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Dynamic conditional correlation models and their application to portfolio risk mitigation

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Čestné prohlášení:

Prohlašuji, že jsem bakalářskou práci na téma "Dynamic conditional correlation models and their application to portfolio risk mitigation" vypracoval samostatně a veškerou použitou literaturu a další prameny jsem řádně označil a uvedl v přiloženém seznamu.

V .... dne ....

Poděkování:

Rád bych na tomto místě poděkoval panu Ing. Lukáši Frýdovi za návrh tématu, vedení mé bakalářské práce a také za podnětné návrhy, které ji obohatily.

Abstrakt

Tato bakalářská práce se zabývá asymetrií ve výnosech kukuřice, zlata a ropy (jak u spotových výnosů, tak i u výnosů futures) a efektivností hedgování těchto komodit pomocí hedge ratia odhadnutého modely ze skupiny DCC. Asymetrie v podmíněném rozptylu byla zjištěna statisticky významnou pouze v případě spotových a futures výnosů ropy a asymetrie v podmíněné korelaci mezi výnosy spot a futures nebyla identifikována statisticky významnou v případě žádné ze studovaných komodit. V rámci efektivnosti hedgování jsem došel k závěru, že rozdíly v eliminaci rozptylu měřeného pomocí Hedging effectiveness indexu jsou mezi jednotlivými modely založenými na DCC a MNČ (slouží jako benchmark) zanedbatelné. Historický Value at Risk naproti tomu identifikoval DCC model s asymetrií v rozptylu (avšak statisticky nevýznamnou) jako potenciálně nejvhodnější z použitých modelů pro hedgovaní kukuřice. V případě ostatních komodit hedge ratio založené na MNČ poskytlo srovnatelný nebo nižší VaR než hedge ratio založené na DCC. Hlavní přínos práce tedy spočívá v empirickém ověření asymetrie ve výnosech vybraných komodit a otestovaní hedgovacích schopností hedge ratia odhadnutého pomocí modelů ze skupiny DCC.

Klíčová slova: DCC, ADCC, GARCH, GJR GARCH,MNČ, Hedge ratio, HE index, VaR

Abstract

This bachelor thesis investigates asymmetry in returns of corn, gold and crude oil (both spot and futures) and hedging effectiveness of these commodities when employing DCC family models for hedge ratio estimation. The asymmetry in conditional variances was found to be significant only in case of crude oil spot and futures returns and asymmetry in conditional correlation of spot and futures returns was not shown to be significant in neither of the investigated commodities. With respect to the hedging performance, we conclude that differences in hedging performance measured by hedging effectiveness index are negligible and thus do not support superiority of DCC family models over OLS, which served as a benchmark. Historical Value at Risk, on the contrary, identified the DCC with asymmetry in conditional variance (despite asymmetry not being significant) to be appropriate for corn hedging, however not for the other two commodities, where the OLS based hedge ratio performed similarly or even better than the DCC family models. The main contribution of the thesis thus lays in empirical investigation of asymmetry in returns of selected commodities and testing hedging potential of DCC family based hedge ratio.

Keywords: DCC, ADCC, GARCH, GJR GARCH,OLS, Hedge ratio, HE index, VaR

Contents

Introduction

This bachelor thesis aims on estimating the hedge ratio by several techniques in order to mitigate the risk attached to holding a physical commodity. The estimation methods namely are ordinary least squares, dynamic conditional correlation, dynamic conditional correlation with asymmetric GARCH and asymmetric dynamic conditional correlation.

There has been an increase in volatility of commodity prices in recent years, which made hedging a necessary part of businesses such as physical commodity trading or airlines (Geman 2005). This change in volatility can be, for instance, attributed to increase in environmental regulation, political instability in certain regions and increase in consumption of commodities in China (Geman 2005).

There are several ways of hedging and this thesis is concerned with hedging using futures contracts as it is probably the most employed strategy among hedgers (Brooks, Henry and Persand 2002). Hedging by futures starts with constructing a portfolio consisting of physical asset and a corresponding opposite position in futures. In order to decrease volatility of portfolio profit in time we employ the minimum variance hedge ratio, which adjusts the futures position so that the P/L from physical asset is offset by the hedge. Within this thesis the hedging of corn, gold and crude oil are investigated, while not accounting for costs arising from futures position adjustment.

The aim of this thesis is twofold. First, it is to investigate whether there is any asymmetry in volatility and correlation. The second hypothesis is concerned with hedging effectiveness where we test whether dynamic conditional correlation family models and accounting for asymmetry leads to higher risk reduction measured by the hedging effectiveness index and the Value at Risk.

The thesis has the following structure. First, we introduce the relevant literature and its main findings linked to the topic of the thesis. Next section contains methodology, which is applied in empirical section. This includes futures pricing, portfolio construction and further econometric techniques employed for volatility and correlation modeling. In next sections, we follow by providing a description of employed data, and finally, we provide empirical results of asymmetries and hedging performance of investigated commodities both in-sample and out-of-sample. The last section of the thesis sums up the main findings and conclusions.

1 Literature review

In paper written by Myers (1991), the hedge ratio of cash (i.e., spot) and futures market of wheat is investigated in period from June 1977 to May 1983. The author calculates the hedge ratio using moving sample variances and covariances of past prediction errors and using GARCH model of Bollerslev (1986). The contribution of the author lays in providing an empirical evidence of time-varying hedge ratio. Despite this, the author emphasizes that the above employed models lead to only slightly better hedging performance than conventional regression techniques.

The hedge ratio has also been investigated by Switzer and El-Khoury (2007) who focused on hedging effectiveness of light sweet crude oil (NYMEX) using futures during periods of high volatility. The data are from 1986 to 2005 and thus contain events such as Iraqi war. The authors test whether including asymmetry in volatility leads to higher variance reduction measured by the Hedging effectiveness index (Ederington 1979). The models employed for hedge ratio modeling include OLS, symmetric BEKK (Engle and Kroner 1995) (BEKK is an acronym of Baba, Engle, Kraft and Kroner, who initially developed the model in 1990) and asymmetric BEKK (Engle and Kroner 1995). The authors conclude that adding asymmetry to conditional volatility improves hedging effectiveness.

The hedging effectiveness of agricultural futures has also been studied by Choudhry (2009), who took cointegration into account and also tested whether there is any difference in hedging performance of storable and non-storable commodities. The author has provided an empirical evidence of negligibly better performance of advanced models compared to traditional OLS estimated models. As another conclusion, the author presents that there is no difference in hedging effectiveness of storable and non-storable commodities.

The hedging performance of several multivariate volatility models including BEKK or CCC (Bollerslev 1990) has been investigated by Chang, McAleer and Tansuchat (2011) on two crude oil benchmarks. The hedging effectiveness is tested on Western Texas Intermediate (WTI) and Brent in period from November 1997 to November 2009. The results of the analysis summarize that the best hedging performance, which is measured by the hedging effectiveness index, provides the diagonal BEKK model, while the DCC is only slightly worse. Other articles concerned with hedge ratio were written by Cecchetti, Cumby and Figlewski (1988) or Chou, Wu and Liu (2009).

2 Methodology

2.1 Spot and Futures markets

With respect to hedging and commodity markets, there are two price quotations, which have to be observed. Those namely are spot and futures prices.

The spot market can be defined as a market, where delivery takes place immediately or with a minimum lag due to technical constraints (Geman 2005). There may be different spot prices on different places, which can, for instance, be caused by transaction costs or quality. Futures presents an agreement between two counterparties, where one party agrees to deliver certain amount of asset of pre-specified quality on pre-specified date (maturity) for a price given today and the other party is obliged to pay for it. Futures are traded on exchanges and thus the contracts are standardized in terms of asset quality, contract maturity and contract size. Forwards, on the contrary, are traded over-the-counter and therefore may be adjusted. Both of these derivatives share similarities, which are reflected in similar pricing techniques.

There are three types of derivatives users (Geman 2005), who namely are speculators, arbitragers and hedgers. Speculators undertake risk in order to bet on either increase or decrease of the underlying asset price. Arbitrageurs seek opportunities to make a riskless profit when no-arbitrage principle is violated. Hedgers are probably the original users of the derivatives and their main objective, on contrary to speculators, is to reduce the risk. For instance, a corn growing farmer may want to protect himself from decrease in corn prices during harvest, therefore he sells a futures contract in spring in order to lock the price in advance. The physical trader can use hedging to eliminate price risks, but he is still exposed to (Geman 2005):

- Delivery risk
- Transportation risk
- Credit risk

The main focus of the thesis is the price risk elimination, which is further described in section 2.3.

2.2 No-arbitrage principle

The price of futures contract is closely linked to the price of the underlying asset. Based on no-arbitrage principle, we can derive a formula for futures pricing (Wilmott 2007) .

Consider this set of variables, t standing for time now, T for time at maturity, F^T for amount of dollars handed at time T and S_t for a spot price in dollars at time t. Now, let us assume we simultaneously sell a unit of an asset for spot price and enter a long forward with maturity at T, which will secure that we will get the asset back for F^T dollars (by definition F^T can be seen as forward). We immediately put the dollars obtained from the short position into a bank account. On maturity, we receive the amount of dollars (RD^T) corresponding to Equation (1), which is time value of money with interest rate i and continuous compounding.

$$RD^T = S_t \times e^{i \times (T-t)} \tag{1}$$

The money obtained from the bank account will be used to fulfill the obligation arising from the forward contract and thus the net position (NP^T) at maturity will be as described in following equation:

$$NP^T = RD^T - F_T^T \tag{2}$$

Based on no-arbitrage principle, our net position at maturity has to be zero, otherwise there exists an opportunity to make riskless profit by either selling spot and going long in forward market or vice versa. Hence, the price of the forward contract at time t maturing at time T is

$$F_t^T = S_t \times e^{i \times (T-t)} \tag{3}$$

The above described formula is however not valid for commodities, where special characteristics have to be taken into account. Most of the commodities are storable and therefore storage theory of Working (1933) has to be applied. Based on this, we compute F_t^T by formula in Equation (4), which accounts for convenience yield (denoted as y) of Brennan (1958) and Telser (1958). The authors claim that holding a commodity can be viewed as a timing option to flexibly react to market changes given the availability of the asset.

$$F_t^T = S_t \times e^{(i-y) \times (T-t)} \tag{4}$$

The introduced forward pricing can be also applied to futures as the differences between them are for purpose of the thesis negligible.

Note that if we put the prices of futures contracts into a graph as a function of maturities, the forward curve is constructed. This curve can be both upward or downward sloping, which depends on risk-free rate and convenience yield. Upward sloping curve is named contango while the downward one is often referred as backwardation. In case of changes in convenience yield or risk-free rate (or both), the forward curve may change. This behavior can be observed, for instance, in case of oil futures (Geman 2005), where the forward curve changes from contango to backwardation and vice versa quite often.

2.3 Hedging and optimal hedge ratio

The initial purpose of derivatives as Geman (2005, p. 4) states is 'vehicle against price risk'. We focus on hedging the price risk using futures and subsequent estimation of hedge ratio.

When futures hedging is employed, an opposite position to the physical trade is taken on futures market. For instance, when a farmer decides to hedge himself against the change in corn prices, he sells a corresponding contract on futures market. This way the farmer locks the price of the corn and his profit/loss from both assets should offset. This however does not work perfectly, since when using naive hedge (i.e., hedge ratio equal one), there still remains the risk that price of futures contract will not be equal to the spot price, when closing the contract before maturity. The risk is named basis risk (Geman 2005) and can be defined as the difference between spot price and futures price at time t of contract maturing at T (see Equation (5)).

$$Basis_t = S_t - F_t^T \tag{5}$$

The hedge ratio aims to capture these differences based on adjustment of futures position so that the final P/L is equal to zero. Hull (2012, p. 68) defines the hedge ratio as 'ratio of the size of the position taken in futures contracts to the size of the exposure'. The naive hedge ratio is not usually sufficient, hence, a hedger may employ a so-called minimum variance hedge ratio, which tries to minimize the variance of the portfolio changes. The portfolio change can be seen in Equation (6a) and hedge ratio (h^*) is subsequently defined in Equation (6b),

$$\Delta PortfolioValue_t = \Delta S_t - h^* \times \Delta F_t^T \tag{6a}$$

$$h^* = \rho_{S,F} \frac{\sigma_S}{\sigma_F} \tag{6b}$$

where $\rho_{S,F}$ stands for correlation between spot and futures returns and σ_S, σ_F represent standard deviations of the spot and futures returns, respectively. The Equation (6b) can be further modified to Equation (7), which is for instance employed by Chang, McAleer and Tansuchat (2011) in slightly different notation:

$$h^* = \frac{Cov(\Delta S, \Delta F)}{Var(\Delta F)} \tag{7}$$

$Cov(\Delta S, \Delta F)$ stands for covariance between spot and futures returns and $Var(\Delta F)$ represents variance of futures returns. This formula however assumes time-invariant hedge ratio, which has been shown to be insufficient (Myers 1991), therefore, a modified conditional hedge ratio is proposed in Equation (8),

$$h_t^* = \frac{Cov(\Delta S, \Delta F|\Omega_{t-1})}{Var(\Delta F|\Omega_{t-1})} \tag{8}$$

where Ω_{t-1} denotes the information set available at $t-1$.

