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Tax impacts of the portfolio rebalancing for the Czech investors

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Declaration of Authenticity

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Prague, August 10, 2018 Signature

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Abstract:

Within the area of portfolio management, this thesis deals with the issue of automated portfolio rebalancing which has gained popularity recently – mainly due to the upswing of so-called robo advisors, which are using modern technologies to automate and streamline processes. The thesis focused on costs and benefits of rebalancing, special attention was paid to tax impacts of rebalancing. The results of the simulation of globally diversified portfolio rebalancing using various strategies on a historical market data from the years 2006-2017 mainly support the findings that rebalancing helps maintaining portfolio allocation while the positive impact on risk-return trade-off in real life environment is debatable. The tax costs of rebalancing are highly dependent on rebalancing frequency and market performance in a particular year. In the observed period, the rebalancing strategies with ten percent drift threshold and either weekly or monthly monitoring proved to generate the best results – they slightly increased risk-adjusted returns and resulted in only on average 3 yearly rebalancing events producing taxable amount worth about one percent of the total average portfolio value. Given various rebalancing strategies, the annual portfolio returns decreased by on average 0,2-0,6 % due to the tax costs related to rebalancing.

Key words:

Rebalancing, tax costs, wealth management, robo advisor

Table of content

1. Introduction

Households and individuals all over the world face many financial issues every day. They include, for example, how to best handle their free money, how to properly invest them and what needs to be done to achieve the set financial goals. That is, precisely, which financial method, service, and product to use. An inexhaustible number of options are available on the market. These issues are addressed by the discipline called wealth management, which combines several sub-activities and services. Unlike asset or money management, wealth management does not just focus on financial planning, investment choices, or the composition of an investment portfolio, but also keeps a close look at the profile of the client. It further examines clients overall financial goals, based on which it adjusts investment strategies and tools over the time. It also deals with planning of retirement funds, insurance and tax optimization of all services used. It puts the client in the center of interest with all aspects of its goals and needs and creates a tailor-made solution. In this way, current authors such as Evensky, Horan, and Robinson (2011) or Hallman and Rosenbloom (2015) are consistently approaching the discipline of wealth management.

In the 2010 CFA Institute study, Horan, Jennings, & Reichenstein (2010), use the term high-net-worth-individuals (HNWIs) when defining wealth management service, suggesting those are the group for whom such service is supposed to be offered. HNWIs are individuals or households with free financial assets over one million dollars (Capgemini, 2017), which in the Czech Republic in the middle of 2018 is about 22 million Crowns. According to the latest study, the Capgemini's World Health Report estimates that there are only about 16 million HNWIs corresponding to about 0.21% of the population. In the Czech Republic, there are approximately 25,400 HNWIs (corresponding to 0.24% of the population). Horan et al. thus anticipate in their definition that wealth management services are not aimed at the general public or that they are unavailable to the vast majority of the population. This unavailability has its reasons. Those reasons are mainly high costs and thus high fees, high minimum investment amounts, but also a relatively small number of physical wealth managers.

However, with the trend of digitalization spreading across industries over the past two decades, many standards have changed. Technology nowadays allows many activities to be done faster, more efficiently and often better than before. This opens up entirely new market opportunities, and companies around the world are catching up quickly. Services previously available to very wealthy individuals can now be available to the general public. And the general public is quickly getting used to it. With the boom of digital transformation, the demand for such services is increasing, too, leaving the traditional versions of the services behind.

This trend, of course, is present also in the financial sector. Combining finance with technology and creating innovative solutions is globally called by the term FinTech (Financial Technology). The area of wealth management might be a good example. Today, this discipline can no longer be defined as targeting exclusively HNWIs, as there is also a form of wealth and asset management that targets the general public. Such form of the service is

called a robo-advisor. At its core, it is an automated version of the traditional wealth management that works on a set of algorithms.

The term robo advisor first appeared at the beginning of the new millennium and has already gone through several phases of development. Thanks to the benefits robo advisors offer - low minimum deposits, low fees or good availability and flexibility - they have quickly found its customers, mainly from the group of technologically savvy, small investors who otherwise would not have access to the traditional services of established wealth managers. (The Rise of Robo Advisors, 2015)

Initially, the development of robo advisors was mainly a domain of startups, especially in the United States. The reasons for it are, for example, that startups can respond more flexibly to new technological trends than traditional banks or financial institutions who can also often be scared of cannibalization of their own services. The fact that development of a robo advisor requires relatively small initial capital expenditures is another reason. The coding of the automated wealth management software is the only significant expense, and then, the robo advisors only need to establish a partnership with a broker or other licensed financial institution who will be able to provide them with trading services.

However, it did not take long, and the development of robo advisors spread to Europe - mostly to Germany and the UK - and even the large banks and traditional financial institutions are increasingly implementing it.

„Whether we build [robo advisor] or buy it, we should have it"

- James Gorman, CEO, Morgan Stanley, Dealbook Conference (2015)

This statement by the director of one of the world's largest investment banks, who spoke at the DealBook conference in New York, only demonstrates how hot a financial issue it is. Traditional institutions either launch their own robo-advisor solution (for example, Charles Schwab or Vanguard) or acquire already-developed startup solutions (as was the case with the acquisition of FutureAdvisor by BlackRock) or build a partnership like SigFig and UBS.

The situation is also illustrated by the following chart showing interest in the term "robo-advisor" in Google's search engine over the last five years. The graph shows the popularity of the search term on the scale 0-100, and we can see the trend is increasing dramatically. Compared to the previous period, from the end of 2014 to the present, the popularity is steadily growing.

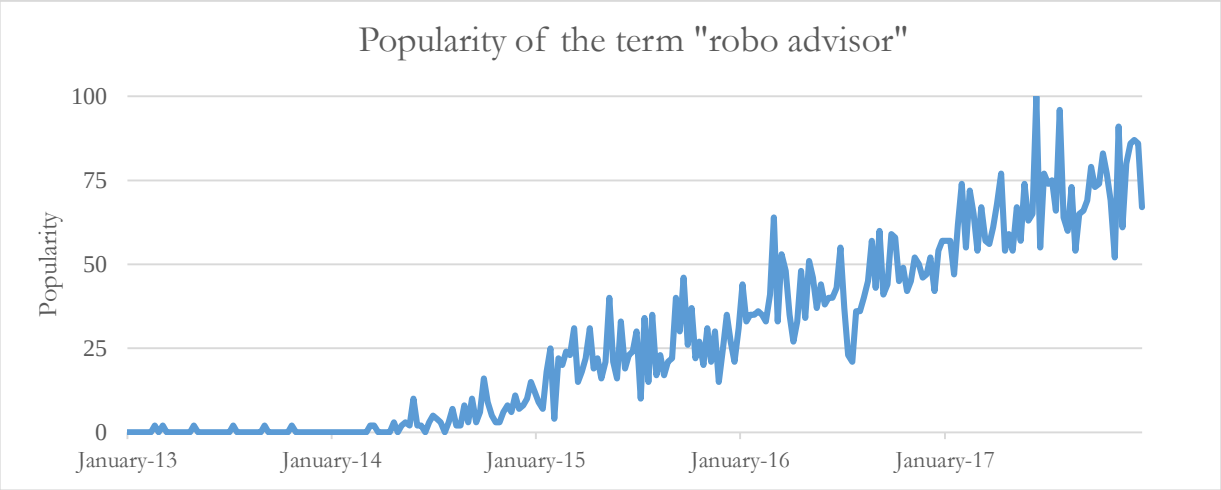

Figure 1 - Popularity of the search term "robo advisor", 2013-2018 (Source: Google Trends, 2018)

The widening trend in the use of technology in the financial sector raises, in a certain sense, doubts about the need for traditional financial services in the way we have known them from past decades. The area of wealth and asset management is a good example. The whole industry is changing along with how new (for example, Wealthfront, Nutmeg) or established companies (such as Charles Schwab, UBS, or BlackRock) are taking steps towards automation and use of technology.

