde{\theta}$ is the parameter constellation yielding the minimum of the loss function. This problem is referred to as a weak identification and is partially important from the perspective of numerical optimization.

Finally, parameters which are one to one related to the unstable root of the system may be unidentifiable upon the observed time series, which obey the transversality condition. For details see Lucke and Gaggermeier (2001). All types of identification problems mentioned above are relative common in the estimation of DSGE models. Their source is often the discrepancy between the model's definition of economic aggregates and the available time series.

Furthermore, some of structural parameters of DSGE model might not be identifiable due to the fact that detrended and seasonally adjusted time series may contain only a little information about the deterministic steady state.

In small scale models the identification issue may generally be resolved by careful inspection of single equations, but in case of larger models there is no possibility how to ex ante detect the which parameters are identifiable.

In addition, identification problems is DSGE models are difficult to detect because the mapping from the vector of structural parameters θ into the state space representation 2.16 - 2.17 that determines the likelihood of Y_T is highly nonlinear.

The diagnosis is also complicated by fact that the likelihood has to be evaluated numerically. Some numerical procedures to detect the identification problems have been proposed. In context of Maximum Likelihood estimation, the weak identification problem may be detected by examination of the Hessian at the optimum or by plotting data likelihood in the neighborhood of the optimum.

As mentioned in subsubsection 2.3.4, the technical reason of the popularity of the Bayesian approach is that by incorporating even a weakly informative prior the curvature into the posterior density surface can be introduced. This, in turn, facilitates numerical maximization and the use of MCMC methods. However, the uncritical use of Bayesian methods, consisting prior distributions which do not truly reflect the existing location uncertainty, whereas data carry no information about parameters, may according to Canova and Sala (2009) hide identification problems instead of highlighting them. There is a simple method for detecting a lack of identification in the Bayesian framework, a diagnostic unavailable in the classical setup.

The identification issue may be examined by estimating the model with more and more diffuse priors, which is referred to in the literature as Bayesian sensitivity analysis. Thus, the posterior of parameters with doubtful identification features will also become more and more diffuse. The concept of Bayesian sensitivity analysis has a broader meaning than solely detecting parameter identifiability. Let consider the following case. Data carry information on estimated parameters but the prior distribution has subjective features or the sample is small. In this case posterior and prior distribution can have different locations. It is important, however, to check how sensitive posterior outcomes are to the choice of prior distributions. A way to assess the robustness of the posterior conclusions is to select an alternative prior density $p_2(\theta)$, with support included in $p(\theta)$ and use it to re-weight posterior draws. Let $w(\theta) = p_2(\theta)/p(\theta)$ so that

$$E_2(h(\theta)) = \int h(\theta)\, p_2(\theta)\, d\theta = \int h(\theta)\, p(\theta)\, w(\theta)\, d\theta,$$

so that

$$h_2(\theta) = \frac{\sum_{i=1}^{n_{sim}} h(\theta)\, w(\theta)}{\sum_{i=1}^{n_{sim}} w(\theta)}.$$

As a general rule, the results are assessed not robust if the means of the $h_2(\theta)$ statistics lie outside the 90% posterior interval constructed for $h(\theta)$. The identification issue in the context of standard closed economy DSGE models is extensively documented in many empirical studies. For details see Smets and Wouters (2003) and Ireland (2004).

In fact, it seems that models such as Smets and Wouters (2003) have some success in exploiting the information contained in the aggregated macroeconomic time series. Lubik and Schorfheide (2006) discuss the identification in context of New Open Economy Macroeconomics (NOEM) DSGE models. They examine the identification issues based on the estimated small-scale two-country model for the US and the EU area. Lubik and Schorfheide (2006) underline that due to the problem with constructing the bilateral current account data, for instance, parameters standing for ineffectiveness of financial markets are in general unidentifiable in open economy models. As a general rule, keeping the theoretical structure of estimated open economy DSGE models simple helps to avoid some of aforementioned problems.

Chapter 3

The New Keynesian DSGE Model and Alternative Monetary Policy Rules in the Czech Republic

3.1 Introduction

This study deals with the impact of alternative monetary policy rules on the economy of Czech Republic. The main parts of this study will be published in Bouda (2014). The monetary policy shock is formulated as a part of New Keynesian model (NKM) where monetary policy is managed using the standard Taylor rule. The NKM maximizes the utility function of households, profit of firms and welfare by central bank. This NKM is taken from Galí (2008). This model is known as a hard worker among models which are used for monetary policy analysis. Equilibrium equations are taken from Galí (2008). Thus derivation of this model is not included in the main part of this thesis. Derivation of this equilibrium equations is in Appendix A. Nevertheless, this NKM model is calibrated and applied on the Czech economy. Moreover, Dynare code which enables replication of this calculation is in Appendix C.

The main goal of this study is to verify the suitability of several modifications of the Taylor rule. This NKM model is estimated with four different Taylor rules. Bayesian comparison technique is used for the assessment of these Taylor rules.

This NKM contains a couple of departures from classical monetary theory. First, imperfect competition in the goods market is introduced by assuming that each firm produces differentiated goods for which it sets the price, instead of taking the price as given. Second, a few constraints are imposed on the price adjusted mechanism by assuming that only a fraction of firms can reset their prices in any given period. In particular, and following much of the literature, a model of staggered price setting due to Calvo (1983) and characterized by random price durations is adopted. The resulting inflation dynamics can also be derived under the assumptions of quadratic costs of price adjustment. For details see Rotemberg (1982). The study is processed as follows.

The section 3.2 describes the general equilibrium equations of the benchmark NKM. These equilibrium equations are taken from the basic version of the NKM which is specified by Galí (2008).The section 3.3 deals with the observed data and calibration of structural parameters of the NKM. This chapter contains description of both GDP and inflation time series which are used in the estimation. The list of all parameters with their economic interpretation and calibrated values is included. This paper is not focused on calibration. Hence, prior values of structural parameters are taken from the relevant studies and sources. The section 3.4 contains a short description of methods and algorithms used for the estimation of the NKM. The section 3.5 covers the results of the estimation as parameter estimates, impulse responses, shock decompositions and predictions. The section 3.6 deals with Taylor rule and its modifications. The section 3.7 contains important results for all modified NKMs. The section 3.8 contains theory about comparison of DSGE models. Four NKMs (benchmark and three modifications) are specified and each contains a different monetary policy rule. These models are compared using the Bayesian techniques. This experiment evaluates whether increasingly sophisticated monetary policy rules bring material improvement to the DSGE model.

3.2 Benchmark model

The NKM consists of economic agents of three types. The households purchase goods for consumption, hold money and bonds, supply labor, and maximize the expected present value of utility. The firms hire labor, produce and sell differentiated products in monopolistically competitive goods markets, and maximize profits. The central bank controls the nominal rate of interest. Figure 3.1 shows the basic structure of the NKM. The NKM consists of six general equilibrium equations and two stochastic shocks definitions. All equations are log-linearized and variables denoted by wave are gap variables. Let's start with the dynamic IS equation

$$\tilde{y}_t = -\frac{1}{\sigma}\left(i_t - E_t\left\{\pi_{t+1}\right\} - r_t^n\right) + E_t\left\{\tilde{y}_{t+1}\right\}, \tag{3.1}$$

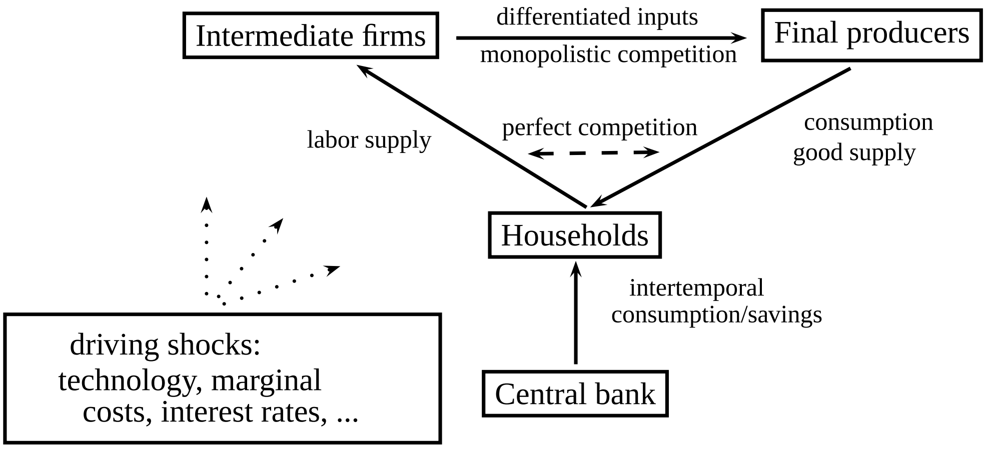

FIGURE 3.1: The structure of the New Keynesian model

where $\tilde{y}_t$ is the output gap[1], i_t is the short term nominal rate, $E_t\{\pi_{t+1}\}$ is the expected inflation in the next period, r_t^n is the natural rate of interest[2], $E_t\{\tilde{y}_{t+1}\}$ represents the expected output gap in the next period and finally the parameter σ is the coefficient of risk aversion. The second equation is called the New Keynesian Phillips curve (NKPC)

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \tilde{y}_t, \tag{3.2}$$

where π_t is the inflation, β is the household discount factor, $\kappa \equiv \lambda\left(\sigma + \frac{\phi+\alpha}{1-\alpha}\right)$ and it is output gap elasticity of inflation, σ is the coefficient of risk aversion, ϕ is the elasticity of labor supply, α is the share of capital and $\lambda = \frac{\theta^{-1}(1-\theta)(1-\beta\theta)(1-\alpha)}{(1-\alpha+\alpha\varepsilon)}$. The third equation shows the evolution of the natural rate of interest

$$r_t^n = \rho + \sigma \psi_{ya}^n E_t\{\Delta a_{t+1}\}, \tag{3.3}$$

where r_t^n is the natural rate of interest, ρ is the real interest rate in the steady state, $\psi_{ya}^n = \frac{1+\phi}{\sigma(1-\alpha)+\phi+\alpha}$ and $E_t\{\Delta a_{t+1}\}$ is the expected change of technology progress in the next period. The fourth equation is the interest rate rule of the central bank, usually called Taylor rule. The rule suggested by Taylor (1993) looks as follows

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + \upsilon_t, \tag{3.4}$$

where i_t is the nominal interest rate, ϕ_π and ϕ_y is the sensitivity of the central bank with respect to inflation and output gap (both are chosen by the central bank), υ_t is an exogenous stochastic component with zero mean. The fifth equation represents the

[1]Output gap is defined as output of basic NKM minus output of Gali's alternative NKM with flexible prices.

[2]Natural rate of interest is defined as result of Gali's alternative NKM with flexible prices.

production function consisting of technology and labor

$$y_t = a_t + (1 - \alpha)\, n_t, \tag{3.5}$$

where y_t is the output, a_t is the level of technology and n_t is the number of worked hours. The sixth equation is the ad-hoc money demand

$$m_t = \pi_t + \tilde{y}_t - \eta i_t, \tag{3.6}$$

where m_t is the money demand, η is the elasticity of the money demand with respect to the nominal interest rate. The last two equations represent the stochastic shocks. The technology shock follows an $AR(1)$ process

$$a_t = \rho_\alpha a_{t-1} + \varepsilon_t^a, \tag{3.7}$$

with the persistence of the technology shock $\rho_\alpha \in \langle 0; 1 \rangle$ and where ε_t^a is a zero mean white noise process. Finally, the monetary policy shock which follows an $AR(1)$ process

$$\upsilon_t = \rho_\upsilon \upsilon_{t-1} + \varepsilon_t^\upsilon, \tag{3.8}$$

with the persistence of the monetary policy shock $\rho_\upsilon \in \langle 0; 1 \rangle$. Positive (negative) realization of ε_t^υ is interpreted as a contractionary (expansionary) monetary policy shock, leading to a rise (decline) in the nominal interest rate, given inflation, and the output gap.

The Czech Republic is rather a small open economy than a closed economy. Thus, there is a need to broaden the interpretation of the monetary policy shock υ_t to exogenous shock as it would be very unreasonable to associate all υ_t realizations with monetary policy shocks. The exogenous shock represents monetary policy shock, fiscal policy shock, shocks to domestic and foreign demand, shock to risk premium, housing shock and many others. If one wanted to distinguish the effects of particular shocks then it would be necessary to specify a more complex DSGE model. Such specification is not necessary for the purpose of this paper and it would make this model less transparent for the reader.

3.3 Data and Calibration

The NKM of the Czech Republic contains two observed variables, see Figure 3.2. The first one is Gross Domestic Product (GDP) and the second is Consumer Price Index

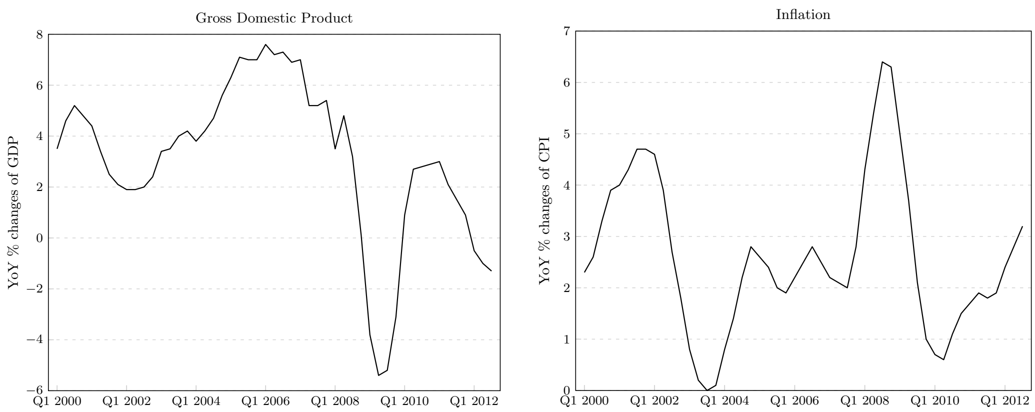

FIGURE 3.2: Observed variables of the NKM

(CPI). Both time series are taken from the database of the Czech National Bank called ARAD.

First, I have to transform both variables into the appropriate form. The GDP is in the constant prices of the year 2005. Next, the GDP is transformed into the year over year percentage changes. In this final form is the GDP inserted into the model. The second observed variable CPI is transformed into the inflation, obtained as a moving year average. It is necessary to evaluate whether all observed variables of a general equilibrium model are stationary, as their stationarity is a very appropriate attribute of the time series. An NKM filled with stationary data is easier to calibrate and then to estimate. The stationarity may be verified just by looking at the data or with the exact statistical tests, see Greene (2011) and Arlt and Arltová (2009). The method used in this paper is KPSS test, for details see Kwiatkowski, Phillips, and Schmidt (1991). The null hypothesis says that the time series is stationary. This hypothesis is not rejected (significance level is 5 %) for both time series. Finally, I have 51 observations and the time series are from Q1 2000 to Q3 2012.

The next important step is the calibration of the structural parameters of our model. The priors of the parameters α, β, ϕ_π, and ϕ_y are taken from the model HUBERT which is used by the Ministry of Finance of the Czech Republic. For details see Štork, Závacká, and Vávra (2009).

The rest of structural parameters are taken from previous studies performed on Czech data. If exists more than one opinion then I take an average value. If such calibration for Czech economy is not available, then calibrations are drawn from Galí, Gertler, and López-Salido (2001). Parameter calibration is a lengthy and difficult process and it is not the main topic of this paper. Thus, I take all prior values from previous studies. Priors of all structural parameters are in Table 3.1.

Parameter	Prior	Description
α	0.5	share of capital
β	0.99	discount factor
ε	1.5	elasticity of substitution, $\log(\varepsilon)/(\varepsilon-1)$, $m=1.1$
θ	0.698	measure of price stickiness, 0 = prices are absolutely flexible
λ	0.154	$\lambda=\theta^{-1}(1-\theta)(1-\beta\theta)(1-\alpha)/(1-\alpha+\alpha\varepsilon)$
ρ	$-\log(\beta)$	real interest rate in the steady state, $\rho=0.0101$
σ	1	coefficient of risk aversion
ϕ	0.80	elasticity of labor supply
ϕ_π	1.5	sensitivity of the central bank with respect to the inflation
ϕ_y	0.25	sensitivity of the central bank with respect to the output gap
ρ_a	0.975	persistence of the technology shock
ρ_υ	0.5	persistence of the exogenous shock
η	4	elasticity of money demand with respect to the nominal interest rate

TABLE 3.1: Priors

3.4 Estimation

The equilibrium of the NKM is characterized by the equations 3.1-3.8. These equations are rewritten into the Dynare. As is mentioned in section 1.7.1 in the Dynare is implemented a comprehensive package of Bayesian techniques which are used for the estimate. These techniques are described in section 2.3.

The main goal is to find the posterior distribution of all unknown parameters (conditional on observed data) and it is performed using the Bayesian rule for the conditional probability. The posterior distribution is obtained by the combination of likelihood function and prior distributions of estimated parameters. The likelihood function is estimated by Kalman filter. The posterior distribution is very often an unknown distribution and thus it is necessary to use a numerical technique to generate random samples. Dynare uses for this purpose Metropolis-Hastings algorithm. This algorithm enables calculation of the basic statistics and moments.

3.5 Results of benchmark model

The results of the Bayesian estimation are shown in Table 3.2. It may be observed that the differences between the prior and the posterior values are not significant. It means that the prior values were correctly calibrated. On the other hand there are very wide confidence intervals. It is caused by the short time series. The log data density is -210.1844 and this statistics is used for comparison of modified versions of DSGE

Parameter	Prior	Posterior	Lower	Upper	Distribution
α	0.50	0.50	0.42	0.58	*beta*
ϕ	0.80	0.79	0.71	0.87	*beta*
ϕ_π	1.50	1.49	1.41	1.58	*norm*
ϕ_y	0.25	0.24	0.16	0.32	*norm*

TABLE 3.2: Estimation results of benchmark model

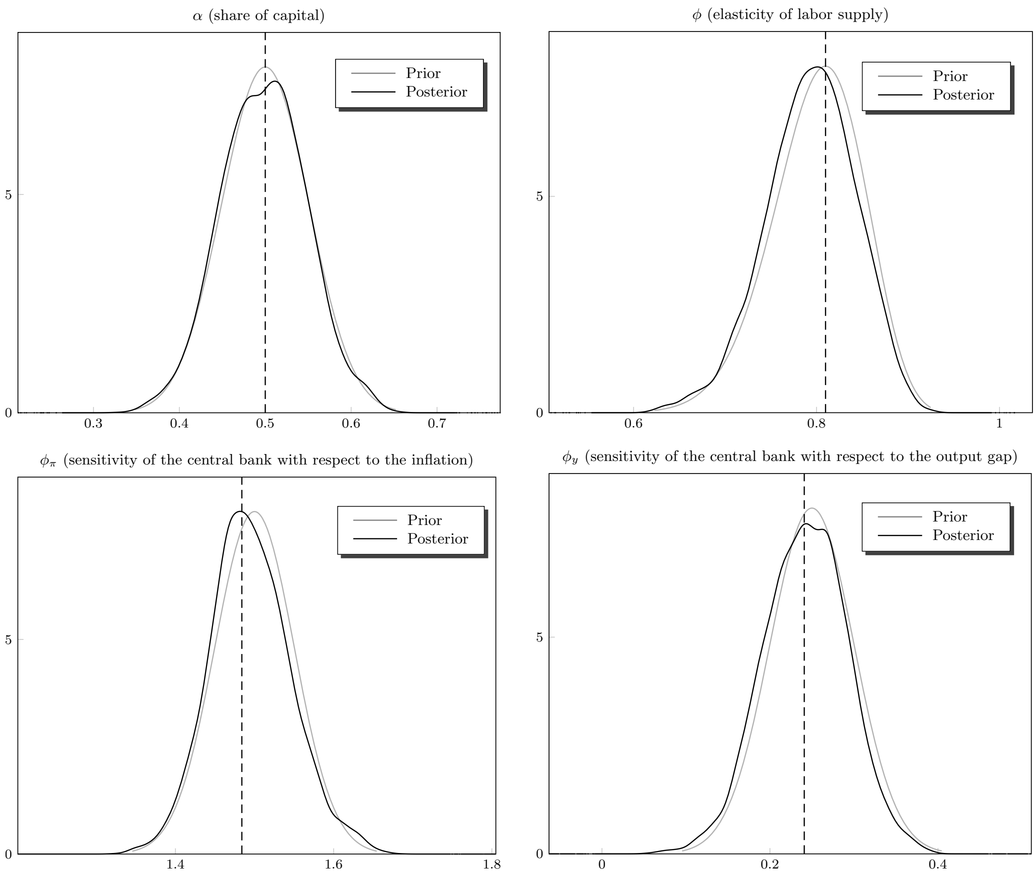

FIGURE 3.3: Priors and Posteriors

models. There are only 51 observations for the Czech economy, which is a very limited dataset as compared to e.g. the U.S. time series data. If the results of the estimation are inserted into 3.4 one can see that the central bank is very sensitive to the level of inflation ϕ_π. On the other hand, the sensitivity to the output gap ϕ_y is not very high. The elasticity of labor supply ϕ is calibrated to one and the posterior mean is the 0.79 that means that the labor supply is not elastic in the Czech Republic. The share of capital is equal to 0.50 which means that share of labor is also 0.50. This knowledge can be used for any future calibrations of a Cobb-Douglas production function.

The Figure 3.3 shows the comparison of the prior distribution and the posterior distribution. Figure 3.4 shows the time series of the GDP. Grey columns represent the influence

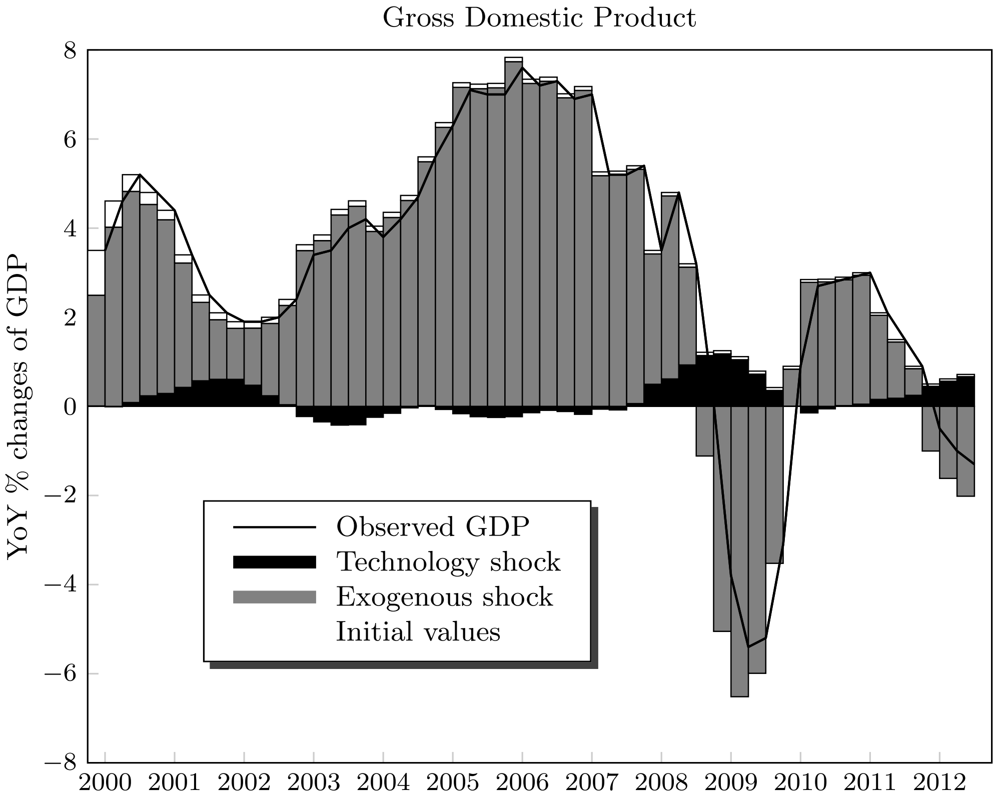

FIGURE 3.4: Shock Decomposition of GDP

of exogenous shocks and the black columns represent technology shocks. Czech Republic has experienced the both good and bad times and the exogenous shocks have played the crucial role in both of them. During good times, Czech economy was supplied by exogenous shocks and very slightly influenced by technology shocks. Bad times are also induced by exogenous shocks and (opposed to good times) technology shocks has a correcting positive effect. Figure 3.5 shows the time series of the inflation and the interpretation of the colors is the same as in the Figure 3.4. The initial values play the minor role as well as in the Figure 3.4. Figure 3.5 may be interpreted as follows: if the inflation tends to grow very fast then the exogenous shocks mitigate the final effects of the technology shocks. Also, as inflation was very low in the years 2003 - 2004, the exogenous shocks caused the inflation to grow. Exogenous shocks had a very positive effect on the Czech economy. Higher level of inflation caused that the GDP growth was not mitigated and the growth of the GDP lasted for another four years.

3.6 Taylor rule and its modifications

The benchmark NKM contains interest rule which is formulated in the spirit of Taylor (1993). This rule is a simple monetary rule that shows how the central bank should

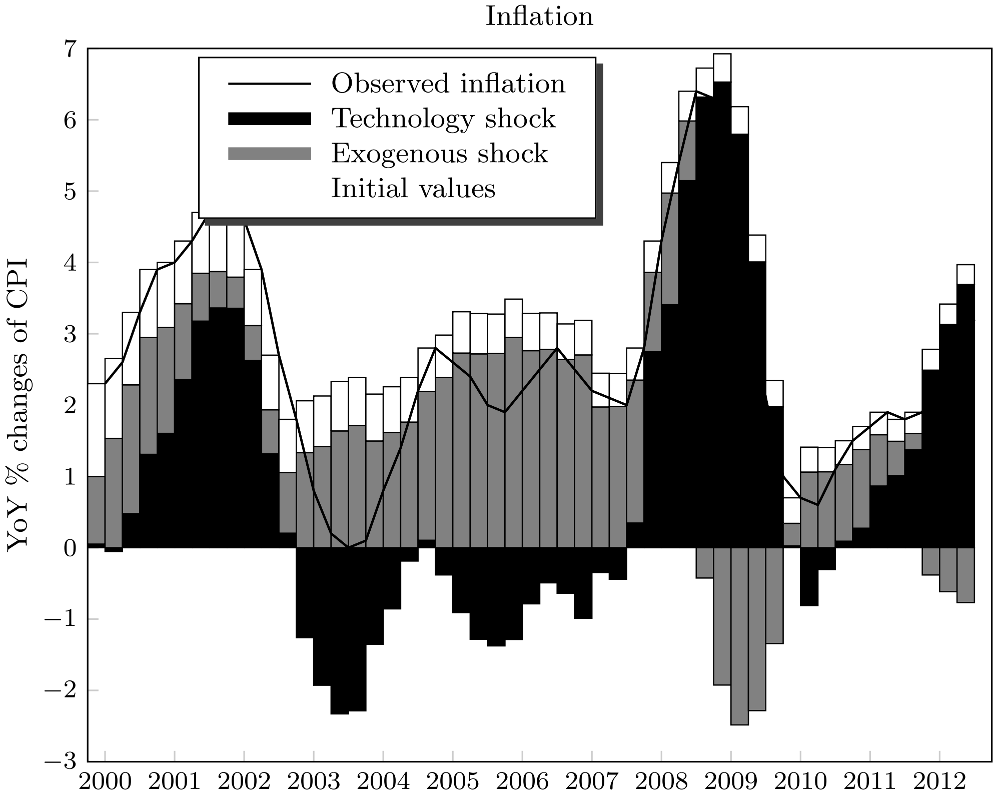

FIGURE 3.5: Shock Decomposition of Inflation

adjust its nominal interest rate in a systematic manner in response to the divergences of actual GDP from its potential level and the divergences of actual inflation rates from the inflation target rate.