2.4 Stationarity and unit root testing

Time series is considered to be strictly stationary if the joint distribution of returns is time invariant, which is a strong assumption, therefore a weaker version of stationarity is usually assumed (Tsay 2010). The time series (r_t) is said to be weakly stationary in case it has constant mean (μ), finite variance and covariance (γ_l) dependent only on lag (l). This is described in Equations (9a) and (9c), respectively.

$$E(r_t) = \mu \tag{9a}$$

$$Var(r_t) < \infty \tag{9b}$$

$$Cov(r_t, r_{t-l}) = \gamma_l \quad l = 1, ..., n \tag{9c}$$

The parameter l stands for order of the lag and thus the value of the covariance changes only with changes of l. The time series which satisfies these conditions is considered to be weakly stationary.

The concept of stationarity (non-stationarity) is closely linked to the so-called unit root testing. In case the time series follows stochastic, non-stationary process, then as Brooks (2008) suggests: Let the process be defined as random walk with a drift,

$$y_t = \mu + y_{t-1} + u_t \tag{10}$$

where u_t is $idd(0, \sigma^2)$. Letting $\Delta y = y_t - y_{t-1}$ and subtracting y_{t-1} from both sides of

Equation (10) we obtain Equation (11a), which can be modified to obtain Equation (11b):

$$y_t - y_{t-1} = \mu + u_t \tag{11a}$$

$$\Delta y = \mu + u_t \tag{11b}$$

Since μ is a constant and u_t is $idd(0, \sigma^2)$ we can conclude that the process is integrated of order one.

Theoretical properties described above can be formally tested using Dickey-Fuller test of Dickey and Fuller (1979), which we employ in its augmented modification since it has certain advantages over the traditional Dickey-Fuller test in terms of error structure in regression (there will be no autocorrelation in u_t) (Brooks 2008). The hypothesis is tested on parameter ψ of Equation (12),

$$\Delta y_t = \psi y_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta y_{t-i} + u_t \tag{12}$$

where ψ is supposed to be less than zero in case of stationary and non-stationary otherwise. The null hypothesis of unit root is tested in favor of alternative hypothesis (stationarity).

2.5 Volatility process

Volatility, which is by its nature unobservable (Chou, Wu and Liu, 2009), plays a vital role in risk management. The commonly used proxy for volatility is standard deviation. Volatility can be modeled as it appears in bunches or clusters. In other words, periods of high volatility are often followed by periods of high volatility and vice versa. Within this thesis two univariate volatility models are described, which namely are GARCH (Bollerslev 1986, Taylor 1987) and GJR GARCH (Glosten, Jagannathan and Runkle 1993).

2.5.1 Generalized autoregressive conditional heteroscedasticity

This model is a generalization of ARCH (Autoregressive conditional heteroskedastic) model of Engle (1982). The conditional variance is modeled using lagged conditional variance estimates and lagged squared error terms, which is depicted in following equation,

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i u_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{13}$$

where α_0 stands for intercept, u_{t-i}^2 represents i-th lag of squared error and σ_{t-j}^2 describes j-th lagged estimate of variance while α_i and β_j are parameters. Parameters p and q stand for order of the GARCH, which is denoted as GARCH(p, q). In order to be stationary, the model has to comply with this condition:

$$\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j < 1 \tag{14}$$

Also note that the parameters α_0, α_i and β_j have to be positive for all i and j. As the estimation technique, we employ the maximum likelihood estimation (MLE).

Before estimating any kind of GARCH family model, it is particularly useful to test ARCH effects (Engle 1982). To test for ARCH effect the residuals of mean equation (example can be seen in Equation (19)) are squared and studied. For instance, the Ljung-Box test (1978) can be used to identify dependence in squared residuals or an approach of Engle (1982) can be followed.

The Ljung-Box test jointly tests autocorrelation on specified squared residual lags with null hypothesis of all autocorrelation functions parameters equal to zero. The test statistics is denoted as Q^* and described in Equation (15),

$$Q^* = T(T+2) \sum_{k=1}^{m} \frac{\hat{\tau}_k^2}{T-k} \sim \chi_m^2 \tag{15}$$

where T stands for sample size, m for maximum lag and $\hat{\tau}_k^2$ represents an estimate of autocorrelation function on k-th lag.

For model selection usually the Schwarz's (1978) Bayesian information criterion (denoted as BIC) is used as proposed by Cappiello, Engle and Sheppard (2006), while paying attention to complexity of the model. The BIC we use is based on log-likelihood function, which is mathematically depicted as

$$BIC = -2\frac{L}{T} + \frac{k}{T} \ln T \tag{16}$$

where L represents log-likelihood function, T denotes number of observations and k stands for number of parameters.

2.5.2 GJR GARCH

For purpose of investigating asymmetry in volatility we also employ the GJR GARCH model of Glosten, Jaganathan and Runkle (1993).

GJR GARCH model accounts for a leverage effect, which has been widely observed in returns of equities. The model targets at capturing different effects of positive and negative shocks (for details see Black (1976) and Christie (1982), or Campbell and Hentschel (1992), Wu 2001). Equities are generally more sensitive to negative than to positive shocks (Engle and Ng 1993). In other words, an unexpected negative information has a larger impact on volatility than its positive counterpart. The model captures this effect by taking into account asymmetry when employing an additional variable, which adjusts volatility estimates in case of negative (positive) past error. The GJR GARCH(p, q, o) is mathematically described in Equation (17),

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i u_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + \sum_{k=1}^{o} \gamma_k u_{t-k}^2 I_{t-k} \tag{17}$$

where $I_{t-k} = 1$ if $u_{t-k} < 0$ and $I_{t-k} = 0$ otherwise. The parameters p, q and o represent order of ARCH effect, GARCH effect and the impact of asymmetry, respectively. The model has to comply with following conditions, which are (Glosten, Jagannathan and Runkle 1993):

$$\alpha_0 > 0 \tag{18a}$$

$$\alpha_i > 0 \quad for \quad i = 1, ..., p \quad and \quad \beta_j > 0 \quad for \quad j = 1, ..., q \tag{18b}$$

$$\alpha_i + \gamma_k > 0 \quad for \quad i = 1, .., p \quad and \quad k = 1, ..., o \tag{18c}$$

$$\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j + \frac{1}{2} \sum_{k=1}^{o} \gamma_k < 1 \tag{18d}$$

where Equation (18d) represents stationarity condition.

For determining the order of the model and estimation the same methodology as for GARCH can be used.

2.6 OLS hedge ratio

Ordinary least square regression model serves as a benchmark for hedging effectiveness in articles written by Ku, Chen and Chen (2007) or Park and Switzer (1995) and therefore we use it for the same purpose. It is worth to note that these authors use a single estimate, i.e., hedge ratio does not change, which we use for in-sample testing. For out-of-sample testing, we have decided to employ rolling estimation since it makes more sense for comparison with other models, which are re-estimated several times in out-of-sample testing. The Equation (19) describes the relationship which will be estimated,

$$\Delta S = \alpha_0 + \beta \Delta F + \epsilon \tag{19}$$

where ϵ is an error term i.e., $\epsilon \sim idd(0, \sigma^2)$. Note that this assumption is often violated due to heteroskedasticity (Choudhry 2009). Both ΔS and ΔF are vectors containing returns corresponding to certain period, called window. This window is then rolled over the out-of-sample data. The β obtained from Equation (19) serves as hedge ratio (i.e., $h^* = \beta$) described in Equation (6a)).

2.7 Dynamic conditional correlation models

Within this section, we describe models, which are used for covariance modeling throughout the paper. First, the DCC model is introduced, than its extension with GJR GARCH (DCCA) and finally its generalization ADCC.

2.7.1 Dynamic conditional correlation

Before deriving formula for dynamic conditional correlation it is worth to prove that conditional correlation of returns is equal to conditional covariance of disturbances under assumptions proposed by Engle (2002). To prove this, let us assume the the returns of two series follow a process described in following equations (Engle 2002),

$$r_{i,t} = \sqrt{h_{i,t}}\epsilon_{i,t}, \quad h_{i,t} = Var(r_{i,t}|\Omega_{t-1}), \quad i = 1, 2, \quad t = 1, 2, .., T \tag{20}$$

where $h_{i,t}$ stands for conditional variance of series i at time t and ϵ is $iid(0, \sigma^2)$ The conditional correlation between two return series r_1 and r_2 with zero means is:

$$\rho_{1,2,t} = \frac{Cov(r_{1,t}r_{2,t}|\Omega_{t-1})}{\sqrt{Var(r_{1,t}|\Omega_{t-1})Var(r_{2,t}|\Omega_{t-1})}} \tag{21}$$

We follow by plugging $r_{i,t} = \sqrt{h_{i,t}}\epsilon_{i,t}$ into Equation (21) and, after cancelling out conditional variances (which are known at time t), we get to:

$$\rho_{1,2,t} = \frac{Cov(\epsilon_{1,t}\epsilon_{2,t}|\Omega_{t-1})}{\sqrt{Var(\epsilon_{1,t}|\Omega_{t-1})Var(\epsilon_{2,t}|\Omega_{t-1})}} \tag{22}$$

Since variance of disturbance term is one, our denominator is equal to one and hence we end up with Equation (23), which says that the correlation between two time series is equal to covariance of its standardized residuals given the process described in Equation (20).

$$\rho_{1,2,t} = Cov(\epsilon_{1,t}\epsilon_{2,t}|\Omega_{t-1}) \tag{23}$$

The main advantage of DCC model over other models such as BEKK is its relatively simple estimation and less parameters. For purpose of deriving DCC model we first introduce the Constant Conditional Correlation model of Bollerslev (1990), in following equations,

$$H_t = D_t R D_t \quad D_t = diag\{\sqrt{h_{i,t}}\} \tag{24}$$

where H_t stands for conditional covariance matrix, R is a constant correlation matrix, i represents particular time series and D_t stands for $n \times n$ diagonal matrix. This model assumes constant correlation and thus approximates it with unconditional correlation estimate.

Equation (24) serves as a decomposition of covariance matrix, where in case of the DCC the time invariant R is replaced with time-varying correlation R_t (Engle 2002).

$$H_t = D_t R_t D_t \tag{25}$$

Under the DCC model of Engle (2002), the correlation is proxied with quasi-correlation (Q_t), which has to be rescaled before being plugged into R_t as otherwise it is not ensured that its value is between -1 and 1 (Engle 2009).

$$R_t = diag\{Q_t\}^{-1/2} Q_t diag\{Q_t\}^{-1/2} \tag{26}$$

We further assume that the process of quasi-correlation is mean reverting and thus can be described by following equation (Engle 2002),

$$Q_t = \bar{Q}(1 - \alpha - \beta) + \alpha(\epsilon_{t-1}\epsilon_{t-1}^{\intercal}) + \beta Q_{t-1} \tag{27}$$

where $\bar{Q}$ is a matrix of unconditional quasi-correlations ($T^{-1}\sum_{t=1}^{T}\epsilon_t\epsilon_t^{\intercal}$), α and β are parameters (scalars) to be estimated, ϵ_{t-1} is a vector of standardized residuals, Q_{t-1} is a quasi-correlation estimate at $t-1$. The parameters α and β drive the speed of mean reversion and Q_t is stationary and positive definite as long as $\alpha + \beta < 1$. The quasi-correlation estimate is obtained via correlation targeting (Engle 2009).

In order to find parameters of DCC, we first have to define the model, where we use the specification merged from Engle (2002) and Ghalanos (2015):

$$r_t|\Omega_{t-1} \sim N(0, H_t) \tag{28a}$$

$$H_t = D_t R_t D_t \tag{28b}$$

$$\epsilon_t = D_t^{-1} r_t \tag{28c}$$

$$D_t^2 = diag\{h_{i,t}\} \tag{28d}$$

$$Q_t = \bar{Q}(1 - \alpha - \beta) + \alpha(\epsilon_{t-1}\epsilon_{t-1}^\intercal) + \beta Q_{t-1} \tag{28e}$$

$$R_t = diag\{Q_t\}^{-1/2} Q_t diag\{Q_t\}^{-1/2} \tag{28f}$$

The parameter i stands for number of series, i.e. $i = 1, 2$ for two time series. The parameters of the model are estimated using maximum likelihood estimation (MLE) in three steps. In the first step the volatility process is described by univariate GARCH models (or any other GARCH family models, which are covariance stationary and assume normal distribution of errors (Engle and Shepard 2001) and in the next step the unconditional correlation is estimated. In the last phase the parameters driving the quasi-correlation are estimated via MLE and then the quasi-correlation is rescaled in order to obtain correlation between -1 and 1. The first equation of the model specification assumes normal distribution of the returns, hence we can maximize the joint probability of the multivariate normal distribution function as written below (Engle 2002):

$$r_t|\Omega_{t-1} \sim N(0, H_t) \tag{29a}$$

$$L = -\frac{1}{2}\sum_{t=1}^{T}(n\log(2\pi) + \log|H_t| + r_t^\intercal H_t^{-1} r_t) \tag{29b}$$

$$L = -\frac{1}{2}\sum_{t=1}^{T}(n\log(2\pi) + \log|D_t R_t D_t| + r_t^\intercal D_t^{-1} R_t^{-1} D_t^{-1} r_t) \tag{29c}$$

$$L = -\frac{1}{2}\sum_{t=1}^{T}(n\log(2\pi) + 2\log|D_t| + \log|R_t| + \epsilon_t^\intercal R_t^{-1}\epsilon_t) \tag{29d}$$

$$L = -\frac{1}{2}\sum_{t=1}^{T}(n\log(2\pi) + 2\log|D_t| + r_t^\intercal D_t^{-1} D_t^{-1} r_t - \epsilon_t^\intercal\epsilon_t + \log|R_t| + \epsilon_t^\intercal R_t^{-1}\epsilon_t) \tag{29e}$$

The function described above can be maximized with respect to its parameters, however in order to provide easier estimation, as Engle (2002) proposes the likelihood function can be split into volatility and correlation part,

$$L(\theta, \phi) = L_v(\theta) + L_c(\theta, \phi) \tag{30a}$$

$$L_v(\theta) = -\frac{1}{2}\sum_{t=1}^{T} n\log(2\pi) + 2\log|D_t| + r_t^{\intercal} D_t^{-1} D_t^{-1} r_t \tag{30b}$$

$$L_c(\hat{\theta}, \phi) = -\frac{1}{2}\sum_{t=1}^{T} -\epsilon_t^{\intercal}\epsilon_t + \log|R_t| + \epsilon_t^{\intercal} R_t^{-1}\epsilon_t \tag{30c}$$

where θ and ϕ are parameters of D and R, respectively. In the first step the volatility parameters are computed, so that in next step it could be used to obtain standardized residuals and perform estimation of dynamic conditional correlation parameters:

$$\hat{\theta} = \arg\max\{L_v(\theta)\} \tag{31a}$$

$$\max_{\phi}\{L_c(\hat{\theta}, \phi)\} \tag{31b}$$

The empirical analysis is conducted using the statistical software R with multiple packages developed for time series. For example, the parameters of quasi-correlation are estimated using the 'rmgarch' package of Alexander Ghalanos (2015) via correlation targeting.