The presence of this trend is also supported by figures on the size of the industry, its growth rate, and future predictions. In 2017, robo-advisor services managed approximately $ 227 billion (assets under management, AUM), serving approximately 13 million clients (Statista.com, 2017). The prospects are also exciting. Statista.com expects that in the next four years, the number of users will increase more than ten times, and that robo advisor's AUM will grow annually by about 38% (2018-2022 CAGR) to $ 1.3 trillion. Some studies assume an even more ambitious growth - for example, according to the global consultancy company A.T. Kearney (2015), robo-advisors would become two trillion industry in 2020. Business Insider Intelligence (2017) in their report then assumes that robo advisors will have $ 4.6 trillion under management in 2022.

Considering such a trend, it could be said that it was only a matter of time when robo advisors had launched in the Czech Republic as well. The first to announce the launch of a robo advisor was ČSOB at the end of January 2017 (Patria.cz, 2017), and the expected date was the year 2018. In August 2017, WOOD & Company also announced the launch of a robo advisor - Portu (Hospodářské noviny, 2017) With its actual start of operations at the end of 2017, Portu is currently the first and only Czech robo advisor.

It can, therefore, be expected that in the coming months or years robo-advisors will also gain popularity in the Czech Republic and will find their customers. However, the purchases and sales of securities might result in tax impacts, according to current Czech legislation. The automated risk management, in the form of rebalancing, can then be a problematic area when it comes to tax liabilities. Rebalancing may result in a relatively frequent sale of securities and

the purchase of others, thus creating a tax liability - and the analysis of this issue is the main topic of this diploma thesis.

Therefore, in the field of increasingly popular automated, algorithmic wealth management and investment portfolio, the thesis deals with the application of such service and its consequences in the Czech Republic. The central area of focus is then the algorithmic risk management - rebalancing (in the sense of performing capital market operations in order to realign the structure of the investment portfolio to the target allocation) as one of the key functionalities of the service. In the context of the rebalancing, the thesis assesses its tax impacts for the end investor. This thesis deals with various risk investment strategies and various rebalancing strategies (drift thresholds, monitoring frequencies).

In the theoretical and methodological part of the thesis, the whole concept of robo-advisors will be presented and put into the context of wealth and asset management. On a theoretical level, the thesis will describe how a general investment process of robo advisors works and what are the specifics of the Czech tax legislation concerning activities on a capital market.

The practical part is then devoted to the analysis of the effect of rebalancing on portfolio performance and its tax implications for a Czech investor. The analysis is based on a simulation of several model portfolios on a real historical data for 2006-2017 using various rebalancing strategies. The aim of the thesis is also to quantify the tax costs of rebalancing for the Czech investor and to tell which rebalancing strategy provided the best results in terms of benefits and costs over observed period. Such conclusion then can be used by robo advisors located in the Czech Republic while forming their investment policy. Moreover, the findings presented in this thesis might serve Czech investors as a base for estimation of what tax costs to expect when choosing certain rebalancing strategy.

2. From traditional wealth management to automization

This thesis focuses mainly on benefits and costs of portfolio rebalancing. However, the portfolio rebalancing is only a small part of a much broader topic – the wealth management. In order to understand well what the rebalancing is and why are we talking about it, the second chapter will present a short introduction to the whole area. It will firstly define the traditional form of the wealth management and will further describe the concept of robo advisors.

2.1. Traditional wealth management

Wealth management is a service provided by banks or other financial institutions to help clients decide what to do with their assets and how to protect or expand their wealth. What distinguishes the service is the fact that its form is tailor-made to each client based on its specific needs, goals, and situation. Because of this, the presence of a human factor in the form of a wealth manager or a consultant who assembles a personalized service was needed, and - with the traditional form of wealth management - still is. This human element, however, makes the service relatively costly. (Swensen, 2009)

The service itself works in the following way: client hires an asset/wealth manager or financial advisor to take care of client's assets - develop a strategy that matches the client's needs, performs operations, and then regularly monitors the situation. The manager then either has full control over the assets and operates the portfolio himself - at his discretion - or contacts the client and discusses with him before any intended action. Full control is generally considered to be more effective as it saves time and allows managers to respond more flexibly to, for example, market situation, thereby achieving better asset protection or higher returns.

Regarding the return for the client of wealth management services, its rate is, apart from the value of the assets held, strongly influenced by the provider's cost or fee structure. Simply put, the end-user return is equal to the return on assets after deduction of all the fees paid for the service to the provider. It means that the higher the fees, the lower the yield. Fees are paid most often in two ways. Either the fee is charged for each transaction (purchase/ sale of an asset), which can motivate asset managers to initiate an excessive number of transactions to maximize their own profits. Or the second standard method is charging clients by the management fee, which covers costs associated with holding assets (custody) and costs of providing investment advice and overall administration. This fee, unlike the trading fee, is charged for the period of time over which the client leaves his assets under management. Most often, this fee is calculated as a percentage of the total asset under management (AUM) and is charged monthly, quarterly or annually. In this context, the study by Cerulli Associates (2011) showed that around 47% of US investors prefer to pay fees for each transaction made and 27% prefer a form of regular fees.

Whatever form the fees take, their amount dramatically affects the total return for the client, especially in the long run. The reason is simple - fees lower the monetary value of the total

assets held, which results in a lower asset value being appreciated. The effect of compounding is being suppressed. And the longer the time horizon, the higher the role of the fee.

High fees are one of the main weaknesses of traditional wealth management services. At the same time, however, the issue of fees is a stream of opportunities for those who can reduce them and thus achieve higher end-user returns. This opportunity is being used by modern asset managers - the robo advisor, who, thanks to the minimization of human factor involvement and automation of their processes leveraging smart algorithms, reduces costs and can offer the service at lower fees. (MyPrivateBanking, 2015)

Where traditional wealth management services are still on top, however, is a degree of personalization. (Kaya, Schildbach, 2017) By replacing the human factor with the computer helps cuting the costs, but artificial intelligence is not yet able to entirely replace personal contact and the benefits of communicating face-to-face with the wealth manager. And, of course, traditional wealth managers charge premiums for it.

2.2. Robo advisors

In the most simplified form, robo advisors can be described as an **automated online portfolio, asset or wealth management service**. It is an investment banking service that combines traditional wealth management approaches and new technologies to make the service more affordable and easy to use. When defining the service, we can start with its two-word name - robo-advisor, which can actually mean a "robotic advisor." MyPrivateBanking (2012)

The first word "robo" is a reference to automation and the use of **technologies and automated processes that completely or partially replace the human factor.** Automation is achieved by using mathematical algorithms and computer programs. The second word "advisor" in the name represents the area of wealth management. That is - services related to client goals and profile evaluation, investment advisory and so on - but in an automated way through online or mobile channels and devices. So if we combine these two terms, we will talk about an online portfolio management solution that allows clients to invest their finances with automated investment advice. (The expansion of Robo-Advisory in Wealth Management, 2016)

Robo advisors basically translate information provided by the client into elements influencing investment decisions and investment choices. These elements include, for example, risk appetite, liquidity requirements, investment horizon, and investment preferences. (Accenture, 2015) Based on these elements, robo advisors then recommend an investment strategy. The investment process will be described in more detail in Chapter 3.

The nature and content of robo advisors are therefore very close to traditional wealth management services. Regarding the range of services, there is nothing new here. However, the **form how these services are being delivered to end clients is innovative.** Unlike traditional investment advisors and wealth or asset managers, where personal contact with a

client is required, robo advisors provide everything online. Clients can obtain the service from anywhere via a computer or mobile phone.

As described in a report from Deutsche Bank (Robo-advice – a true innovation in asset management, 2017), what robo advisors have in common is the aspect of simplicity, ease of use and user-friendly interface. Robo advisors provide clients with information about investing in a comprehensible way and are intuitive. By that, they gained popularity among clients from the general public who may find the traditional form of wealth management to be too complicated.