The principal objectives of monetary policy are to dampen business cycle fluctuations, to maintain price stability and to achieve the maximum sustainable growth. The rule suggested by Taylor (1993) used the interest rates as policy instrument to achieve these objectives. The greatest strength and weakness of the Taylor rule is its simplicity. Indeed, its simple structure pushed many economists to criticize it and leads many central bankers to conclude that the coefficients in the rule and the equilibrium interest rate must differ across countries and overtime.

The first limit concerns the choice of the three variables: inflation, output gap and the neutral (equilibrium) real interest rate. The robustness of estimation results can be sensitive to data selection and depend on the estimation of trend GDP, the measure of inflation.

The second limit is the timing of the information used by the rule. Taylor used contemporaneous observations of inflation and output in his rule while in reality central banks must rely on lagged information. The structure of the Taylor rule assumes that policy makers consider only current information when making policy decisions and this view is at odds with the forward-looking nature of central banks.

Finally, many economists demonstrated that the parameterization chosen by Taylor reflected exactly the preference of the American monetary authority but they must differ across countries. It may be observed that there are some limitations therefore it is necessary to perform some modifications of the Taylor rule. Subsequently, the NKM is estimated with modified Taylor rule. The NKM with modified Taylor rule is formulated in the same way as the benchmark model. Finally, models are compared using technique called Bayesian comparison. The first modification is for the sake of simplification. Here, I suppose that central bank takes into account only the level of inflation and the output gap is not relevant for central bank's decision making process. The simple monetary policy rule can be written as follows

$$i_t = \rho + \phi_\pi \pi_t + \upsilon_t. \tag{3.9}$$

This simple rule is an alternative to the Taylor rule used in benchmark model.

The second modification is based on Taylor (1993) and discussed by Svensson (2000). According to Srour (2001), there are many reasons for interest rate smoothing. First, the behaving of the central bank is important for investors and smoothing of interest rates can reduce volatility of a term premium and therefore volatility of long-term interest rates and other financial market instruments. Second, the central bank has usually limited information about the shocks hitting the economy. Third, many shocks are serially correlated. The Svensson specification takes the following form

$$i_t = (1 - \phi_i)\left[\bar{i} + \phi_\pi \pi_t + \phi_y \tilde{y}_t\right] + \phi_i i_{t-1} + \upsilon_t. \tag{3.10}$$

where ϕ_i is the interest rate smoothing parameter and $\bar{i}$ is the steady state value of short-term interest rate.

The third modification is not taken from previous papers. This modification is proposed by the author and its goal is simple. It is desirable to incorporate forward looking information into the interest rate rule. It may be observed that basic versions of the NKM do not take into account forward looking information. Two important modifications of the Taylor rule have received wide acceptance. First, Clarida, Gali, and Gertler (1997) departed from the original backward looking Taylor rule to a forward looking specification, which arguably better represents the objectives of central banks. In order to control inflation, the policy instrument would respond to the deviation of the inflation forecast from its assumed target. Second, Orphanides (2000) stressed the importance of policy rules being operational by showing that there are significant differences between

Parameter	**Prior**	**Posterior**	**Lower**	**Upper**	**Distribution**
α	0.50	0.50	0.41	0.57	*beta*
ϕ	0.80	0.80	0.73	0.89	*beta*
ϕ_π	1.50	1.49	1.40	1.57	*norm*

TABLE 3.3: Estimation results with the simple Taylor rule

monetary policy evaluated over revised data and over the real time data (i.e. data available to policymakers at the time they are making decisions). Undoubtedly, this is an important topic. This is supported by fact that National Bureau of Economic Research dedicated one chapter in volume about monetary policy rules to the forward looking rules for monetary policy. For details see Batini and Haldane (1999). The main goal of the third modification is to specify the forward looking Taylor rule which is not specified in previous papers. This modifications is based on 3.10 which was specified by Svensson. The expected level of inflation and output gap plays a crucial role as well. The interest rule contains expectations of the central bank and is specified as follows

$$i_t = (1 - \phi_i)\left\{\bar{i} + E\left[\phi_\pi \pi_{t+1}\right] + E\left[\phi_y \tilde{y}_{t+1}\right]\right\} + \phi_i i_{t-1} + \upsilon_t, \tag{3.11}$$

where E is the operator for rational expectations.

3.7 Results of modified models

This chapter describes results of all modified NKM. The shock decomposition is not commented because this type of chart is the same for all modified NKM. For brevity, only the results of posterior distribution are presented. Model evaluation is performed using marginal likelihood which is discussed in 2.3.7.1.

The marginal likelihood $p\,(Y_T|M)$ is calculated as follows

$$p\,(Y_T|M) = \int p\,(Y_T|\theta, M)\, p\,(\theta|M) d\theta, \tag{3.12}$$

where Y_T are the observations until period T, M stands for specific model and θ represents the parameters of model M. Marginal likelihood $p\,(Y_T|M)$ measures how well model M predicts the observed data Y_T.

Posterior results for simple Taylor rule 3.9 are shown in the Table 3.3. Marginal likelihood is -209.4762 which is in absolute value lower benchmark model (-210.1844). It means that NKM with simple Taylor rule does not fit data as good as benchmark model.

Parameter	Prior	Posterior	Lower	Upper	Distribution
α	0.50	0.49	0.41	0.57	*beta*
ϕ	0.80	0.80	0.72	0.89	*beta*
ϕ_π	1.50	1.48	1.40	1.57	*norm*
ϕ_y	0.25	0.27	0.19	0.35	*norm*
ϕ_i	0.70	0.60	0.54	0.65	*norm*

Table 3.4: Estimation results with the Svensson's Taylor rule

Parameter	Prior	Posterior	Lower	Upper	Distribution
α	0.50	0.49	0.41	0.57	*beta*
ϕ	0.80	0.80	0.72	0.89	*beta*
ϕ_π	1.50	1.48	1.40	1.57	*norm*
ϕ_y	0.25	0.27	0.19	0.35	*norm*
ϕ_i	0.70	0.60	0.54	0.65	*norm*

Table 3.5: Estimation results with the forward looking Taylor rule

Next, the results of Svensson specification 3.10 of the Taylor rule are shown in the Table 3.4. Marginal likelihood is -209.5511 which is in absolute value lower than benchmark (-210.1844) and it means that Svensson modification does not fit data as good as benchmark model.

In this paper is specified forward looking version 3.11 of Taylor rule. This specification has not been tested in any previous papers about this topic and results of posterior distribution are shown in the Table 3.5. Marginal likelihood is -214.1717 and it is in absolute value higher than benchmark (-210.1844). It means that forward looking version of Taylor rule fits data better than benchmark model. It may be observed that posterior estimates are almost the same for all modifications of Taylor rule.

However, even if parameter estimates are almost the same, the fit of model to data can be different (especially in case of forward looking modification of Taylor rule because estimated parameters have different economic interpretation than in previous cases).

3.8 Bayesian comparison

A natural method to assess the empirical validity of DSGE model is to compare its predictive performance, measured by the marginal likelihood, with other available models or perhaps an even larger class of non-structural linear reduced-form models. Having computed the approximation of 3.12 Bayesian model selection is done by pairwise comparison of the models through a posterior odds ratio 2.43.

M_i/M_j	**benchmark**	**simple**	**svensson**	**forward**
benchmark		1.94	1.80	0.02
simple	0.52		0.93	0.01
svensson	0.56	1.07		0.01
forward	56.95	110.29	102.59	

Table 3.6: Bayes ratio

The interpretation of Bayes factor in Table 2.1. For more details see 2.3.7.1. Bayes Ratio comparison results may be observed from Table 3.6. The best modification to a Taylor rule is the forward looking version. From the forward looking point of view there is strong evidence against the NKM with the benchmark Taylor rule and some even more decisive evidence against the NKM with simple and Svensson's Taylor rules. Generally, results may be interpreted as follows: Forward looking Taylor rule has the best fit to data. All other versions of the Taylor rule have almost the same fit to data.

3.9 Conclusion

This study deals with alternative monetary policy (Taylor) rules. First, the benchmark NKM which is based on Galí (2008) is specified, calibrated and estimated using Bayesian techniques. The estimation of benchmark model is complete. It means that there are figures of prior and posterior distributions and shock decomposition of GDP and inflation.

Subsequently, the first contribution of this study is as follows. The specification of three alternative monetary policy rules is performed. For the alternative monetary policy rules the charts of prior and posterior distributions and shock decomposition are not presented due to very high coincidence with benchmark model. Parameter estimates and marginal density is presented for each modification of monetary policy rule. This study contains two interesting outputs. First, the shock decomposition of GDP comes with the finding that changes to GDP are caused by exogenous shocks. This is valid for both good and bad times as well. On the other hand, technology shocks play a minor role in GDP formation. The shock decomposition of inflation comes with following findings. If the inflation tends to grow very fast then the exogenous shock mitigates the final effect of the technology shock. On the other hand, the inflation was very low in the years 2003 - 2004 and using the exogenous shock the inflation started to grow. It had a very positive effect to the Czech economy.

Next, the second contribution of this study is as follows. The parameter estimates of NKM with different monetary policy rules are almost the same. But still there are techniques which can be used for the comparison of DSGE models. Section 3.7 introduces these techniques and using these Bayesian techniques one is able to compare the predictive power of individual DSGE models. Four NKM with different monetary policy rules are compared. The benchmark model contains standard monetary policy rule which is proposed by Galí (2008). Next, the simple, Svensson and forward looking monetary policy rules are specified and estimated as a part of NKM. Secondly, the main result of this paper is that the NKM with the forward looking monetary policy rule fits data much better than the NKM with benchmark, simple and Svensson monetary policy rule. This finding confirms that general opinion on monetary policy rules is valid for the Czech Republic. Relevant foreign studies, e.g. Woodford (2003) or Clarida, Gali, and Gertler (1997) also prove that forward looking monetary policy rules are much better than their backward looking counterparts.

Chapter 4

The Small Open Economy Model of the Czech Republic with Housing sector

4.1 Introduction

This chapter deals with development of the Small Open Economy (SOE) DSGE model of the Czech Republic with housing sector. The housing sector extends SOE model by many features. One of them is the link between loans and living. Recent financial crisis was caused largely by subprime mortgages which were provided in the U.S. These mortgages were normally approved to clients (borrowers) with lower credit ratings. A conventional mortgage is not offered because lender views the borrower as having a larger than average risk of defaulting on the loan. Lending institutions often charge interest on subprime mortgages at a rate that is higher than a conventional mortgage in order to compensate them for carrying more risk. These borrowers are very sensitive to any economic downturn. When U.S. economy started to fall into recession then exactly this kind borrowers became defaults. Next, lenders (banks and other financial institutions) repossessed collateral (house) which was subject of subprime mortgage. When workout departments of all lenders start to sell repossessed collaterals then house prices starts to decline. This has impact also on non-defaulted clients and market price of their houses. In a very extreme case that market house prices are halved compare to value valid to time of application, the borrowers lose motivation to pay the principal and borrowers let financial institution to repossess their house. This situation happened in the U.S. after financial crisis.

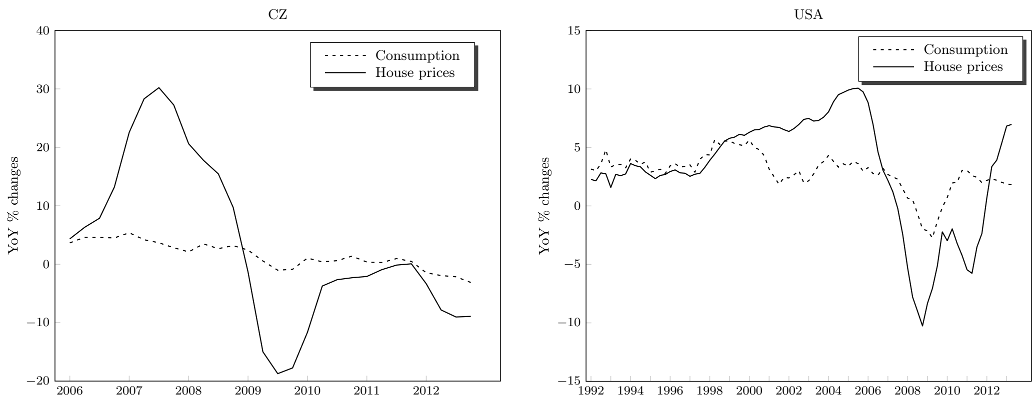

FIGURE 4.1: House prices and consumption in the USA and the Czech Republic

DSGE models are very often criticized that well-developed housing sector is missing. This critique and all facts above brought me to the idea to incorporate the housing market features into the SOE DSGE model. This incorporation is inspired by study Aoki, Proudman, and Vlieghe (2002) which deals with housing sector but only in closed economy environment. I believe that this extension of SOE model can effectively improve parameter estimates and its fit to data as well as the theoretical understanding to our domestic economy.

Figure 4.1 shows the relationship between house prices and household consumption in the Czech Republic and USA. The USA data shows that there is a very strong relationship between house prices and household consumption. In case of growth of house prices one may see slight increase of consumption and in case of decline of house prices one may see fall of consumption. YoY (Year over Year) changes in house prices are much greater than YoY changes in household consumption. Given that the financial accelerator forms part of the wider housing-consumption relationship. This financial accelerator is largely inspired by Bernanke, Gertler, and Gilchrist (1999) (BCG). House prices affect the level of collateral of each consumer. It means that in case of high house prices one is able to borrow more money because his loan is secured by higher value of collateral.

As one can see on Figure 4.2 a majority (80%) of households are owners of their houses. Thus, I see a very high potential for the application of financial accelerator extended by housing sector. The Figure 4.2 shows the proportion of households which are owners or tenants. One can see that there are a significant difference between proportion of owners in the Czech Republic (CR) and in the EU(15)[1]. Figure 4.3 shows the big difference between CR and EU(15) in the potential of rentals. In the CR the share of owners with mortgage is much lower than in the EU(15). Assuming, that the target share of owners with mortgage in the CR is the same as now in the EU(15). Under

[1]EU(15) consists of the following countries: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, Sweden, United Kingdom.

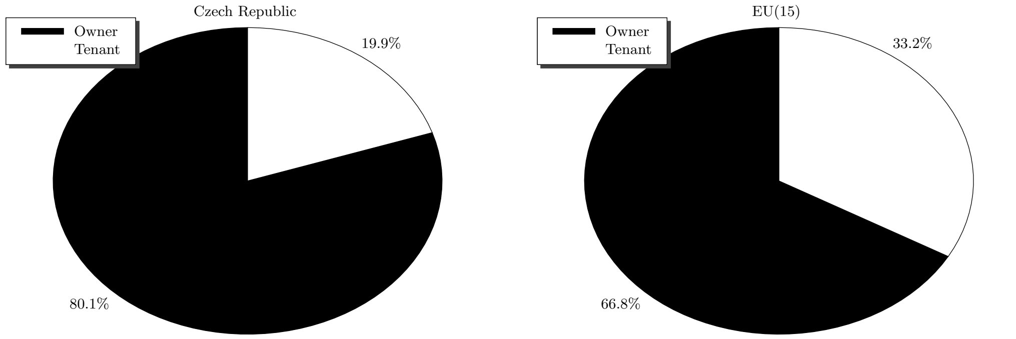

Figure 4.2: Distribution of households by tenure status

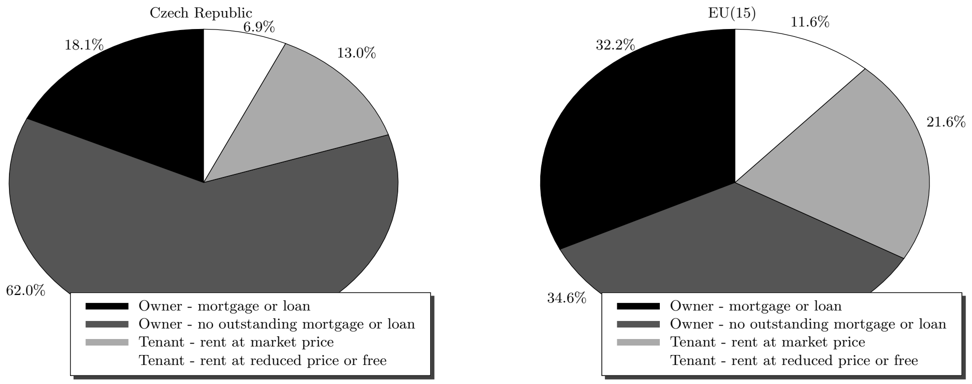

Figure 4.3: Detailed distribution of households by tenure status

this assumption, consumers who have an access to financial market will be in the future more important than ever before and it is necessary to take this finding into account during the development of DSGE model. Thus, I suppose that housing sector is one of the key features of Czech economy and influence of this sector will be much bigger in a few next years.

As is shown in Figure 4.4 the household debt to housing still grows. This time series show that the growth rate rapidly declines from 2008 to 2013. It is mainly caused by short history of mortgages in the Czech Republic. The first mortgage was introduced into the Czech Republic in 1995 but this introduction was very lengthy. An illustrative example demonstrates that between years 1995 and 2000 the same volume of mortgages as in June 2007 (19 billion CZK) was provided. The Figure 4.4 also shows the default rate of secured loans to housing (mainly mortgages). One can see in Figure 4.4 that default rate of clients who have mortgages increased rapidly from 2008 to 2010. This is the effect of economic recession. After this recession there is a stable default rate from 2010 to 2013. Finally, one may observe that volume of loans grows and default rate is now quite stabilized.

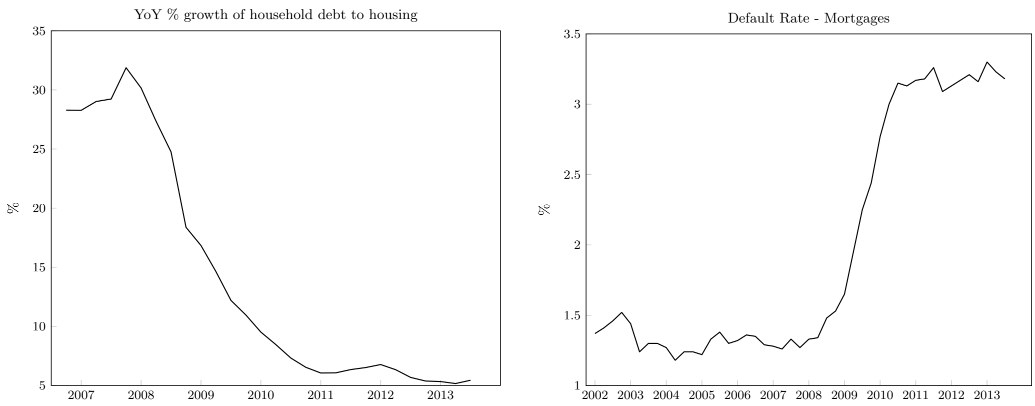

FIGURE 4.4: Household debt to housing and default rate

Going back to "credit/collateral channel" which is a potential role for a financial accelerator, interest rates reflect, inter alia, credit risk and serve as a means to ration credit. When house prices rise, the increase in the value accrues to the house owner and thus provides additional equity. This additional equity may be either used to reduce the loan to value ratio of the next house purchase or is available to fund consumption via housing equity withdrawal. Theory says that a lower credit risk entails a lower interest rate. When risk is mitigated by increased collateral value then bank approves a lower mortgage interest rate.

Bernanke, Gertler, and Gilchrist (1999) (BCG) introduced the framework for a financial accelerator mechanism. Authors show how this mechanism makes economic cycles more extreme in a closed economy model. The main feature of BCG is the incorporation of credit market frictions. Whereas economic models typically use the risk-free interest rate as the interest variable. In reality, borrowers often have to pay significantly higher interest rates. Banks know that there is a risk that borrower fall into default and it is very easy to calculate this risk. As a result, lenders will typically charge a premium over the risk free rate. BCG incorporate this as a risk premium paid by borrowers (firms). In reality, the size of risk premium depends on a myriad of factors. BCG simplify this relationship to one of dependence on net asset. As a borrower's net assets increase in value, the likelihood that the lender will recover monies owed also increases, allowing for a reduction in the risk premium. Conversely events that cause a reduction in the borrower's net assets serve to drive up the interest rate faced by the borrowers. Thus to the extent that net assets are co-cyclical with order economic shocks, the financial accelerator mechanism serves to amplify the effect of the original shock.

Aoki, Proudman, and Vlieghe (2002) and Aoki, Proudman, and Vlieghe (2004) (APV) apply the concept of a financial accelerator to the housing market and household consumption in a closed economy model. Abstracting from explicit financial assets, APV set up a model in which household's net assets are driven by the value of the housing stock and the debt required to purchasing the housing stock. As the net value of the housing stock increases, the interest premium charged by lenders on mortgage decreases. House owners thus face both an increase in the equity in their homes and a lower mortgage cost. Thus they have the ability and incentive to withdraw some of the change in equity and use it to fund consumption. The main feature of APV model is the heterogeneous approach for modeling of households. A portion of households is able to freely access the debt market while the remaining households are constrained by current income although both types are endowed with housing assets. The main finding of APV is that the financial accelerator amplifies the effect of shocks.

Iacoviello and Neri (2010) introduced a closed economy DSGE model that also includes land as an explicit factor of production for housing. Their version of financial accelerator is incorporated in the budget constraint of homeowners by setting a debt limit for impatient households that is tied to the nominal value of the housing stock. It means that rise in the values of the housing stock allows impatient households access to a greater amount of debt and fund consumption. As with APV, both patient and impatient households have housing assets.

4.2 Model

The Aoki, Proudman, and Vlieghe (2002) (APV) model was constructed as a closed economy model. In this study, I specify a modified version which incorporates an open economy extension and also contains government. This inclusion of both foreign sector and government makes the model more applicable to a small open economy such as Czech Republic. The key open economy features are as follows. The production function contains an imported intermediate good. The goods producing sector sells to foreign consumers in addition to home consumers. A proportion of consumers are able to access foreign capital markets. The model is extended by a specific fiscal authority called government. Intuitively, one would expect that the stimulatory effect of government spending on the economy would naturally increase the level of economic activity. Before I present the model in details it is necessary to mention that model focuses on certain features which are present in the Czech economy. In particular, I follow APV which means that productive capital is neglected. Similarly I ignore international trade in services, consumer imports and cross-border investment.

Following the framework used by APV, there are two types of households. The first type of consumers is able to access the capital markets and they can smooth consumption across time by buying or selling financial assets. These households follow the permanent income hypothesis (PIH). The other type of household uses rule of thumb (ROT) consumption, spending all their income on consumption. ROT consumers are effectively completely credit constrained as they do not have any access to the credit markets. This dual differentiation of consumers is based on Campbell and Mankiw (1989).

Unlike the APV model, in this study ROT households are further characterized by the fact that they do not own any housing assets. This guarantees consistency so that households which do not have access to the credit markets to smooth consumption are also unable to purchase a house (it means that mortgage would not be granted to them). Given that residential mortgage lending institutions would be unwilling to extend credit to potential borrowers who have no assets, not to mention insufficient funds for a deposit on a house, this is not only reasonable but also arguably more realistic than positing credit-constrained consumers repeatedly accessing the mortgage market. Both types of consumers purchase goods from firms each period, receive wage income from labor supplied to firms and pay rental to the homeowners.

PIH households are divided into two complementary components: a homeowner and a consumer. The homeowner transacts in the housing market each period, selling the housing stock and purchasing the stock anew. Against the net worth of housing stock, the homeowner borrows to meet any shortfall between the price of the housing stock bought at the end of the period and the price realized on sale of the existing housing stock. Net worth is defined as the value of the housing stock less outstanding debt and less any dividends paid to consumers. This dividend is the mechanism by which the housing equity withdrawal is captured. Homeowners also charge a rental fee to consumers. Thus the housing stock is completely owned by the PIH consumers and the ROT consumers pay rental to their PIH landlords.

Firms are monopolistically competitive and produce a continuum of consumer goods. Each period they hire labor from households and also purchase an intermediate input from abroad. These imports are used by firms each period and capital is effectively assumed to be constant. The output of firms is consumed by either household or government, exported or used to produce additional housing stock. The conversion of consumer goods to housing stock is assumed to follows q-type investment theory which is explained later.

The monetary authority has a Taylor rule reaction function (with lagged inflation and the output gap as indicators of inflationary pressure) and uses the nominal interest rate as its lever subject to a smoothing parameter.

The government collects lump sum taxes from consumers and purchases consumer goods. The difference between these two is either funded through the sale of government bonds or, where taxes exceed expenditure, is used to retire debt. For simplicity government

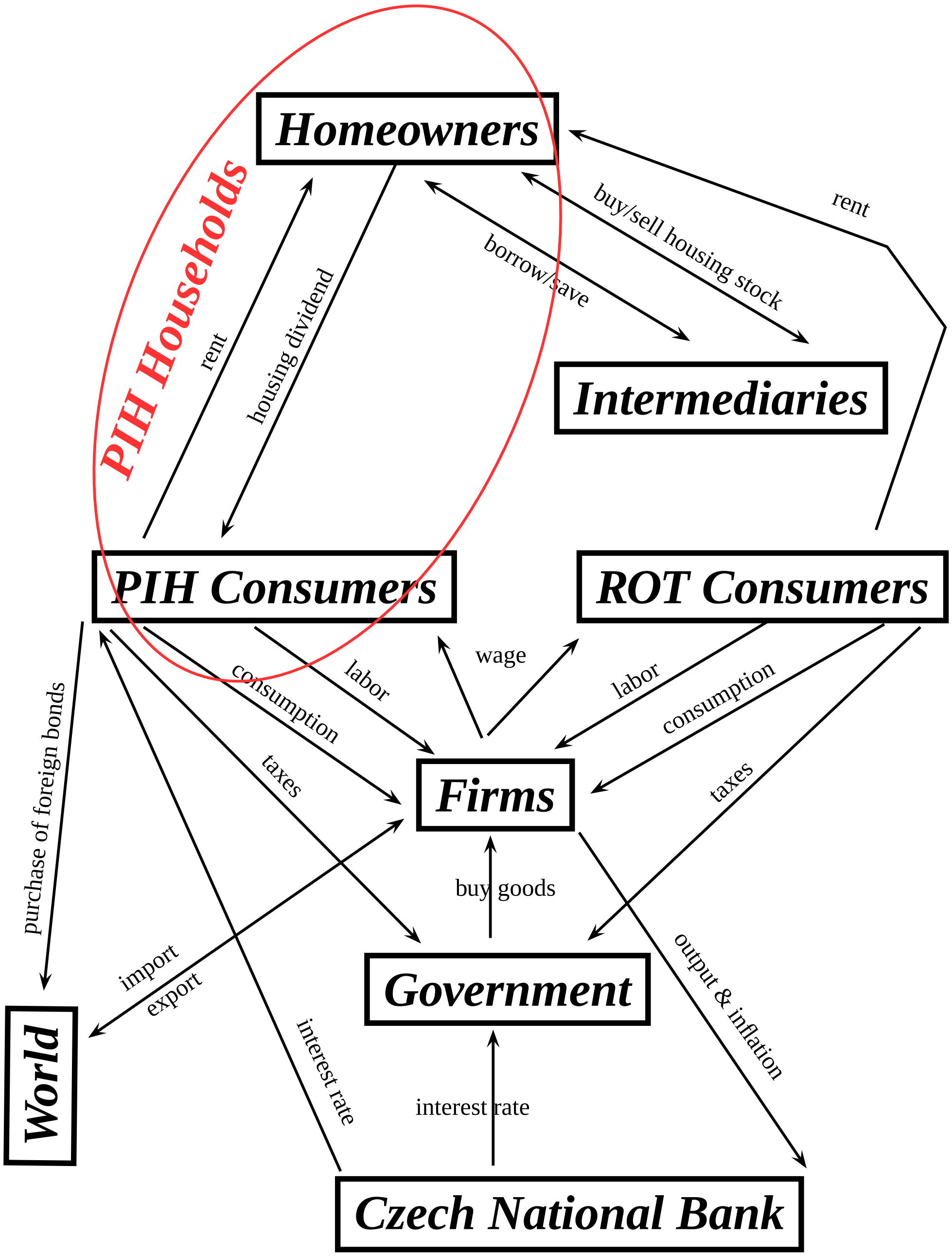

FIGURE 4.5: The structure of the SOE DSGE model with housing

expenditure does not impact households directly (in other models it is done in the form of transfer) but rather is a source of final demand for consumer goods and thence labor demand and imports. Following Galí and Gertler (2007), fiscal policy is modeled as the combination of exogenous government spending, government debt and lump sum taxes.