2.7.2 Dynamic conditional correlation with asymmetric GARCH

The DCC model supports using various variance models for modeling volatility of the time series (for details, see section 2.7.1) and therefore we have decided to extend the analysis by employing GJR GARCH, which accounts for asymmetry in variance of a time series. In further analysis we denote this model as DCCA for simplicity.

2.7.3 Asymmetric dynamic conditional correlation

Although asymmetry in volatility has been widely studied, there has been given a little attention to asymmetry in covariances and particularly in correlations. This was the initial trigger of Cappiello, Engle and Sheppard (2006), who generalized DCC model of Engle (2002).

The economic reasoning behind the asymmetry in correlations lays in time-varying risk premium (Cappiello, Engle and Sheppard 2006). Under assumption of CAPM world the negative shocks lead to an increase in volatility of the assets and thus the covariance has to be adjusted in order to keep the beta invariant.

The asymmetric dynamic conditional correlation model is described by the following equations,

$$r_t|\Omega_{t-1} \sim N(0, H_t) \tag{32a}$$

$$H_t = D_t R_t D_t \tag{32b}$$

$$\epsilon_t = D_t^{-1} r_t \tag{32c}$$

$$D_t^2 = diag\{h_{i,t}\} \tag{32d}$$

$$Q_t = (\bar{Q} - \alpha\bar{Q} - \beta\bar{Q} - \gamma\bar{N}) + \alpha\epsilon_{t-1}\epsilon_{t-1}^\intercal + \gamma n_{t-1}n_{t-1}^\intercal + \beta Q_{t-1} \tag{32e}$$

$$R_t = diag\{Q_t\}^{-1/2} Q_t diag\{Q_t\}^{-1/2} \tag{32f}$$

where up to the third equation the specification remains the same as in case of DCC. The only modification is in Equation 4, where parameter γ is added. The $\bar{Q}$ stands for unconditional correlation and $\bar{N}$ equals $T^{-1}\sum_{t=1}^{T} n_t n_t^\intercal$, where $n_t = I[\epsilon_t < 0] \circ \epsilon_t$ ($\circ$ stands for Hadamard product i.e., element-wise product). The necessary and sufficient condition for covariance matrix to be positive and semi-definite can be found in Cappiello, Engle and Sheppard (2006).

2.8 Hedging performance measures

The models described above have to be properly compared in order to decide, which of them leads to greatest risk reduction. For this purpose we employ two measures employed in similar articles, those namely are Hedging effectiveness index and the Value at Risk.

2.8.1 Hedging effectiveness

Ku, Chen and Chen (2007) suggest that volatility models should be compared by variance and thus employs the hedging effectiveness ratio of Ederington (1979), which is also used in paper written by Chang, McAleer and Tansuchat (2011). The hedging effectiveness index is described in following equation,

$$HE = \frac{\sigma^2_{unhedged} - \sigma^2_{hedged}}{\sigma^2_{unhedged}} \tag{33}$$

where $\sigma^2_{unhedged}$ and σ^2_{hedged} stand for variance of unhedged portfolio and variance of hedged portfolio, respectively. The index therefore says how many percent of unhedged portfolio variance are reduced by using the hedge.

2.8.2 Value at Risk

As another measure of portfolio risk, we employ the Value at Risk (VaR), which was developed by J. P. Morgan as part of the RiskMetrics$^{\text{TM}}$. VaR as a concept lays in a quantile function of a distribution, which can be formally written as (Eydeland and Wolyniec 2003),

$$Pr(\Delta P \leq -VaR) = -\alpha \tag{34}$$

where ΔP stands for change in portfolio value and α for given quantile of the distribution.

The usage of VaR is suggested also by Engle (2002) for comparing hedging performance of different models. Despite Engle used parametric VaR with underlying assumption of normality of returns, we employ the historical VaR. The main reason for this is that it describes the actually observed data and, although it is backward looking, it is not our ambition to predict the portfolio risk, but rather evaluate observed risk.

For a given period, the changes in portfolio value, which are determined in Equation (6a), are calculated. Thus a new distribution of portfolio returns is obtained. The distribution is than ordered from smallest to highest portfolio change so that the cumulative probability density function can be created. Then the corresponding quantile of the empirical cumulative distribution is multiplied by initial portfolio value and daily Value at Risk is computed (for details, see Hull 2012).

$$PortfolioValue \times VaR^T_{1-\alpha} \tag{35}$$

T and α stand for period and level of significance, respectively. For purpose of our analysis we use the $VaR^{1D}_{0.95}$, i.e. daily VaR on 5% level of significance. VaR as a measure of risk provides an answer to the question how big the loss can at maximum be in selected time period with given probability.

3 Data, its descriptives and sources

In empirical part of the thesis there are three commodities investigated including both futures and spot time series. Those commodities namely are corn, gold and oil. All of the data come from quandl.com, except for oil spot data, which were obtained from eia.gov (website of US Energy Information Administration). The oil data correspond to US crude oil benchmark Western Texas Intermediate (WTI).

Futures data are not continuous in nature, since there are many contracts with different maturities. However, for purpose of the analysis a single time series is needed, therefore, an artificial time series had to be created. There are two key elements of continuous futures construction (Quandl Inc. 2017) and those namely are the date of rolling and adjustment made to prices (for details, see for instance Masteika, Rutkauskas and Alexander 2012).

For the purpose of the analysis we were forced to construct the futures time series on our own, as we were not able to find it for free. We employed the recommended approach for economic forecasting and regression (Quandl Inc. 2017) in order to perceive economic properties of the series. This method uses a first-day-of-the-month as the date of rolling and the calendar-weighted rolling as the price adjustment. The technique relies on assumption that the portfolio of futures contracts is rolled in last 4 days (each day 20%) of the month before maturing month. The remaining 20% are rolled on the first day of the month, when the contract expires. The price adjustment in this case is simply represented by weighted average of prices of 2 consecutive contracts, where weights are the fractions of contracts held in each contract month.

In Table 1 a simple characteristic of futures contracts can be found. With respect to the spot prices, all the data come from the USA and specific location can be found (if available) in Table 2.

Table 1: Futures specification

Commodity	Exchange	Contract Months	Contract Size
Corn	CBOT	Mar, May, Jul, Sep, Dec	5000 bushels
Gold	COMEX	Jan-Dec	100 troy ounces
WTI	NYMEX	Jan-Dec	1000 barrels

Source: http://www.cmegroup.com/

Table 2: Spot specification

Commodity	Location
Corn	Central Illinois, USA
Gold	USA
WTI	Cushing, Oklahoma, USA

Source: Own creation

The time series dates from 4th December 2000 to 1st September 2016, which corresponds to almost 4000 observation (exact values for each series can be found in Table 4). In order to provide sound solution for hedging effectiveness, we have decided to also provide out-of-sample testing, therefore, the original dataset was split by rule of thumb – 80% of observations was used for model development (training set), while the remaining 20% served as a testing set:

Table 3: Number of observations in training and testing set

	Training set	Testing set
Corn	3144	787
Gold	3162	791
WTI	3161	791

Source: Own calculations

We follow by providing summary descriptives of log returns of all inspected commodities in Table 4. The means of all time series are positive, but close to zero, which is as expected. The highest mean was observed in case of gold and the smallest, on the contrary, in case of oil and corn. Standard deviations indicate that the most dispersed returns around the mean are those of WTI spot, where the standard deviation reached 0.025, which is roughly double the standard deviation of gold (0.012). Higher moments indicate that none of these series is normal, for instance, all of the series have negative skew except for corn futures, where it is slightly positive. The highest kurtosis has been identified in case of gold spot time series with value around 8 suggesting fat tails of the distribution. Corn spot and futures time series on the contrary have the smallest kurtosis, with values around 5.37 and 5.06, respectively. Histogram of each distribution and its comparison with Gaussian distribution can be found in appendix.

Table 4: Description of returns

	Obs	Mean	SD	Min	Max	Skewness	Kurtosis
Corn Futures	3,931	0.000	0.018	-0.079	0.090	0.001	5.008
Corn Spot	3,931	0.000	0.019	-0.124	0.093	-0.134	5.376
Gold Futures	3,953	0.000	0.012	-0.098	0.086	-0.328	7.988
Gold Spot	3,953	0.000	0.012	-0.096	0.068	-0.282	8.100
WTI Futures	3,952	0.000	0.023	-0.165	0.148	-0.156	6.222
WTI Spot	3,952	0.000	0.025	-0.171	0.164	-0.109	7.483

Source: Own calculations

In Table 5 we provide the unconditional correlation between spot and futures log returns of each of the inspected time series. We clearly see that the relationship between spot and futures returns of gold is weaker than in case of other commodities.

Table 5: Correlations between spot and futures

	Corn	Gold	WTI
Log differences	0.9487	0.6966	0.9159

Source: Own calculations

4 Empirical results

Within this section, we present the results of asymmetry and hedging performance of above specified models on corn, gold and oil (WTI) in the same order.

As described in methodology section, we start the time series analysis with formally testing the stationarity using the augmented Dickey-Fuller (ADF) test with lag equal to 15. Based on the ADF test we conclude that all of the inspected commodity prices are integrated of order one. We provide the ADF test statistics in following table:

Table 6: Augmented Dickey-Fuller test statistics results

	Corn Spot	Corn Futures	Gold Spot
Levels	-1.792	-1.902	-1.300
Log returns	-14.711*	-14.782*	-15.982*

	Gold Futures	WTI Spot	WTI Futures
Levels	-1.355	-1.547	-1.487
Log returns	-16.003*	-14.727*	-14.390*

Source: Own calculations; *The p-value is less than 1%

We follow by looking at mean processes of the time series, where we conclude that of all the investigated time series are found to be noise and thus the specification in Equation (32c) remains valid.

4.1 Corn

4.1.1 Corn volatility process

The first commodity we take a look at is corn, where we start by describing the volatility process of futures and spot returns. The Ljung-Box test unveils significant linear dependence in squared log returns and thus suggests that the GARCH family models may be suitable for volatility modeling.

Although selection of the model based on BIC leads to GARCH(5,5) for both spot and futures, we rather employ GARCH(1,1), which sufficiently describes the volatility process while remaining parsimonious (Brooks 2008).The processes are mainly driven by lagged volatility estimates as β_1^F and β_1^S reach values 0.931 and 0.936, respectively. All the parameters, except for intercepts, are significant. The parameter estimates are compliant with stationarity condition as well as non-negativity constraint. Precise parameter estimates and standard errors can be found in Table 7.

In the next step we estimate the parameters of the GJR GARCH(1,1,1), which is subsequently used as an input to the DCCA and ADCC models. The asymmetric terms of GJR GARCH models (γ_1^F and γ_1^S) have not been found to be significant and hence we conclude that there is no asymmetric response to negative errors in volatility, which holds for futures and for spot as well. Again the volatility processes are mainly driven by autoregressive effect as β_1^F and β_1^S are 0.930 and 0.934, respectively. The impact of lagged squared residuals is, on the contrary, minor when parameters α_1^F and α_1^S are only slightly above zero. Both of the volatility models satisfy the stationarity and non-negativity constraint. The sum of the parameter estimates α_1^F, β_1^F and $\frac{1}{2}\gamma_1^F$ is below unity, which holds for parameters corresponding to spot as well. Constraints described in Section (2.5.2) are satisfied as well. All the parameter estimates and corresponding standard errors can be found in Table 8. The comparison of estimated volatility and observed volatility (conditional standard deviations vs absolute returns) can be found in plots in appendix.