Generally speaking, both the form of the service and the channels through which the service is delivered to clients are tailored to their target group. The target group for robo advisors is no longer just high-net-worth-individuals (HNWI). Rather, they target people from the general public who do not have either time, education, or interest in taking care of their investments personally. The amount of wealth no longer plays such a role. Unlike the traditional wealth management target group who are generally being rich and wanting to be treated adequately, the people who are using robo advisors are no longer looking for personal contact with a private financial advisor in the offices of a large financial institution, but rather, they prefer accessibility from anywhere, cost-effectiveness and simplicity.

Clients of robo advisors are often people who are either disappointed with the services of their advisor or asset manager and are looking for an alternative, or people who have not been able to reach the services of private wealth management for any reason. However, as Planet of Finance notes in its report, we can find also experienced investors and young professionals who are starting to give some money aside among the clients of robo advisors (The Rise of Robo-advisory, 2016).

History and development

As mentioned in the introduction, the term robo advisor is a relatively new term. The first platforms, which can be called robo advisors, emerged in 2008 in the United States of America. (Naryanan, 2016) As described by the Corporate Insight (2016) report, their inception was de facto response to the changing demands of wealth management clients and the need to innovate financial products. The current clientele of the so-called baby boomers began gradually retiring, limiting their investment activity and withdrawing rather than investing new money. New clients from younger generations already had other requirements for their wealth managers - especially in the areas of technology use and the level of fees charged for services.

An important milestone for the robo advisory industry was then the year 2010 in which the Betterment was launched. This robo advisor greatly increased the popularity of these services and is still the world's most widely used robo advisor portal. (EY, 2016)

However, robo advisors experienced the most significant boom over the last five years, when they began extensive expansion beyond the United States. In 2016, Deloitte stated that there are approximately 100 different robo advisors across fifteen countries (The expansion of

Robo-Advisory in Wealth Management, 2016). Among the most well-known are the American Betterment, Wealthfront, Sigfig, Stash and Motif, British Nutmeg, German Scalable Capital or Vaamo. In the Czech Republic, the first robo advisor portal was launched at the beginning of 2018 under the name Portu.

Although the history of robo advisors is not long, they have already gone through several phases of development. This development is well documented by Deloitte (2016), which divided it into four generations or versions labeled 1.0-4.0. This division is based on an analysis of both current and historical robo advisors on the market. Different generations often exist on the market at the same time, and they differ from each other by the range of functionalities or functions they cover. The newer generation always includes all the features of the previous one, and adds new ones to them.

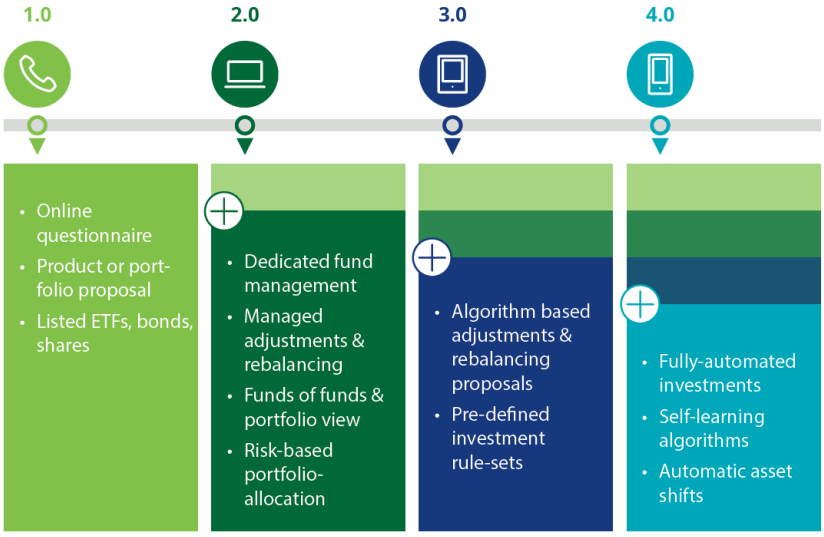

Figure 2 - Generations of robo advisors (Source: The expansion of Robo-Advisory in Wealth Management, 2016)

The following four paragraphs briefly introduce each generation. The text is based on the paper called The expansion of Robo-advisory in Wealth Management from Deloitte (2016). Robo advisors of the 1.0 generation are such platforms whose scope of activities is very limited. This is the oldest version, which essentially works only as a filter of current investment products and investment strategies based on the results obtained using a very simple online questionnaire. Based on the client responses, these robo advisors show appropriate investment products using an on-line interface, but there is no link to any bank or broker that would provide the product. Clients have to make their own asset purchases after getting recommendations, and they also have to carry out future portfolio management themselves.

The Robo advisors of the 2.0 generation are such platforms that no longer use the online questionnaire to filter out existing products, but rather to select among pre-set investment strategies that differ in their risk. Therefore, clients are also recommended a target portfolio allocation that is most suitable for them. The pre-constructed portfolios are constructed manually by a team of portfolio managers who also make adjustments over time. Asset trading and subsequent portfolio management is a semi-automated process in which

portfolio managers define rules for algorithms, supervise the process, and manually decide on adjustments.

Robo advisors of the third generation (3.0) additionally offer automated risk management in the form of algorithmic rebalancing. They use mathematical algorithms to evaluate whether the portfolio needs to be realigned to the target allocation to maintain the desired risk level. Full evaluation, purchase and portfolio management are therefore automated based on predefined algorithms. The role of portfolio managers is mainly supervisory.

Robo advisors of the 4.0 generation are currently the most advanced solutions. Compared to the previous generation, they differ mainly in the use of artificial intelligence (AI) - in the form of self-learning algorithms. Such robo advisors then provide the entire investment process and the administration themselves, with minimum involvement of a human asset manager. In addition to higher level of automation of all the functionalities also provided by previous generations, they are able to monitor the market situation continually and reflect it in real time in the investment strategy of each individual client. The human factor in this version of the robo advisory is almost eliminated.

As the Delloite (2016) report notes, about 80% of all current robo advisors are generation 3.0, but a large shift towards the broader use of artificial intelligence is expected. Some robo advisors are already trying to implement algorithms that are able to predict future market developments (for example, volatility) and adjust investment strategies accordingly or employing self-learning mechanisms of client profile assessment that improve themselves over time based on past evaluations they made.

2.3. Robo advisors versus traditional wealth management

The following chapter compares the traditional concept of wealth management services with what robo advisor offers. It firstly compares the range of activities offered and, subsequently, the procedures applied. It looks at whether, and to what extent, can robo advisors be a fully-fledged alternative to traditional wealth management services.

In the book The Private Wealth Management, Hallman and Rosenbloom (2014) map areas which are generally covered by traditional wealth management. These include investment management, tax optimization, risk management, personal finance planning, real estate financing planning, behavioral finance, and the ability to make the right technical decisions quickly. The following figure illustrates the intersection of these areas.

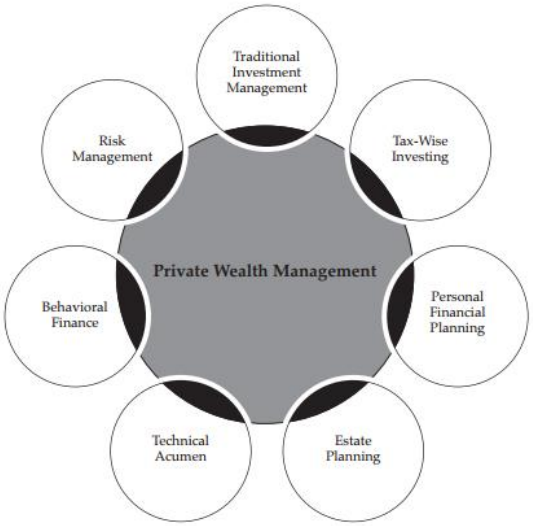

Figure 3 - A view of private wealth management (Source: Hallman, Rosenbloom, 2014)

The following part will take the most important areas separately and compare the traditional approach to the approach of robo advisors.