Figure 4.5 shows the structure and dynamics of SOE DSGE model with housing.

Each agent mentioned above maximizes its utility function. The goal is to obtain a set of first order conditions characterizing the equilibrium of this SOE model.

Variable	Description
C	Aggregate consumption
q	Real house price
I	Housing investment
h	Housing stock
X_c	Relative price of consumption good
Y	Real output
IM	Imports
A	Technology
L	Aggregate labor
w	Real wage
mc	Real marginal cost
C^r	ROT consumption
L^r	ROT labor supply
C^p	PIH consumption
L^p	PIH labor supply
T	Lump-sum taxes (in real terms)
R^n	Nominal interest rate
R	Real domestic interest rate
π	Overall inflation
$\tilde{y}$	Output gap
X_{ii}	Monetary policy shock
N	Net worth
R_h	Return on housing
D	Housing dividend
EX	Net exports
RS	Real exchange rate
Y^f	Foreign output
R^f	Foreign interest rate
S	Nominal exchange rate
b	The borrowing undertaken to finance the purchase of housing stock
Y^{flex}	Flexible price output
B^G	Government debt
X_h	Relative price of renting
G	Government spending
π_c	Consumption good inflation
c	Goods consumption
y	GDP (Y-IM)
Exogenous variables representing stochastic shocks	
ϵ_A	Technology shock
$\epsilon_{X_{ii}}$	Domestic real interest rate shock
ϵ_{R^f}	Foreign real interest rate shock
ϵ_{Y^f}	Foreign demand shock
ϵ_G	Government spending shock

Table 4.1: Variables

Parameter	Prior	Description
ϕ	0.7	Net worth ratio
Ω	-0.1	Sensitivity of interest rate premium to net worth ratio
$\frac{\chi'(\phi)}{\chi(\phi)}\phi$	3	Sensitivity of dividend to net worth ratio
n	0.5	Proportion of consumers that are PIH
Γ_d	0.52	q-theory sensitivity
η	0.9999	Consumer substitution between housing and goods coefficient
δ	0.005	Housing depreciation rate
υ	0.81	Steady state goods consumption as a proportion of overall consumption
ρ_i	0.70	Interest rate smoothing parameter
ρ_A	0.90	Autocorrelation for technology shock
ρ_G	0.70	Autocorrelation of fiscal spending shock
ρ_{R^f}	0.80	Autocorrelation of foreign interest rate shock
ρ_{Y^f}	0.80	Autocorrelation of of foreign demand shock
$\rho_{X_{ii}}$	0.80	Autocorrelation of domestic interest rate shock
γ_π	1.50	Coefficient on inflation in monetary policy rule
γ_y	0.25	Coefficient on output gap in monetary policy rule
θ	0.50	$1-\theta$ is a probability of firm resetting its price
α	0.65	Import weight in production function
γ	-0.2	Labor-imports substitution coefficient in production function
ξ	1.1097	Leisure coefficient in utility function
β	0.99	Discount rate
δ_b	-0.001	The cost of intermediation in the foreign currency bond market
ϑ	1	Export sensitivity to real exchange rate
ζ	1	Export sensitivity to foreign demand
ϕ_B	0.33	Distribution of fiscal imbalances with respect to the government debt
ϕ_G	0.1	Distribution of fiscal imbalances with respect to the government exogenous spending
G/Y	0.2	Government spending/output ratio
EX/Y	0.6	Exports/output ratio
IM/Y	0.7	Imports/output ratio

Table 4.2: Priors

The log-linear equations are derived in the following subsections. Variables with a hat represent the percentage change from their steady state value and variables without subscript denote the steady state values of those. Table 4.1 shows the short description of all variables which are used in this DSGE model. Table 4.2 shows the description of parameters and their prior values which are explained later. Equations mentioned in the section 4.3 appear in the Dynare code which is in Appendix D.

In order to make model more compatible with Czech economy the steady state is derived as well in Appendix B. This model is estimated using the DSGE framework which is in detail described in chapter 2. In order to estimate model properly I had to diverge several times from standard DSGE framework. These diversions are described in section 4.6 (Bayesian estimation). For sake of clarity Dynare code which contains log-linearized equations is in Appendix D. This code enables replication of this study.

4.2.1 Firms

Goods are produced by firms. The goods are differentiated so that there is monopolistic competition and each producer has a certain degree of market power in the short run. The market power consist in the ability to set their prices. Firms are assumed to be domestically owned. This assumption is arguably strong because in the Czech Republic we have high degree of foreign ownership. Thus, I neglect the need to consider equity investment flows. Next, no firm dividend is explicitly modeled to keep the model as simple and clear as possible. Logically, ownership of firms would be on the shoulders of PIH households as ROT households are unable to build up savings in order to purchase an equity stake. However, if PIH households do not view changes to firm dividends as permanent income changes, then they will not alter their consumption behavior in response. The combination of labor and imported intermediate goods is used together with an exogenously given level of technology as inputs by firms. The CES production function us used for a production of consumption goods. The output of firm z is given by

$$y_t(z) = \left[\alpha IM_t(z)^\gamma + (1-\alpha)\left(A_t L_t(z)\right)^\gamma\right]^{\frac{1}{\gamma}}, \tag{4.1}$$

where $IM_t(z)$ is the quantity of intermediate imports used by firm z, $L_t(z)$ is labor demand, A_t is the level of technology, α is the weight given to imports and $1-\alpha$ is the weight on labor and $\frac{1}{1-\gamma}$ is the elasticity of substitution between labor and imports. Log-linearizing and aggregating across all firms

$$\begin{aligned}
\log Y_t &= \log\left[\alpha IM_t^\gamma + (1-\alpha)\left(A_t L_t\right)^\gamma\right]^{\frac{1}{\gamma}} \\
\hat{Y}_t &= \frac{1}{Y_t}\frac{1}{\gamma}Y_t^{1-\gamma}\left[\alpha IM_t^\gamma \widehat{IM}_t + (1-\alpha)\left(A_t L_t\right)^\gamma\left(\hat{A}_t - \hat{L}_t\right)\right]
\end{aligned}$$

which around the steady state reduces to

$$\hat{Y}_t = \varphi\widehat{IM}_t + (1-\varphi)\left(\hat{A}_t - \hat{L}_t\right), \tag{4.2}$$

where

$$\varphi = \frac{\alpha IM^\gamma}{\alpha IM^\gamma + (1-\alpha)\left(AL\right)^\gamma}.$$

4.2.2 Input demand determination

The cost minimization problem for the firms is defined as follows

$$\mathcal{L} = W_t L_t + \left(S_t P_t^f\right) IM_t + \lambda_t \left(y_t - [\alpha IM_t^\gamma + (1-\alpha)(A_t L_t)^\gamma]^{\frac{1}{\gamma}}\right), \tag{4.3}$$

where W_t is the nominal wage rate, S_t is the nominal exchange rate (measured in domestic currency units per foreign currency unit) and P_t^f is the price level of foreign goods.

Differentiating 4.3 with respect to L_t

$$\begin{aligned}
\frac{d\mathcal{L}}{dL_t} &= W_t - \lambda_t \frac{1}{\gamma}[\alpha IM_t^\gamma + (1-\alpha)(A_t L_t)^\gamma]^{\frac{1}{\gamma}-1}(1-\alpha)\gamma(A_t L_t)^{\gamma-1} A_t \\
&= W_t - \lambda_t y_t^{1-\gamma}(1-\alpha)(A_t L_t)^{\gamma-1} A_t \\
&= W_t - \lambda_t y_t^{1-\gamma}(1-\alpha) A_t \left\{\frac{y_t}{(A_t L_t)}\right\}^{1-\gamma} \\
\lambda_t &= \frac{W}{(1-\alpha) A_t^\gamma \left\{\frac{y_t}{L_t}\right\}^{1-\gamma}}. \qquad (4.4)
\end{aligned}$$

4.4 is interpreted as the marginal cost condition aggregating across firms with respect to labor. Differentiating 4.3 with respect to IM_t

$$\begin{aligned}
\frac{d\mathcal{L}}{dIM_t} &= \left(S_t P_t^f\right) - \lambda_t \frac{1}{\gamma}[\alpha IM_t^\gamma + (1-\alpha)(A_t L_t)^\gamma]^{\frac{1}{\gamma}-1}\alpha\gamma IM^{\gamma-1} \\
&= \left(S_t P_t^f\right) - \lambda_t \alpha \left\{\frac{y_t}{IM_t}\right\}^{1-\gamma} \\
\lambda_t &= \frac{S_t P_t^f}{\alpha \left\{\frac{y_t}{IM_t}\right\}^{1-\gamma}}. \qquad (4.5)
\end{aligned}$$

Hence equating marginal costs across labor and intermediate imports yields

$$\begin{aligned}
\frac{W_t}{(1-\alpha) A_t^\gamma \left\{\frac{y_t}{L_t}\right\}^{1-\gamma}} &= \frac{S_t P_t^f}{\alpha \left\{\frac{y_t}{IM_t}\right\}^{1-\gamma}} \\
\frac{w_t}{RS_t} &= \frac{(1-\alpha) A_t^\gamma \left\{\frac{y_t}{L_t}\right\}^{1-\gamma}}{\alpha \left\{\frac{y_t}{IM_t}\right\}^{1-\gamma}},
\end{aligned}$$

where $w_t = \frac{W_t}{P_t}$ is the real wage rate and RS_t is the real exchange rate. The y_t can be canceled and then this is expressed as follows

$$\left(\frac{IM_t}{L_t}\right)^{1-\gamma} A_t^\gamma = \frac{\alpha}{(1-\alpha)} \frac{w_t}{RS_t}.$$

Log-linearizing

$$\begin{aligned}
(1-\gamma)(\log IM_t - \log L_t) + \gamma \log A_t &= \log\alpha - \log(1-\alpha) + \log w_t - \log RS_t \\
(1-\gamma)\left(\frac{dIM_t}{IM_t} - \frac{dL_t}{L_t}\right) + \gamma\frac{dA_t}{A_t} &= \frac{dw_t}{w_t} - \frac{dRS_t}{RS_t} \\
(1-\gamma)\left(\widehat{IM}_t - \hat{L}_t\right) + \gamma\hat{A}_t &= \hat{w}_t - \widehat{RS}_t \\
\widehat{IM}_t &= \frac{1}{(1-\gamma)}\left(\hat{w}_t - \widehat{RS}_t - \gamma\hat{A}_t\right) + \hat{L}_t. \qquad (4.6)
\end{aligned}$$

4.2.3 Marginal cost

From the production function 4.1, note that

$$\begin{aligned}
\left(\frac{y_t}{L_t}\right)^\gamma &= \frac{\alpha IM_t^\gamma + (1-\alpha)(A_tL_t)^\gamma}{L_t^\gamma} \\
&= \alpha\left(\frac{IM_t^\gamma}{L_t^\gamma}\right) + (1-\alpha)A_t^\gamma \\
\frac{y_t}{L_t} &= \left[\alpha\left(\frac{IM_t}{L_t}\right)^\gamma + (1-\alpha)A_t^\gamma\right]^{\frac{1}{\gamma}}.
\end{aligned}$$

Log-linearizing the marginal cost 4.4 is obtained

$$\begin{aligned}
\log\frac{\lambda_t}{P_{c,t}} &= \log\frac{W_t}{P_{c,t}} - \log(1-\alpha) - \gamma\log A_t - (1-\gamma)\log\frac{y_t}{L_t} \\
\log mc_t &= \log w_t - \log(1-\alpha) - \gamma\log A_t - \\
&\quad -\frac{(1-\gamma)}{\gamma}\log\left(\alpha\left(\frac{IM_t}{L_t}\right)^\gamma + (1-\alpha)A_t^\gamma\right). \qquad (4.7)
\end{aligned}$$

Differentiating the last element of 4.7

$$\begin{aligned}
d\left[\alpha\left(\frac{IM_t}{L_t}\right)^\gamma + (1-\alpha)A_t^\gamma\right] &= \alpha\gamma\left(\frac{IM_t}{L_t}\right)^\gamma\left(\frac{L_t dIM_t - IM_t dL_t}{L_t^2}\right) + (1-\alpha)\gamma A_t^{\gamma-1}dA_t \\
&= \alpha\gamma\left(\frac{IM_t}{L_t}\right)^\gamma\left(\frac{dIM_tL_t}{IM_tL_t} - \frac{IM_t}{L_t}\frac{L_t}{IM_t}\frac{dL_t}{L_t}\right) + (1-\alpha)\gamma A_t\gamma\frac{dA_t}{A_t} \\
&= \alpha\gamma\left(\frac{IM_t}{L_t}\right)^\gamma\left(\widehat{IM}_t - \hat{L}_t\right) + (1-\alpha)\gamma A_t^\gamma\hat{A}_t.
\end{aligned}$$

Parameter φ can be expressed as

$$\varphi = \frac{\alpha\left(\frac{IM}{L}\right)^\gamma}{\alpha\left(\frac{IM}{L}\right)^\gamma + (1-\alpha)A^\gamma}.$$

Then

$$d\log\left(\alpha\left(\frac{IM_t}{L_t}\right)^\gamma + (1-\alpha)A_t^\gamma\right) = \varphi\left(\widehat{IM}_t - \hat{L}_t\right) + (1-\varphi)\hat{A}_t$$

and

$$\begin{aligned}\widehat{mc_t} &= \hat{w}_t - \gamma \hat{A}_t - (1-\gamma)\left[\varphi\left(\widehat{IM_t} - \hat{L}_t\right) + (1-\varphi)\hat{A}_t\right] \\ &= \hat{w}_t - (1-\gamma)\varphi\left(\widehat{IM_t} - \hat{L}_t\right)\left[(1-\gamma)(1-\varphi)+\gamma\right]\hat{A}_t. \end{aligned} \tag{4.8}$$

4.2.4 Resource constraint

Output of firms is consumed by domestic consumers, government and foreign consumers or utilized in the production of housing

$$\begin{aligned} Y_t &= c_t + I_t + G_t + EX_t \\ \frac{dY_t}{Y_t} &= \frac{c_t}{Y_t}\frac{dc_t}{c_t} + \frac{I_t}{Y_t}\frac{dI_t}{I_t} + \frac{G_t}{Y_t}\frac{dG_t}{G_t} + \frac{EX_t}{Y_t}\frac{dEX_t}{EX_t} \\ \hat{Y}_t &= \frac{c_t}{Y_t}\hat{c}_t + \frac{I_t}{Y_t}\hat{I}_t + \frac{G_t}{Y_t}\hat{G}_t + \frac{EX_t}{Y_t}\widehat{EX}_t. \end{aligned}$$

Because of steady state one can drop the time subscripts for the levels variables

$$\hat{Y}_t = \frac{c}{Y}\hat{c}_t + \frac{I}{Y}\hat{I}_t + \frac{G}{Y}\hat{G}_t + \frac{EX}{Y}\widehat{EX}_t, \tag{4.9}$$

where c_t represents domestic consumption of consumption goods, I_t is investment in housing, G_t represents government spending and EX_t is consumption goods exported to foreign consumers.

4.2.5 Export demand

Foreign demand for firm z's output is given as follows

$$\begin{aligned} EX_t(z) &= \left(\frac{S_t^{-1}P_t(z)}{S_t^{-1}P_{c,t}}\right)^{-\epsilon} EX_t \\ &= \left(\frac{P_t(z)}{P_{c,t}}\right)^{-\epsilon} EX_t. \end{aligned}$$

Following McCallum and Nelson (1999), I assume that aggregate export demand is given by

$$\begin{aligned} EX_t &= \left(\frac{S_t P_t^f}{P_t}\right)^{\upsilon}\left(Y_t^f\right)^{\varsigma} \\ \widehat{EX}_t &= \vartheta \widehat{RS}_t + \varsigma \hat{Y}_t^f, \end{aligned} \tag{4.10}$$

where $RS_t = \frac{S_t P_t^f}{P_t}$ is the real exchange rate, Y_t^f is foreign output, and $\vartheta > 0$ and $\zeta > 0$.

4.2.6 Prices

The composite price index is a combination of the prices of consumption goods and rental

$$P_t = \left[\upsilon P_{c,t}^{1-\eta} + (1-\upsilon) P_{h,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}, \tag{4.11}$$

where υ is the share of expenditure on consumer goods, η is the elasticity of substitution between consumption goods and housing, $P_{c,t}$ is price of consumption goods, $P_{h,t}$ is price of rental services.

Taking differences

$$\begin{aligned} 0 &= \upsilon(1-\eta) X_{c,t}^{-\eta} dX_{c,t} + (1-\upsilon)(1-\eta) X_{h,t}^{-\eta} dX_{h,t} \\ &= \frac{dX_{c,t}}{X_{c,t}} X_{c,t}^{1-\eta} dX_{c,t} + (1-\upsilon)(1-\eta)\upsilon(1-\eta)\frac{dX_{h,t}}{X_{h,t}} X_{h,t}^{1-\eta} \\ &= \hat{X}_{c,t} + \left(\frac{1-\upsilon}{\upsilon}\right)\left(\frac{X_h}{X_c}\right)^{1-\eta} \hat{X}_{h,t}. \end{aligned}$$

Thus,

$$\hat{X}_{c,t} = -\left(\frac{1-\upsilon}{\upsilon}\right)\left(\frac{X_h}{X_c}\right)^{1-\eta} \hat{X}_{h,t}, \tag{4.12}$$

where $X_{c,t} = \frac{P_{c,t}}{P_t}$ is the relative price of the representative consumption good and $X_{h,t} = \frac{P_{h,t}}{P_t}$ is the relative price of rental services.

Price stickiness is incorporated into the model through limiting the ability of firms to reset their prices every period. Each firm has a 50% chance to set the price for its good, $p_t(z)$, each period, giving an average duration for its price for four quarters. This value is taken from DSGE model of the Ministry of Finance of the Czech Republic. Firms maximize their profits by choosing $p_t(z)$ and thereby indirectly labour and import quantities (minimizing their cost given the price level).

4.2.7 Households

The households can be divided into groups according to their consumption behavior. Rule of thumb (ROT) consumers do not accumulate financial assets or liabilities and thus they cannot accumulate wealth so as to afford a deposit on a house, let alone purchase one outright. In contrast, permanent income hypothesis (PIH) consumers are able to access the capital markets and thus they are able to save or borrow. PIH households are owners of the housing stock. The homeowner component of the households rents the housing stock to both ROT and PIH consumers. PIH households sell and repurchase the entire housing stock each period and depending upon their net worth relative to the value of the housing stock, will pay a dividend to the PIH consumers.

Treatment of preferences is standard. Consumer's j intra-period utility may be expressed as follows

$$\log C_t^j + \xi \log \left(1 - L_t^j\right),$$

where $\xi > 0$ and represents the disutility of labor coefficient, L_t^j denotes labor and C_t^j denotes a CES consumption aggregator of form

$$C_t^j = \left[\upsilon^{\frac{1}{\eta}} \left(c_t^j\right)^{\frac{\eta-1}{\eta}} + (1-\upsilon)^{\frac{1}{\eta}} \left(h_t^j\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}},$$

where υ is the share of expenditure on consumer goods, η is the elasticity of substitution between consumption goods and housing, c_t^j is a Dixit-Stiglitz aggregator of differentiated goods and h_t^j denotes housing services. The differentiated goods is indexed by $z \in (0,1)$ and the Dixit-Stiglitz aggregator for consumption goods is defined as

$$c_t^j = \left[\int_0^1 c_t^j(z)^{\frac{\epsilon-1}{\epsilon}} dz \right]^{\frac{\epsilon}{\epsilon-1}},$$

and the corresponding price index for consumption goods is given by

$$P_{c,t} = \left[\int_0^1 p_t(z)^{1-\varepsilon} dz \right]^{\frac{1}{1-\varepsilon}}.$$

Given a level of composite consumption C_t^j, intra-period utility maximization implies the following demand function for each good

$$c_t^j = \upsilon \left(\frac{P_{c,t}}{P_t} \right)^{-\eta} C_t^j,$$

$$h_t^j = (1-\upsilon) \left(\frac{P_{h,t}}{P_t} \right)^{-\eta} C_t^j,$$

where $P_{c,t}$ and $P_{h,t}$ denote prices of consumption goods and rental price of housing, respectively. The composite price index, P_t, is defined in 4.11. Demand for each of the consumption goods is given by

$$c_t^j(z) = \left(\frac{p_t(z)}{P_{c,t}} \right)^{-\epsilon} c_t^j.$$

Aggregating across all consumers leads to

$$\begin{aligned}
c_t &= \upsilon\left(\frac{P_{c,t}}{P_t}\right)^{-\eta} C_t \\
&= \upsilon X_{c,t}^{-\eta} C_t \\
\log c_t &= \log \upsilon - \eta \log X_{c,t} + \log C_t \\
\hat{c}_t &= \hat{C}_t - \eta \hat{X}_{c,t} \qquad (4.13)
\end{aligned}$$

and

$$\begin{aligned}
h_t &= (1-\upsilon)\left(\frac{P_{h,t}}{P_t}\right)^{-\eta} C_t \\
&= (1-\upsilon) X_{h,t}^{-\eta} C_t \\
\log h_t &= \log \upsilon - \eta \log X_{h,t} + \log C_t \\
\hat{h}_t &= \hat{C}_t - \eta \hat{X}_{h,t}. \qquad (4.14)
\end{aligned}$$

4.2.7.1 PIH consumer's first order conditions

Each period all consumers face a budget constraint. The left side of equation contains expenditures and the right side comprises income of PIH consumers. Moreover, consumers may in each period purchase or sell financial assets either domestically or abroad. Following Paoli (2009) in real terms the budget constraint is as follows

$$C_t^P + B_{H,t-1} + RS_t B_{F,t-1} = \frac{B_{H,t}}{R_{t+1}} + \frac{RS_t B_{F,t}}{\left(R_{t+1}^f\right)\psi\left(RS_t B_{F,t}\right)} + \frac{W_t}{P_t} + D_t,$$

where C_t^P is the PIH consumption (housing investments are included here), L_t^P is the PIH labor supply, RS_t is the real exchange rate, $B_{H,t-1}$ is the amount of real domestic currency bonds purchased ($B_{H,t-1} < 0$) or issued ($B_{H,t-1} > 0$) at time $t-1$ with maturity t, $B_{F,t-1}$ is similarly real foreign currency denominated bonds, RS_t is the real exchange rate, R_{t+1} is the real domestic interest rate from period t to $t+1$, R_{t+1}^f is similarly the foreign interest rate, $\frac{W_t}{P_t}$ is the real wage, and D_t is the dividend paid by tenants.

The maximization problem of PIH consumers is as follows

$$\begin{aligned}
\mathcal{L} = \sum_{k=0}^{\infty} \beta^k E_t \Bigg\{ & \log C_{t+k}^P + \xi \log\left(1 - L_{t+k}^P\right) \\
& + \lambda_s \Bigg[C_{t+k}^P + B_{H,t+k-1} + RS_{t+k} B_{F,t+k-1} \qquad (4.15) \\
& - \frac{B_{H,t+k}}{R_{t+k+1}} - \frac{RS_{t+k} B_{F,t+k}}{\left(R_{t+k}^f\right)\psi\left(RS_{t+k} B_{F,t}\right)} - \frac{W_{t+k}}{P_{c,t+k}} L_{t+k}^P - D_{t+k} \Bigg] \Bigg\},
\end{aligned}$$

where β^k is the discount factor applied in period k.