4.1.2 Dynamic conditional correlation models

In order to obtain the hedge ratio (see Equation (8)), the DCC models are estimated. For all of the DCC based models we employ multivariate GARCH(1,1).

The first estimated model is the DCC model of Engle (2002), where we find the parameter driving impact of lagged residuals ($\alpha_1^{F,S}$) to be significant with value 0.064. The parameter $\beta_1^{F,S}$ suggests that the process of correlation is mainly driven by past values of quasi-correlation as the $\beta_1^{F,S}$ is 0.917 and significant. Note that the stationarity condition is satisfied. Precise values of the model parameters and corresponding standard errors can be found in the following table:

Table 7: DCC parameters - Corn

	Estimate	Std. Error	t value	Pr(>|t|)
α_0^F	0.000	0.000	1.002	0.317
α_1^F	0.058	0.015	4.016	0.000
β_1^F	0.931	0.020	45.586	0.000
α_0^S	0.000	0.000	0.880	0.379
α_1^S	0.053	0.012	4.608	0.000
β_1^S	0.936	0.019	50.210	0.000
$\alpha_1^{F,S}$	0.064	0.012	5.561	0.000
$\beta_1^{F,S}$	0.917	0.016	55.809	0.000

Source: Own calculations

As the next model we present the results of DCCA in Table 8, where as already noted in section 4.1.1 there is no asymmetry in the second moment. The parameters $\alpha_1^{F,S}$ and $\beta_1^{F,S}$ remain similar as in case of DCC model.

Table 8: DCCA parameters - Corn

	Estimate	Std. Error	t value	Pr(>|t|)
α_0^F	0.000	0.000	0.924	0.356
α_1^F	0.051	0.012	4.224	0.000
β_1^F	0.930	0.020	47.617	0.000
γ_1^F	0.016	0.016	1.000	0.317
α_0^S	0.000	0.000	0.507	0.612
α_1^S	0.044	0.011	3.974	0.000
β_1^S	0.934	0.025	36.852	0.000
γ_1^S	0.022	0.022	1.020	0.308
$\alpha_0^{F,S}$	0.064	0.011	5.664	0.000
$\beta_1^{F,S}$	0.917	0.016	56.278	0.000

Source: Own calculations

As the last of the models applied to estimating covariance of corn spot and futures, we provide the results of asymmetric dynamic conditional correlation model in Table 9. The p-values (Pr) of γ_1^F, γ_1^S and $\gamma_1^{F,S}$ suggest that there are no significant asymmetries in conditional variances nor in conditional correlation.

Table 9: ADCC parameters - Corn

	Estimate	Std. Error	t value	Pr(>|t|)
α_0^F	0.000	0.000	0.908	0.364
α_1^F	0.051	0.012	4.195	0.000
β_1^F	0.930	0.020	47.407	0.000
γ_1^F	0.016	0.017	0.968	0.333
α_0^S	0.000	0.000	0.508	0.612
α_1^S	0.044	0.011	3.939	0.000
β_1^S	0.934	0.026	36.524	0.000
γ_1^S	0.022	0.021	1.035	0.301
$\alpha_1^{F,S}$	0.064	0.016	4.048	0.000
$\beta_1^{F,S}$	0.917	0.017	53.054	0.000
$\gamma_1^{F,S}$	0.000	0.036	0.000	1.000

Source: Own calculations

Although the best model could be selected using information criterion, we rather compare the models by its hedging performance both in-sample and out-of-sample as it is our main goal to compare models in terms of their hedging potential (Chang, McAleer and Tansuchat 2011).

4.1.3 In-sample hedging performance

As a benchmark for in-sample hedging performance we have decided to use unconditional hedge ratio (denoted as OLS) (see Equation (7) for details). For model comparison we employ the Hedging effectiveness index (HE index) and daily Value at Risk (VaR) with underlying assumption of initial capital equal to $1000,000$ USD, which is an arbitrarily chosen value. The results of hedging performance are presented in the following table:

Table 10: In-sample hedging performance - Corn

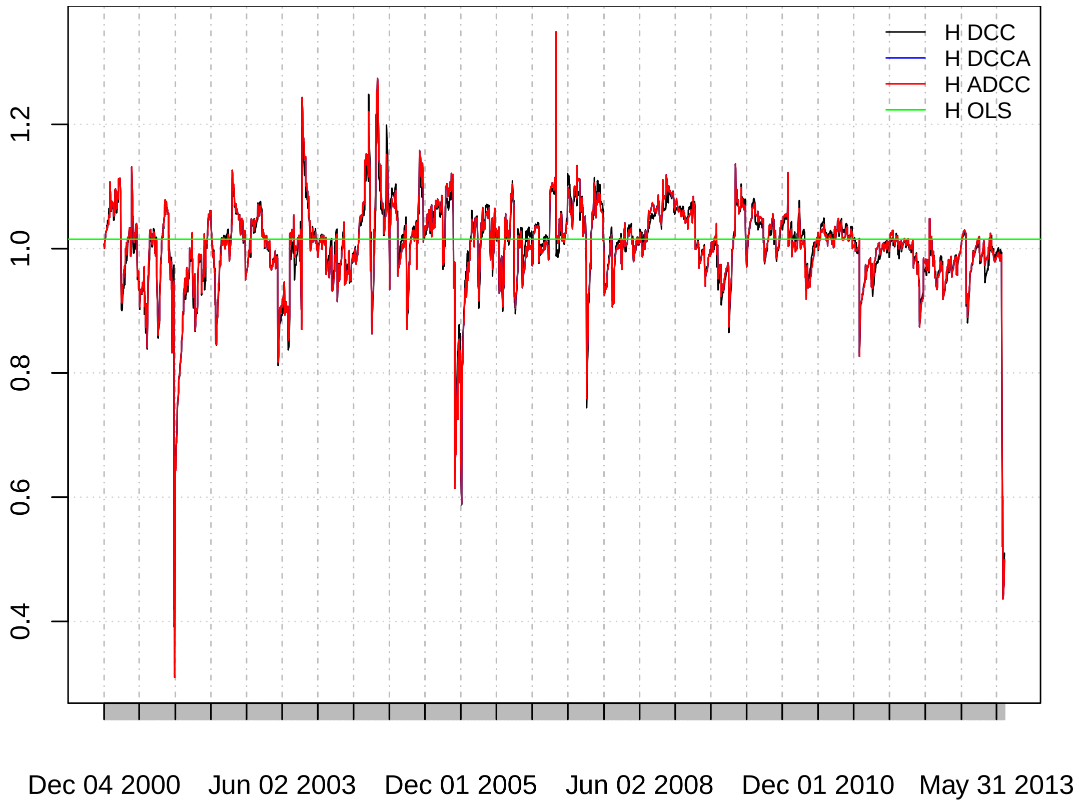
	OLS	DCC	ADCC	DCCA
HE index	91.917%	92.015%	92.013%	92.013%
VaR	$8,233.789$	$8,045.546$	$8,085.119$	$8,085.119$

Source: Own calculations

Based on the HE index we conclude that the hedge ratio based on the DCC model leads to the highest variance reduction as more than 92% of the spot variance is reduced. The differences among HE indexes of all the models are however tiny. In terms of Value at Risk, the DDC based hedge ratio outperforms the rest of the models with value around 2.5% less than the OLS benchmark.

Figure 1 describes the in-sample development of hedge ratio, where H DCC, H DCCA, H ADCC, H OLS denote hedge ratio based on the DCC model, on the DCCA model, on the ADCC model and the unconditional hedge ratio, respectively. It can be observed how the conditional hedge ratio changes in time from values below 1 to values above 1 and returns to its unconditional value. It is also worth to note that there are only minor differences between each of the DCC based models.

Figure 1: Corn in-sample hedge ratio estimates



Source: Own calculations

4.1.4 Out-of-sample hedging performance

For out-of-sample testing we employ the rolling estimation of parameters with moving window of 5 days. We have selected 5 days as it corresponds to a business week and the estimation still remains feasible in terms of computing capacity. The assumptions of VaR presented in previous section are valid for out-of-sample testing as well. The models, where rolling was applied are denoted as 'model roll' (for instance, OLS roll is the unconditional hedge ratio estimate re-estimated every fifth day). The out-of-sample results are presented in the following table:

Table 11: Out-of-sample hedging performance - Corn

	OLS	DCC	ADCC	DCCA
HE index	81.375%	80.870%	80.858%	80.858%
VaR	$6,662.091$	$6,555.777$	$6,208.403$	$6,208.403$

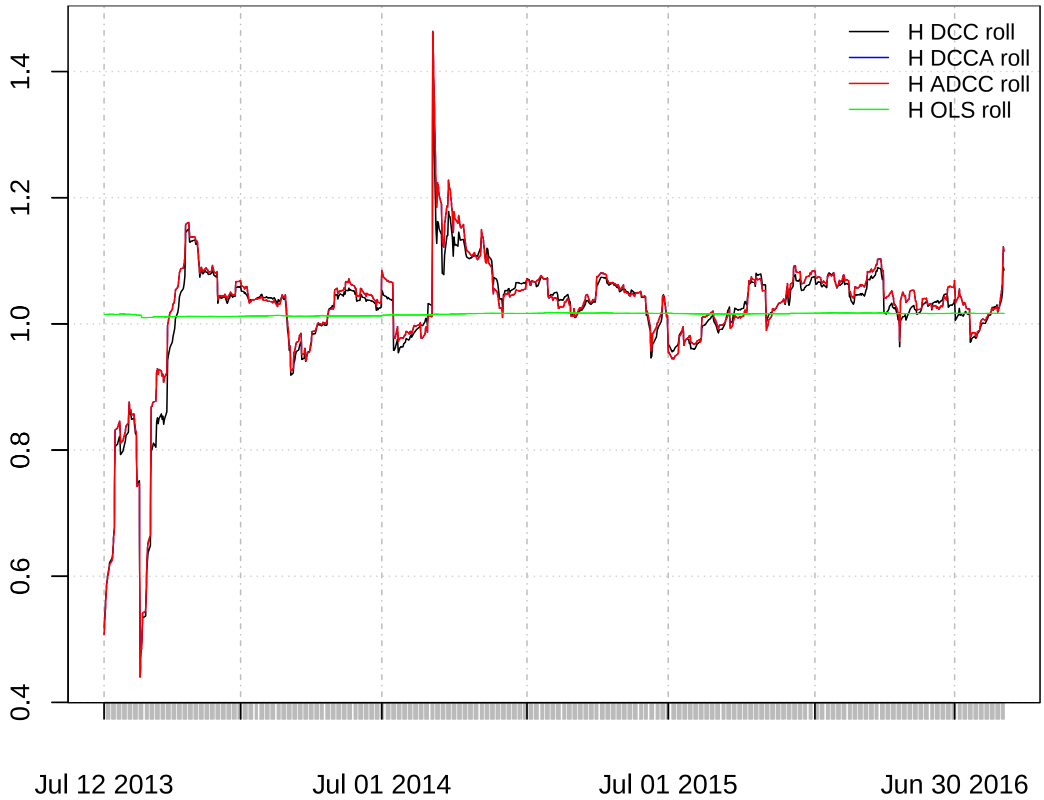
	OLS roll	DCC roll	ADCC roll	DCCA roll
HE index	81.370%	80.839%	80.779%	80.779%
VaR	$6,640.860$	$6,577.727$	$6,198.549$	$6,198.549$

Source: Own calculations

The results suggest that DCC family models are not able to provide higher variance reduction than simple OLS models as the highest variance reduction is obtained with OLS roll model (81.370%). Note that the differences in the HE indices are again only marginal. The highest reduction within out-of-sample testing is around 10% less than during the in-sample testing, but this is as expected since models usually perform worse on testing sets than on development sets. In terms of VaR the DCCA (also ADCC) roll model outperforms other models as it leads to value which is around 6.5% less than in case of OLS rolling benchmark. The out-of-sample VaR is smaller than in-sample VaR for all the models, which may possibly be result of less extreme events within the testing sample and shorter period.

In Figure 2 the development of hedge ratios based on rolling models is shown (Plot of hedge ratios based on unrolled models can be found in appendix). H DCC roll, H DCCA roll, H ADCC roll and H OLS roll represent rolling estimation of the hedge ratio based on the DCC model, on the DCCA model, on the ADCC model and the unconditional hedge ratio, respectively. All of the hedge ratios estimated by the DCC based models sort of co-move with only minor differences within a range from 0.5 to 1.4. Rolling OLS hedge ratio, on the contrary, enjoys only small changes compared to the DCC based models.

Figure 2: Corn out-of-sample hedge ratio estimates of rolling models



Source: Own calculations

4.2 Gold

4.2.1 Volatility

We follow by analyzing the gold, where we again start by describing its volatility process. Both of the time series exhibit significant autocorrelation in squared residuals and hence the volatility modeling is feasible. For the same reasons as in case of corn we employ GARCH(1,1). We have found the α_1^F parameter to be insignificant, which is mainly result of a large standard error. The rest of the parameters remains significant (except for intercepts), while all corresponding conditions are satisfied for both futures and spot models. Results can be found in Table 12.