Investment management

Within the investment management - the key section of wealth management - we can say that the level of coverage of this area by robo advisors is very similar to the traditional form. It may, however, be true that the degree of customization of an investment plan to an individual client may be higher in the traditional form of the service which involves personal contact. Limited customization is an aspect for which robo advisors are often criticized. (Mullainathan, 2012) However, robo advisor platforms do a lot to be able to understand the client profile and its goals and offer a solution that would be most suitable for a group of all clients with a similar profile. Fein (2015) further adds:

'Rather than considering themselves as a provider of an individual personal investment advice, robo advisors should describe themselves as providers of online tools that allow clients to assess their own risk tolerance and investment preferences and then choose an investment strategy that has been recommended for investors with a similar profile. It would be a mistake for retail investors to consider robo advisors to be providers of comprehensive investment advice that will accurately match individual needs.'

Regarding methodologies and procedures for choosing investment strategies and portfolios, the differences between robo advisors and traditional wealth managers are mainly of a systematic nature. While robo advisor portals have, in most cases, a well-established investment methodology based on portfolio and economic theory (see for example Betterment's, Wealthfront's or Scalable Capital's Whitepapers), and apply it to the creation of their pre-prepared investment strategies, we can't always say the same about traditional wealth managers. For example, studies show that traditional wealth managers do not always act systematically. The reason, as presented by Mullainathan (2012), may be that wealth managers might be biased and use assumptions while collecting information about the client.

Some questions may be omitted (for example, the questions about age or income level are often omitted when asking women). That might result in building an incomplete client profile and providing subsequent investment advice that does not take into account all information. Similarly, Mullainathan (2012) points out, for example, that traditional wealth managers are more likely to advise women to hold more liquid assets than men, or that with the increasing volume of an investment, the recommended stock allocation is generally decreased. On the other hand, if the process is automated, as is the case with robo advisors, this inconsistency caused by a human factor can be avoided.

Risk management

The domain of most robo advisors is an automated rebalancing that ensures dynamic risk management. Rebalancing algorithms ensure that the actual portfolio allocation does not deviate by more than the specified limit (drift) from the target one to maintain certain risk level (whether measured by standard deviation or VaR). In the long run, rebalancing can increase returns. (Swensen, 2009) The rebalancing and its aspects are described in more detail in Chapter 4. However, the difference between traditional wealth management and robo advisors is their ability to offer rebalancing services to all clients at moderate costs. While robo advisors use technologically advanced algorithms for rebalancing, and providing it to all clients, traditional wealth managers usually do rebalancing processes manually. Manual processing is very costly, and the rate of fees reflects such a fact. Rebalancing is therefore offered only to the most affluent clients where the costs of rebalancing are marginal compared to the whole size of the portfolio.

Tax optimization

Robo platform advisor and traditional wealth management are not, in principle, tax advisors. However, both traditional wealth management services and robo advisors often offer a form of investment that can be tax-optimized. They can, for example, do some trades in order to reduce the tax burden by using tax-loss harvesting. This is possible for example in the United States. The details and description of this form of tax optimization are usually described by robo-advisers in their White Papers, which are available on their websites (for example, Betterment, 2018; Wealthfront, 2018; Charles Schwab, 2018). In the Czech environment, however, this form of tax optimization in the form of tax-loss harvesting is not possible. The Czech Tax Legislation is different and will be discussed in more detail in Chapter 5.

Personal financial planning and estate planning

This is an area where the personal contact of the client with the wealth manager is probably the most valuable. Although many robo advisor platforms offer the possibility of goal-based investing (for example investing with a goal to buy a house), the aspect of personal contact and communication of additional information about the situation is a significant advantage of the traditional wealth management service. (Hallman, Rosenbloom, 2014)

2.4. Advantages and disadvantages of robo advisors

Interest in robo advisor portals has grown in recent years, not only because it is a new innovative type of investment that attracts technological enthusiasts, but also because of the many benefits this service offers. Some of these were already outlined or mentioned in previous chapters. However, the following chapter presents a comprehensive list of benefits and evaluates its significance. The enumeration is based on both linking insights from robo advisor industry reports (for example Deloitte, 2016; EY, 2016; The Planet of Finance, 2017; Accenture, 2015) and author's analysis of tens of robo-advisor platforms operating primarily in American and European markets. For the author's analysis, the accounts at portals like for example Betterment, Wealthfront, Vaamo, Scalable Capital, Wealthify, Nutmeg, WealthHorizon, Motif and Portu were created and the service was tested from the user's point of view.

Mentioning the list of benefits is needed in this thesis mainly as a supporting argument for the assumption that even people in the Czech Republic will most likely start using services of this type in the near future.

Advantages

One of the key reasons why robo advisors are so popular are their fees. Thanks to the use of modern technologies and automation of the processes, companies can cut costs and thus offer their services only at a **very low fee**. Fees are most often charged as a percentage of the total amount of money they manage (AUM). Fees typically range from 0.4-1.8% pa. Companies then either charge one rate to all clients or distinguish clients according to the amount of money invested, the more the client is invested in the whole, the lower the fee is. Exceptionally, we can find a fixed monthly or annual fee. Entry or exit fees, which are common for, for example, mutual funds, are hardly ever found among robo advisors. (The Rise of Robo Advisors, 2015)

Low fees are of interest to all investors as they considerably increase total net returns for the client. More money is being appreciated and the return is rising in absolute terms. In particular, low fees are welcomed by small investors, for which the level of fees for traditional asset management services was a major barrier to their use.

The second advantage is the ability to **set up an account and start investing via an online interface** from anywhere, without having to process documents and personally visit a branch or a consultant. (Lam, 2016) The identity of the customer is being verified on the basis of photocopies of - most likely - two identity documents uploaded by users during a registration process. Registration is always required before the investment service is provided, therefore it is impossible that an unverified person would invest. The easier the steps between the phases of getting to know the service and actively using it, the higher the conversion rate.

Another advantage of the robo advisors is also their **clarity and user-friendliness**. A simple user interface plays a role, especially if the target audience is the general public that often has no deep knowledge of the investment vehicles, finance and is not well oriented to a

technically-looking environment. And the general public is most often the target group for robo advisors. Therefore, it is necessary to take care of the simplicity and also the visual appearance of the web pages. However, it is not the case that the content or important information on robo advisors' pages are missing. Their organization and form are usually well thought out - investors from the general public are not burdened by exhaustive technical information, but a more sophisticated audience has good access to all the information they need.

The common feature of many robo advisors is also the ubiquitous offer of **help and support**. A customer help-line, communication with the support using online chat directly from the web page and of course by email is an industry standard. Again, it is a very important factor contributing to the positive perception of the benefits of these platforms - especially if we take into account the target audience, who appreciate and use the opportunity to ask or get reassurance.

From an investment point of view, the advantage of robo advisors is their **ability to minimize impulsive human decision-making** and the impact of emotions on investing. (Lam, 2016) Investments in robo advisors are controlled by algorithms, according to predefined rules and inputs from both the company and client. Recommended investment strategies are algorithmically created based on the client's goals and risk profile, ignoring the current short-term market trends that may be attractive to self-investors, but incompatible with the recommended investment strategy.

Last but not least, the benefits include the fact that clients **invest their money directly into instruments traded on the capital markets**. Most robo portal advisors build portfolios of ETFs that are publicly traded on stock exchanges. This is what distinguishes robo advisors from, for example, mutual funds where investors buy non-publicly traded certificates. In the case of bankruptcy or default of the mutual fund, there is a risk that investors will not be able to sell the certificates since the only one who can buy them back is the mutual fund. The robo advisors are different. If they get into trouble, clients can move their ETFs and sell them directly at the market via another broker.

Disadvantages

It is not true, however, that robo advisors bring only benefits. We can also find several things that play against them.