Maximizing 4.15 with respect to C^P_{t+k} is expressed as

$$\frac{\partial \mathcal{L}}{\partial C^P_{t+k}} = \beta^k \left(\frac{1}{C^P_{t+k}} + \lambda_{t+k} \right).$$

Setting to zero,

$$\begin{aligned} \beta^k \left(\frac{1}{C^P_{t+k}} + \lambda_s \right) &= 0 \\ \lambda_t &= -\frac{1}{C^P_t}. \end{aligned} \tag{4.16}$$

Maximizing 4.15 with respect to $B_{H,t+k}$ is expressed as

$$\frac{\partial \mathcal{L}}{\partial B_{H,t+k}} = -\beta^k \lambda_{t+k} \frac{1}{R_{t+k+1}} + \beta^{k+1} E_t \lambda_{t+k+1}.$$

Setting to zero,

$$\begin{aligned} \beta^k \lambda_{t+k} \frac{1}{R_{t+k+1}} - \beta^{k+1} E_t \lambda_{s+1} &= 0 \\ \lambda_t &= R_{t+1} \beta E_t \lambda_{t+1}. \end{aligned} \tag{4.17}$$

Substituting 4.16 into 4.17 yields

$$\frac{1}{C^P_t} = \beta R_{t+1} E_t \frac{1}{C^P_{t+1}}.$$

Log-linearizing

$$\begin{aligned} \log \frac{1}{C^P_t} &= \log \beta + \log R_{t+1} + \log E_t \frac{1}{C^P_{t+1}} \\ -\log \frac{1}{C^P_t} &= \log \beta + \log R_{t+1} - \log E_t \frac{1}{C^P_{t+1}} \\ \frac{dC^P_t}{C^P_t} &= E_t \frac{dC^P_{t+1}}{C^P_{t+1}} - \frac{dR_{t+1}}{R_{t+1}} \\ \hat{C}^P_t &= E_t \hat{C}^P_{t+1} - \hat{R}_{t+1}. \end{aligned} \tag{4.18}$$

Maximizing 4.15 with respect to $B_{F,t+k}$

$$\frac{\partial \mathcal{L}}{\partial B_{F,t+k}} = -\beta^k \lambda_{t+k} \frac{RS_{t+k}}{\left(R^f_{t+k+1}\right) \psi\left(RS_{t+k} B_{F,t+k}\right)} + \beta^{k+1} E_s \lambda_{t+k+1} RS_{t+k+1}.$$

Setting to zero,

$$\frac{\lambda_{t+k} RS_{t+k}}{\left(R^{f}_{t+k+1}\right)\psi\left(RS_{t+k}B_{F,t+k}\right)} = \beta E_t\left(\lambda_{t+k+1}RS_{t+k+1}\right).$$

Substituting 4.16 into this yields

$$\frac{1}{C^P_t} = \left(R^f_{t+1}\right)\psi\left(RS_t B_{F,t}\right)\beta E_t\left(\frac{1}{C^P_{t+1}}\frac{RS_{t+1}}{RS_t}\right). \tag{4.19}$$

Substituting 4.18 into 4.19 yields

$$\begin{aligned}
\beta R_{t+1} E_t \frac{1}{C^P_{t+1}} &= \left(R^f_{t+1}\right)\psi\left(RS_t B_{F,t}\right)E_t\left[\frac{1}{C^P_{t+1}}\frac{RS_{t+1}}{RS_t}\right]\\
\beta R_{t+1} &= \left(R^f_{t+1}\right)\psi\left(RS_t B_{F,t}\right)E_t\frac{RS_{t+1}}{RS_t}\\
\hat{R}_{t+1} &= \hat{R}^f_{t+1} - \delta_b \hat{b}_t + E_t\widehat{RS}_{t+1} - \widehat{RS}_t,
\end{aligned} \tag{4.20}$$

where

$$\begin{aligned}
\hat{b}_t &= \left(\frac{S_t P^f_t B_{F,t}}{P_t} - b\right)\frac{1}{C}\\
\delta_b &= -\psi' C\\
b &= \text{steady state level of foreign currency debt.}
\end{aligned}$$

4.2.7.2 Wage determination

Maximizing 4.15 with respect to L^P_s (labor)

$$\frac{\partial \mathcal{L}}{\partial L^P_s} = \beta^{s-t}\left[-\frac{\xi}{1-L^P_s} - \lambda_s\frac{W_s}{P_{c,s}}\right] = 0.$$

Substituting in 4.16

$$\begin{aligned}
\frac{1}{C^P_t} &= \frac{\xi}{w_t\left(1-L^P_t\right)}\\
\xi C^P_t &= w_t\left(1-L^P_t\right)\\
\log\xi + \log C^P_t &= \log w_t + \log\left(1-L^P_t\right)\\
\frac{dC^P_t}{C^P_t} &= \frac{dw_t}{w_t} - \frac{dL^P_t}{1-L^P_t}\frac{dL^P_t}{L^P_t}\\
\hat{C}^P_t &= \hat{w}_t - \frac{L^P}{1-L^P}\hat{L}^P_t,
\end{aligned} \tag{4.21}$$

where w_t is the real wage.

4.2.7.3 Rule of thumb consumption

Rule of thumb consumers maximize following utility function

$$\log C_t^r + \xi \log \left(1 - L_t^r\right),$$

subject to

$$C_t^r = w_t L_t^r - (1-n) T_t,$$

where C_t^r is aggregate consumption by ROT consumers, L_t^r is the labor supplied by ROT consumers and $-(1-n) T_t$ is ROT consumers' share of lump sum tax to be paid. The introduction of taxes creates a wedge between the fixed supply of labor that would normally entail with ROT consumers and the supply of labor in the fiscal model. This wedge stems directly from the fact that ROT consumers supply labor over and above the level that would be supplied in the absence of a tax.

Combining the definition of ROT consumption with the first order conditions for consumption and labor supply one obtains

$$\begin{aligned}
w_t L_t^r - (1-n) T_t &= \frac{w_t \left(1 - L_t^r\right)}{\xi} \\
w_t L_t^r \left(1 + \frac{1}{\xi}\right) &= \frac{w_t}{\xi} + (1-n) T_t \\
L_t^r &= \frac{1}{1+\xi} + \frac{\xi}{w_t} (1-n) T_t \\
\hat{L}_t^r &= \frac{\xi (1-n) T}{w L^r} \left(\hat{T}_t - \hat{w}_t\right).
\end{aligned} \tag{4.22}$$

Changes to the aggregate labor supply are driven only by changes in PIH labor supply.

$$\begin{aligned}
L_t &= n L_t^p + (1-n) L_t^r \\
dL_t &= n dL_t^p + (1-n) dL_t^r \\
\frac{dL_t}{L} &= \frac{n L^p}{L} \frac{dL_t^p}{L^p} + (1-n) \frac{n L^r}{L} \frac{dL_t^r}{L^r} \\
\hat{L}_t &= n_L \hat{L}_t^p + (1-n) \hat{L}_t^r \\
\hat{L}_t &= n_L \hat{L}_t^p,
\end{aligned}$$

where $n_L = \frac{nL^p}{L}$. Rule of thumb consumers do not smooth their income with any investments and they consume all their income each period. Derivation is performed as

$$\begin{aligned}
C_t^r &= w_t L_t^r - (1-n) T_t \\
dC_t^r &= w_t dL_t^r + L_t dw_t - (1-n) dT_t \\
\hat{C}_t^r &= \frac{w L^r}{C^r} \left(\hat{L}_t^r + \hat{w}_t\right) - \frac{(1-n) T}{C^r} \hat{T}_t.
\end{aligned} \tag{4.23}$$

4.2.8 Aggregate consumption

Aggregate consumption is the weighted consumption of PIH and ROT consumers

$$
\begin{aligned}
C_t &= nC_t^P + (n-1)\,C_t^r \\
dC_t &= ndC_t^P + (n-1)\,dC_t^r \\
\frac{dC_t}{C_t} &= \frac{nC_t^P}{C_t}\frac{ndC_t^P}{C_t^P} + \frac{(n-1)\,C_t^r}{C_t}\frac{dC_t^r}{C_t^r} \\
\frac{dC_t}{C_t} &= \frac{nC_t^P}{C_t}\frac{ndC_t^P}{C_t^P} + \frac{(n-1)\,C_t - nC_t^P}{C_t}\frac{dC_t^r}{C_t^r} \\
\hat{C}_t &= n_C\hat{C}_t^P + \frac{C-nC^P}{C}\hat{C}_t^r \\
\hat{C}_t &= n_C\hat{C}_t^P + (1-n_C)\,\hat{C}_t^r, \qquad (4.24)
\end{aligned}
$$

where $n_C = n\frac{C^P}{C}$ and n is the proportion of PIH consumers.

4.2.9 The mortgages and risk-free interest rate

In accordance with Bernanke, Gertler, and Gilchrist (1999), the cost of borrowing for the purchase of housing capital is at a premium to the risk-free interest rate. This premium varying inversely with the extent that the borrower has net positive wealth. Bernanke, Gertler, and Gilchrist (1999) outlined agency problem, mortgage lenders will only lend to homeowners at a premium to the risk-free rate. Homeowners expect that they make again on their capital as well as the rental from consumers. They will borrow up to the point that the cost of borrowing is just equal to the expected return from the housing asset. Thus,

$$E_tR_{h,t+1} = f\left(\frac{N_{t+1}}{q_th_{t+1}}\right)R_{t+1}, \qquad (4.25)$$

where $E_tR_{h,t+1}$ represents the rate of return on owning housing asset from period t to $t+1$, N_{t+1} is the net worth of the household determined at the end of period t and thus carried over into period $t+1$, q_t is the real price of housing in period t, $f' < 0$ and h_{t+1} is the stock of housing determined at the end of period t and carried over into period $t+1$. Log-linearizing of 4.25 one obtains

$$
\begin{aligned}
\log E_tR_{h,t+1} &= \log f\left(\frac{N_{t+1}}{q_th_{t+1}}\right) + \log R_{t+1} \\
E_t\frac{dR_{h,t+1}}{R_{h,t+1}} &= \frac{f'(.)}{f(.)}\frac{N_{t+1}}{q_th_{t+1}}\left(\frac{dN_{t+1}}{N_{t+1}} - \frac{dq_t}{q_t} - \frac{dh_{t+1}}{h_{t+1}}\right) + \frac{dR_{t+1}}{R_{t+1}} \\
E_t\hat{R}_{h,t+1} &= \Omega\left(\hat{N}_{t+1} - \hat{q}_t - \hat{h}_{t+1}\right) + \hat{R}_{t+1}, \qquad (4.26)
\end{aligned}
$$

where $\Omega = \frac{f'(\phi)}{f(\phi)}\phi$ and $\phi = \frac{N}{qh}$ is the steady state net worth ratio.

4.2.10 Ex-post gross return on housing

Homeowners expect that they will have a return on housing asset. Housing stock bought in period t is rented to consumers in period $t+1$. The price of housing stock may change thus providing a capital gain or loss. Depreciation is assumed to erode some of the value of the existing housing stock.

$$E_t R_{h,t+1} = E_t \left(\frac{X_{h,t+1} + (1-\delta)\, q_{t+1}}{q_t} \right),$$

where q_t is the real house price and δ represents the housing depreciation rate. Ex-post this becomes

$$\begin{aligned} \log R_{h,t} &= \log\left(X_{h,t} + (1-\delta)\, q_t\right) - \log q_{t-1} \\ d\log R_{h,t} &= \frac{1}{X_{h,t} + (1-\delta)\, q_t} \left(dX_{h,t} + (1-\delta)\, dq_t\right) - d\log q_{t-1}. \end{aligned}$$

Defining $\mu = \frac{(1-\delta)q}{X_h + (1-\delta)q}$ one obtains

$$\hat{R}_{h,t} = (1-\mu)\, \hat{X}_{h,t} + \mu \hat{q}_t - \hat{q}_{t-1}. \tag{4.27}$$

4.2.11 Net worth accumulation

The value of homeowners is given at the end of each period by

$$V_t = R_{h,t} q_{t-1} h_t - f\left(\frac{N_t}{q_{t-1} h_t}\right) R_t b_t, \tag{4.28}$$

where b_t represents the borrowing undertaken to finance the purchase of the housing stock. It is equal to $q_t h_{t+1} - N_{t+1}$. D_t is the dividend payed to consumers, the net worth of homeowners is given by

$$\begin{aligned} N_{t+1} &= V_t - D_t \\ \log N_{t+1} &= \log V_t - \log D_t \\ d\log N_{t+1} &= \frac{1}{V_t - D_t} \left(V_t \frac{dV_t}{V_t} - D_t \frac{dD_t}{D_t} \right) \\ \frac{dN_{t+1}}{N_{t+1}} &= \frac{1}{N_{t+1}} \left(V_t \frac{dV_t}{V_t} - D_t \frac{dD_t}{D_t} \right). \end{aligned}$$

4.28 can be simplified as follows

$$
\begin{aligned}
V_t &= R_{h,t}q_{t-1}h_t - f\left(\phi_{t-1}\right)R_t\left(q_{t-1}h_t - N_t\right) \\
V_t &= R_{h,t}q_{t-1}h_t - f\left(\phi_{t-1}\right)R_tq_{t-1}h_t + f\left(\phi_{t-1}\right)R_tN_t \\
V_t &= R_{h,t}q_{t-1}h_t - R_{h,t}q_{t-1}h_t + R_{h,t}N_t - N_{t-1} \\
V_t &= R_{h,t}N_t.
\end{aligned}
$$

Thus,

$$
\begin{aligned}
\frac{dN_{t+1}}{N_{t+1}} &= \frac{1}{N_{t+1}}\left[R_{h,t}N_t\hat{V}_t - \left(R_{h,t}N_t - N_{t+1}\right)\hat{D}_t\right] \\
\frac{dN_{t+1}}{N_{t+1}} &= \frac{R_{h,t}N_t}{N_{t+1}}\hat{V}_t - \frac{R_{h,t}N_t - N_{t+1}}{N_{t+1}}\hat{D}_t
\end{aligned}
$$

and the steady state is defined as $N_{t+1} = N_t = N$. Then one obtains

$$
\hat{N}_{t+1} = R_h\hat{V}_t - \left(R_h - 1\right)\hat{D}_t.
$$

Starting from the expanded definition of V_t

$$
\begin{aligned}
V_t &= R_{h,t}q_{t-1}h_t - f\left(\phi_{t-1}\right)R_tq_{t-1}h_t + f\left(\phi_{t-1}\right)R_tN_t \\
d\left(R_{h,t}q_{t-1}h_t\right) &= R_hqh\left(\hat{R}_{h,t} + \hat{q}_{t-1} + \hat{h}_t\right) \\
d\left(f\left(\phi_{t-1}\right)R_tq_{t-1}h_t\right) &= Rqhf'\left(\phi\right)\phi\left(\hat{N}_t - \hat{q}_{t-1} - \hat{h}_t\right) + f\left(\phi\right)Rqh\left(\hat{R}_t + \hat{q}_{t-1} + \hat{h}_t\right) \\
d\left(f\left(\phi_{t-1}\right)R_tN_t\right) &= RNf'\left(\phi\right)\phi\left(\hat{N}_t - \hat{q}_{t-1} - \hat{h}_t\right) + f\left(\phi\right)RN\left(\hat{N}_t + \hat{R}_t\right).
\end{aligned}
$$

Hence

$$
\begin{aligned}
\frac{dV_t}{V} &= \frac{dV_t}{R_hN} \\
&= \frac{qh}{N}\left(\hat{R}_{h,t} + \hat{q}_{t-1} + \hat{h}_t\right) \\
&- \frac{qh}{N}\frac{f'\left(\phi\right)}{f\left(\phi\right)}\phi\left(\hat{N}_t - \hat{q}_{t-1} - \hat{h}_t\right) - \frac{qh}{N}\left(\hat{R}_t + \hat{q}_{t-1} + \hat{h}_t\right) \\
&+ \frac{f'\left(\phi\right)}{f\left(\phi\right)}\phi\left(\hat{N}_t - \hat{q}_{t-1} - \hat{h}_t\right) + \hat{N}_t + \hat{R}_t \\
&= \left(1 + bn\right)\hat{R}_{h,t} - bn\hat{R}_t + \left(1 - bn\Omega\right)\hat{N}_t + bn\Omega\left(\hat{q}_{t-1} + \hat{h}_t\right),
\end{aligned}
$$

where $\Omega = \frac{f'(\phi)}{f(\phi)}\phi$ is interpreted as the sensitivity of dividend to net worth ratio and $bn = \frac{1}{\phi} - 1$. It can be then rewritten as

$$
\hat{N}_{t+1} = R_h\left[\left(1 + bn\right)\hat{R}_{h,t} - bn\hat{R}_t + \left(1 - bn\Omega\right)\hat{N}_t + bn\Omega\left(\hat{q}_{t-1} + \hat{h}_t\right)\right] - \left(R_h - 1\right)\hat{D}_t. \tag{4.29}
$$

4.2.12 Housing equity withdrawal

Homeowners pay a dividend to consumers at the end of each period, the size of dividend is determined by the homeowner's net worth.

$$
\begin{aligned}
D_t &= \chi\left(\frac{N_{t+1}}{q_t h_{t+1}}\right) \\
dD_t &= \chi'(.)\,\chi\left(\frac{N_{t+1}}{q_t h_{t+1}}\right)\left[\hat{N}_{t+1} - \hat{q}_t - \hat{h}_{t+1}\right] \\
\hat{D}_t &= \frac{\chi'(\phi)}{\chi(\phi)}\phi\left[\hat{N}_{t+1} - \hat{q}_t - \hat{h}_{t+1}\right].
\end{aligned} \tag{4.30}
$$

where $\chi'(\phi) > 0$.

4.2.13 Housing capital accumulation

Housing stock behaves as follows

$$
\begin{aligned}
h_{t+1} &= h_t + I_t - \delta h_t \\
dh_{t+1} &= (1-\delta)\,dh_t + dI_t \\
\frac{dh_{t+1}}{h_{t+1}} &= (1-\delta)\frac{h_t}{h_{t+1}}\frac{dh_t}{h_t} + \frac{I_t}{I_{t+1}}\frac{dI_t}{I_t},
\end{aligned}
$$

where I_t is the consumption goods purchased by house producers in order to produce h_{t+1} units of housing.

We know following steady states $h_{t+1} = h_t = h$ and $\frac{I_t}{h_{t+1}} = \frac{I}{h} = \delta$. After substitution one obtains

$$
\hat{h}_{t+1} = (1-\delta)\,\hat{h}_t + \delta\hat{I}_t. \tag{4.31}
$$

4.2.14 House prices and investment

Investments and house prices are linked by a q-theory[2]. House producers purchase I_t of the consumption goods and use this to produce h_{t+1} units of housing. The law of motion of the capital stock is

$$
h_{t+1} = I_t + (1-\delta)\,h_t.
$$

[2]Tobin (1969) writes "One, the numerator, is the market valuation: the going price in the market for exchanging existing assets. The other, the denominator, is the replacement or reproduction cost: the price in the market for the newly produced commodities. We believe that this ratio has considerable macroeconomic significance and usefulness, as the nexus between financial markets and markets for goods and services." The formula may be for purposes of this study rewritten as follows $q = \frac{\text{value of housing stock}}{\text{net worth}}$.

Bernanke, Gertler, and Gilchrist (1999) assume that there are increasing marginal adjustment costs in the production of housing stock so that

$$h_{t+1} = \Phi\left(\frac{I_t}{h_t}\right) h_t + (1-\delta)\, h_t,$$

where $\Phi' > 0$.

I other words, homeowners invest I_t but only the fraction $\Phi\left(\frac{I_t}{h_t}\right) h_t$ of this investment becomes actual gross investment. The production function of homeowners is

$$I_t^h = \Phi\left(\frac{I_t}{h_t}\right) h_t,$$

where I_t^h is the production of new housing units. This function is increasing and concave in I_t, which can be conceived of as raw materials the homeowner is using to produce new housing stock together with undepreciated housing stock h_t.

Each unit of I_t has cost equal to $X_{c,t}$. The marginal cost of housing unit is $X_{c,t}$/Marginal Product of Investment, i.e.

$$\frac{X_{c,t}}{\Phi'\left(\frac{I_t}{h_t}\right)},$$

where $X_{c,t}$ is the price of consumption goods relative to the composite price index.

Assuming perfect competition on the market the equilibrium condition

$$\frac{q_t}{X_{c,t}} = \Phi'\left(\frac{I_t}{h_t}\right),$$

where q_t represents real house prices.

Log-linearizing

$$\begin{aligned}
\log q_t - \log X_{c,t} &= \log \Phi'\left(\frac{I_t}{h_t}\right) \\
d\log q_t - d\log X_{c,t} &= \frac{1}{\Phi'(.)}\Phi''\left[\frac{h_t dI_t - I_t dh_t}{h_t^2}\right] \\
&= \frac{\Phi''(.)}{\Phi'(.)}\left[\frac{I_t}{h_t}\left(\hat{I}_t - \hat{h}_t\right)\right] \\
\hat{q}_t &= \Gamma_d\left(\hat{I}_t - \hat{h}_t\right) + \hat{X}_{c,t}, \qquad (4.32)
\end{aligned}$$

where $\Gamma_d = \frac{\Phi''(.)}{\Phi'(.)}\left(\frac{I}{h}\right)$ and may be interpreted as q-theory sensitivity.

4.2.15 Current account

The of the resource constraints for all agents in the economy is represented by current account. Foreign households trade only foreign currency denominated debt. This is assumed by Paoli (2009). Then the current account reduces to

$$\frac{S_t P_t^f B_{F,t}}{P_t} = \frac{S_t P_t^f B_{F,t}}{P_t R_{t+1}^f \psi(.)} + EX_t - \frac{S_t P_t^f}{P_t} IM_t, \tag{4.33}$$

where S_t is the nominal exchange rate, P_t^f is the foreign price of imports, $B_{F,t}$ is the issuance or purchase of real foreign currency debt, P_t is the given price level, R_{t+1}^f is the foreign interest rate, ψ is the interest rate premium, EX_t represents foreign purchases and IM_t represents imports.

Differentiating of the second expression in 4.33

$$\begin{aligned}
d & \left(\frac{S_t P_t^f B_{F,t}}{P_t R_{t+1}^f \psi\left(\frac{S_t P_t^f B_{F,t}}{P_t}\right)} \right) = \frac{1}{R^f \psi\left(\frac{SP^f B_F}{P}\right)} d\left(\frac{S_t P_t^f B_{F,t}}{P_t}\right) \\
& + \frac{SP^f B_F}{P} \frac{1}{R^f} \left[-\psi\left(\frac{SP^f B_F}{P}\right)^{-2} \psi'\left(\frac{SP^f B_F}{P}\right) d\left(\frac{S_t P_t^f B_{F,t}}{P_t}\right) \right] \\
& + \frac{SP^f B_F}{P} \frac{1}{\psi\left(\frac{SP^f B_F}{P}\right)} \left[-\left(R^f\right)^{-2} dR_t^f \right] \\
= & \ \beta d\left(\frac{S_t P_t^f B_{F,t}}{P_t}\right) \\
& - \beta \frac{SP^f B_F}{P} \frac{\psi'\left(SP^f B_F\right)}{\psi\left(SP^f B_F\right)} d\left(\frac{S_t P_t^f B_{F,t}}{P_t}\right) \\
& - \beta \frac{SP^f B_F}{P} \frac{dR_{t+1}^f}{R_{t+1}^f}
\end{aligned}$$

Using the assumption that in the steady state the premium factor is equal to 1.

$$= \ \beta \left\{ d\left(\frac{S_t P_t^f B_{F,t}}{P_t}\right) \left[1 - \frac{SP^f B_F}{P} \psi'\left(\frac{SP^f B_F}{P}\right) \right] - \frac{SP^f B_F}{P} \frac{dR_{t+1}^f}{R_{t+1}^f} \right\}.$$

Defining $b_t = \frac{\left(\frac{S_t P_t^f B_{F,t}}{P_t} - b\right)}{C}$, $a = \frac{b}{C}$ and $\delta_b = -\psi'(b)\,C \Rightarrow a\delta = -\psi'(b)\,b$ one obtains following relations

$$
\begin{aligned}
d\left(\frac{S_t P_t^f B_{F,t}}{P_t R_{t+1}^f \psi\left(\frac{S_t P_t^f B_{F,t}}{P_t}\right)}\right) &= \beta\left[b_t C\left(1 - a\delta_b\right) - b\hat{R}_{t+1}^f\right] \\
&= \beta\left[b_t C\left(1 - a\delta_b\right) - aC\hat{R}_{t+1}^f\right] \\
&= \beta C\left[b_t C\left(1 - a\delta_b\right) - a\hat{R}_{t+1}^f\right].
\end{aligned}
$$

Differentiating of the first expression in 4.33

$$
\frac{S_t P_t^f B_{F,t-1}}{P_t} = \frac{S_{t-1} P_{t-1}^f B_{F,t-1}}{P_{t-1}} \frac{S_t}{S_{t-1}} \frac{P_t^f}{P_{t-1}^f} \frac{P_{t-1}}{P_t}.
$$

$$
\begin{aligned}
d\left(\frac{S_t P_t^f B_{F,t-1}}{P_t}\right) &= d\left(\frac{S_{t-1} P_{t-1}^f B_{F,t-1}}{P_{t-1}}\right) + \frac{SP^f B_F}{P}\left[d\left(\frac{S_t}{S_{t-1}}\right) + d\left(\frac{P_t^f}{P_{t-1}^f}\right) + d\left(\frac{P_{t-1}}{P_t}\right)\right] \\
&= b_{t-1}C + b\left[\Delta\hat{S}_t + \hat{\pi}_t^f - \hat{\pi}_t\right]
\end{aligned}
$$

Differentiating of the third expression in 4.33

$$
\begin{aligned}
d\left(EX_t\right) &= \left(\frac{SP^f}{P}\right)^{\vartheta} \varsigma\left(Y^f\right)^{\varsigma-1} dY_t^f + \left(Y^f\right)^{\varsigma} \vartheta\left(\frac{SP^f}{P}\right)^{\vartheta-1} d\left(\frac{S_t P_t^f}{P_t}\right) \\
&= EX\left[\varsigma\frac{dY_t^f}{Y^f} + \vartheta\frac{d\left(\frac{S_t P_t^f}{P_t}\right)}{\left(\frac{SP^f}{P}\right)}\right] \\
&= EX\left[\varsigma\hat{Y}_t^f + \vartheta\widehat{RS}_t\right],
\end{aligned}
$$

where ς is the export sensitivity to foreign demand, ϑ is the export sensitivity to real exchange rate and Y_t^f is the foreign output.

Differentiating of the fourth expression in 4.33

$$
\begin{aligned}
d\left(IM_t \frac{S_t P_t^f}{P_t}\right) &= \frac{SP^f}{P} dIM_t + IMd\left(\frac{S_t P_t^f}{P_t}\right) \\
&= \frac{SP^f}{P} dIM_t + IM\left[\frac{dIM_t}{IM} + \frac{d\left(\frac{S_t P_t^f}{P_t}\right)}{\frac{SP^f}{P}}\right] \\
&= \frac{SP^f}{P} dIM_t + IM\left[\widehat{IM}_t + \widehat{RS}_t\right]
\end{aligned}
$$

Hence

$$\begin{aligned}\beta C\left[b_t\left(1+a\delta\right)-a\hat{R}^f_{t+1}\right]=b_{t-1}C\quad &+\quad b\left[\Delta\hat{S}_t+\hat{\pi}^f_t-\hat{\pi}_t\right]\\ -EX\left[\varsigma\hat{Y}^f_t+\upsilon\widehat{RS}_t\right]\quad &+\quad \frac{SP^f}{P}IM\left[\widehat{IM}_t+\widehat{RS}_t\right]\end{aligned}\tag{4.34}$$

4.2.16 New Keynesian Phillips curve

Prices are sticky according to Calvo (1983). Firms reset their prices each period with a probability $1-\theta$. Hence firm z that is able to reset its price in period t will seek to optimize

$$\begin{aligned}\mathcal{L} &= E_t\sum_{k=0}^{\infty}\left(\theta\beta\right)^k\lambda_k\left[p^*_t\left(z\right)y_{t+k}\left(z\right)-TC_{t+k}\left(y_{t+k}\left(z\right)\right)\right]\\ &= E_t\sum_{k=0}^{\infty}\left(\theta\beta\right)^k\lambda_k\left[p^*_t\left(z\right)\left(\frac{p^*_t\left(z\right)}{P_{c,t+k}}\right)^{-\epsilon}Y_{t+k}-TC_{t+k}\left(\left(\frac{p^*_t\left(z\right)}{P_{c,t+k}}\right)^{-\epsilon}Y_{t+k}\right)\right]\\ &= E_t\sum_{k=0}^{\infty}\left(\theta\beta\right)^k\lambda_k\left[p^*_t(z)^{1-\epsilon}\left(\frac{1}{P_{c,t+k}}\right)^{-\epsilon}Y_{t+k}-TC_{t+k}\left(\left(\frac{p^*_t\left(z\right)}{P_{c,t+k}}\right)^{-\epsilon}Y_{t+k}\right)\right],\end{aligned}$$

where θ is the probability that firms do not reset their prices, β is the discount rate,$p^*_t\left(z\right)$ is the optimal price of firm z at time t, $y_{t+k}\left(z\right)$ is the output of firms z at time $t+k$, TC_{t+k} represents total costs at time $t+k$, ϵ is the price elasticity of demand faced by each firm, $P_{c,t+k}$ is the aggregate price for all consumer goods at time $t+k$ and Y_{t+k} is the aggregate goods output of all firms.