The similar properties as described above also hold for the asymmetric model, where GJR GARCH(1,1,1) is chosen for both of the time series. The parameters driving the asymmetry (γ_1^F and γ_1^S) in conditional volatility are not found to be statistically significant. The condition imposed on γ_1^F is satisfied as the sum of α_1^F and γ_1^F is less than 1. The same also holds for γ_1^S. Results of GJR GARCH models for futures and spot time series are presented in Table 13.

4.2.2 Dynamic Conditional Correlation models

In next step we estimate the unconditional hedge ratio and subsequently the DCC model, where we find parameter $\beta_1^{F,S}$ not to be significant, which suggests that the DCC model may not be able to capture the time-varying correlation. The parameter $\alpha_1^{F,S}$, on the contrary, remains significant. The DCC parameter estimates and corresponding standard errors can be seen in the following table:

Table 12: DCC parameters - Gold

	Estimate	Std. Error	t value	Pr(>|t|)
α_0^F	0.000	0.000	0.446	0.656
α_1^F	0.047	0.040	1.181	0.238
β_1^F	0.940	0.047	20.191	0.000
α_0^S	0.000	0.000	1.213	0.225
α_1^S	0.057	0.017	3.283	0.001
β_1^S	0.926	0.019	48.178	0.000
$\alpha_1^{F,S}$	0.075	0.018	4.157	0.000
$\beta_1^{F,S}$	0.275	0.226	1.215	0.224

Source: Own calculations

In next step we estimate the DCCA model, where similar properties as above were investigated. The parameter $\beta_1^{F,S}$ still remains insignificant.

Table 13: DCCA parameters - Gold

	Estimate	Std. Error	t value	Pr(>|t|)
α_0^F	0.000	0.000	0.134	0.894
α_1^F	0.054	0.131	0.413	0.680
β_1^F	0.941	0.137	6.875	0.000
γ_1^F	-0.012	0.028	-0.437	0.662
α_0^S	0.000	0.000	0.961	0.337
α_1^S	0.062	0.017	3.561	0.000
β_1^S	0.927	0.024	39.284	0.000
γ_1^S	-0.010	0.024	-0.423	0.672
$\alpha_1^{F,S}$	0.074	0.018	4.027	0.000
$\beta_1^{F,S}$	0.276	0.229	1.202	0.229

Source: Own calculations

As the last model employed for spot and futures correlation of gold, we present the results of the asymmetric dynamic conditional correlation model, where we conclude, that there is no asymmetry in correlation ($\gamma_1^{F,S}$).

Table 14: ADCC parameters - Gold

	Estimate	Std. Error	t value	Pr(>|t|)
α_0^F	0.000	0.000	0.134	0.894
α_1^F	0.054	0.131	0.412	0.680
β_1^F	0.941	0.137	6.873	0.000
γ_1^F	-0.012	0.028	-0.437	0.662
α_0^S	0.000	0.000	0.951	0.342
α_1^S	0.062	0.017	3.553	0.000
β_1^S	0.927	0.024	39.362	0.000
γ_1^S	-0.010	0.024	-0.430	0.668
$\alpha_1^{F,S}$	0.074	0.034	2.209	0.027
$\beta_1^{F,S}$	0.276	0.435	0.634	0.526
$\gamma_1^{F,S}$	0.000	0.086	0.000	1.000

Source: Own calculations

4.2.3 In-sample hedging performance

Hedging performance results on the development sample indicate that the DCC family models are outperformed by the unconditional hedge ratio in terms of both the HE index and the VaR. The unconditional OLS model leads to highest risk reduction (around 45% of unhedged position) and also contributes to lower VaR by around 0.5% compared to the ADCC and DCCA models. The poor performance of dynamic conditional correlation based models may be caused by comparatively smaller correlation, which is in case of gold only around 70%, while both corn and WTI have correlation between spot and futures above 90%.

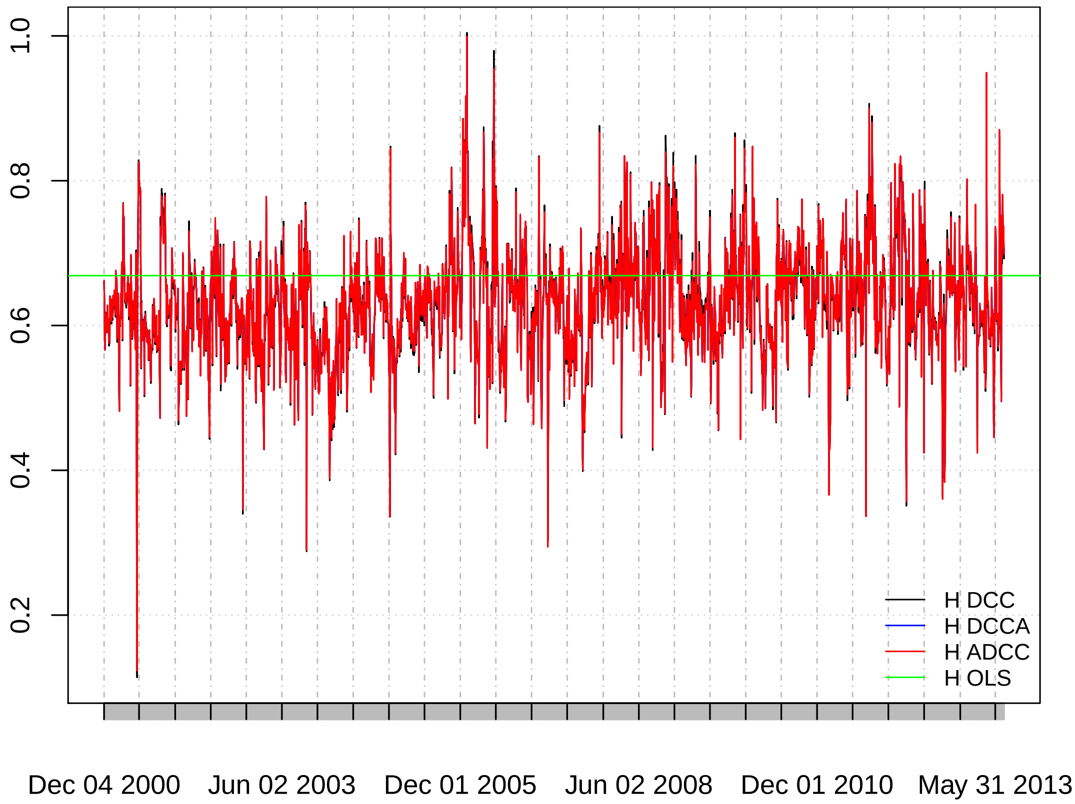
Table 15: In-sample hedging performance - Gold

	OLS	DCC	ADCC	DCCA
HE index	45.801%	44.624	44.602%	44.602%
VaR	$13,512.990$	$13,647.480$	$13,596.340$	$13,596.340$

Source: Own calculations

As the model parameters suggest, there are only minor differences among the DCC family models, which is clearly seen in Figure 3. The hedge ratio is on average slightly smaller compared to corn as the values range between 0.1 and 1. Note that the hedge ratio development appears to be quite noisy compared to the one obtained in case of corn.

Figure 3: Gold in-sample hedge ratio estimates



Source: Own calculations

4.2.4 Out-of-sample hedging performance

The highest risk reduction is obtained with the ADCC model, which performs a bit better than the rolling OLS. In terms of VaR, the smallest value is obtained when employing the unconditional hedge ratio estimated on development sample, but the difference is less than 1% compared to the DCC model (the best out of the DCC based models). One can notice higher values of the HE index, which are around one half larger than within the sample.

Table 16: Out-of-sample hedging performance - Gold

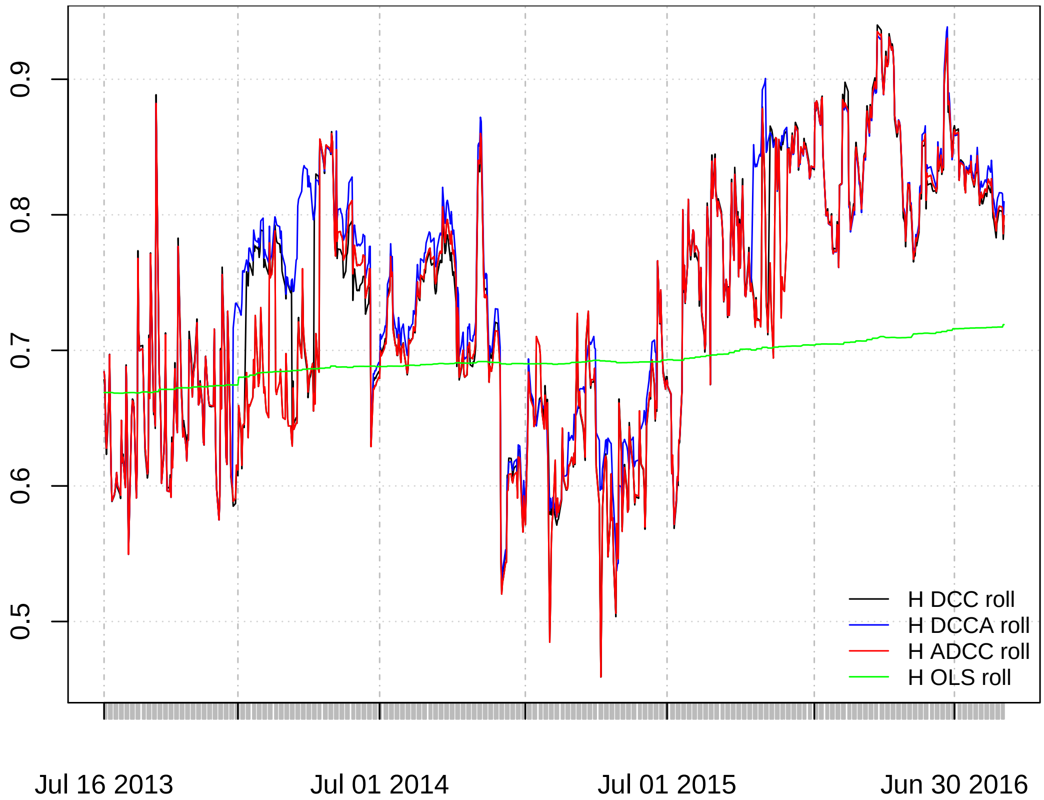
	OLS	DCC	ADCC	DCCA
HE index	64.110%	63.534%	63.525%	63.525%
VaR	$9,151.221$	$9,223.591$	$9,311.795$	$9,311.742$

	OLS roll	DCC roll	ADCC roll	DCCA roll
HE index	64.636%	64.437%	64.657%	64.284%
VaR	$9,311.692$	$9,562.350$	$9,397.357$	$9,549.050$

Source: Own calculations

In Figure 4 we provide the development of rolling hedge ratio estimates, where it can be seen how the ratio changes from approximately 0.4 to 1. There are only minor differences among the DCC family models and the series seems to be quite similar (in terms of noisiness) to the one obtained in case of corn. Note how the unconditional rolled correlation estimate is growing through time, which was not observed in case of corn. The development of rolling hedge ratios can be found in the following figure:

Figure 4: Gold out-of-sample hedge ratio rolling estimates



Source: Own calculations

4.3 Western Texas Intermediate

4.3.1 Volatility process

The analysis of WTI is started by looking at squared returns, which indicate a strong evidence of volatility clustering and therefore we again employ the GARCH family models in order to describe the volatility process. Namely, we use the univariate GARCH(1,1) and GJR GARCH(1,1,1), which both sufficiently describe the second moment of the series. All the parameters in both of the model types are significant except for α_0^F and α_0^S in the GARCH model. The significance of γ_1^F and γ_1^S confirms the asymmetric response of volatility to negative past errors. For all the parameter estimates and corresponding standard errors, see Table 17 and Table 18 for GARCH and GJR GARCH, respectively.

4.3.2 Dynamic conditional correlation models

In the next step we estimate the DCC model, where we find the parameter driving impact of lagged standardized residuals ($\alpha_1^{F,S}$) to be insignificant. Exact values of the model parameters can be found in the following table:

Table 17: DCC parameters - WTI

	Estimate	Std. Error	t value	Pr(>|t|)
α_0^F	0.000	0.000	0.631	0.528
α_1^F	0.058	0.005	12.572	0.000
β_1^F	0.928	0.018	50.990	0.000
α_0^S	0.000	0.000	1.301	0.193
α_1^S	0.067	0.013	4.993	0.000
β_1^S	0.916	0.007	126.079	0.000
$\alpha_1^{F,S}$	0.099	0.061	1.615	0.106
$\beta_1^{F,S}$	0.892	0.077	11.557	0.000

Source: Own calculations

We follow by estimating the DCCA model, where asymmetric parameters of volatility are found to be significant as noted in section 4.3.1. Again the parameter $\alpha_1^{F,S}$ is found to be insignificant.