Firstly, it is their limitation of the ability to modify the service directly for a particular client. Using algorithms to automate the entire process helps to reduce costs and make service more affordable, but on the other hand, the whole investment process and clients are unified to a certain extent. Although each client is evaluated individually, in the end, robo advisors recommend an investment strategy that is just one of a number of pre-prepared strategies. Algorithms are not able to take into account all factors to the extent that a person - a portfolio manager would do. It is therefore important to expect certain rigidity of the whole system when using services of this type.

For clients who do not have strong confidence in online technology, the absence of any personal contact may be actually a disadvantage. Some people need to have the personal contact with a real person while doing their financial decision.

Further, some clients can see the risks of personal data frauds or hacker attacks. This is also connected with the ability of the general public to adopt an entirely new concept of financial services. The current clientele of robo advisors is composed mainly of so-called innovators and early adopters. The question will be if more conservative users -who still represent the majority in the market - will adopt the service. If the pace of innovations outpaces the ability to accept them, it could be a disadvantage of robo portal advisors (Robo-Advisors 2.0, 2015)

3. Investment process

The second chapter has defined the concept of wealth management and presented its development – from the traditional version to automatedone. Moreover, the reange of activities and areas offered by wealth managers was outlined. The following chapter, on the other hand, will describe the investment process performed by wealth managers and robo advisors. The insights and knowledge described in this chapter will also be used in the practical part when constructing the portfolios used in the simulation.

The following chapter is based on the theoretical insight of the investment process of traditional wealth management, as offered by, for example, Evensky and Horan (2011). Such insights are then compared with the way robo advisory platforms work, using both secondary sources and knowledge obtained through the personal analysis of several foreign, and domestic platforms.

Evensky and Horan (2011) clearly outline the entire investment process that wealth managers follow with their clients. It is an endless cycle of several concurrent and successive activities designed to provide the client with such asset management which is based on his wishes but also to the needs and limitations. New and new data (both on client and market data) are being collected and evaluated regularly. New findings dynamically affect the form of the entire delivered service, investment strategy, and plan.

The goal of the wealth management investment process is to get to know the client first - in several areas. Firstly, investor's personal goals - both short-term and long-term- are assessed. Also, investor's available assets, but also liabilities are examined. An equally important area is the client's risk profile, which examines both the risk appetite and the risk capacity. Moreover, last but not least, client expectations are considered.

All the information collected, together with an analysis of the current market environment, enters the investment policy making, by which a specific investment portfolio is constructed and assets are distributed among the individual assets. However, the process does not end, as it is followed by regular monitoring, evaluation of the chosen strategy, rebalancing and eventual acceptance of changes in investment policy. A summary of the whole process can be seen in the following figure.

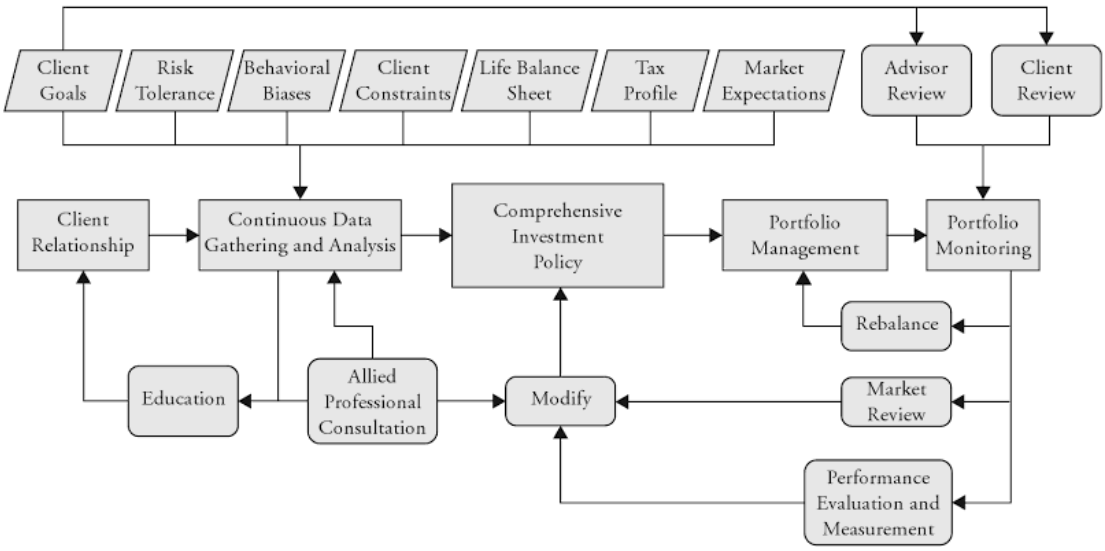

Figure 4 - The wealth management investment process (Source: Evensky, Horan, 2011)

It is important to mention that the investment process of wealth management should be independent from investors wealth and should always follow all the steps. Despite the fact that the final recommended investment strategy might, for example, differ for investors with various wealth levels, none of the elements of wealth management should be skipped.

How does the investment process differ for robo advisors

In the next part, we will compare the steps taken in the traditional wealth management investment process to those covered by robo advisors in their investment process. For the comparison, I use both research papers on robo advisors and my personal insights obtained while testing several online advisors – mainly Betterment, Wealthfront, Motif Investing, Scalable Capital, Wealthify.

Generally said, robo advisors are doing good job covering most of the steps of traditional investment process. Of course, human touch is missing which makes the results less individualized but that is on the other hand offset by the marginal fees to investors. Robo advisors, as their nature already suggests, are aiming for maximal automization – within all the elements of investment process.

Such automization can in some areas bring only benefits, however, in some areas, it comes hand in hand with some drawbacks, too. We can take portfolio monitoring, rebalance and trade execution as a positive example. That is an area where human activity is only needed in a form of surveillance and most of the actions can be fully automated. In contrast, investment profile assessment, goal setting and strategy selection is an area where automation does not really add superior quality. The main reason to support this statement is that there are two contradictory goals robo advisors want to reach at the same time. Similarly to traditional wealth management process, robo advisors aim to get to know their investors well in order to recommend them the best investment plan or solution. And obviously, you need to spend some time with the investor and ask lots of questions. At the same time, however, there is a

huge pressure put on UX design and making the whole service as simple as possible so everyone understands and can use it. This leads to overly simplified client onboarding process. As a research study from Deutsche Bank states that robo advisors often strive to prepare one-size-fits-all short questionnaire to assess the client. Moreover, robo advisors assume that investors with similar profile will respond to subjective questions similarly – which might not be true. (Kaya and Schildbach, 2017)

Such simplification might lead to inaccurate investor assessment and, therefore, inaccurate recommendation on which strategy to choose. Of course, robo advisors can employ much longer questionnaires and ask exhaustive set of questions, but then, another issue comes in play – investors' tendency to loose concentration and provide robo advisors with fast, ill-considered questions.

3.1. Risk, diversification and return

In the practical part of the thesis, we will run a simulation using three portfolios of different risk levels. To understand the concept of risk well, the following chapter will present an introduction in this topic. The chapter will explain various concepts of risk and will define the sources and forms of risk. Moreover, the concept of diversification and the relation of risk and return will be described.

Every investment in capital markets is inherently linked to yield and risk. Kevin (2015) states that people are investing to achieve future wealth appraisal. However, as the future itself is uncertain, the appraisal is uncertain too. And it is precisely the uncertainty associated with investing that represents the risk.

We can distinguish between the expected yield and the realized yield. The expected yield is such a future return that the investor expects to receive. The realized return - as the name suggests - is then the actual return of the investor by his investment. Investors make investment decisions based on expectations of expected returns of the intended investment. However, the realized yield may not correspond to the expected return. So the possibility of this mismatch of the expected and realized yield is called by the term risk. If the reality corresponds precisely to what the investor expects, the risk would not exist. (Kevin, 2015)

Thus, the higher the uncertainty that actual future earnings will be close to the expected yield, the higher the risk. Investments with a stable yield over time and with the yield approaching the expected yield, can, therefore, be considered as low-risk or conservative. On the other hand, investments that are highly volatile over time are considered to be very risky.