Differentiating with respect to $p^*_t\left(z\right)$ yields

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial p^*_t\left(z\right)} &= E_t\sum_{k=0}^{\infty}\left(\theta\beta\right)^k\lambda_k\left[\left(1-\epsilon\right)p^*_t(z)^{1-\epsilon}\left(\frac{1}{P_{c,t+k}}\right)^{-\epsilon}Y_{t+k}\right.\\ &\qquad\left.-\,MC_{t+k}\left(.\right)\left(-\epsilon\right)p^*_t(z)^{-\epsilon-1}\left(\frac{1}{P_{c,t+k}}\right)^{-\epsilon}Y_{t+k}\right]\\ &= E_t\sum_{k=0}^{\infty}\left(\theta\beta\right)^k\lambda_k\left[\left(1-\epsilon\right)\left(\frac{p^*_t\left(z\right)}{P_{c,t+k}}\right)^{-\epsilon}Y_{t+k}-MC_{t+k}\left(.\right)\frac{\epsilon}{p^*_t\left(z\right)}\left(\frac{p^*_t\left(z\right)}{P_{c,t+k}}\right)^{-\epsilon}Y_{t+k}\right],\end{aligned}$$

where MC_{t+k} represents marginal costs at time $t+k$.

Setting to zero

$$0=E_t\sum_{k=0}^{\infty}\left(\theta\beta\right)^k\frac{1-\epsilon}{p^*_t\left(z\right)}\lambda_k y_{t+k}\left(z\right)\left[p^*_t\left(z\right)-\frac{\epsilon}{\epsilon-1}MC_{t+k}\left(.\right)\right]$$

$$p^*_t\left(z\right)E_t\sum_{k=0}^{\infty}\left(\theta\beta\right)^k\lambda_k y_{t+k}\left(z\right)=\frac{\epsilon}{\epsilon-1}E_t\sum_{k=0}^{\infty}\left(\theta\beta\right)^k\lambda_k y_{t+k}\left(z\right)MC_{t+k}\left(z\right).$$

Aggregating and differentiating with respect to $P^*_{c,t}$ and $\lambda_{t+k}Y_{t+k}$

$$\begin{aligned} dP^*_{c,t}\lambda Y \sum_{k=0}^{\infty}(\theta\beta)^k + P_c E_t \sum_{k=0}^{\infty}(\theta\beta)^k d\left(\lambda_{t+k}Y_{t+k}\right) &= \frac{\epsilon}{\epsilon-1} MCE_t \sum_{k=0}^{\infty}(\theta\beta)^k d\left(\lambda_{t+k}Y_{t+k}\right) \\ &+ \frac{\epsilon}{\epsilon-1}\lambda Y E_t \sum_{k=0}^{\infty}(\theta\beta)^k dMC_{t+k}. \end{aligned}$$

Dividing through by $\lambda Y \sum\limits_{k=0}^{\infty}(\theta\beta)^k$

$$\begin{aligned} dP^*_{c,t} + P_c \frac{E_t \sum\limits_{k=0}^{\infty}(\theta\beta)^k d\left(\lambda_{t+k}Y_{t+k}\right)}{\lambda Y \sum\limits_{k=0}^{\infty}(\theta\beta)^k} &= \frac{\epsilon}{\epsilon-1} MC \frac{E_t \sum\limits_{k=0}^{\infty}(\theta\beta)^k d\left(\lambda_{t+k}Y_{t+k}\right)}{\lambda Y \sum\limits_{k=0}^{\infty}(\theta\beta)^k} \\ &+ \frac{\epsilon}{\epsilon-1} \frac{E_t \sum\limits_{k=0}^{\infty}(\theta\beta)^k dMC_{t+k}}{\lambda Y \sum\limits_{k=0}^{\infty}(\theta\beta)^k}. \end{aligned}$$

Dividing through by $P_c = \frac{\epsilon}{\epsilon-1}MC$

$$\begin{aligned} \hat{P}^*_{c,t} + \frac{E_t \sum\limits_{k=0}^{\infty}(\theta\beta)^k d\left(\lambda_{t+k}Y_{t+k}\right)}{\lambda Y \sum\limits_{k=0}^{\infty}(\theta\beta)^k} &= \frac{E_t \sum\limits_{k=0}^{\infty}(\theta\beta)^k d\left(\lambda_{t+k}Y_{t+k}\right)}{\lambda Y \sum\limits_{k=0}^{\infty}(\theta\beta)^k} \\ &+ \frac{E_t \sum\limits_{k=0}^{\infty}(\theta\beta)^k dMC_{t+k}}{\lambda Y \sum\limits_{k=0}^{\infty}(\theta\beta)^k MC}. \end{aligned}$$

$$\hat{P}^*_{c,t} = (1-\theta\beta)\, E_t \sum_{k=0}^{\infty}(\theta\beta)^k \widehat{MC_{t+k}}. \tag{4.35}$$

Expanding 4.35 one obtains

$$\begin{aligned} \hat{P}^*_{c,t} &= (1-\theta\beta)\, E_t \left[\widehat{MC_t} + (\theta\beta)\,\widehat{MC_{t+1}} + (\theta\beta)^2\widehat{MC_{t+2}} + \cdots\right] \\ &= (1-\theta\beta)\,\widehat{MC_t} + (1-\theta\beta)(\theta\beta)\, E_t \sum_{k=0}^{\infty}(\theta\beta)^k \widehat{MC_{t+k+1}} \\ &= (1-\theta\beta)\,\widehat{MC_t} + (\theta\beta)\, E_t \hat{P}^*_{c,t+1}. \end{aligned}$$

Subtracting $\hat{P}_{c,t}$ from both sides yields

$$\hat{P}^*_{c,t} - \hat{P}_{c,t} = (1-\theta\beta)\left(\widehat{MC_t} - \hat{P}_{c,t}\right) + (\theta\beta)\left[E_t\hat{P}^*_{c,t+1} - \hat{P}_{c,t}\right]. \tag{4.36}$$

Define the fraction of firms whose price was set j periods ago as w_j such that

$$w_j = (1-\theta)\,\theta^j.$$

Thus the current aggregate price may be specified as follows

$$P_{c,t} = \left[\sum_{j=0}^{\infty} w_j p^*_{t-j}(z)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}},$$

where $p^*_{t-j}(z)$ is effectively the common price set by firms j periods ago. Hence

$$\hat{P}^*_{c,t} = (1-\theta\beta)\,E_t \sum_{k=0}^{\infty} (\theta\beta)^k \widehat{MC_{t+k}}. \tag{4.37}$$

Expanding 4.37 one obtains

$$\begin{aligned}
P^{1-\epsilon}_{c,t} &= \sum_{j=0}^{\infty} w_j p^*_{t-j}(z)^{1-\epsilon} \\
&= (1-\theta)\,p^*_t(z)^{1-\epsilon} + (1-\theta)\,\theta p^*_{t-1}(z)^{1-\epsilon} + (1-\theta)\,\theta^2 p^*_{t-2}(z)^{1-\epsilon} + \cdots .
\end{aligned}$$

Thus

$$\begin{aligned}
P^{1-\epsilon}_{c,t-1} &= (1-\theta)\,p^*_{t-1}(z)^{1-\epsilon} + (1-\theta)\,\theta p^*_{t-2}(z)^{1-\epsilon} + (1-\theta)\,\theta^2 p^*_{t-3}(z)^{1-\epsilon} + \cdots \\
P^{1-\epsilon}_{c,t} &= (1-\theta)\,p^*_t(z)^{1-\epsilon} + \theta P^{1-\epsilon}_{c,t} \\
P_{c,t} &= \left[(1-\theta)\,p^*_t(z)^{1-\epsilon} + \theta P^{1-\epsilon}_{c,t}\right]^{\frac{1}{1-\epsilon}} \\
\log P_{c,t} &= \frac{1}{1-\epsilon}\log\left[(1-\theta)\,p^*_t(z)^{1-\epsilon} + \theta P^{1-\epsilon}_{c,t}\right] \\
\frac{dP_{c,t}}{P_{c,t}} &= \frac{1}{1-\epsilon}\frac{1}{P^{1-\epsilon}_{c,t}}\left\{(1-\theta)(1-\epsilon)\,p^*_t(z)^{1-\epsilon}\frac{dp^*_t(z)}{p^*_t(z)} + \theta(1-\epsilon)\,P^{1-\epsilon}_{c,t-1}\frac{dP_{c,t-1}}{P_{c,t-1}}\right\}.
\end{aligned}$$

Noting that in the steady state $P_c = p^*(z)$ and that $p^*_t(z)$ is equivalent to the aggregate $P^*_{c,t}$

$$\begin{aligned}
\hat{P}_{c,t} &= (1-\theta)\,\hat{P}^*_{c,t} + \theta P_{c,t-1} \\
\hat{P}^*_{c,t} &= \frac{\hat{P}_{c,t}}{1-\theta} - \frac{1}{1-\theta}P_{c,t-1}.
\end{aligned} \tag{4.38}$$

Subtracting $\hat{P}_{c,t}$ from 4.38

$$\hat{P}^*_{c,t} - \hat{P}_{c,t} = \frac{1}{1-\theta}\hat{P}_{c,t} - \frac{1}{1-\theta}\hat{P}_{c,t-1} - \hat{P}_{c,t}.$$

Which is equal to 4.36

$$
\begin{aligned}
\frac{1}{1-\theta}\hat{P}_{c,t}-\frac{\theta}{1-\theta}\hat{P}_{c,t-1}-\hat{P}_{c,t} &= (1-\theta\beta)\left(\widehat{MC_t}-\hat{P}_{c,t}\right)+\theta\beta\left[E_t\hat{P}^*_{c,t+1}-\hat{P}_{c,t}\right] \\
&= (1-\theta\beta)\left(\widehat{MC_t}-\hat{P}_{c,t}\right)+\theta\beta\left[E_t\frac{1}{1-\theta}\hat{P}_{c,t+1}-\frac{\theta}{1-\theta}\hat{P}_{c,t}-\hat{P}_{c,t}\right] \\
\frac{1-1+\theta}{1-\theta}\hat{P}_{c,t}-\frac{\theta}{1-\theta}\hat{P}_{c,t-1} &= (1-\theta\beta)\left(\widehat{MC_t}-\hat{P}_{c,t}\right)+\frac{\theta\beta}{1-\theta}\left[E_t\hat{P}_{c,t+1}-\theta\hat{P}_{c,t}-(1-\theta)\hat{P}_{c,t}\right] \\
\frac{\theta}{1-\theta}\left(\hat{P}_{c,t}-\hat{P}_{c,t-1}\right) &= (1-\theta\beta)\left(\widehat{MC_t}-\hat{P}_{c,t}\right)+\frac{\theta\beta}{1-\theta}\left[E_t\hat{P}_{c,t+1}-\hat{P}_{c,t}\right] \\
\hat{P}_{c,t}-\hat{P}_{c,t-1} &= \frac{(1-\theta)(1-\theta\beta)}{\theta}\left(\widehat{MC_t}-\hat{P}_{c,t}\right)+\beta\left[E_t\hat{P}_{c,t+1}-\hat{P}_{c,t}\right] \\
\hat{\pi}_{c,t} &= \frac{(1-\theta)(1-\theta\beta)}{\theta}\left(\widehat{MC_t}-\hat{P}_{c,t}\right)+\beta E_t\hat{\pi}_{c,t+1},
\end{aligned}
$$

where $\hat{\pi}_{c,t}$ represents the consumption good inflation.

$\widehat{MC_t}$ is nominal marginal cost and thus $\widehat{mc_t}=\widehat{MC_t}-\hat{P}_{c,t}$. Phillips curve may be written as

$$
\hat{\pi}_{c,t}=\frac{(1-\theta)(1-\theta\beta)}{\theta}\widehat{mc_t}+\beta E_t\hat{\pi}_{c,t+1},
$$

where mc_t represents real marginal cost.

The final form of the New Keynesian Phillips Curve is as follows

$$
\hat{\pi}_{c,t}=\kappa\widehat{mc_t}+\beta E_t\hat{\pi}_{c,t+1}, \tag{4.39}
$$

where $\kappa=\frac{(1-\theta)(1-\theta\beta)}{\theta}$.

4.2.17 Flexible price output

Following expression represents production function which is extended by imports and real exchange rate compare to APV closed economy version. Flexible price output may be interpreted as potential output. The relationship for output assuming the flexible prices may be expressed as follows

$$
\begin{aligned}
\hat{Y}_t^{flex} &= \frac{-\alpha\left(\frac{IM}{Y}\right)^{\gamma}}{(1-\gamma)\left[(1-\alpha)\left(\frac{AL}{Y}\right)^{\gamma}-\left(\frac{1-\gamma}{1-\gamma+\tau}\right)\right]}\widehat{RS_t} \\
&+ \frac{(1-\alpha)\left(\frac{1-\gamma}{1-\gamma+\tau}\right)\left(\frac{AL}{Y}\right)^{\gamma}}{(1-\alpha)\left(\frac{AL}{Y}\right)^{\gamma}-\left(\frac{1-\gamma}{1-\gamma+\tau}\right)}\hat{A}_t,
\end{aligned}
\tag{4.40}
$$

where $\tau=\frac{L}{1-L}$.

4.2.18 Real exchange rate identity

$$\begin{aligned}
RS_t &= \frac{S_t P_t^f}{P_t} \\
\frac{E_t RS_{t-1}}{RS_t} &= \frac{E_t\left(\frac{S_{t+1}P_{t+1}^f}{P_{t+1}}\right)}{\frac{S_t P_t^f}{P_t}} \\
&= E_t \frac{\frac{S_{t+1}}{S_t}\pi_{t+1}^f}{\pi_{t+1}} \\
E_t\widehat{RS_{t+1}} - \widehat{RS_t} &= E_t\hat{S}_{t+1} - \hat{S}_t + E_t\hat{\pi}_{t+1}^f - E_t\hat{\pi}_{t+1}. \qquad (4.41)
\end{aligned}$$

4.2.19 Nominal interest rate identity

$$R_{t+1}^n = R_{t+1}\frac{E_t P_{t+1}}{P_t}.$$

Define

$$\pi_t = \frac{P_t}{P_{t-1}}.$$

Hence,

$$\begin{aligned}
\pi_t \frac{P_{c,t-1}}{P_{c,t}} &= \frac{X_{c,t-1}}{X_{c,t}} \\
\frac{\pi_t}{\pi_{c,t}} &= \frac{X_{c,t-1}}{X_{c,t}} \\
\hat{\pi}_t - \hat{\pi}_{c,t} &= \hat{X}_{c,t-1} - \hat{X}_{c,t}. \qquad (4.42)
\end{aligned}$$

$$\begin{aligned}
\log R_{t+1}^n &= \log R_{t+1} + \log E_t\pi_{t+1} \\
\hat{R}_{t+1}^n &= \hat{R}_{t+1} + E_t\hat{\pi}_{t+1}. \qquad (4.43)
\end{aligned}$$

4.2.20 Monetary authority

The monetary policy is driven using the Taylor rule comprising the nominal interest rate for current period, the last period's goods price inflation and an output gap measure. The Taylor rule takes the following form

$$\hat{R}_{t+1}^n = \rho_i \hat{R}_t^n + (1-\rho_i)\,\gamma_\pi\hat{\pi}_{t-1} + (1-\rho_i)\,\gamma_y\tilde{y}_t, \qquad (4.44)$$

where ρ_i is the smoothing coefficient, γ_π is the weight on lagged inflation and γ_y is the weight on the output gap. The output gap is defined as the difference between current output and the potential output (which is defined as output level achievable under fully flexible prices).

4.2.21 Government

The government collects lump-sum taxes from consumers and purchases consumer goods. It can be funded through the sale of government bonds or, where taxes exceed expenditure, is used to retire debt. In order to keep model as simple as possible the government expenditure does not impact households directly in form of a transfer. Government expenditure is a source of final demand for consumer goods and support labor demand and imports. Government expenditure does not impact households directly[3], but is rather a source of demand for producers and hence labor demand. Following Galí, López-Salido, and Valles (2007) fiscal policy is modeled as the combination of exogenous government spending G_t, government debt B_t^G and lump-sum taxes T_t. $R_{t+1}^{-1} B_{t+1}^G$ represents the quantum of funds borrowed in period t with B_{t+1}^G repaid in period $t+1$. Hence the government budget constraint can be specified as follows

$$\begin{aligned} T_t + R_{t+1}^{-1} B_{t+1}^G &= B_t^G + G_t \\ dT_t + \frac{R dB_{t+1}^G - B^G dR_{t+1}}{R^2} &= dG_t + dB^G. \end{aligned}$$

Given that in the steady state $T + B = B + G \Leftrightarrow T = G$ and in addition assuming a steady state of $B^G = 0$

$$\begin{aligned} \frac{G}{Y}\frac{dT_t}{T} + \frac{1}{R}\left(\frac{dB_{t+1}^G}{Y} - \frac{B^G}{Y}\frac{dR_{t+1}}{R}\right) &= \frac{dB_t^G}{Y} + \frac{G}{Y}\frac{dG_t}{G} \\ \frac{dB_{t+1}^G}{Y} &= R\left[\frac{dB_t^G}{Y} + \frac{G}{Y}\left(\hat{G}_t - \hat{T}_t\right)\right] \\ B_t^G &= R B_{t-1}^G + R\frac{G}{Y}\hat{G}_t - R\frac{G}{Y}\hat{T}_t. \end{aligned} \tag{4.45}$$

The fiscal rule determines how expenditure is funded, initially as a rule of lump-sum taxes and consequently how much new debt is issued

$$\begin{aligned} \frac{T_t - T}{T}\frac{T}{Y} &= \phi_B \frac{B_t^G}{Y} + \phi_G \frac{G_t - G}{G}\frac{G}{Y} \\ \hat{T}_t &= \left(\frac{G}{Y}\right)^{-1} \phi_B \frac{B_t^G}{Y} + \phi_G \hat{G}_t, \end{aligned} \tag{4.46}$$

[3]Government expenditure which has direct impact on households is typically represented by transfer.

where ϕ_B represents the distribution of fiscal imbalances with respect to the government debt and ϕ_G is the distribution of fiscal imbalances with respect to the government exogenous spending.

Then one can express government expenditure on good z as follows

$$G_t(z) = \left(\frac{p_t(z)}{P_{c,t}}\right)^{-\epsilon} G_t.$$

4.2.22 Stochastic shocks

The model contains four different shocks, a technology shock, a domestic nominal interest rate shock, a foreign real interest rate shock and a foreign demand shock. The shock processes are stationary and have similar forms. For each shock k, its log difference is given by

$$\hat{k}_t = \rho_k \hat{k}_{t-1} + \epsilon_{k,t},$$

where $0 < \rho_k < 1$ is a necessary condition which guarantees that processes are stationary. Stationarity of the technology shock is a strong assumption. However, this also means that it is not necessary to deflate real variables by the technology process A_t.

4.3 Log-linear model

In order to keep this model as transparent as possible the short description and reference to the original equation for each log-linearized equation are added.

Cobb-Douglas production function (4.2), under the assumption that capital is fixed may be written as follows

$$\hat{Y}_t = \varphi \widehat{IM}_t + (1-\varphi)\left(\hat{A}_t + \hat{L}_t\right),$$

where Y_t is the real output, IM_t represents imports, A_t is the technology and L_t is the aggregate labor, $\varphi = \frac{\alpha IM^\gamma}{\alpha IM^\gamma + (1-\alpha)(AL)^\gamma}$, α is the import weight in production function and γ is the labor-imports substitution coefficient in production function.

Input demand (4.6) is determined as

$$\widehat{IM}_t = \frac{1}{(1-\gamma)}\left(\hat{w}_t - \widehat{RS}_t - \gamma \hat{A}_t\right) + \hat{L}_t,$$

where w_t is the real wage and RS_t is the real exchange rate.

Labor market equilibrium (4.8) is characterized as

$$\widehat{mc_t} = \hat{w}_t - (1-\gamma)\varphi\left(\widehat{IM_t} - \hat{L}_t\right)[(1-\gamma)(1-\varphi)+\gamma]\hat{A}_t,$$

where mc_t is the real marginal cost.

Resource constraint (4.9) is represented by

$$\hat{Y}_t = \frac{c}{Y}\hat{c}_t + \frac{I}{Y}\hat{I}_t + \frac{G}{Y}\hat{G}_t + \frac{EX}{Y}\widehat{EX}_t,$$

where c_t is the goods consumption, I_t is the housing investment, G_t is the government spending and EX_t is the net export.

Export demand (4.10) is defined as

$$\widehat{EX}_t = \vartheta\widehat{RS}_t + \varsigma\hat{Y}_t^f,$$

where ϑ is the export sensitivity to real exchange rate and Y_t^f is the foreign output.

Following expression (4.12) is an identity.

$$\hat{X}_{c,t} = -\left(\frac{1-\upsilon}{\upsilon}\right)\left(\frac{X_h}{X_c}\right)^{1-\eta}\hat{X}_{h,t},$$

where $X_{c,t}$ is the relative price of the representative consumption good, υ is the steady state goods consumption as a proportion of overall consumption and $X_{h,t}$ is the relative price of rental services.

Demand for consumption goods (4.13) and housing services (4.14) are defined as

$$\hat{c}_t = \hat{C}_t - \eta\hat{X}_{c,t},$$

$$\hat{h}_t = \hat{C}_t - \eta\hat{X}_{h,t},$$

where C_t is the aggregate consumption, η is the consumer substitution between housing and goods coefficient and h_t is the housing stock.

Consumption of PIH consumers (4.18) follows

$$\hat{C}_t^P = E_t\hat{C}_{t+1}^P - \hat{R}_{t+1},$$

where C_t^P is the PIH consumption, $E_t.$ is the rational expectation operator and R_t is the real domestic interest rate.

Equilibrium condition considering domestic and foreign investments to financial assets (4.20) is represented by

$$\hat{R}_{t+1} = \hat{R}^f_{t+1} - \delta_b \hat{b}_t + E_t \widehat{RS}_{t+1} - \widehat{RS}_t,$$

where R^f_t is the real foreign interest rate, δ_b is the cost of intermediation in the foreign currency bond market and b_t is the borrowing undertaken to finance the purchase of housing stock.

Wage of PIH consumers (4.21) is defined as

$$\hat{C}^P_t = \hat{w}_t - \frac{L^P}{1 - L^P} \hat{L}^P_t,$$

where L^P_t is the PIH labor supply.

Consumption of ROT consumers (4.23) is specified as

$$\hat{C}^r_t = \frac{wL^r}{C^r} \left(\hat{L}^r_t + \hat{w}_t \right) - \frac{(1-n)\,T}{C^r} \hat{T}_t,$$

where L_t is the aggregate labor, n is the proportion of consumers that are PIH and T_t is the lump-sum tax (in real terms).

Aggregate consumption (4.24) follows

$$\hat{C}_t = n_C \hat{C}^P_t + (1 - n_C)\, \hat{C}^r_t,$$

where $n_C = n\frac{C^p}{C}$.

Housing investment demand (4.26, 4.27 and 4.32) is characterized by

$$E_t \hat{R}_{h,t+1} = \Omega \left(\hat{N}_{t+1} - \hat{q}_t - \hat{h}_{t+1} \right) + \hat{R}_{t+1},$$

$$\hat{R}_{h,t} = (1-\mu)\, \hat{X}_{h,t} + \mu \hat{q}_t - \hat{q}_{t-1},$$

$$\hat{q}_t = \Gamma_d \left(\hat{I}_t - \hat{h}_t \right) + \hat{X}_{c,t},$$

where R_h is the return on housing, Ω is the sensitivity of interest rate premium to the networth ratio, N_t is the net worth, q_t is the real house price, $\mu = \frac{(1-\delta)q}{X_h + (1-\delta)q}$, δ is the housing depreciation rate and Γ_d is the q-theory sensitivity.

The evolution of net worth (4.29) depends on the net return from housing investment minus dividend payments. This may be written as follows

$$\hat{N}_{t+1} = R_h \left[(1+bn)\, \hat{R}_{h,t} - bn\hat{R}_t + (1-bn\Omega)\, \hat{N}_t + bn\Omega \left(\hat{q}_{t-1} + \hat{h}_t \right) \right] - (R_h - 1)\, \hat{D}_t,$$

where $bn = \frac{1}{\phi} - 1$, ϕ is the net worth ratio and D_t is the housing dividend.

Dividend rule (4.30) is defined as

$$\hat{D}_t = \frac{\chi'(\phi)}{\chi(\phi)}\phi\left[\hat{N}_{t+1} - \hat{q}_t - \hat{h}_{t+1}\right],$$

where $\frac{\chi'(\phi)}{\chi(\phi)}\phi$ is the sensitivity of dividend to net worth ratio.

Accumulation of housing capital (4.31) is represented by

$$\hat{h}_{t+1} = (1-\delta)\,\hat{h}_t + \delta\hat{I}_t.$$

Resource constraint for all agents in this model (4.34) is specified as

$$\begin{aligned}\beta C\left[b_t(1+a\delta) - a\hat{R}^f_{t+1}\right] = b_{t-1}C \quad &+ \quad b\left[\Delta\hat{S}_t + \hat{\pi}^f_t - \hat{\pi}_t\right] \\ -EX\left[\varsigma\hat{Y}^f_t + \vartheta\widehat{RS}_t\right] \quad &+ \quad \frac{SP^f}{P}IM\left[\widehat{IM}_t + \widehat{RS}_t\right],\end{aligned}$$

where β is the discount rate, b_t is the borrowing undertaken to finance the purchase of housing stock, S_t is the nominal exchange rate, π^f_t is the foreign inflation, π_t is the domestic inflation,P^f is the steady state of the foreign price level and P is the steady state of the domestic price level.

A variant of the New Keynesian Phillips Curve (4.39) is defined as

$$\hat{\pi}_{c,t} = \kappa\widehat{mc}_t + \beta E_t\hat{\pi}_{c,t+1},$$

where $\pi_{c,t}$ is the consumption good inflation, $\kappa = \frac{(1-\theta)(1-\theta\beta)}{\theta}$ and $1-\theta$ is a probability of firm resetting its price.

Output under the assumption of flexible prices (4.40) follows

$$\begin{aligned}\hat{Y}^{flex}_t &= \frac{-\alpha\left(\frac{IM}{Y}\right)^\gamma}{(1-\gamma)\left[(1-\alpha)\left(\frac{AL}{Y}\right)^\gamma - \left(\frac{1-\gamma}{1-\gamma+\tau}\right)\right]}\widehat{RS}_t \\ &+ \frac{(1-\alpha)\left(\frac{1-\gamma}{1-\gamma+\tau}\right)\left(\frac{AL}{Y}\right)^\gamma}{(1-\alpha)\left(\frac{AL}{Y}\right)^\gamma - \left(\frac{1-\gamma}{1-\gamma+\tau}\right)}\hat{A}_t,\end{aligned}$$

where $\tau = \frac{L}{1-L}$.