Table 18: DCCA parameters - WTI

	Estimate	Std. Error	t value	Pr(>|t|)
α_0^F	0.000	0.000	4.701	0.000
α_1^F	0.025	0.008	3.214	0.001
β_1^F	0.924	0.007	142.090	0.000
γ_1^F	0.065	0.019	3.476	0.001
α_0^S	0.000	0.000	6.270	0.000
α_1^S	0.035	0.009	3.791	0.000
β_1^S	0.909	0.002	442.699	0.000
γ_1^S	0.067	0.026	2.547	0.011
$\alpha_1^{F,S}$	0.103	0.056	1.857	0.063
$\beta_1^{F,S}$	0.888	0.071	12.529	0.000

Source: Own calculations

As the last model we estimate the ADCC model, where significant asymmetry in correlation is not found ($\gamma_1^{F,S}$). The parameter estimates can be found in the following table:

Table 19: ADCC parameters - WTI

	Estimate	Std. Error	t value	Pr(>|t|)
α_0^F	0.000	0.000	4.699	0.000
α_1^F	0.025	0.008	3.258	0.001
β_1^F	0.924	0.007	138.430	0.000
γ_1^F	0.065	0.018	3.526	0.000
α_0^S	0.000	0.000	6.065	0.000
α_1^S	0.035	0.009	4.111	0.000
β_1^S	0.909	0.003	353.872	0.000
γ_1^S	0.067	0.024	2.788	0.005
$\alpha_1^{F,S}$	0.092	0.040	2.279	0.023
$\beta_1^{F,S}$	0.876	0.081	10.796	0.000
$\gamma_1^{F,S}$	0.041	0.053	0.769	0.442

Source: Own calculations

4.3.3 In-sample hedging performance

The OLS model outperforms the DCC family models in terms of both HE index and VaR, where reaches values 82.234% and 9800.760, respectively. The unconditional hedge ratio is able to reduce the VaR by approximately 4% more than hedge ratio estimated by the ADCC model.

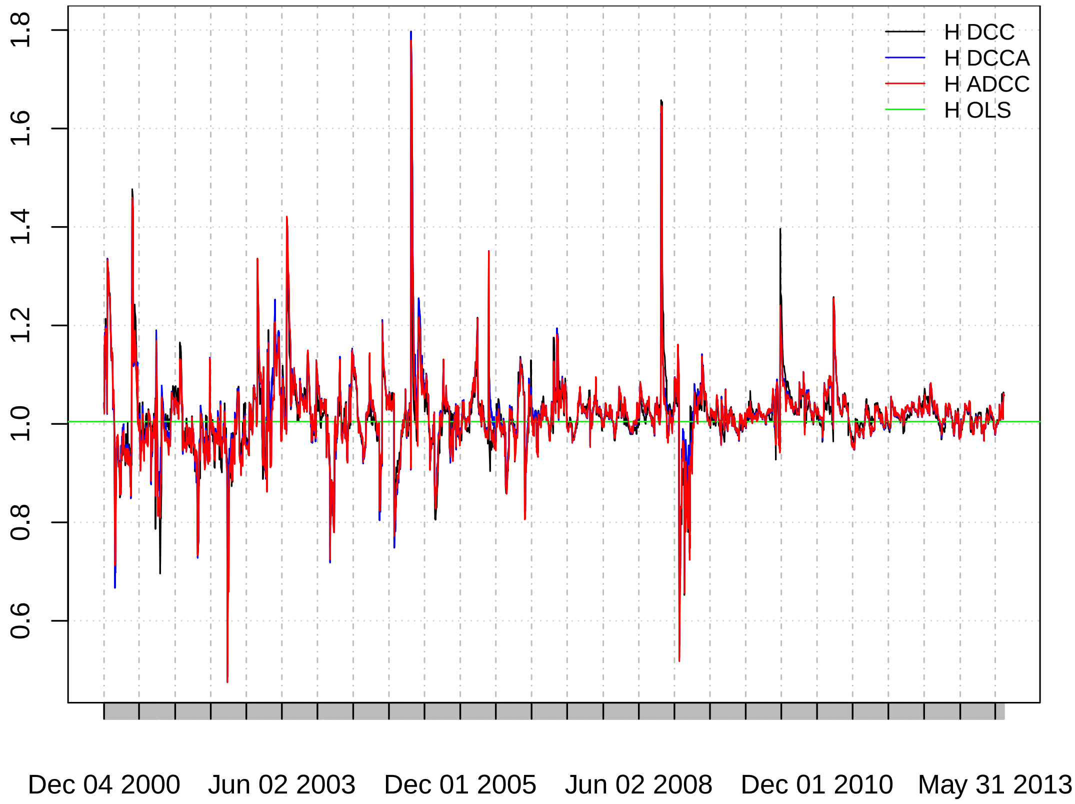
Table 20: In-sample hedging performance - WTI

	OLS	DCC	ADCC	DCCA
HE index	82.234%	81.111%	81.119%	81.047%
VaR	$9,800.760$	$10,249.160$	$10,097.720$	$10,235.500$

Source: Own calculations

The hedge ratio estimates range from 0.5 to 1.8, which can be seen in Figure 5.

Figure 5: WTI in-sample hedge ratio estimates



Source: Own calculations

4.3.4 Out-of-sample hedging performance

In terms of out-of-sample performance the unconditional OLS delivers the largest risk reduction and also leads to the smallest VaR, which is around 7% less than the VaR of the hedge ratio based on the DCC model.

Table 21: Out-of-sample hedging performance - WTI

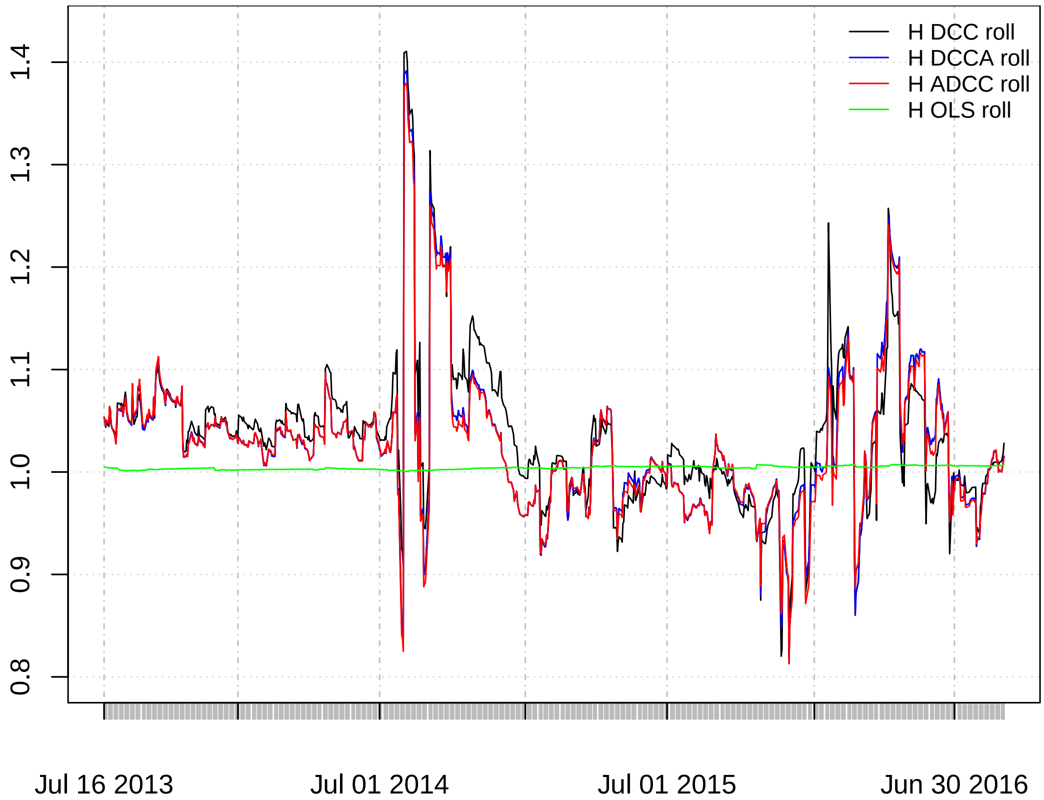
	OLS	DCC	ADCC	DCCA
HE index	90.213%	89.602%	89.761%	89.735%
VaR	7,777.112	8,382.802	8,498.027	8,566.046

	OLS roll	DCC roll	ADCC roll	DCCA roll
HE index	90.207%	89.708%	89.790%	89.768%
VaR	7,782.419	8,557.701	8,494.863	8,466.193

Source: Own calculations

The development of out-of-sample hedge ratios can be found in the following figure:

Figure 6: WTI Out-of-sample hedge ratio rolling estimates



Source: Own calculations

Conclusion

This thesis has investigated existence of asymmetries in volatility and correlation and their impact on hedge ratio estimation and hedging effectiveness of corn, gold and crude oil.

Regarding the results of asymmetry, we have investigated that there is no asymmetric response to volatility in case of corn and gold. The same does not hold for crude oil, where significant asymmetry in the variances of both spot and futures returns exists. The asymmetry in correlation has not been found to be significant in neither of the time series.

With respect to the hedging performance, we conclude that the variance reduction does not greatly differ among employed models and thus the DCC family models may not necessarily be superior to OLS, which is consistent with Moosa (2003), Myers (1991) or Chang, McAleer and Tansuchat (2011), who came to similar results. The Value at Risk results, on the contrary, suggest that DCC based models, which account for asymmetry, may notably reduce the Value at Risk as has been observed in case of corn. The same however has been rejected for gold and crude oil, where the OLS hedge ratio outperformed the more complicated DCC family models.

Within this thesis the obtained distributions of portfolios have been studied by HE index and VaR, thus a potential further research could focus on studying these distributions more profoundly and possibly provide explanation why the DCC models did not lead to higher risk reduction than the OLS based hedge ratios in general. Research could also be extended by taking into account transaction costs.

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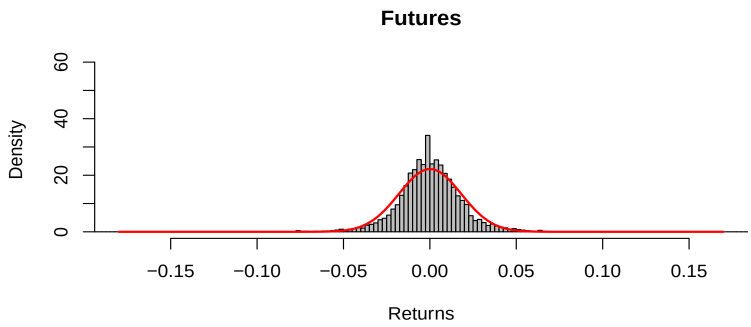
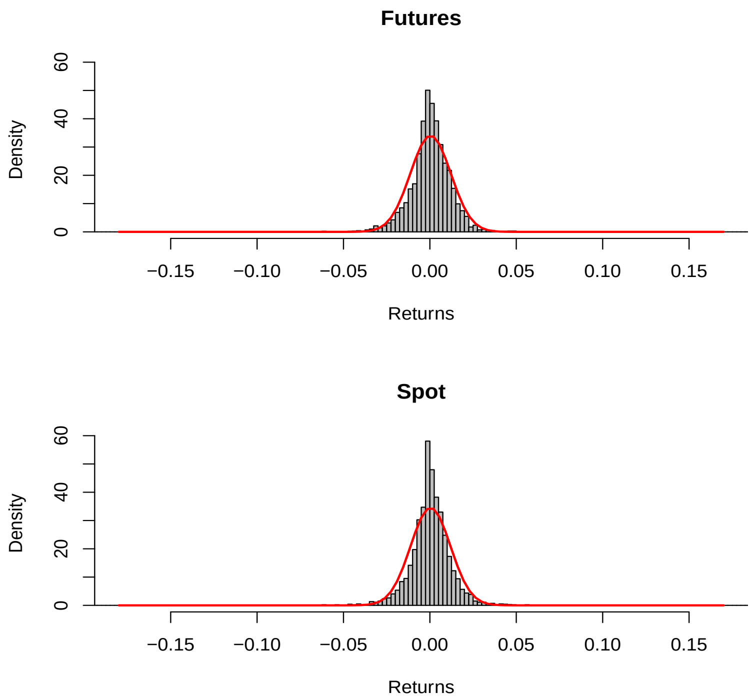
Appendix A