Types of Risk

There si not only one risk. The overall risk of the investment can be decomposed into several elements. Firsly, we have to distinguish between systematic and unsystematic risk, often called specific risk. Both of these groups have then their subgroups. The overall risk of the investment is then influenced by all of the subgroups. Following diagram summarizes all the types of risk clearly:

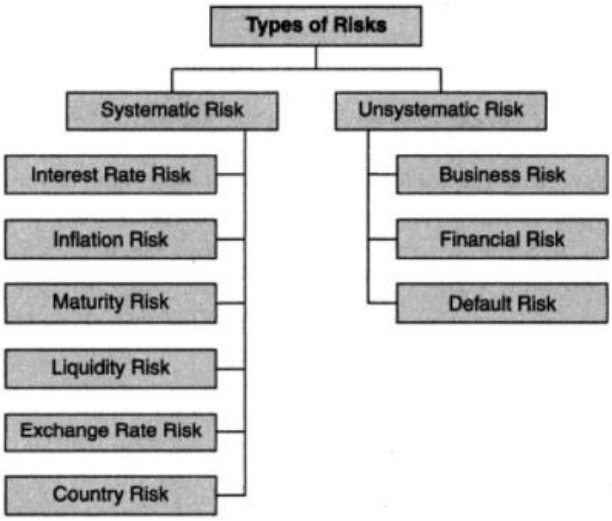

Figure 5 - Types of risk (Source: Dun, Bradstreet, 2009)

First, the distinction between systematic and specific risk needs to be defined. As described by Dun & Bradstreet (2009), systemic risk arises from changes in the economy as a whole, such as inflation or unemployment. **Systematic risk** affects all companies on the market or within the particular sector, but the impact on individual companies varies (for example, depending on whether or not the products and services are considered as essential). Often, this risk is also called market risk.

Specific risk is associated with the activities of individual companies. These may include management decisions, employee strikes, litigation, and many other unforeseen events. This risk does not affect all companies at the same time but is individual for each company and therefore specific. (Dun & Bradstreet, 2009)

The primary sources of systematic risk, as stated by Dun & Bradstreet (2009), include:

- **Interest rate risk** - risk from changes in market interest rates that affect, for example, the cost of operating companies
- **Inflation risk** - the risk of money devaluation affecting purchasing power
- **Liquidity risk** - the risk of inability to quickly swap assets held for money without significant loss of value
- **Exchange rate risk** - risk arising from volatile market rates that may cause depreciation of one currency against others
- **Regional risk** - the risk associated with unforeseeable events related to a particular area or state (for example, political upheaval or natural disasters)

The primary sources of specific risk, as stated by Dun & Bradstreet (2009), include:

- **Corporate risk** - risk associated with the company's business activity. Uncertainty in the company's ability to generate profits (EBIT)

- **Financial risk** - the uncertainty associated with the use of different company funding methods, expressed as pre-tax revenue variability (EBT)
- **Default risk** - risk associated with the ability to meet its obligations

The risk can also be seen as the volatility of assets' value or price in the market.

Measuring risk

There are several methods to measure the risk of an investment or portfolio. Each of them provide a slightly different, unique perspective. The following text briefly introduces the three primary and most commonly used methods. Those are standard deviation, beta, and the concept called value at risk.

Standard deviation

Probably the most fundamental concept of portfolio risk measuring is the one using standard deviation. The volatility of the portfolio is determined by the variance of its return from the mean value (expected return). We calculate the spread using historical portfolio values. As a measure of risk then we use the standard deviation, which is obtained as the root of the calculated variance. If the return of the portfolio never deviated from the expected return, its standard deviation would be zero, and we would consider it as a risk-free investment. Generally, those portfolios with the higher standard deviation are generally considered to be riskier. (Berk, DeMarzo, 2014)

Beta

Systematic risk - meaning the variability of realized earnings due to market or economic factors - can be measured for individual stocks using a statistical indicator called beta. Beta gives us information on how the stock variability of a given action is sensitive to the performance of the entire market (or, respectively, the market portfolio). The input data for the calculation of the beta are both the historical data of the return of the given asset, but also the historical data of the index return representing the entire market. Subsequently, a beta coefficient is determined using the regression and correlation analysis to indicate to what extent the variability of the asset value follows the variability of the entire market. Stocks or portfolios that have beta larger than one are more volatile than the market portfolio and the assets with beta below one are being less volatile compared to market. (Kevin, 2015)

Value at Risk

An alternative way to measure the risk of an investment or portfolio is a newer method formulated at the beginning of the 1990s of the twentieth century - value at risk. Value at risk (VaR) measures the maximal loss (in the percentage of the portfolio) that an investor with a probability of 95 % can experience for a given period. Unlike the variance analysis, VaR focuses only on measuring the downside risk of the investment - the risk of loss. VaR measures the worst possible outcome of a portfolio that can occur with the probability of 95 %. Therefore, it provides information that, with only five percent probability, the loss of the

investment over certain period will be higher than the calculated value. Value at risk is then defined by three parameters: (Kevin, 2015)

1. Potential loss size (determined either in absolute terms or as a percentage of the investment)
2. Likelihood of loss (level of confidence, usually 95% probability)
3. Timeframe (or horizon)

Therefore, if we use the VaR method, we try to come up with the following wording: "I am X percent sure that the loss of the portfolio will not exceed the Y crown/ percent in the next N days" (Hull, 2015)

Historical relationship between risk and return

Investors are willing to take the risk only if compensated for it. The higher the risk they have to bear, the higher the compensation they require. The return on investment, which could be considered risk-free, would be minimal. In fact, we can not find any investment or portfolio that would be completely free of risk. Nonetheless, academic publications often consider, for example, the purchase of US government bonds, whether with a long or short maturity, as a risk-free investment. The long-run average annual yield for the last 1926-2011 period of such government bonds is 3.6%, as stated by Berk and DeMarzo (2014). If investors want to achieve a higher return, they must accept an extra risk.

Looking at historical data, we can see that the different classes of assets show different combinations of risk and return. There are asset classes that can generally be considered as conservative, low-risk, but also asset classes whose volatility is significantly higher, but reward investors with higher returns. The following table shows a comparison of the average annual returns and volatility of the four major classes of assets traded on the US market over a period of 85 years - shares of large companies, small company shares, government bonds and corporate bonds.

Asset class	Average annual return	Average standard deviation
Large companies	18,70%	39,20%
Small companies	11,70%	20,30%
Corporate bonds	6,60%	7,00%
Government bonds	3,60%	3,10%

Figure 6 - Comparison of the average risk and return for different asset classes, US, 1926-2011(Source: Berk, DeMarzo, 2014)

Looking at the values in the table above, it might seem that there is some direct dependence on return on risk or vice versa. However, as Berk and DeMarzo (2014) states, there is no clear relationship between stock volatility and their returns in the real world. On the theoretical level, Harry Markowitz (1952) came out to describe the expected return of the stock as a function of the beta (sensitivity of stock performance to market). Therefore, he

defined the relationship between the expected return of the individual stock as a function of the market return and the stock beta.

Generally speaking, the shares of large companies tend to be less volatile than small ones. At the same time, however, we can observe that portfolios of shares of large companies are less volatile than individual stocks of the largest companies. It can be explained by the term of diversification, which is discussed in the next chapter.

Diversification

If we talk about diversification in relation to investment, we generally mean allocating available funds among a wider variety of assets. It can be distributed to different assets within one class (e.g. shares) or across several different classes (e.g. bonds, real estate, etc.). As stated by Ross (1976), the portfolio is well diversified if the return on the portfolio given by its specific risk approximates zero. Thus, it implies that a diversified portfolio carries only the systematic risk and corresponding return.