Real exchange rate identity (4.41) follows

$$E_t\widehat{RS_{t+1}} - \widehat{RS_t} = E_t\hat{S}_{t+1} - \hat{S}_t + E_t\hat{\pi}^f_{t+1} - E_t\hat{\pi}_{t+1}.$$

Overall inflation (4.42) is calculated as

$$\hat{\pi}_t - \hat{\pi}_{c,t} = \hat{X}_{c,t-1} - \hat{X}_{c,t}.$$

Nominal interest rate identity (4.43) is defined as

$$\hat{R}^n_{t+1} = \hat{R}_{t+1} + E_t\hat{\pi}_{t+1}.$$

A variant of monetary policy (Taylor) rule (4.44) follows

$$\hat{R}^n_{t+1} = \rho_i \hat{R}^n_t + (1-\rho_i)\,\gamma_\pi \hat{\pi}_{t-1} + (1-\rho_i)\,\gamma_y \tilde{y}_t + X_{ii_t},$$

where R^n_t is the nominal interest rate, ρ_i is the interest rate smoothing parameter, γ_π is the coefficient on inflation in monetary policy rule, γ_y is the coefficient on output gap in monetary policy rule, y_t is the Gross Domestic Product and X_{ii_t} is the artificial variable which helps us to insert the stochastic shock into this equation.

Government debt (4.45) is driven by

$$B^G_t = RB^G_{t-1} + R\frac{G}{Y}\hat{G}_t - R\frac{G}{Y}\hat{T}_t,$$

where B^G_t is the government debt.

Fiscal rule (4.46) determines how expenditure is funded. This rule is specified as

$$\hat{T}_t = \left(\frac{G}{Y}\right)^{-1} \phi_B \frac{B^G_t}{Y} + \phi_G \hat{G}_t,$$

where ϕ_B is the distribution of fiscal imbalances with respect to the government debt and ϕ_G is the Distribution of fiscal imbalances with respect to the government exogenous spending.

Output gap is defined as

$$\tilde{y}_t = \hat{Y}_t - \hat{Y}^{flex}_t.$$

Gross Domestic Product is in this model represented by

$$\hat{y}_t = \frac{Y}{y}\hat{Y}_t - \frac{IM}{y}\hat{IM}_t.$$

Technology shock, domestic interest rate shock, a foreign real interest rate shock, a foreign demand shock and government spending shock are driven as follows

$$\hat{A}_t = \rho_A \hat{A}_{t-1} + \epsilon_A,$$

$$X_{ii} = \rho_{X_{ii}} X_{ii_{t-1}} + \epsilon_{X_{ii}},$$

$$\hat{R}^f_t = \rho_{\hat{R}^f_t} \hat{R}^f_{t-1} + \epsilon_{\hat{R}^f_t},$$

$$\hat{Y}^f_t = \rho_{\hat{Y}^f_t} \hat{Y}^f_{t-1} + \epsilon_{\hat{Y}^f_t},$$

$$\hat{G}_t = \rho_G \hat{G}_{t-1} + \epsilon_G.$$

Log-linearized equations contain many deep structural parameters which are calculated as follows

$$\varphi = \frac{\alpha IM^{\gamma}}{\alpha IM^{\gamma} + (1-\alpha)(AL)^{\gamma}},$$

$$n_C = n\frac{C^p}{C},$$

$$n_L = n\frac{L^p}{L},$$

$$\delta_b = -\psi'(RS)(B_F)C,$$

$$\Omega = \frac{f'(\phi)}{f(\phi)}\phi,$$

$$\mu = \frac{(1-\delta)q}{X_h + (1-\delta)q},$$

$$bn = \frac{1}{\phi} - 1,$$

$$\phi = \frac{N}{qh},$$

$$\Gamma_d = \frac{\Phi''\frac{I}{h}}{\Phi'\frac{I}{h}}\frac{I}{h},$$

$$\kappa = \frac{(1-\theta)(1-\theta\beta)}{\theta}.$$

4.4 Calibration

Calibration of many parameters is based on a computed steady state which is in turn consistent with empirical data. This study contains the real price of housing and the stock of housing as the key variables upon which the rest of steady state may be calculated. The steady state relationships are derived in Appendix B and numbered equations may be seen also in Dynare code in Appendix D. Using these relationships, in combination with the assumed parameter values in Table 4.2 allow for the key steady state values to be computed.

For determination of the steady state, I use parameter and variable values which are consistent with data of the Czech Republic. The introduction of the open economy requires calibration of the import and export related parameters. This model neglects services and also consumer goods imports, two possible treatments suggest themselves in order to establish steady state imports. One can use aggregate data on exports and imports.

Model contains five exogenous variables (the real exchange rate, technology, real house price, housing stock and aggregate labor) which are used to calculate the rest of steady state variables (endogenously).

Calibrated parameters may be split into two groups. The first group contains parameters which are calibrated on my own. The second group of parameters is taken from other related studies.

Following parameters are based purely on data of the Czech Republic. G/Y is set to 0.2, EX/Y is set to 0.6 and IM/Y is set to 0.7.

The discount rate is set to 0.99 which is synchronized value other DSGE models. E.g. see Galí (2008).

Parameter α is set to 0.65 which corresponds to weight of imports in CES production function of the Czech Republic.

The leisure coefficient in utility function ξ is set so that the steady state provision of labor is equal to 0.33. It means that ξ is set to 1.1097.

The weight on consumption goods υ is set to 0.81 which is consistent with the expenditure weights obtained from Czech Statistical Office.

Export sensitivity to real exchange rate ϑ and export sensitivity to foreign demand ζ are both set to 1. Czech Republic is export oriented economy and these two structural parameters should highlight this feature.

The elasticity of substitution between imports and labor γ is set to -0.2, reflecting a relatively modest degree of complementarity between labor and imports. The choice of γ is constrained by the need to ensure that the flexible price output reacts sensibly to a technology shock. For details see 4.40.

Autocorrelations of all stochastic shocks are based on simple linear regression. Thus, they are calibrated as follows: $\rho_A = 0.90$, $\rho_G = 0.70$, $\rho_{Rf} = 0.80$, $\rho_{Yf} = 0.80$ and $\rho_{X_{ii}} = 0.80$.

Interest rate smoothing parameter is based on linear regression, too. Parameter ρ_i is set to 0.70.

The proportion of PIH consumers n is set to 0.5. This values is in accordance with research of Pánková (in press). This parameter has not been used in any DSGE model of the Czech Republic. This parameter is calibrated to 0.7 in case of New Zealand. I decided to set this parameter more conservative.

The coefficients ρ_i, γ_π and γ_y which are present in monetary policy rule are calibrated in accordance with DSGE model of the Ministry of Finance of the Czech Republic. Interest rate smoothing parameter ρ_i is set to 0.7. Coefficient on inflation γ_π is set to 1.5 and coefficient on output gap γ_y is set to 0.25.

$1 - \theta$ is a probability of firm resetting its prices. θ is set to 0.5 which is the value used by the Ministry of Finance of the Czech Republic.

The cost of intermediation in the foreign currency bond market δ_b is set to -0.001 which is in line with Thoenissen (2004).

Parameter representing distribution of fiscal imbalances with respect to government debt ϕ_B is set to 0.33 and parameter representing the distribution of of fiscal imbalances with respect to government exogenous spending ϕ_G is set to 0.1. Parameter representing the consumer substitution between housing and goods η is se to 0.9999. These three parameters are taken from Khan and Reza (2013).

The net worth to housing ratio $\phi = \frac{N}{qh}$ is set to 0.7. Which is taken from previous study performed by Lees (2009).

The rest of parameters is calibrated according to Aoki, Proudman, and Vlieghe (2002). It means that sensitivity of interest rate premium to net worth ratio Ω is set to -0.1, sensitivity of dividend to net worth ratio $\frac{\chi'(\phi)}{\chi(\phi)}\phi$ is set to 3. Q-theory sensitivity Γ_d is set to 0.52. Housing depreciation rate δ is set to 0.005.

4.5 Data

The presence of five stochastic shocks allows verification of how this DSGE model fits observed data. Each stochastic shock enables application of one observed variable. Thus the following five observed variables are introduced: The Gross Domestic Product

(GDP), real house prices, nominal exchange rate, inflation and nominal interest rate. The time series of these variables are taken from International Monetary Fund (IMF), database of the Czech Statistical Office (CSO) and database of the Czech National Bank (ARAD). Time series are on quarterly basis and due to limitation of house prices data I use the period from Q1 2006 to Q3 2013 which means 31 quarterly observations.

Real GDP is taken from the database of the IMF. GDP is in constant prices of the year 2005 and data are seasonally adjusted. Next, GDP is transformed to the YoY (Year over Year) percentage changes and it is outlined in Figure 4.6.

Calculation of inflation is based on CPI (Consumer Price Index) which is in real terms (year 2005 = 100). CPI is taken from IMF database. This index is transformed to the YoY percentage changes which represent the inflation. For details see Figure 4.6.

Nominal exchange rate is represented by the Nominal Effective Exchange Rate of the Czech Republic which is taken from database IMF. This is an index (year 2005 = 100). YoY percentage changes of this index may be interpreted as changes of competitiveness. The final time series may be seen in Figure 4.7.

Nominal interest rate is represented by Prague Inter Bank Offered Rate (PRIBOR). Inter Bank rate is the rate of interest charged on short-term loans made between banks. Banks borrow and lend money in the interbank market in order to manage liquidity and meet the requirements placed on them. The quarterly data is used in this study thus I decided to apply 3 months PRIBOR rate which is published in ARAD. The time series of PRIBOR is in Figure 4.7.

Real house prices are taken from the database of the CSO. House prices are in index (year 2005 = 100). House prices are represented by realized prices of flats. This index is then transformed to the YoY percentage changes. Next, these YoY % changes (housing price inflation) minus inflation (CPI inflation) represent the approximation of the real house prices. This time series is outlined in Figure 4.8.

4.6 Bayesian estimation

The DSGE model which is specified in section 4.2, summarized in section 4.3 and calibrated according to section 4.4 is estimated using the Bayesian techniques described in the chapter 2. The estimation is departed from the standard framework when I need to obtain estimation of log likelihood by Kalman filter. Time series are not stationary and it means that diffuse Kalman filter is applied. Next departure from standard framework is applied when I need to obtain mode of the posterior distribution. The Quasi-Newton

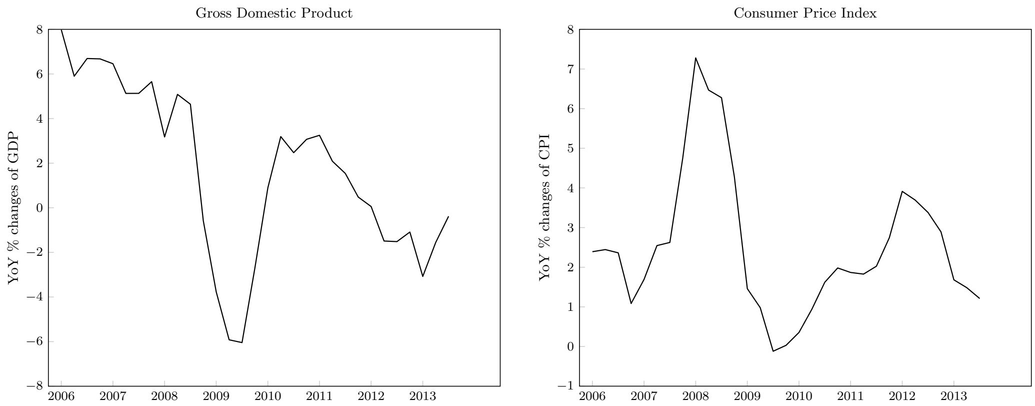

FIGURE 4.6: Gross Domestic Product and Consumer Price Index

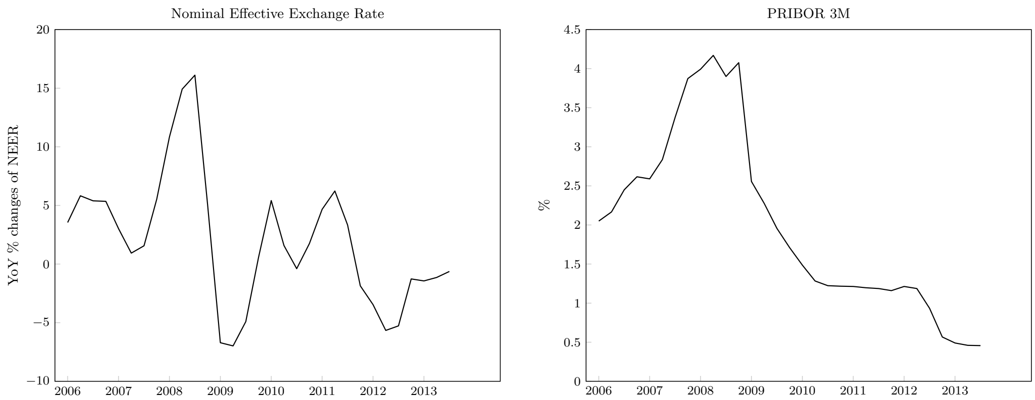

FIGURE 4.7: Exchange Rate & PRIBOR 3M

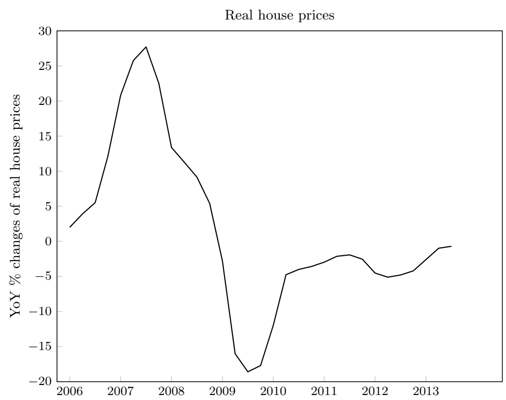

FIGURE 4.8: Real house prices

Parameter	Prior	Posterior	Lower	Upper	Distribution
η	1.000	0.999	0.983	1.015	*norm*
δ	0.005	0.005	0.003	0.006	*beta*
n	0.500	0.501	0.485	0.518	*beta*
γ_π	1.500	1.499	1.482	1.515	*norm*
γ_y	0.250	0.250	0.235	0.266	*norm*
ρ_i	0.700	0.700	0.685	0.717	*norm*
ϕ	0.700	0.700	0.684	0.717	*norm*
Ω	-0.100	-0.096	-0.111	-0.082	*norm*
$\frac{\chi'(\phi)}{\chi(\phi)}\phi$	3.000	3.000	2.983	3.017	*norm*
θ	0.500	0.500	0.483	0.517	*beta*
ξ	1.110	1.109	1.093	1.126	*norm*
β	0.990	0.990	0.988	0.992	*beta*

TABLE 4.3: Estimation results for SOE with housing sector

method with line search is recommended by standard DSGE framework (algorithm 8). Unfortunately, this optimization routine does not work due to absence of positive definite matrix. This is the most likely caused by fact that priors which are defined in section 4.4 are too narrow. There are two options how one can deal with this trouble. First, the recalibration of parameters may be helpful but this move does not guarantee success. Second, one can use another algorithm for the calculation of posterior mode. I decided for the second option because I do not want to change prior values which are carefully calibrated in order to fit the Czech economy. The way out of this situation is Monte Carlo Optimization. For details see algorithm 9.

4.6.1 Parameter estimates

Table 4.3 shows the parameter estimates. Prior is the calibrated value of chosen distribution, posterior is the value obtained as a result of Bayesian estimation. Lower and upper represent 90% confidence intervals. One can see that all parameters are consistent with prior values and lower and upper intervals are not very wide. This means that priors which are chosen according to economic theory fit well the economy of the Czech Republic. The results of Bayesian estimation may be used for further calibration of similar models or for predictions, impulse response functions or shock decomposition.

4.6.2 Impulse responses

Figures 4.9, 4.10 and 4.11 show the posterior distribution of impulse response functions, also called Bayesian Impulse Response Functions (Bayesian IRFs). Model produces

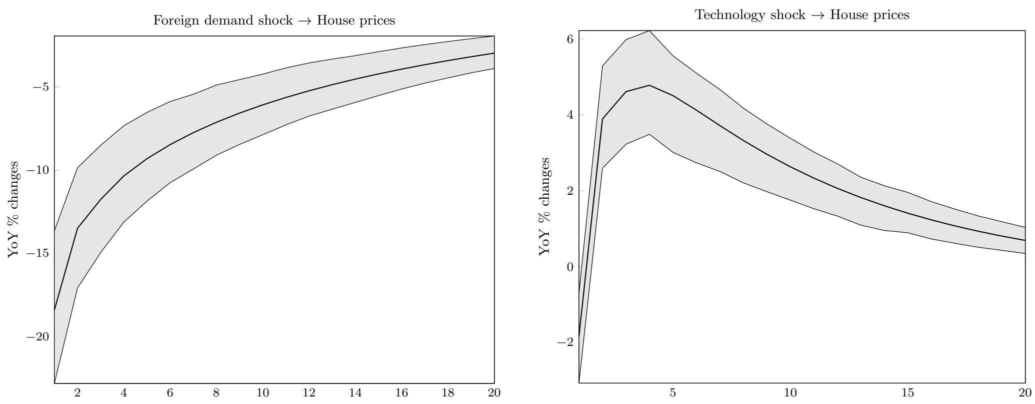

FIGURE 4.9: Bayesian impulse response functions of foreign demand shock and technology shock

many Bayesian IRFs, but this application is aimed on housing. Thus, I decided to print only Bayesian IRFs which are relevant for the real house prices.

Bayesian IRFs are triggered as follows. The exogenous variables take random values in each period. These random values follow a normal distribution with zero mean. But variability of these shock is specified. I followed manual of Dynare and I set for all shocks standard error equal to 0.05.

The intensity of these stochastic shocks is dimensionless. But one may interpret these shocks as positive, because of this positive standard error.

Foreign demand shock is responsible for rapid increase of exports and thus GDP. The sudden increase of GDP causes increase of wealth and inflation which causes short-term decrease of real house prices.

Technology shock enters the model through the production function. The responses of the model to a positive technology stochastic shock increase the real house prices which are then gradually decreasing for 20 periods. This increase is caused by fact that technology shock increases the GDP first and this effect causes increase of house prices after a few periods. In the open economy model the real exchange rate affects the cost of imports and thus also the maximizing behavior of producers, there is no such effect in the original APV model.

Domestic interest rate shock enters the model through the equation which represents Taylor rule. This shock causes an increase of nominal interest rate. This affects increase of GDP and inflation. These facts temporary causes increase of wealth which causes increase of house prices because of stronger domestic demand.

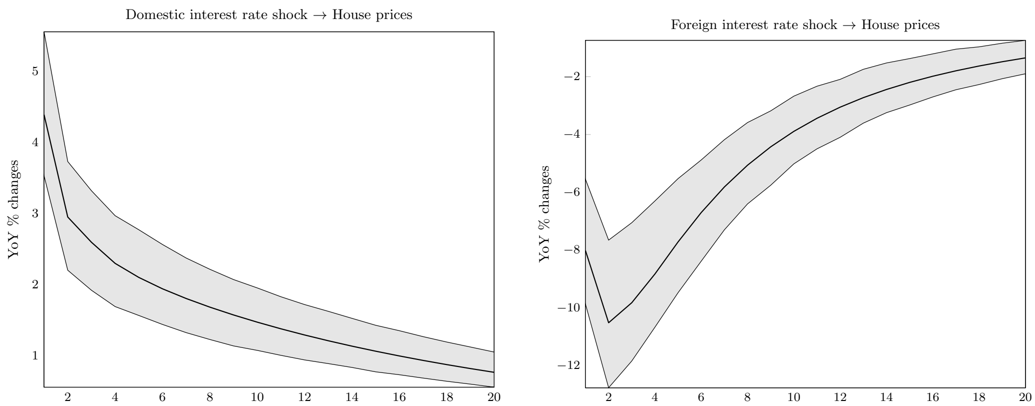

FIGURE 4.10: Bayesian impulse response functions of domestic interest rate shock and foreign interest rate shock

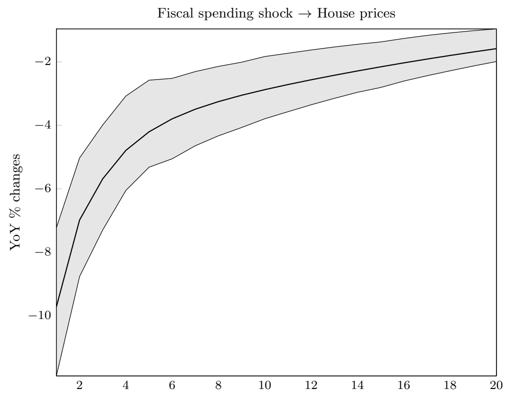

FIGURE 4.11: Bayesian impulse response functions of fiscal spending shock

Foreign interest rate shock is responsible for increase of interest rates in abroad. This means that financial capital flows abroad because of higher potential profit. PIH consumers prefer investments abroad in contrast with domestic investments. The domestic economy is lack of housing investments and this fact causes decrease of real house prices.

Fiscal spending shock is responsible for increase of government expenditure and GDP. The rapid increase of GDP and wealth causes inflation which is responsible for short-term decrease of the real house prices.

Most of these stochastic shocks are fully absorbed after longer time than 5 years. This is quite interesting finding because the politicians in Czech Republic are voted for four years. This means that politicians should study impacts of their decisions very carefully because impacts are more long-term.

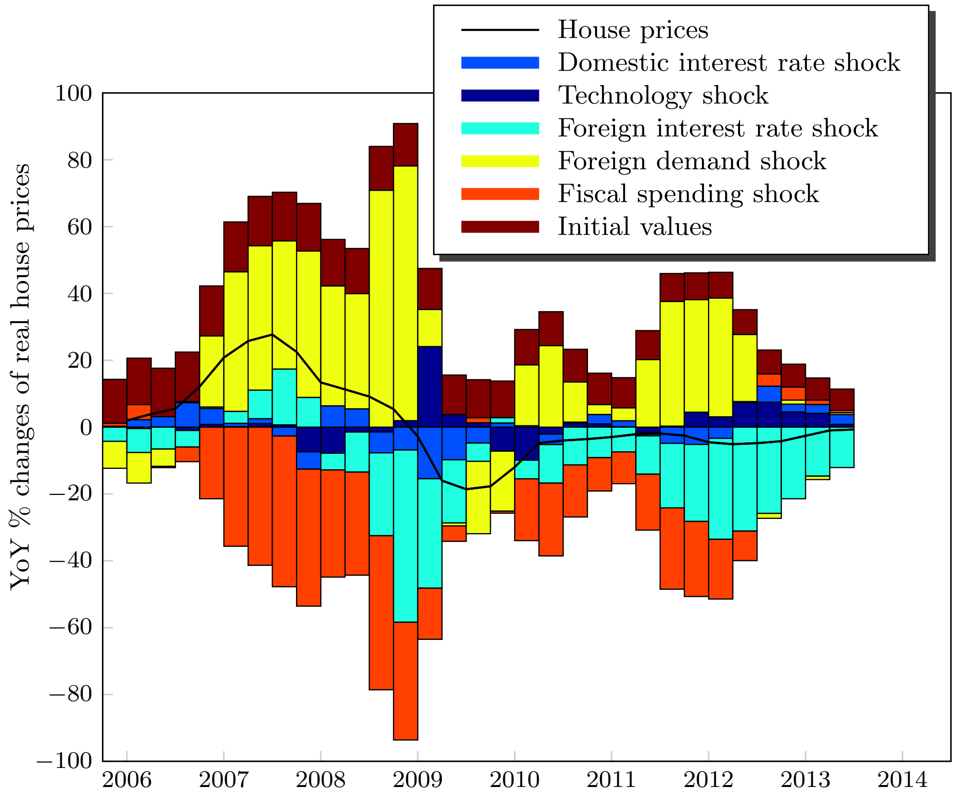

FIGURE 4.12: Shock decomposition of real house prices

4.6.3 Shock decomposition

Figure 4.12 shows how each stochastic shock affect the YoY % change of real house prices. One may see that housing boom between the years 2006 - 2009 was mainly caused by the foreign interest rate and foreign demand. Subsequent decrease of house prices started in 2009 and it was caused by decline foreign interest rate and foreign demand. One can also see a significant decrease of fiscal spending which should cause increase of real house prices. Nevertheless, this decline is compensated by foreign demand which is the real driver of YoY % changes of real house prices.

4.6.4 Forecast

Figure 4.13 shows the ex-ante prediction of GDP and real house prices. The black line may be interpreted as mean forecast and green lines represent the borders of the distribution of forecasts where the uncertainty about shocks is averaged out. The distribution of forecasts therefore only represents the uncertainty about parameters and may be also interpreted as 90% confidence interval.

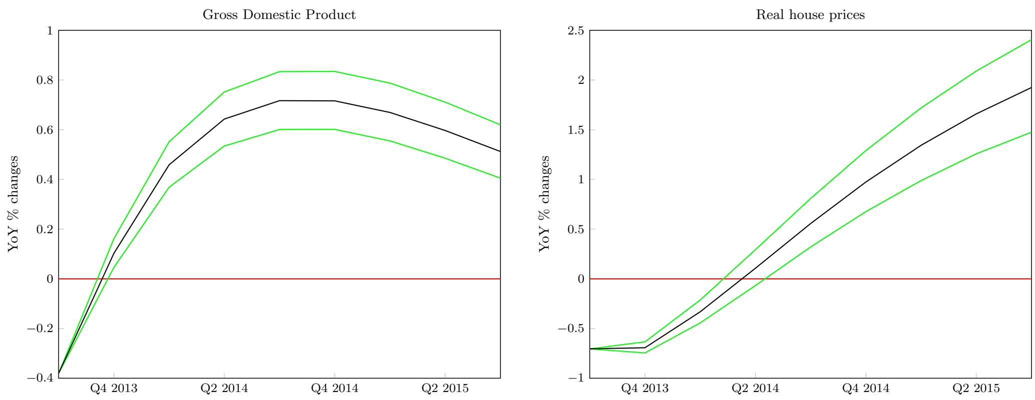

FIGURE 4.13: Ex-ante prediction of GDP and house prices

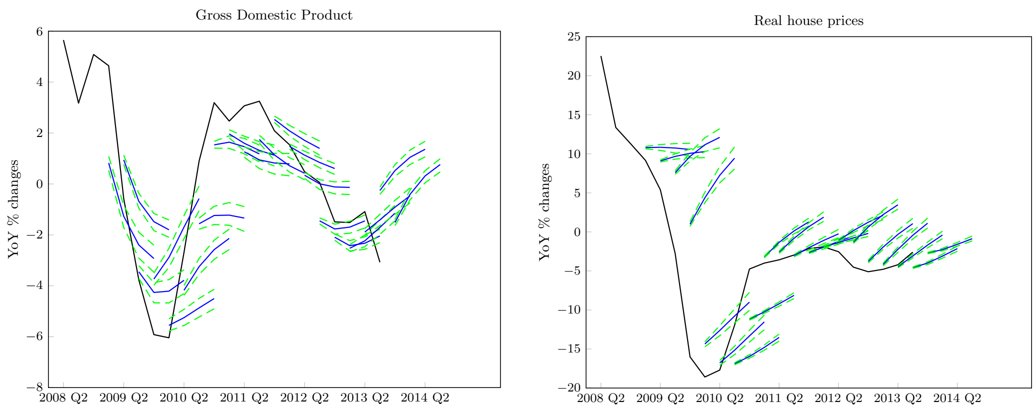

FIGURE 4.14: Ex-post prediction of GDP and house prices

One may see that small open economy model extended by government and housing sector which is specified and estimated in this study predicts the gradual incline of GDP. This increase presents the end of recession in the year 2014. Unfortunately, the GDP is going to growth much slower from year 2015. It is important to mention that model was estimated before interventions of CNB which performed in November 2013. This is the main reason why model predicts lower GDP growth than the most up to date prediction of CNB.