Figure A.1: Corn histogram of returns vs normal distribution



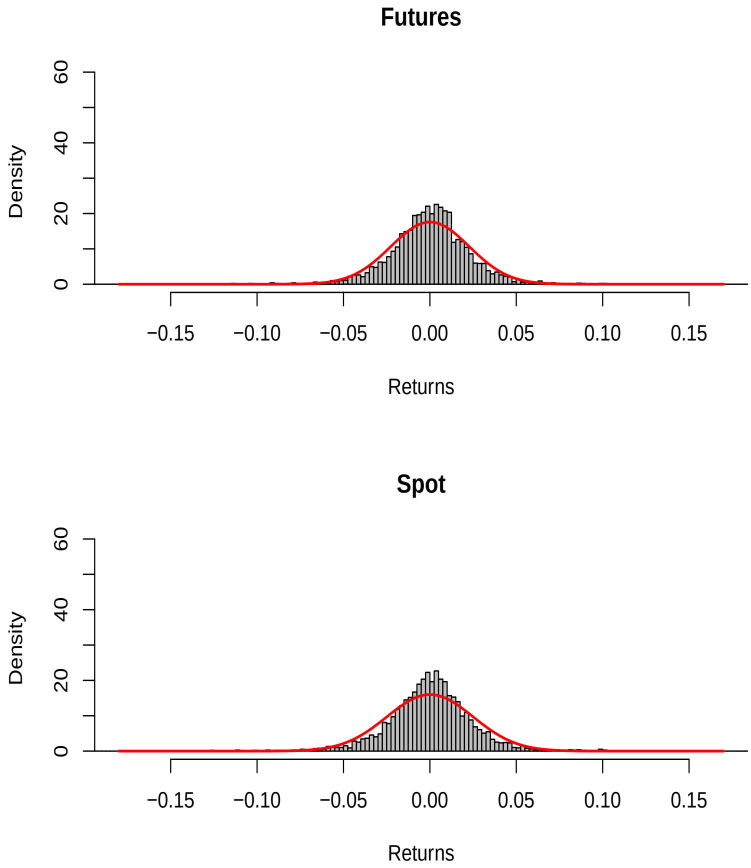
Source: Own calculations

Figure A.2: Gold histogram of returns vs normal distribution



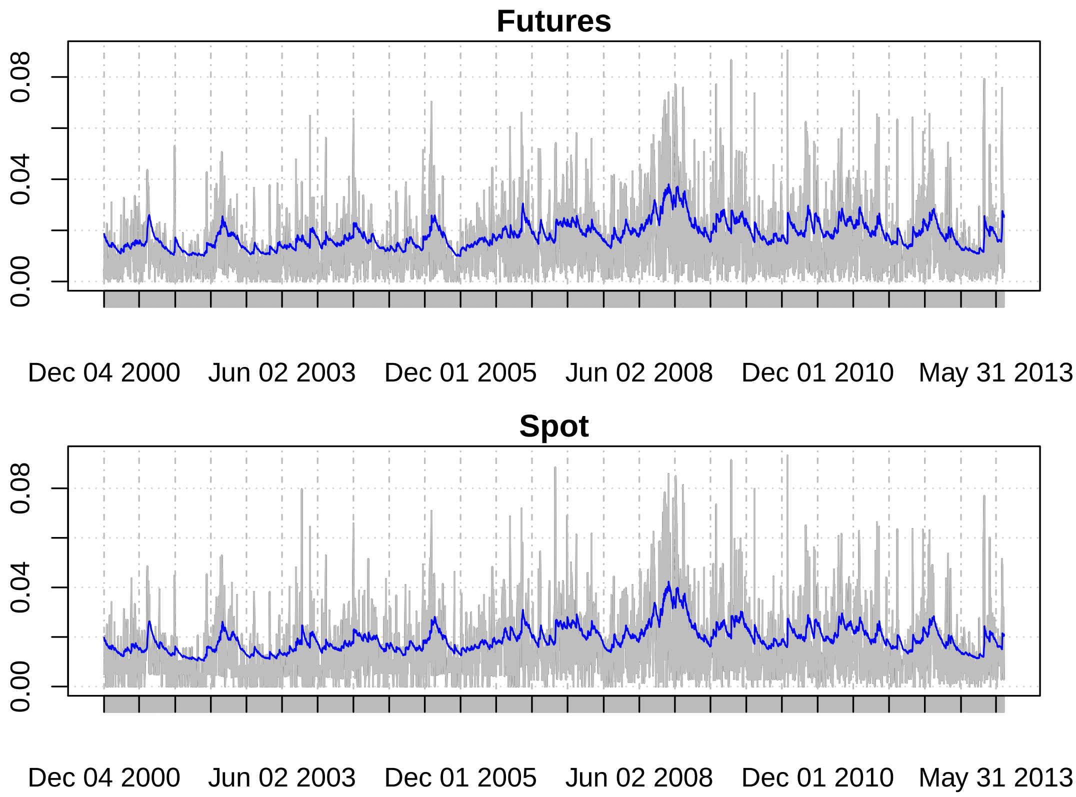
Source: Own calculations

Figure A.3: WTI histogram of returns vs normal distribution



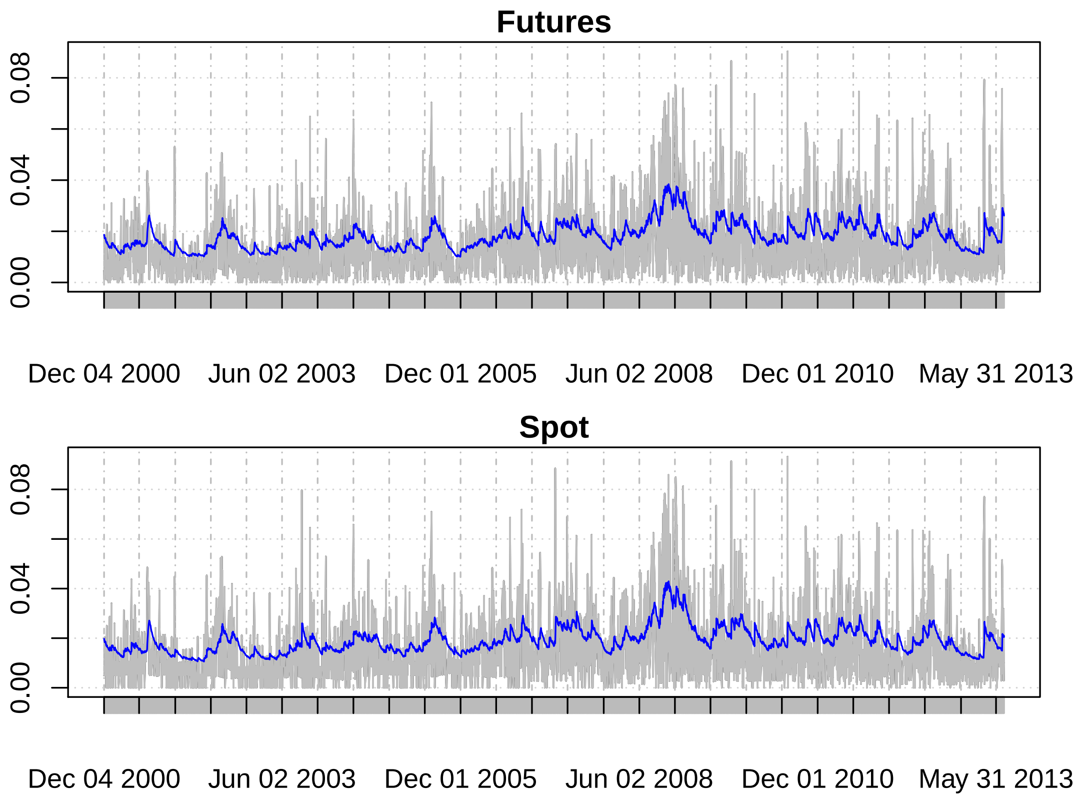
Source: Own calculations

Figure A.4: Corn in-sample observed vs estimated volatility GARCH



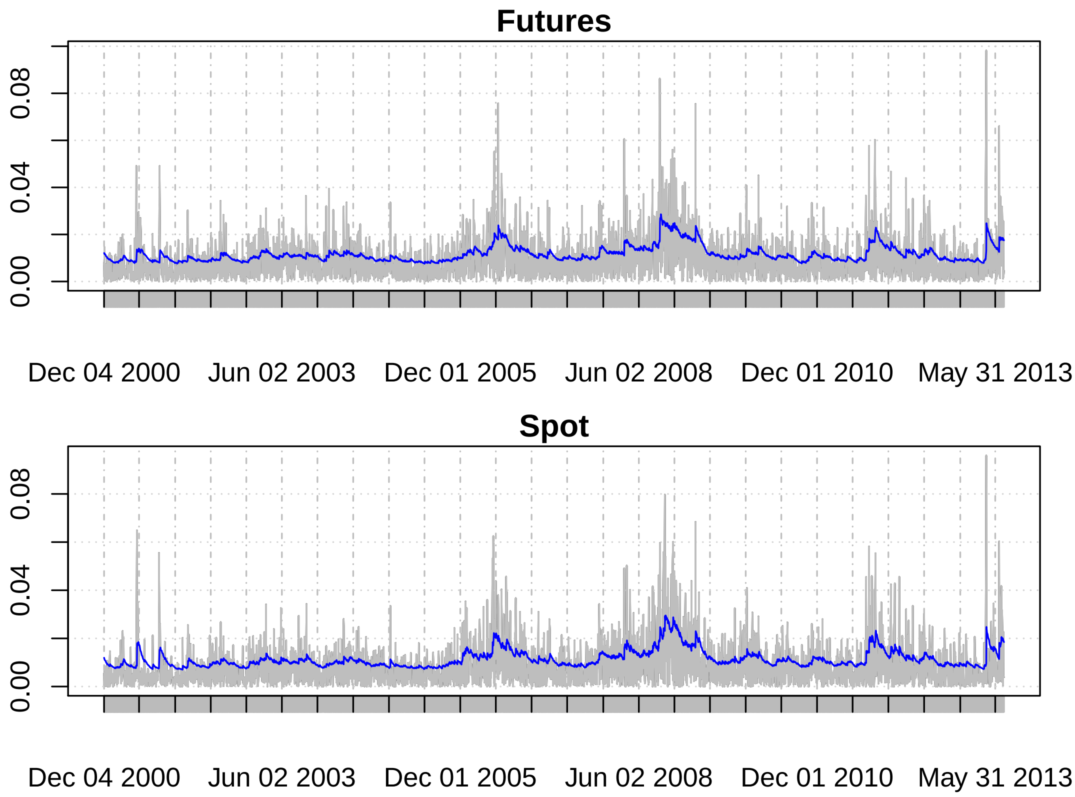
Source: Own calculations

Figure A.5: Corn in-sample observed vs estimated volatility GJR GARCH



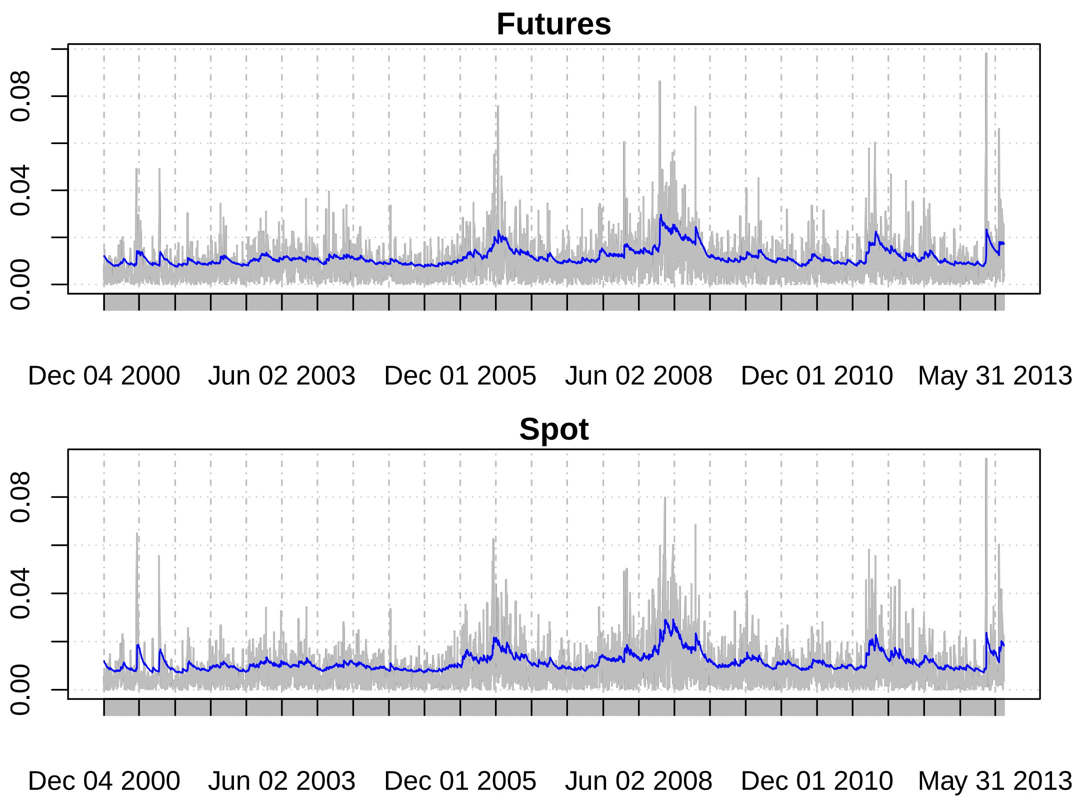
Source: Own calculations

Figure A.6: Gold in-sample observed vs estimated volatility GARCH



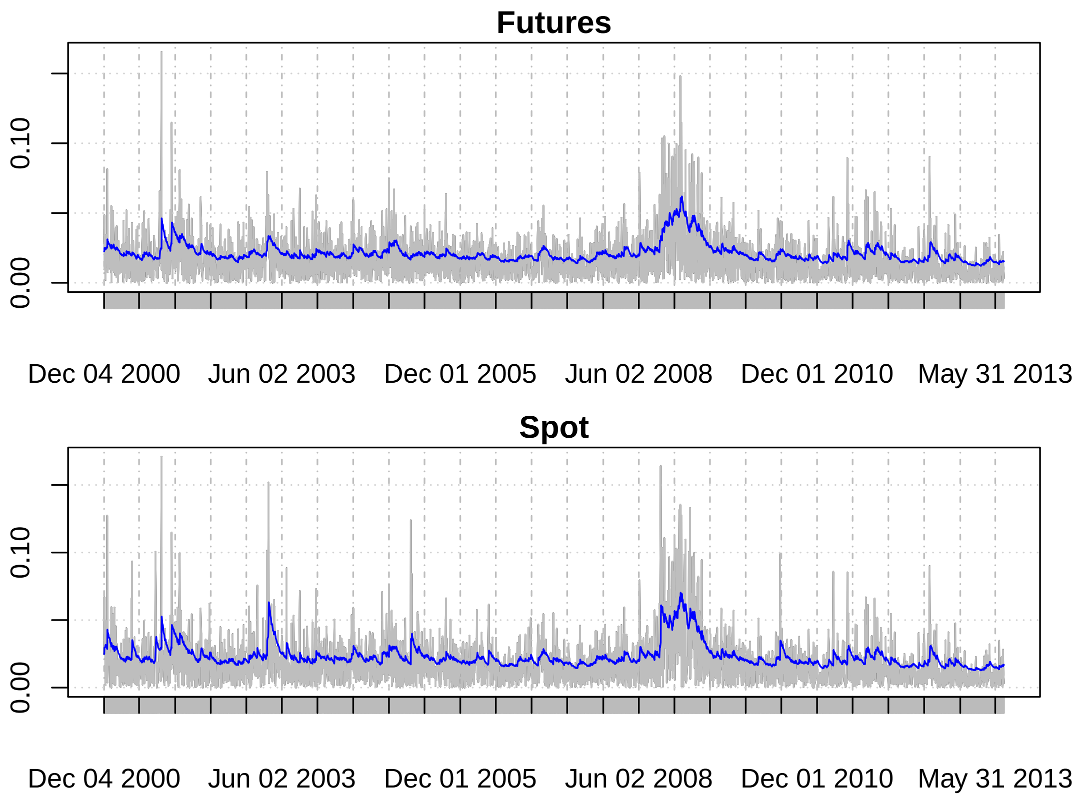