However, to better understand what diversification is, let us first look at one table comparing the risk of individual stocks and the local market portfolio. It is a comparison of the standard deviations of selected companies over a period of five years between 2013-2017 with standard deviations of indices representing the entire market in which the company's shares are traded. The relative standard deviation of return is calculated from the daily closing prices of each share.

Market	Equity	Standard Deviation (equity)	Index	Standard deviation (index)
Czech Republic	ČEZ	15,69%	PX	5,94%
Germany	Volkswagen	21,86%	DAX	14,44%
India	TATA	30,83%	BSE SENSEX	15,72%
Japan	Sony	33,95%	Nikkei 225	15,69%
Netherlands	Heineken	19,79%	AEX	12,88%
France	LVMH	24,71%	CAC 40	10,26%
Swiss	Nestlé	8,34%	SMI	6,23%
Spain	Banco Santander	19,26%	MADX	10,53%

Figure 7 - Comparison of the volatility of equity with the volatility of corresponding market index (Source: Bloomberg data 2013-2017, author)

From the table, we see that individual stocks and markets have different volatilities and therefore different risk levels. However, in all cases, the volatility of the individual share exceeds the volatility of the market index, which is itself part of it. Thus, individual stocks are riskier than portfolios created of these stocks (like for example, indexes). If we create a portfolio of several individual shares, its volatility does not equal to the average volatility of its components. The resulting volatility will be lower, as diversification reduces volatility - risk. (Brealey, Myers, Allen, 2014)

The conclusions that diversification reduces portfolio risk has historically been proved in many studies, such as the one by Booth and Fama (1992) or Garvy and Hannon (1998).

By distributing disposable funds between multiple shares, it is possible to reduce the risk as the prices of the individual shares do not move identically. Statistically said, they are not

perfectly correlated which allows reaching the same level of expected return with lower volatility if the right assets are combined. As Brealey and Mayers (2014) also state, "*even small diversification can provide a significant reduction in portfolio volatility*."

Such finding can be illustrated by the following chart of stock portfolio volatility (measured by its standard deviation) and the number of shares from which the portfolio is composed part from the relationship between the number of stocks in the portfolio and its volatility, the chart compares portfolios of shares of the Typical firms, S-type shares, and Type I-type shares. The current company's risk consists of both a systematic risk (non-diversifiable) and a specific (diversified) risk. S and I companies are theoretical concepts of companies that do not carry any specific risk, or no systematic risk, respectively.

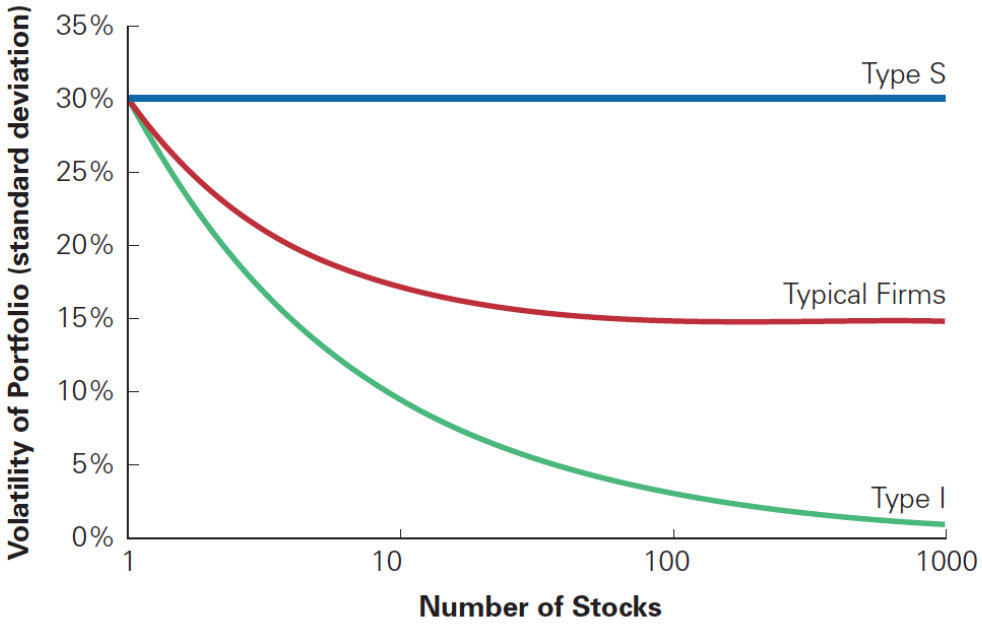

Figure 8 - Portfolio volatility as a function of number of stocks (Source: Berk, DeMarzo, 2014)

As noted aslo by Berk and DeMarzo (2014), several insights can be taken out of the graph:

1. The volatility of the portfolio of companies with only non-diversified systematic risk (S-type companies) does not change with the increase in the number of shares contained in this portfolio and remains only affected by a systematic risk
2. On the other hand, the volatility of the portfolio of companies that contains only a specific risk that can be diversified (Type I companies) is gradually decreasing until it is close to zero
3. With the growing number of companies in the portfolio (composed of companies with at least some systematic risk), its volatility declines quickly first; then the pace slows down until it comes to a close to just the volatility given by systematic risk. It confirms the fact that even a small diversification can provide a significant reduction in portfolio volatility if we add additional shares to a yet little-diversified portfolio. In the case of a broadly diversified portfolio, the effect of adding another share to its overall volatility is smaller.

Regarding the **number of different shares needed for sufficient diversification**, Francis and Ibbotson (2002) offer the exact number. They point out that the specific risk can be diversified almost to zero if the portfolio contains at least 36 random shares as the non-systematic negative and positive effects of those shares in the portfolio tend to cancel each other.

However, the Campbell, Lettau, Malkie, and Xu (2001) study points out the fact that the average correlation declines over the time and, at the same time, firms' specific risk increases. Therefore, the higher number of shares is required to reach fully diversified portfolio.

This is perhaps the reason why Malkiel (2003) in his book A Random Walk Down The Wall Street speaks of a bigger number of shares. The "golden number," which ensures full diversification of the specific risk of the investment portfolio, is based on him 50 shares.

In addition to diversification among individual companies, diversification across sectors (geographical, sectoral), as well as other non-equity classes of assets such as bonds, real estate, commodities, foreign currencies and others, must be kept in mind. Returns of these asset classes may be even less correlated than the returns of individual stocks or equity sectors, and thus may provide additional opportunities for portfolio volatility to decrease. (Dun, Bradstreet, 2009)

Globalisation plays its role in diversification too. On one hand, it has greatly increased the investment opportunities available to investors. On the other hand led to increased correlation among many – often very distant – asset classes. Nevertheless, as stated by Goetzmann, Li and Rouwenshorst (2005) the diversification benefit from adding emerging markets into portfolio has not changed much. Therefore, importance of emerging markets as a diversification driver is increasing.

After all, the famous saying "do not put all the eggs in one basket", is highly applicable even in investing.

How well do people dversify

Polkovichenko (2005), for example, shows that among US investors holding shares, 90 % hold less than 10. On average, they only hold four. Such number is way below the number recommended by academics to reach full diversification.

In addition, the research mentions that the shares held by individual investors are mainly shares of companies in the same or a similar sector. Similarly, a study conducted in Norway showed that investors tend to hold highly underdiversified portfolios. (Ødegaard, 2009)

There are several potential explanations for these findings. One of them, for example, is that investors buy shares of those companies that are well known to them. With this fundamental insight of the bias towards the well known came in his study Huberman (2001).

Another explanation, based on findings of Kaniel, Kremer, and DeMarzo (2004), may be that people compare their portfolio primarily with the portfolios and the performance of the

people in their surrounding - whether friends or family. Moreover, they aim to match their performance to the performance of their friends. In a way, we can again say, they are biased. Therefore, if someone is holding a heavily sub-diversified portfolio in the investor group, it is likely that the other investors around it will be affected and will build up their portfolio very similarly.

Measuring risk-adjusted return

As an addition to the issue of risk and return relationship, we will just briefly introduce the topic of measuring the risk-adjusted return because this measure will be used in the practical part of the thesis.