Next, model predicts the future development of the real house prices and one may see the rapid increase. This increase is predicted by many analysts for a long time. The reason is that construction has been with their profits at the bottom since the economic recession started. Thus, I believe that this increase of house prices will be caused by recovery in demand.

Moreover, Figure 4.14 shows recursive ex-post prediction. These forecasts are result of an iterative process, where one data point observation is added to the time series each time. One may see the ability of this DSGE model to predict future dynamics and break-points in the data.

4.7 Macroprudential policy

The recent financial crisis has demonstrated the necessity of introducing policies and regulations that adapt to changes in the financial environment. In a fragile global economy, traditional macroprudential actions have not seemed to be sufficient to avoid the crisis and have a fast effective recovery. The crisis and its consequences have opened a real debate about the reforms that need to be made in the financial and regulatory banking system, and in the policy instruments that have to be used in order to avoid similar events. The new direction of policy interventions may be so-called macroprudential approach to mitigate the risk of the financial system as a whole that is the systemic risk. The term macroprudential refers to the use of prudential tools to explicitly promote the stability of the financial system in a global sense, not just the individual institutions (banks). The goal of this kind of regulation and supervision would be to avoid the transmission of financial shocks to the broader economy.

In this section of my thesis one macroprudential experiment is performed. The goal is to investigate what are the impacts of introduction less strict and stricter Loan to Value[4] (LTV) rule. Introduction of lower (stricter) LTV ratio should mitigate the risk which is connected with default of counterparty. On the other hand, higher (less strict) LTV ratio supports economy in good times but also tends to deepen crisis in bad times.

This model does not contain such thing as LTV ratio. On the other hand, model comprises the net worth ratio

$$\phi = \frac{N}{qh}, \tag{4.47}$$

which can be perceived as $1 - LTV$, but not on household level but on economy level. N is the net worth which is calculated as housing stock less outstanding debt less dividends paid to consumers, q represents the real house prices and h is the housing stock. The economic interpretation is as follows. The lower net worth ratio means that current level of housing stock is mainly financed by housing loans. On the other hand, the higher net worth ratio means that current level of housing stock is not financed by housing loans and is almost fully owned by PIH households.

Following 4.47 one obtain

$$LTV = 1 - \phi = 1 - \frac{N}{qh}.$$

Now, the LTV ratio has the usual interpretation. The lower LTV means that there is a high level of collateralization in our economy. On the other hand, higher LTV means that majority of housing stock is covered by housing loans.

[4]The Loan to Value (LTV) ratio is a financial term used by lenders to express the ratio of Mortgage Amount to Appraised Value of the Property.

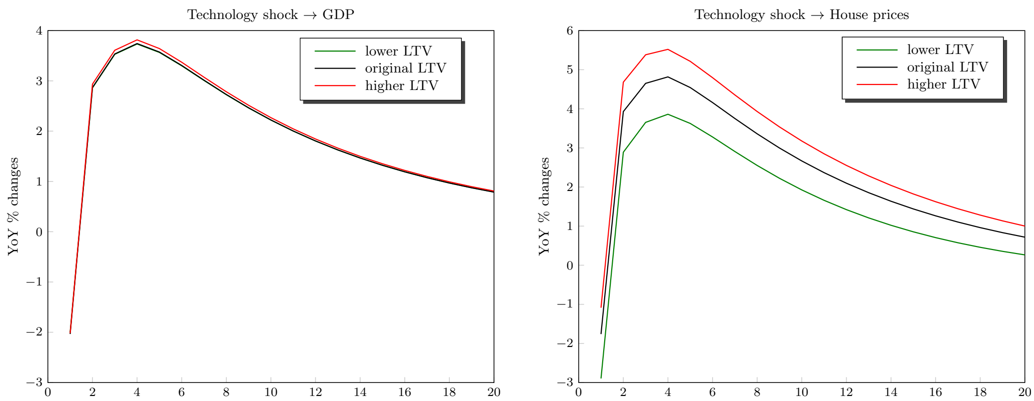

FIGURE 4.15: Impulse response functions of technology shock

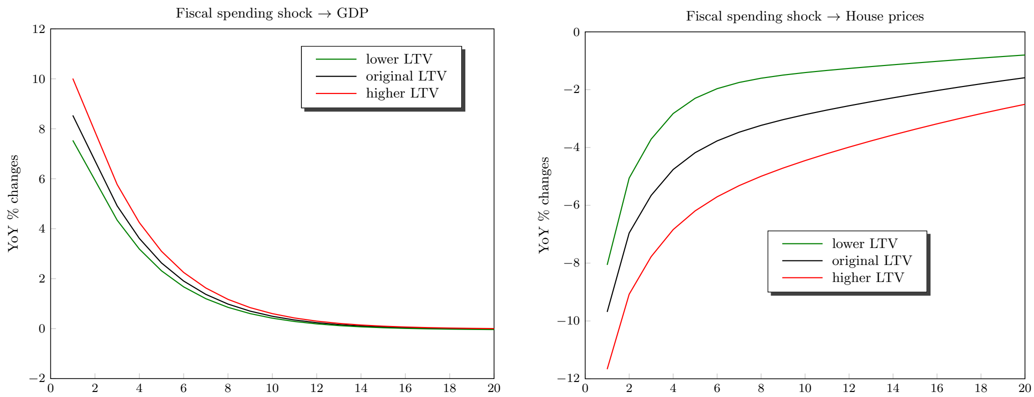

FIGURE 4.16: Impulse response functions of government spending shock

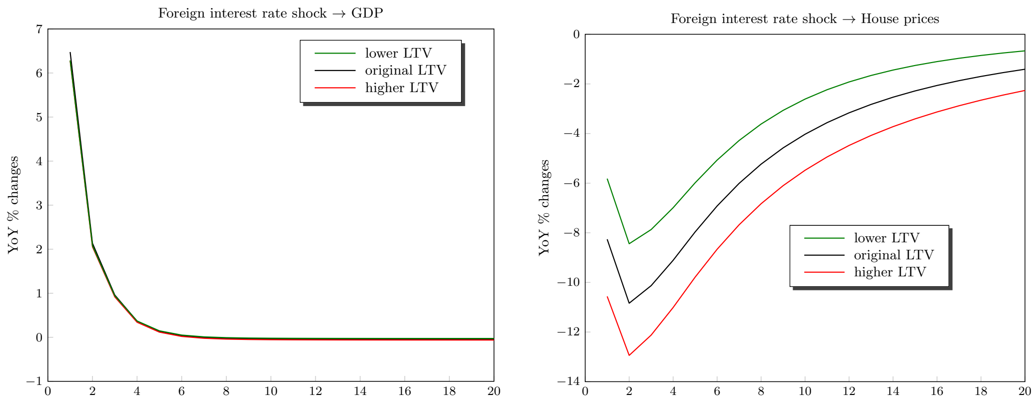

FIGURE 4.17: Impulse response functions of foreign interest rate shock

The experiment performed in this chapter is as follows. Figures 4.15, 4.16, 4.17, 4.18 and 4.19 show behavior of impulse response functions to technology shock, government spending shock, foreign interest rate shock, domestic interest rate shock and foreign demand shock. The original LTV is in SOE model set to 0.3. The higher LTV is set to 0.4 and lower LTV is set to 0.2.

One may see that behavior of GDP is almost the same for all three levels of LTV. In

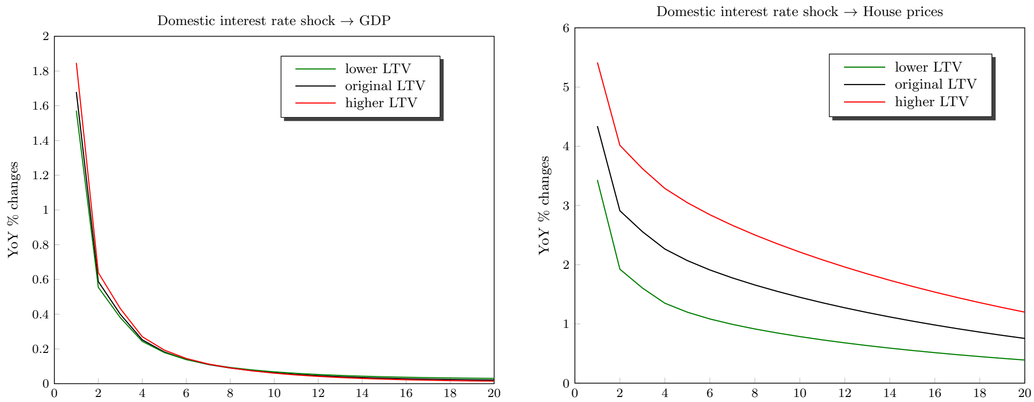

FIGURE 4.18: Impulse response functions of domestic interest rate shock

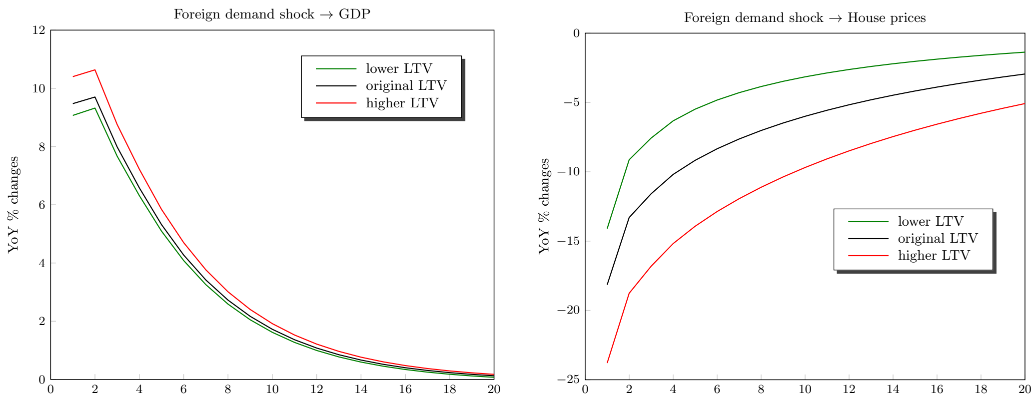

FIGURE 4.19: Impulse response functions of foreign domestic demand shock

contrast with GDP one can see that house prices are more sensitive to level of LTV. This reality is in accordance with Iacoviello and Neri (2010). The results are very conclusive. Higher LTV ratio in the economy causes higher and more volatile house prices than in case of original or lower LTV. This fact is observed for all shocks. In order to keep house prices less volatile it is recommended to implement rather more restrictive LTV. This recommendation is based on fact that investors prefer stable economic environment.

4.8 Conclusion

This study deals with Small Open Economy (SOE) model with housing sector. Model is based on research of Aoki, Proudman, and Vlieghe (2002) (APV) which is constructed as a closed economy model. I incorporated into the APV model open economy extensions and government. These extensions make model more applicable to SOE such as Czech Republic. Following the framework set by APV, there are two types of households. The first type of consumers is able to access the capital markets and they can smooth consumption across time by buying or selling financial assets. These households follow the permanent income hypothesis (PIH). The other type of household uses rule of thumb (ROT) consumption, spending all their income on consumption. ROT consumers are effectively completely credit constrained as they do not have any access to the credit markets. This dual differentiation of consumers is based on Campbell and Mankiw (1989). Unlike the APV model, in this study ROT households are further characterized by the fact that they do not own any housing assets. This guarantees consistency so that households which do not have access to the credit markets to smooth consumption are also unable to purchase a house (it means that mortgage would not be granted to them). Both types of consumers purchase goods from firms each period, receive wage income from labor supplied to firms and pay rental to the homeowners. PIH households are divided into two complementary components: a homeowner and a consumer. The homeowner transacts in the housing market each period, selling the housing stock and purchasing the stock anew. Against the net worth of housing stock, the homeowner borrows to meet any shortfall between the price of the housing stock bought at the end of the period and the price realized on sale of the existing housing stock. Net worth is defined as the value of the housing stock less outstanding debt and less any dividends paid to consumers. This dividend is the mechanism by which the housing equity withdrawal is captured. Homeowners also charge a rental fee to consumers. Thus the housing stock is completely owned by the PIH consumers and the ROT consumers pay rental to their PIH landlords.

Firms are monopolistically competitive and produce a continuum of consumer goods. Each period they hire labor from households and also purchase an intermediate input from abroad. These imports are used by firms each period and capital is effectively assumed to be constant. The output of firms is consumed by either household or government, exported or used to produce additional housing stock. The monetary authority has a Taylor rule reaction function (with lagged inflation and the output gap as indicators of inflationary pressure) and uses the nominal interest rate as its lever subject to a smoothing parameter. The government collects lump sum taxes from consumers and purchases consumer goods. The difference between these two is either funded through the sale of government bonds or, where taxes exceed expenditure, is used to retire debt. Following Galí and Gertler (2007), fiscal policy is modeled as the combination of exogenous government spending government debt and lump sum taxes.

There are many outputs which come as a result of Bayesian estimation. One may check Bayesian impulse response functions which show how much stochastic shocks affect house prices. Next, the shock decomposition of real house prices is performed. The main outcome of this analysis shows that real house prices are mainly driven by foreign interest rate shock and foreign demand shock. Successful estimation of all parameters enables to perform prediction of observed variables to next two years. The main findings are that one may expect recovery of Gross Domestic Product (GDP) in next two years and the nominal interest rate (PRIBOR 3M) will continue in a long-term decline. On the other hand, prediction of house prices shows that one may expect recovery in housing market and gradual increase of real house prices.

Final section deals with macroprudential policy experiment and tries to come up with answer on the following question: is the higher/lower Loan to Value (LTV) ratio better for the Czech Republic? This experiment is very conclusive and shows that level of LTV does not affect GDP. On the other hand, house prices are very sensitive to this LTV ratio. The recommendation for the Czech National Bank could be summarized as follows. In order to keep house prices less volatile and attractive for investors CNB should implement rather lower LTV ratio than higher.

Conclusions

The term DSGE refers to a special class of dynamic stochastic macroeconomic models which feature a sound micro-founded general equilibrium framework, characterized by the optimizing behavior of rational agents subject to technology, budget, and institutional constraints. DSGE models received wide support not only among researchers, but also from policy making circles, supporting, for instances, the monetary decision-making processes at central banks around the world because of the ability to fit these structural models to the data.

This thesis is divided into four chapters and each chapter has its specific contribution.

The first chapter describes the motivation of DSGE models construction. Two schools of DSGE modeling are described and their main differences are stressed. To be consistent with current economic research I provide the most up to date criticism of DSGE models, too. Finally, two useful software packages which may be used for simulation or estimation of DSGE models are presented.

The second chapter is a synthesis of many relevant papers into one integrated framework. The main contribution of this chapter lies in the creation of one comprehensive econometric framework which enables to simulate or estimate DSGE models.

The third chapter deals with the impact of alternative monetary policy rules on the economy of Czech Republic. The New Keynesian DSGE model is derived and fully calibrated to fit the Czech data. Parameters of the model are estimated using the Bayesian techniques. This model is estimated several times but with different version of monetary policy rule. Three monetary policy rules are taken from previous studies but the fourth forward looking monetary policy rule is introduced by myself. The forward looking version of the monetary policy rule is chosen as the best performing one. The main contribution of this chapter is both didactic and experimental including the results.

The fourth chapter discusses the possibility of incorporating the Czech Republic housing sector into the DSGE models. Two types of households are comprised. First type of household is able to access the capital markets and can smooth consumption across time by buying or selling financial assets. These households follow the permanent income hypothesis (PIH). Second type of household uses rule of thumb (ROT) consumption, spending all their income on consumption. ROT consumers are effectively completely credit constrained as they do not have any access to the credit markets. Again, the data sources and calibration of all parameters are performed. After estimating, the shock decomposition of the real house prices is performed. The main outcome of this analysis shows that the biggest changes in the real house prices are mainly driven by foreign interest rate shock and foreign demand shock. The successful estimation of all parameters enables to perform prediction of observed variables for next two years. Prediction of the real house prices shows that one may expect gradual increase in next two years. Finally, the macroprudential policy experiment is performed. This experiment is very conclusive and shows that level of LTV (loan amount to appraised value of the property) does not affect GDP. On the other hand, house prices are very sensitive to this LTV ratio. The recommendation for the Czech National Bank could be summarized as follows. In order to keep the real house prices less volatile implement rather lower LTV ratio than higher. The main contribution of this chapter lies in the inclusion of housing sector into the SOE DSGE model and its application to the Czech Republic.

Appendix A

Derivation of the New Keynesian model

A.1 Households

Assume a representative infinitely-lived household, seeking to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right),$$

where β^t is discount factor, N_t denotes hours of work and C_t is a consumption index given by

$$C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where $C_t\left(i\right)$ representing the quantity of good i consumed by the household in period t. Assume the existence of a continuum of goods represented by the interval $[0,1]$. Parameter ε denotes the elasticity of substitution. The period budget constraint now takes the form

$$\int_0^1 P_t\left(i\right) C_t\left(i\right) di + Q_t B_t \leq B_{t-1} + W_t N_t + T_t$$

for $t = 0, 1, 2 \ldots$, where $P_t\left(i\right)$ is the price of good i, W_t is the nominal wage, B_t represents purchases of one period bonds at a price Q_t, and T_t is the lump sum component of income which may include for example dividends from ownership of firms. The households must decide how to allocate its consumption expenditures among the different goods. This requires that the consumption index C_t being maximized for any given level of expenditures $\int_0^1 P_t\left(i\right) C_t\left(i\right) di$. The solution of that problem provides Galí (2008) and yields the set of demand equations

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t \tag{A.1}$$

for all $i \in [0,1]$, where $P_t \equiv \left[\int_0^1 P_t(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}$ is an aggregate price index. Furthermore, and conditional on such optimal behavior

$$\int_0^1 P_t(i)\, C_t(i) di = P_t C_t$$

i.e., total consumption expenditures can be written as a product of the price index times the quantity index. Plugging the previous expression into the budget constraint yields

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + T_t.$$

The optimal consumption/savings and labor supply decisions are described by the following conditions

$$\begin{aligned} -\frac{U_{n,t}}{U_{c,t}} &= \frac{W_t}{P_t}, \\ Q_t &= \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\}. \end{aligned}$$

Under the assumption of a period utility given by

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi}$$

and the resulting log-linear versions of the above optimality conditions take the form

$$\begin{aligned} w_t - p_t &= \sigma c_t + \phi n_t, \\ c_t &= E_t\{c_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - \rho\right), \end{aligned} \tag{A.2}$$

where $i_t \equiv -\log Q_t$ is the short term nominal rate and $\rho \equiv -\log \beta$ is the discount rate, and where lowercase letters are used to denote the logs of the original variables. The previous conditions are supplemented with an ad-hoc log-linear money demand equation of the form

$$m_t - p_t = y_t - \eta i_t.$$

A.2 Firms

Assume a continuum of firms indexed by $i \in [0,1]$. Each firm produces a differentiated good, but they all use an identical technology, represented by the production function

$$Y_t(i) = A_t N_t(i)^{1-\alpha}, \tag{A.3}$$

where A_t represents the level of technology, assumed to be common to all firms and to evolve exogenously over time. All firms face an identical isoelastic demand schedule given by A.1, and take the aggregate price level P_t and aggregate consumption index C_t as given. Following the formalism proposed in Calvo (1983), each firm may reset its price only with probability $1-\theta$ in any given period, independent of the time elapsed since the last adjustment. Thus, each period a measure $1-\theta$ of producers reset their prices, while a fraction θ keep their prices unchanged. As a result, the average duration of a price is given by $(1-\theta)^{-1}$. In the context, θ becomes a natural index of price stickiness. The aggregate price dynamics is described by the equation

$$\Pi_t^{1-\varepsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon},$$

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is the gross inflation rate between $t-1$ and t, and P_t^* is the price set in period t by firms re-optimizing their price in that period. Furthermore, a log-linear approximation to the aggregate price index around that steady state yields

$$\pi_t = (1-\theta)\left(p_t^* - p_{t-1}\right). \tag{A.4}$$

A firm re-optimizing in period t will choose the price P_t^* that maximizes the current market value of the profits generated while that price remains effective. Formally, it solves the problem

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \left(P_t^* Y_{t+k|t} - \psi_{t+k}\left(Y_{t+k|t}\right)\right)\right\}$$

subject to the sequence of demand constraints

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon} C_{t+k}$$

for $k = 0, 1, 2, \ldots$, where $Q_{t,t+k} \equiv \beta^k (C_{t+k}/C_t)^{-\sigma} (P_t/P_{t+k})$ is the stochastic discount factor for nominal payoffs, $\psi(\cdot)$ is the cost function, and $Y_{t+k|t}$ denotes output in period $t+k$ for a firm that last reset its price in period t. The first order condition takes the form

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k|t} \left(P_t^* - \mathcal{M} \psi_{t+k|t} \right) \right\} = 0, \tag{A.5}$$

where $\psi_{t+k|t} \equiv \Psi_{t+k}^{'} \left(Y_{t+k|t} \right)$ denotes the marginal cost in period $t+k$ for a firm which last reset its price in period t and $M \equiv \frac{\varepsilon}{\varepsilon - 1}$. Note that in the limiting case of no price rigidities ($\theta = 0$), the previous condition collapses to the familiar optimal price-setting condition under flexible prices

$$P_t^* = \mathcal{M} \psi_{t|t}$$

which allows us to interpret $\mathcal{M}$ as the desired markup in the absence of constraints on the frequency of price adjustment. Henceforth, $\mathcal{M}$ is referred to as the desired or frictionless markup. Next, the optimal price-setting condition A.5 is linearized around the zero inflation steady state. Before doing so, however, it is useful to rewrite it in terms of variables that have a well-defined value in that steady state. In particular, dividing by P_{t-1} and letting $\Pi_{t,t+k} \equiv P_{t+k}/P_t$, expression A.5 can be rewritten as

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k|t} \left(\frac{P_t^*}{P_{t-1}} - \mathcal{M} C_{t+k|t} \right) \right\} = 0, \tag{A.6}$$

where $MC_{t+k|t} \equiv \psi_{t+k|t}/P_{t+k}$ is the real marginal cost in period $t+k$ for a firm whose price was last set in period t. In the zero inflation steady state, $P_t^*/P_{t-1} = 1$ and $\Pi_{t-1,t+k} = 1$. Furthermore, constancy of the price level implies that $P_t^* = P_{t+k}$ in that steady state, from which it follows that $Y_{t+k|t} = Y$ and $MC_{t+k|t} = MC$, because all firms will be producing the same quantity of output. In addition, $Q_{t,t+k} = \beta^k$ must hold in that steady state. Accordingly, $MC = 1/\mathcal{M}$. A first order Taylor expansion of A.6 around the zero inflation steady state yields A.5 can be rewritten as

$$p_t^* - p_{t-1} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \widehat{mc}_{t+k|t} + (p_{t+k} - p_{t-1}) \right\}, \tag{A.7}$$

where $\widehat{mc}_{t+k|t} \equiv mc_{t+k|t} - mc$ denotes the log deviation of marginal cost from its steady state value $mc = -\mu$, and where $\mu \equiv \log \mathcal{M}$ is the log of the desired gross markup. In order to gain some intuition about the factors determining a firm's price-setting decision it is useful to rewrite A.7 as

$$p_t^* = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \widehat{mc}_{t+k|t} + p_{t+k} \right\}.$$

Hence, firms resetting their prices will choose a price that corresponds to the desired markup over a weighted average of their current and expected marginal costs, with the weights being proportional to the probability of the price remaining effective at each horizon θ^k.