Source: Own calculations

Figure A.7: Gold in-sample observed vs estimated volatility GJR GARCH



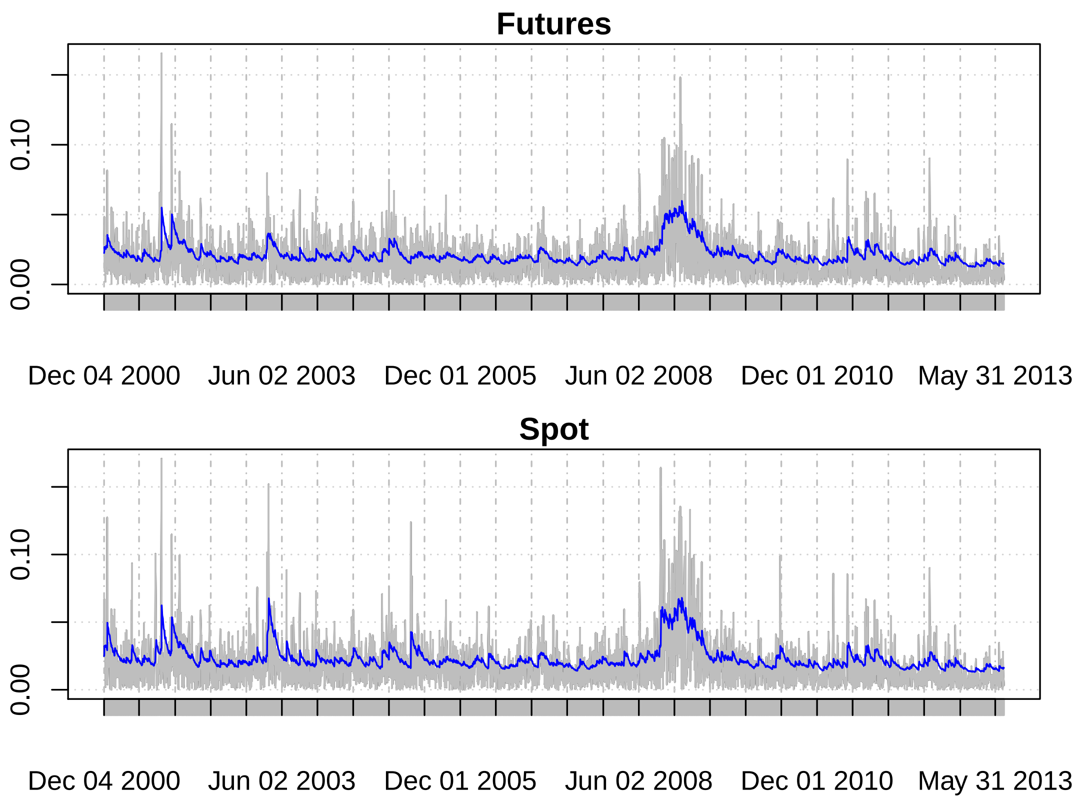
Source: Own calculations

Figure A.8: WTI in-sample observed vs estimated volatility GARCH



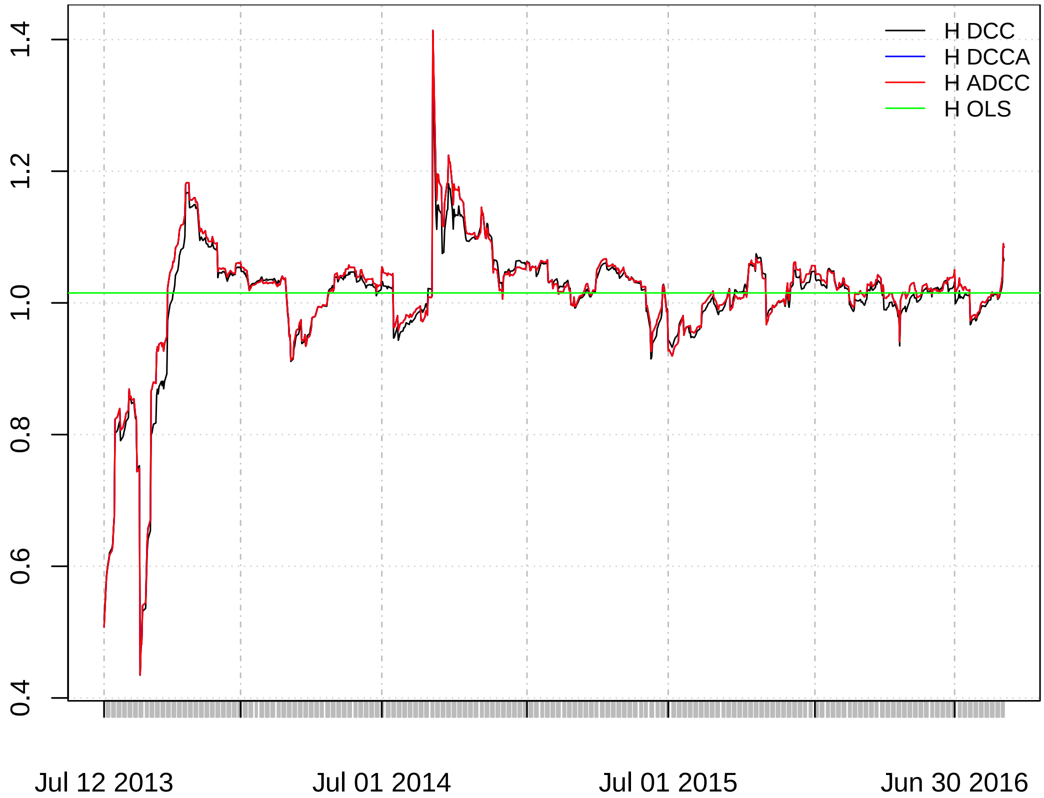
Source: Own calculations

Figure A.9: WTI in-sample observed vs estimated volatility GJR GARCH



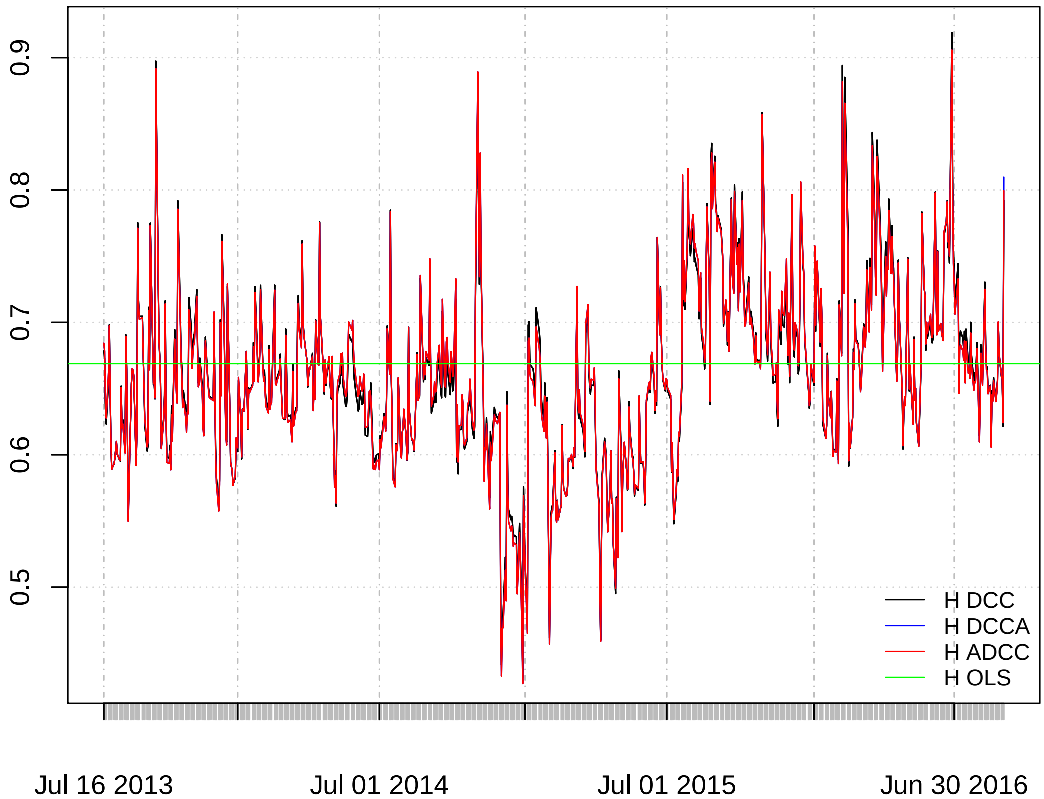
Source: Own calculations

Figure A.10: Corn out-of-sample unrolled hedge ratio estimates



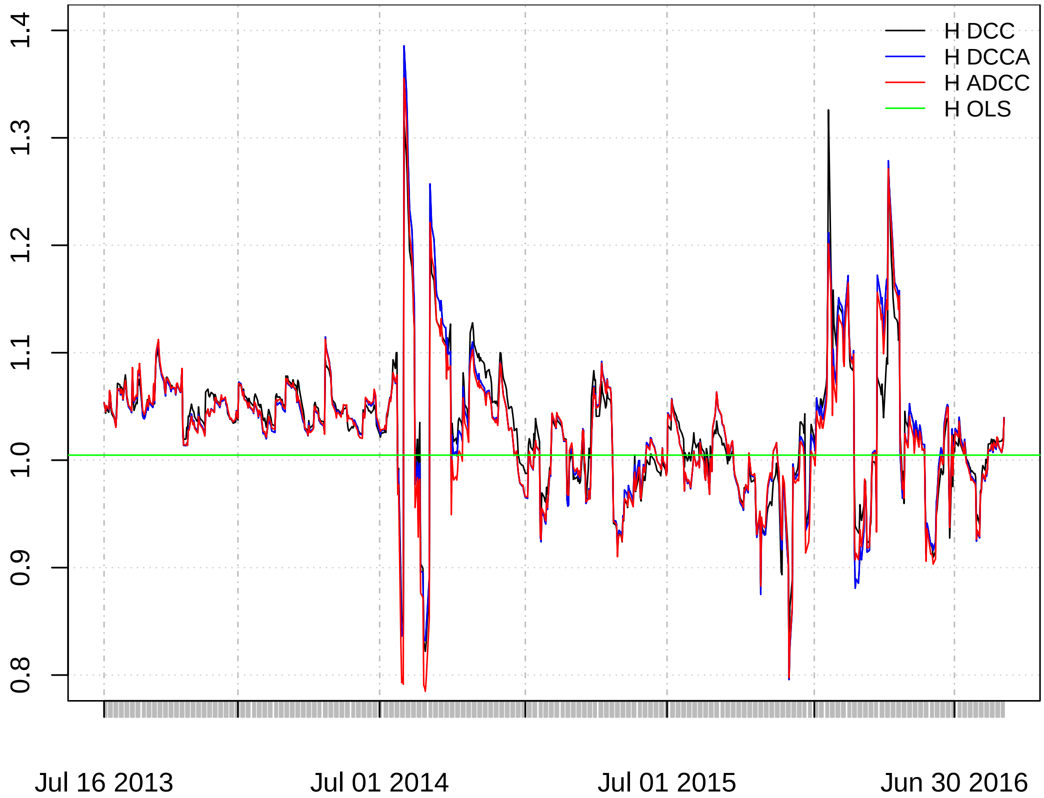
Source: Own calculations

Figure A.11: Gold out-of-sample unrolled hedge ratio estimates



Source: Own calculations

Figure A.12: WTI out-of-sample unrolled hedge ratio estimates



Source: Own calculations