A.3 Equilibrium

$$Y_t(i) = C_t(i)$$

for all $i \in [0,1]$ and all t. Letting aggregate output to be defined as $Y_t \equiv \left(\int_0^1 Y_t(i)^{1-\frac{1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}}$ it follows that

$$Y_t = C_t$$

must hold for all t. One can combine the above goods market clearing condition with the consumer's Euler equation to yield the equilibrium condition

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - \rho\right). \tag{A.8}$$

Market clearing in the labor market requires

$$N_t = \int_0^1 N_t(i)\, di.$$

Using A.3 one obtain

$$N_t = \int_0^1 \left(\frac{Y_t(i)}{A_t}\right)^{\frac{1}{1-\alpha}} di = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\varepsilon}{1-\varepsilon}} di,$$

where the second equality follows from A.1 and the goods market clearing condition. Taking logs,

$$(1-\alpha)\, n_t = y_t - a_t + d_t,$$

where $d_t \equiv (1-\alpha)\log\int_0^1 \left(P_t(i)/P_t\right)^{-\frac{\varepsilon}{1-\varepsilon}}$ and di is a measure of price dispersion across firms. Thus we can write the following approximate relation between aggregate output, employment, and technology as

$$y_t = a_t + (1-\alpha)\, n_t. \tag{A.9}$$

Next an expression is derived for an individual firm's marginal cost in terms of the economy's average real marginal cost. The latter is defined by

$$\begin{aligned} mc_t &= (w_t - p_t) - mpn_t \\ &= (w_t - p_t) - (a_t - \alpha n_t) - \log(1-\alpha) \\ &= (w_t - p_t) - \frac{1}{1-\alpha}(a_t - \alpha y_t) - \log(1-\alpha) \end{aligned}$$

for all t, where the second equality defines the economy's average marginal product of labor, mpn_t, in a way of consistent with A.9. Using the fact that

$$\begin{aligned} mc_{t+k|t} &= (w_{t+k} - p_{t+k}) - mpn_{t+k|t} \\ &= (w_{t+k} - p_{t+k}) - \frac{1}{1-\alpha}\left(a_{t+k} - \alpha y_{t+k|t}\right) - \log(1-\alpha) \end{aligned}$$

then

$$\begin{aligned} mc_{t+k|t} &= mc_{t+k} + \frac{\alpha}{1-\alpha}\left(y_{t+k|t} - y_{t+k}\right) \\ &= mc_{t+k} + \frac{\alpha\varepsilon}{1-\alpha}\left(p_t^* - p_{t+k}\right), \end{aligned} \tag{A.10}$$

where the second equality A.10 follows A.1 combined with the market clearing condition $c_t = y_t$. Notice that under the assumption of constant returns to scale ($\alpha = 0$), $mc_{t+k|t} = mc_{t+k}$, i.e., marginal cost is independent of the level of production and, hence, it is common across firms. Substituting A.10 into A.7 and rearranging terms yields

$$\begin{aligned} p_t^* - p_{t-1} &= (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\Theta\widehat{mc}_{t+k} + (p_{t+k} - p_{t-1})\right\} \\ &= (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\Theta\widehat{mc}_{t+k} + \sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\pi_{t+k}\right\}\right\}, \end{aligned}$$

where $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\varepsilon} \leq 1$. Notice that the above discounted sum can be rewritten more compactly as the difference equation

$$p_t^* - p_{t-1} = \beta\theta E_t\left\{p_{t+1}^* - p_t\right\} + (1-\beta\theta)\,\Theta\widehat{mc}_t + \pi_t. \tag{A.11}$$

Finally, combining A.4 and A.11 yields the inflation equation

$$\pi_t = \beta E_t\left\{\pi_{t+1}\right\} + \lambda\widehat{mc}_t, \tag{A.12}$$

where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta$ is strictly decreasing in the index of price stickiness θ, in the measure of decreasing returns α, and in the demand elasticity ε. Solving A.12 forward, inflation is expressed as the discounted sum of current and expected future deviations of real marginal costs from steady state

$$\pi_t = \lambda\sum_{k=0}^{\infty}\beta^k E_t\left\{\widehat{mc}_{t+k}\right\}.$$

Next, a relation is derived between the economy's real marginal cost and a measure of aggregate economic activity. Notice that independent of the nature of price setting, average real marginal cost can be expressed as

$$
\begin{aligned}
mc_t &= (w_t - p_t) - mpn_t \\
&= (\sigma y_t + \varphi n_t) - (y_t - n_t) - \log(1-\alpha) \\
&= \left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) y_t^n - \frac{1+\varphi}{1-\alpha} a_t - \log(1-\alpha),
\end{aligned} \tag{A.13}
$$

where derivation of the second and third equalities make use of the household's optimality condition A.2 and the approximate aggregate production relation A.9. Defining the natural level of output, denoted by y_t^n, as the equilibrium level of output under flexible prices

$$mc = \left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) y_t^n - \frac{\varphi + \alpha}{1-\alpha} a_t - \log(1-\alpha), \tag{A.14}$$

thus implying

$$y_t^n = \psi_{ya}^n a_t + \vartheta_y^n, \tag{A.15}$$

where $\upsilon_{ya}^n \equiv -\frac{(1-\alpha)(\mu - \log(1-\alpha))}{\sigma(1-\alpha)+\varphi+\alpha} > 0$ and $\psi_{ya}^n \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$. The presence of market power by firms has the effect of lowering that output level uniformly over time, without affecting its sensitivity to changes in technology. Subtracting A.14 to A.13 one obtains

$$\widehat{mc}_t = \left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right)(y_t - y_t^n) \tag{A.16}$$

i.e., the log deviation of real marginal cost from steady state is proportional to the log deviation of output from its flexible price counterpart. Following convention, henceforth that deviation is referred to as the output gap, and it is denoted by $\tilde{y} \equiv y_t - y_t^n$. By combining A.16 and A.12 one obtains an equation relating inflation to its one period ahead forecast and the output gap

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \kappa \tilde{y}_t, \tag{A.17}$$

where $\kappa \equiv \lambda\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$. A.17 is often referred to as the New Keynesian Phillips curve (NKPC), and constitutes one of the key building blocks of the basic New Keynesian model. The second key equation describing the equilibrium of the New Keynesian model can be obtained by rewriting A.8 in terms of the output gap as

$$\tilde{y}_t = -\frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - r_t^n\right) + E_t\{\tilde{y}_{t+1}\},$$

where r_t^n is the natural rate of interest, given by

$$
\begin{aligned}
r_t^n &\equiv \rho + \sigma E_t\left\{\Delta y_{t+1}^n\right\} \\
&\equiv \rho + \sigma \psi_{ya}^n E_t\{\Delta a_{t+1}\}.
\end{aligned}
$$

Appendix B

Derivation of the steady state equations of the Small Open Economy model of the Czech Republic with Housing sector

In order to calculate recursive law of motion for the log-linear model, a number of steady state values are required. Thus in order to make the model consistent, structural parameters should be based on a computed steady state which is in turn consistent with empirical data.

The steady state relationship implied by the model equations mean that it is possible to derive the steady state by using only a small subset of the model variables. Model contains five exogenous variables (the real exchange rate, technology, real house price, housing stock, and aggregate labor) which are used to calculate the rest of steady state variables (endogenously).

The steady state equations are based on model equations. Time indexes are not included in these equations because steady state expresses the long-term relationship.

B.1 Ex-ante return on housing

The long term return from housing asset may be expressed as follows

$$\begin{aligned} E_t R_{h,t+1} &= f\left(\frac{N_{t+1}}{q_t h_{t+1}}\right) R_{t+1} \\ R_h &= f\left(\frac{N}{qh}\right) R, \end{aligned} \tag{B.1}$$

where R_h is the return on housing, N is the net worth, q is the real house price, h is the housing stock and R is the real domestic interest rate.

B.2 Relative price of rental

Relative price of renting may be derived from model equation as follows

$$\begin{aligned} E_t R_{h,t+1} &= E_t\left(\frac{X_{h,t+1} + (1-\delta)\, q_{t+1}}{q_t}\right) \\ R_h &= \left(\frac{X_h}{q}\right) + (1-\delta) \\ X_h &= \left[R_h - (1-\delta)\right] q, \end{aligned} \tag{B.2}$$

where X_h is the relative price of renting which is in long-term period derived using the R_h the return on housing, δ the housing depreciation rate and q the real house price.

B.3 Relative price of consumption goods

The relative price of consumption goods X_c is derived as follows

$$\begin{aligned} P_t &= \left[\upsilon P_{c,t}^{1-\eta} + (1-\upsilon)\, P_{h,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} \\ 1 &= \left[\upsilon X_{c,t}^{1-\eta} + (1-\upsilon)\, X_{h,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} \\ 1 &= \left[\upsilon X_{c}^{1-\eta} + (1-\upsilon)\, X_{h}^{1-\eta}\right]^{\frac{1}{1-\eta}}, \end{aligned} \tag{B.3}$$

where X_h is the relative price of rental services and υ represents the share of expenditure on consumer goods.

B.4 Dividend

The size of divided which is paid by homeowners to consumers is derived as follows

$$\begin{aligned} N_{t+1} &= V_t - D_t \\ &= R_{h,t}q_{t-1}h_t - f\left(\frac{N_t}{q_{t-1}h_t}\right)R_t b_t - D_t \\ N &= R_h qh - f\left(\frac{N}{qh}\right)R\left(qh - N\right) - D \\ &= R_h N - D \\ N\left(1 - R_h\right) &= -D \\ D &= N\left(R_h - 1\right), \end{aligned} \tag{B.4}$$

where R_h is the return on housing, N is the net worth and D is the size of dividend.

B.5 Aggregate consumption

Aggregate consumption C may be expressed by h housing stock, X_h the relative price of rental services and η the elasticity of substitution between consumption goods and housing as follows

$$\begin{aligned} h_t &= \left(1 - \upsilon\right)\left(\frac{P_{h,t}}{P_t}\right)^{-\eta} C_t \\ &= \left(1 - \upsilon\right)X_h^{-\eta}C \\ C &= \frac{h}{\left(1 - \upsilon\right)X_h^{-\eta}}. \end{aligned} \tag{B.5}$$

B.6 Household consumption of goods

Consumption of goods by households is equal to log-linearized model equation but without the time indexes. Consumption of goods by households may be expressed as follows

$$\begin{aligned} c_t &= \upsilon\left(\frac{P_{c,t}}{P_t}\right)^{-\eta} C_t \\ c &= \upsilon X_c^{-\eta}C, \end{aligned} \tag{B.6}$$

where c is the goods consumption, υ is the share of expenditure on consumer goods, X_c is the relative price of the representative consumption good, η is the elasticity of substitution between consumption goods and housing and C is the aggregate consumption.

B.7 Investement

Investments into the housing stock I are determined by the housing depreciation rate δ and h housing stock.

$$\begin{aligned} h_{t+1} &= h_t + I_t - \delta h_t \\ I &= \delta h. \end{aligned} \tag{B.7}$$

B.8 Resource constraint

Resource constraint says that output of firms Y is consumed by domestic consumers c, government G, foreign consumers EX or utilized in the production of housing.

$$\begin{aligned} Y_t &= c_t + I_t + G_t + EX_t \\ Y &= c + I + \frac{G}{Y}Y + \frac{EX}{Y}Y \\ Y\left(1 - \frac{G}{Y} - \frac{EX}{Y}\right) &= c + I \\ Y &= \frac{c+I}{1 - \frac{G}{Y} - \frac{EX}{Y}}. \end{aligned} \tag{B.8}$$

B.9 Aggregate labor

Aggregate labor is expressed using the Y output, γ labor-imports substitution coefficient, IM imports and α import weight in the production function.

$$\begin{aligned} Y_t(z) &= \left[\alpha IM_t(z) + (1-\alpha)(A_t L_t(z))^{\gamma}\right]^{\frac{1}{\gamma}} \\ L &= \left(\frac{Y^{\gamma} - IM^{\gamma}}{1-\alpha}\right). \end{aligned} \tag{B.9}$$

B.10 Goods producers first order conditions

This relationship is equivalent to the log-linearized model equation.

$$\begin{aligned} \left(\frac{IM_t}{L_t}\right)^{1-\gamma} &= \frac{\alpha}{1-\alpha}\frac{w_t}{RS_t} \\ \left(\frac{IM}{L}\right)^{1-\gamma} &= \frac{\alpha}{1-\alpha}\frac{w}{RS}. \end{aligned} \tag{B.10}$$

B.11 Wage rate

Wages w are determined by the aggregate consumption C, ξ the leisure coefficient and L labor.

$$\begin{aligned} \frac{1}{C_t} &= \frac{\xi}{w_t\left(1-L_t\right)} \\ w &= \frac{C\xi}{1-L}. \end{aligned} \tag{B.11}$$

B.12 Rule of thumb labor supply

This relationship is equivalent to the log-linearized model equation.

$$\begin{aligned} w_t L_t^r - \left(1-n\right) T_t &= \frac{w_t\left(1-L_t^r\right)}{\xi} \\ w_t L_t^r \left(1+\frac{1}{\xi}\right) &= \frac{w_t}{\xi} + \left(1-n\right) T_t \\ L^r &= \frac{1}{1+\xi} + \frac{\xi}{w}\left(1-n\right) T. \end{aligned} \tag{B.12}$$

B.13 Permanent income hypothesis labor supply

PIH labor supply is calculated as labor supply L minus the labor supply of rule of thumb consumers L^r.

$$\begin{aligned} L_t &= nL_t^p + \left(1-n\right) L_t^r \\ L^p &= \frac{L-\left(1-n\right) L^r}{n}. \end{aligned} \tag{B.13}$$

B.14 Rule of thumb consumption

Consumption of rule of thumb consumers C^r is driven by wage w and their labor supply L^r.

$$\begin{aligned} C_t^r &= w_t L_t^r \\ C^r &= wL^r. \end{aligned} \tag{B.14}$$

B.15 Permanent income hypothesis consumption

Consumption of PIH consumers C^p is calculated as aggregate consumption C minus consumption of ROT consumers C^r.

$$\begin{aligned} C_t &= nC_t^p + (1-n)\,C_t^r \\ C^p &= \frac{C-(1-n)\,C^r}{n}, \end{aligned} \tag{B.15}$$

where n is the proportion of PIH consumers.

Appendix C

Dynare code of the New Keynesian model

```
// Typed and annotated by Milan Bouda, September 2013
// Dynare 4.3.3

var y, pi, i, a, rn, n, m, v;
varexo e_a e_v;

parameters alpha, beta, theta, sigma, phi, rho, phi_pi, phi_y, rho_a, rho_v,
           lambda, kappa, psi, epsilon, eta;

alpha=0.5;     // If =0 we have a CRS production technology. Else it's
                  decreasing returns to scale .
epsilon=1.5;   // Elasticity of substitution derived from the markup
                  forumla m=log(epsilon/(epsilon-1)). Using m=1.1.
beta=0.99;     // The discount factor.
theta=0.698;   // Measure of price stickiness. If =0 then prices are flexible.
lambda=(theta^(-1))*(1-theta)*(1-beta*theta)*(1-alpha)/(1-alpha+alpha*epsilon);
rho=-log(beta);  // Real interest rate in the steady state (no shocks).
sigma=1;       // Coefficient of risk aversion.
phi=0.8;       // Elasticity of labor supply.
phi_pi=1.5;    // Sensitivity of the central bank with respect to inflation.
phi_y=0.25;    // Sensitivity of the central bank with respect to the output gap.
rho_a=0.975;   // Persistence of the technology shock.
rho_v=0.5;     // Persistence of the monetary policy shock.
eta=4;         // Elasticity of the money demand with respect to the nominal
```

```
                 interest rate.
kappa=lambda*(sigma+(phi+alpha)/(1-alpha));
psi=(1+phi)*((sigma+phi+alpha*(1-sigma))^(-1));

model;
y=y(+1)-1/sigma*(i-pi(+1)-rn); // Eq. 1: The Dynamic IS equation.
pi=beta*pi(+1)+kappa*y;        // Eq. 2: The New Keynesian Philips Curve.
rn=rho+sigma*psi*(rho_a-1)*a;  // Eq. 3: The evolution of the natural
                                  rate of interest.
i=rho+phi_pi*pi+phi_y*y+v;     // Eq. 4: The interest rate rule of
                                  the central bank.
y=a+(1-alpha)*n;               // Eq. 5: The production function consisting
                                  of technology and labor. This relationship
                                  is only true up to a 1st order approximation.
m=pi+y-eta*(i);               // Eq. 6: Ad-hoc money demand.
a=rho_a*a(-1)+e_a;             // Eq. 7: Technology shocks follow an AR(1)
                                  process with persistence rho_a.
v=rho_v*v(-1)+e_v;             // Eq. 8: Monetary policy follow an AR(1)
                                  process with persistence rho_v.
end;

initval;
y=0;
m=0;
n=0;
pi=0;
i=rho;
rn=rho;
a=0;
v=0;
e_a=0;
e_v=0;
end;

steady;
check;

shocks;
var e_a;
```

```
stderr 0.01;
var e_v;
stderr 0.01;
end;

estimated_params;
alpha, beta_pdf, 0.5, 0.05;
phi, beta_pdf, 0.8, 0.05;
phi_pi, normal_pdf, 1.5, 0.05;
phi_y, normal_pdf, 0.25, 0.05;
stderr e_a, inv_gamma_pdf, 0.01, inf;
stderr e_v, inv_gamma_pdf, 0.01, inf;
end;

varobs y pi;

estimation(datafile=data_cz);
identification;
dynare_sensitivity;
shock_decomposition y pi;
```

Appendix D

Dynare code of the SOE model with Housing sector

```
// Typed and annotated by Milan Bouda, December 2013
// Dynare 4.3.3

// Define variables
var
C     // Aggregate consumption
q     // Real house price
I     // Housing investment
h     // Housing stock
Xc    // Relative price of consumption good
Y     // Real output
IM    // Imports
A     // Technology
L     // Aggregate labor
w     // Real wage
mc    // Real marginal cost
Cr    // ROT consumption
Lr    // ROT labor supply
Cp    // PIH consumption
Lp    // PIH labor supply
T     // Lump-sum taxes (in real terms)
Rn    // Nominal interest rate
R     // Real domestic interest rate
pi    // Overall inflation
```

```
gap   // Output gap
Xii   // Monetary policy shock
N     // Net worth
Rh    // Return on housing
D     // Housing dividend
EX    // Exports
RS    // Real exchange rate
Yf    // Foreign output
Rf    // Foreign interest rate
S     // Nominal exchange rate
b     // The borrowing undertaken to finance the purchase of housing stock
yflx  // Flexible price output
BG    // Government debt
Xh    // Relative price of renting
G     // Government spending
pic   // Consumption good inflation
gdcon // Goods consumption
gdp;  // GDP (Y-IM)

// Define exogenous variables
varexo eps_a eps_xii eps_rf eps_yf eps_g;

// Define parameters
parameters
nkl sp Omega adj h_ss npih Gammad q_ss eta delta nu rhoi rhoa rhorf rhog
rhoxii gammapi gammay theta alpha gamma xi beta bn kappa R_ss Rh_ss Xh_ss
Xc_ss mu I_ss gy exy imy RS_ss N_ss D_ss C_ss gdcon_ss Y_ss T_ss L_ss tau
w_ss Lr_ss Lp_ss digamma Cr_ss Cp_ss phi_g phi_b iml varphi vartheta zeta
ncp nlp u c1 c2 v gdp_ss rhoyf b_ss ab_ss deltab G_ss;

// Calibration
nkl=0.7;          // Net worth ratio
sp=0.026;         // Contribution of financial accelerator
Omega=-0.1;       // Sensitivity of interest rate premium
adj=3;            // Sensitivity of dividend to net worth ratio
npih=0.5;         // Proportion of consumers that are PIH
Gammad=0.52;      // q-theory sensitivity
eta=0.9999;       // Consumer substitution between housing and goods coefficient
delta=0.005;      // Housing depreciation rate
```

```
nu=0.81;          // Steady state goods consumption as a proportion of overall
                     consumption
rhoi=0.7;         // Interest rate smoothing parameter
rhoa=0.9;         // Autocorrelation for technology shock
rhorf=0.8;        // Autocorrelation of foreign interest rate shock
rhog=0.7;         // Autocorrelation of fiscal spending shock
rhoxii=0.8;       // Autocorrelation of domestic interest rate shock
rhoyf=0.8;        // Autocorrelation of of foreign demand shock
gammapi=1.5;      // Coefficient on inflation in monetary policy rule
gammay=0.25;      // Coefficient on output gap in monetary policy rule
theta=0.5;        // 1-theta = probability of firm resetting its price
alpha=0.65;       // Import weight in production function
gamma=-0.2;       // Labor-imports substitution coefficient in production
                     function
xi=1.1097;        // Leisure coefficient in utility function
beta=0.99;        // Discount rate
deltab=-0.001;    // The cost of intermediation in the foreign currency
                     bond market
vartheta=1;       // Export sensitivity to real exchange rate
zeta=1;           // Export sensitivity to foreign demand
phi_g=0.1;        // Distribution of fiscal imbalances with respect to
                     the government exogenous spending
phi_b=0.33;       // Distribution of fiscal imbalances with respect to
                     the government debt
gy=0.2;           // Government spending/output ratio
exy=0.6;          // Exports/output ratio
imy=0.7;          // Imports/output ratio
bn=(1/nkl)-1;
kappa=(1-theta)*(1-beta*theta)/theta;
q_ss=5.035602055;// Steady state of real house price
h_ss=0.5;         // Steady state of housing stock
R_ss=1/beta;      // Steady state of real domestic interest rate
Rh_ss=R_ss+sp/4; // Steady state of return on housing
Xh_ss=(Rh_ss-(1-delta))*q_ss;// Steady state of relative price of renting
Xc_ss=((1-(1-nu)*Xh_ss^(1-eta))/nu)^(1/(1-eta));// Steady state of relative
                                                    price of consumption good
mu=(1-delta)*q_ss/(Xh_ss+(1-delta)*q_ss);
I_ss=delta*h_ss; // Steady state of housing investment
RS_ss=1;          // Steady state of real exchange rate
```

```
N_ss=nkl*q_ss*h_ss;// Steady state of net worth
D_ss=N_ss*(Rh_ss-1);// Steady state of housing dividend
C_ss=h_ss/((1-nu)*Xh_ss^-eta);// Steady state of aggregate consumption
gdcon_ss=nu*(Xc_ss^-eta)*C_ss;// Steady state of goods consumption
Y_ss=(gdcon_ss+I_ss)/(1-exy-gy);// Steady state of real output
T_ss=gy*Y_ss;    // Steady state of taxes
L_ss=0.33;       // Steady state of aggregate labor
tau=L_ss/(1-L_ss);
w_ss=((1-alpha)/alpha)*(imy*Y_ss/L_ss)^(1-gamma);// Steady state of real wage
Lr_ss=1/(1+xi)+(xi*(1-npih)*T_ss)/w_ss;// Steady state of ROT labor supply
Lp_ss=(L_ss-(1-npih)*Lr_ss)/npih;// Steady state of PIH labor supply
digamma=(w_ss/xi)*(1-L_ss)/C_ss;
Cr_ss=w_ss*Lr_ss-(1-npih)*gy*Y_ss;// Steady state of ROT consumption
Cp_ss=(C_ss-(1-npih)*Cr_ss)/npih;// Steady state of PIH consumption
iml=(alpha/(1-alpha)*w_ss/RS_ss)^(1/(1-gamma));
varphi=(alpha*iml^gamma)/((alpha*iml^gamma)+(1-alpha));
ncp=npih*Cp_ss/C_ss;
nlp=npih*Lp_ss/L_ss;
u=((1-alpha)*(L_ss/Y_ss)^gamma)-(1-gamma)/(1-gamma+tau);
c1=-alpha*imy^gamma/(1-gamma);
c2=(1-alpha)*(L_ss/Y_ss)^gamma*(1+tau)/(1-gamma+tau);
v=nu*(1-eta)*Xc_ss^(1-eta);
gdp_ss=Y_ss*(1-imy);// Steady state of GDP (Y-IM)
b_ss=1/(beta-1)*(exy*Y_ss/RS_ss-imy*Y_ss);// Steady state of the borrowing
                                            undertaken to finance the purchase
                                            of housing stock
ab_ss=b_ss/C_ss;
G_ss=gy*Y_ss;      // Steady state of government spending

model(linear);

// LOG-LINEARIZED EQUAITONS

0  =  Rh-(1-mu)/eta*C+(1-mu)/eta*h(-1)-mu*q+q(-1);
0  =  -q+Gammad*I-Gammad*h(-1)+Xc;
0  =  -Y+varphi*IM+(1-varphi)*A+(1-varphi)*L;
0  =  w-(1-gamma)*varphi*IM+(1-gamma)*varphi*L-((1-gamma)*(1-varphi)+
      gamma)*A-mc;
0  =  -h+delta*I+(1-delta)*h(-1);
```

```
0  =  -Cr+(w_ss*Lr_ss/Cr_ss)*w+(w_ss*Lr_ss/Cr_ss)*Lr-(1-npih)*G_ss/Cr_ss*T;
0  =  Rn-rhoi*Rn(-1)-(1-rhoi)*gammapi*pi(-1)-(1-rhoi)*gammay*gap+Xii;
0  =  -C+ncp*Cp+(1-ncp)*Cr;
0  =  -N+Rh_ss*(bn+1)*Rh+Rh_ss*(bn*Omega)*q(-1)+Rh_ss*(bn*Omega)*
      h(-1)-Rh_ss*bn*R(-1)+Rh_ss*(1-bn*Omega)*N(-1)-(Rh_ss-1)*D;
0  =  Cp-w+(Lp_ss/(1-Lp_ss))*Lp;
0  =  -C+xi*(1-npih)^2*(gy*Y_ss/C_ss)*T+(w_ss*(1-L_ss))/(xi*C_ss)*
      w-(w_ss*L_ss/(xi*C_ss))*L;
0  =  -IM+L+(1/(1-gamma))*w-(gamma/(1-gamma))*A-(1/(1-gamma))*RS;
0  =  -EX+vartheta*RS+zeta*Yf;
0  =  -beta*(1+ab_ss*deltab)*b+beta*ab_ss*Rf+b(-1)+ab_ss*S-ab_ss*
      S(-1)-ab_ss*pi-(exy*Y_ss*zeta/C_ss)*Yf-((exy*Y_ss*vartheta-RS_ss*
      imy*Y_ss)/C_ss)*RS+(RS_ss*imy*Y_ss/C_ss)*IM;
0  =  Xc+(1-nu)/nu*(Xh_ss/Xc_ss)^(1-eta)*Xh;
0  =  -pi+pic-Xc+Xc(-1);
0  =  -Y+(gdcon_ss/Y_ss)*gdcon+(I_ss/Y_ss)*I+gy*G+exy*EX;
0  =  -L+nlp*Lp+(1-nlp)*Lr;
0  =  gap-Y+yflx;
0  =  gdcon-C-eta*Xc;
0  =  gdp-(Y_ss/gdp_ss)*Y+(imy*Y_ss/gdp_ss)*IM;
0  =  yflx-(c2/u)*A-(c1/u)*RS;
0  =  -Xh+(1/eta)*C-(1/eta)*h(-1);
0  =  -BG+R_ss*BG(-1)+R_ss*gy*G-R_ss*gy*T;
0  =  -gy*T+phi_b*BG(-1)+phi_g*gy*G;
0  =  -D+adj*N-adj*q-adj*h;

// Expectations

0  = Cp+R-Cp(+1);
0  = -Rn+R+pi(+1);
0  = -R+Rf+deltab*b+RS(+1)-RS;
0  = -Rh(+1)+R+Omega*N-Omega*q-Omega*h;
0  = RS(+1)-RS-S(+1)+S+pi(+1);
0  = pic-kappa*mc-beta*pic(+1);

// Log linearized law of motion for exogenous shocks

A  =  rhoa*A(-1)+eps_a;
Xii  =  rhoxii*Xii(-1)+eps_xii;
```

```
Rf  =  rhorf*Rf(-1)+eps_rf;
Yf  =  rhoyf*Yf(-1)+eps_yf;
G  =  rhog*G(-1)+eps_g;
end;

steady;
check;

shocks;
var eps_a; stderr 0.05;
var eps_xii; stderr 0.05;
var eps_rf; stderr 0.05;
var eps_yf; stderr 0.05;
var eps_g; stderr 0.05;
end;

// BAYESIAN ESTIMATION

varobs Y pic q S Rn;

estimated_params;
eta, normal_pdf, 0.9999, 0.01;    // Consumer substitution between housing
                                     and goods coefficient
delta, beta_pdf, 0.005, 0.001;    // Housing depreciation rate
npih, beta_pdf, 0.5, 0.01;        // Proportion of consumers that are PIH
gammapi, normal_pdf, 1.5, 0.01;   // Coefficient on inflation in monetary
                                     policy rule
gammay, normal_pdf, 0.25, 0.01;   // Coefficient on output gap in monetary
                                     policy rule
rhoi, normal_pdf, 0.7, 0.01;      // Interest rate smoothing parameter
nkl, normal_pdf, 0.7, 0.01;       // Net worth ratio
Omega, normal_pdf, -0.1, 0.01;    // Sensitivity of interest rate premium
adj, normal_pdf, 3, 0.01;         // Sensitivity of dividend to net worth
                                     ratio
theta, beta_pdf, 0.5, 0.01;       // 1-theta = probability of firm resetting
                                     its price
xi, normal_pdf, 1.1097, 0.01;     // Leisure coefficient in utility function
beta, beta_pdf, 0.99, 0.001;      // Discount rate
```

```
stderr eps_a, inv_gamma_pdf, 0.01, inf;
stderr eps_xii, inv_gamma_pdf, 0.01, inf;
stderr eps_rf, inv_gamma_pdf, 0.01, inf;
stderr eps_yf, inv_gamma_pdf, 0.01, inf;
stderr eps_g, inv_gamma_pdf, 0.01, inf;
end;

estimation(datafile=data_cz, mode_compute=6, plot_priors=0, diffuse_filter,
mh_replic=200000, mh_nblocks=2, irf=20, bayesian_irf, forecast=8);
identification;
dynare_sensitivity;
shock_decomposition Y pic q S Rn;
```

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