Contrary to other types of return measurement (like for example average annual return), risk-adjusted return includes risk in its calculation of return, providing an investor information about how much return does the investment made in a relation to the amount of risk investor bears with that investment.

There are several ways to calculate risk-adjusted return. Among others, we can mention for example Treynor's ratio, Jensen's alpha or the well known Sharpe ratio. Due to the fact Shapre ratio will be used in the practical part, we will present a little more detail on it.

Sharpe (1994) defined its ratio as way to measure expected return of the investment per unit of risk. Basically, it is and excess return of the investment over the risk free rate per the unit of risk (measured as a standard deviation. The risk-adjusted return can be therefore calculated using following formula.

$$Sharpe\ ratio = \frac{r_p - r_f}{\sigma_p}$$

where:

$$r_p = portfolio\ return,$$
$$r_f = risk\ free\ retunr\ and$$
$$\sigma_p = portfolio\ standard\ deviation$$

3.2. Asset classes

The following part will introduce the issue of division assets into various classes. Explaining this topic is important for good understanding of the rebalancing principle, which is based on keeping the actual asset class allocation of the portfolio in line with the target. If an investor would not distinguish between asset classes or would not diversify its portfolio among various asset classes, the whole purpose of rebalancing would be lost.

Individual assets can be divided into several classes and subclasses by their nature. Generally, they differ from each other in many aspects. For example, they differ in: who issued them,

whether they paid any fixed interest or dividends or, for example, from which geographical area they come from. There are many criteria under which assets can be divided. In general, however, there is no universal approach to designating assets to classes, and each investment firm, portfolio manager or robo-adviser approaches the division issue by to their own. Moreover, there is no exact definition of how detailed the division should be, too.

Therefore, finding a portfolio constructed of only 2-3 asset classes is as common as finding a portfolio of even 20 or more asset classes. For example, the American robo advisor Charles Schwab uses 28 different classes and subclasses of assets to build their portfolios.

Concerning this topic, in his book Pioneering portfolio management, Swensen (2009, p.101) notes following:

In general, the experts do not agree on the appropriate number of classes of assets that should be included in the portfolio. However, the number of classes should not be too large, so each class still have the ability to influence the performance of the overall portfolio, but it should be large enough to prevent individual classes from influencing the portfolio too much. Some classes with the allocation lower than 5 or 10% of the total portfolio do not have much importance - such a small allocation has little potential to influence the overall portfolio performance. Contrary, the representation of any class greater than 25 or 30 percent represents a particular risk of excessive concentration of assets. Swensen then further mentions that most portfolios work best if they consist of roughly six asset classes.

However, when based on an analysis of various robo advisors, some classes and subclasses can be found more often in divisions, and therefore I will introduce them more in detail:

The basic distribution of assets is between stocks, bonds, commodities, real estate, and cash. Shares, in simple terms, represent ownership of part of the company. Bonds are securities that, in most cases, carry a predetermined fixed return to the investor and a promise that their nominal value will also be repaid after the maturity date. Commodities are raw materials (such as precious metals or oil) that are traded without any difference in quality. Real estate represents assets such as land and buildings. Cash is then thought of as all money, including account balances - in different currencies.

Shares and bonds, as the two major classes of assets, are very often further divided into subclasses based on other different criteria. In following part, definitions by Hallman and Rosenbloom (2015) will be used.

<u>The most commonly used subclasses of shares include:</u>

Growth stocks - are shares of such companies whose sales and returns (and therefore total value) are growing faster than the overall economy and other companies in the market. These companies are mostly well-run, focused on research and innovation, and most of the profits reinvest into development. Therefore, they usually pay no or only very small dividends. Most often these are new companies with great potential to influence their entire industry.

Dividend stocks - are shares that pay more-than-average dividends. Investors resort to them especially in times when the expected growth of markets is uncertain.

Value stocks - are those shares whose current market value is, based on fundamental and other analysis, below their intrinsic value, that is, under its theoretical price taking into account all publicly available information. Those are, for example, shares of companies whose price is affected in the short term by information that is unlikely to have a long-term effect on the company's condition, such as the changes of some people in management.

Blue-Chips - are the shares of the largest and most profitable companies - companies that have steady growth and regularly pay dividends.

Emerging market stocks - are shares of companies in the developing markets. For these shares, a higher rate of appreciation is expected, but they carry greater risk and volatility.

Large, Mid and Small cap stocks - Shares are often distributed even by the size of their market capitalization to large, medium and small.

Bonds are most often distinguished at:

Government bonds - are bonds issued by national governments. Bonds issued by advanced countries are generally considered to be a very safe form of investment. However, this relative safety is balanced by a relatively low yield. They can be further sub-divided by the length of maturity.

Municipal bonds - are bonds issued either by commercial banks or by various territorial units. They are generally considered safe but more risky than government.

Corporate bonds - are bonds issued by individual companies and their risk is based on the creditworthiness of the company. In general, they are considered to be riskier than, for example, government bonds.

Bonds are often further divided according to the issuer's quality (in terms of the risk of failure to meet its obligations). The quality of issuers and bonds is determined by rating agencies such as Standard & Poor, Moody's, or Fitch. Depending on the quality, we distinguish: **Investment-grade bonds** - high and medium grade bonds (S&P ratings AAA-BBB) - and so-called **High-Yield bonds** - more or less speculative bonds that do not reach the investment grade (S&P ratting BB and worse).

Quality	Standard & Poor's	Moody's	Fitch
Invetment grade bonds			
Prime	AAA	Aaa	AAA
High grade	AA	Aa	AA
Upper medium grade	A	A	A
Lower medium grade	BBB	Baa	BBB
Non-investment grade bonds			
(High Yield bonds)			
Slightly speculative	BB	Ba	BB
Speculative	B	B	B
Highly speculative	CCC	Caa	CCC
Lowest quality bonds	CC, C, D	Ca, C	DDD,DD,D

Figure 9 - Bonds ratting system (Source: Hallman a Rosenbloom, 2015)

Asset classes used by several robo advisors

As already mentioned in the introduction of this chapter, the number of asset classes and their labels differ from company to company. The following table lists the classes of assets that are used to build their portfolios for selected robo advisor portals. These are the three leading American companies – Betterment, Wealthfront a Charles Schwab – and, for comparison, one Czech representative – Portu.

Betterment (13)	Wealthfront (11)	Charles Schwab (28)	Portu (10)
US stocks	US stocks	US large cap	US stocks
US Large-cap stocks	International stocks	US large cap - fundamental	European stocks
US Mid-cap stocks	Emerging markets stocks	US small cap	Asian stocks
US Small-cap stock	Dividend stocks	US small cap - fundamental	Emerging markets stocks
International stocks	US government bonds	International large cap	Global bonds
Emerging markets stocks	Corporate bonds	International large cap - fundamental	EU Corporate bonds
Short-term government bonds	Emerging market bonds	International small cap	US Corporate bonds
TIPS	Municipal bonds	International small cap - fundamental	Emerging market bonds
US investment grade bonds	TIPS	Emerging market stocks	US High-Yield bonds
US municipal bonds	Real estate	Emerging market stocks - fundamental	Cash
US corporate bonds	Commodities	US REITS	
International bonds		US dividend stocks	
Emerging market bonds		International dividend stocks	
		MLP	
		US government bonds	
		US investment grade bonds	
		Securitized bonds	
		REITS	
		US High-Yield bonds	
		International government bonds	
		Emerging markets bonds	
		Priority stocks	
		Bank loans	
		Municipal bonds	
		California municipal bonds	
		Gold	
		Cash	

Figure 10 - Asset classes used by selected robo advisors (Source: web pages of Betterment, Wealthfront, Charles Schwab and Portu, 2018)

Regarding the number of asset classes used by our four selected robo advisors to construct the portfolio, we can state that the most - 28 - have Charles Schwab, which means the finest division of assets. The remaining three representatives divide between assets into between ten and thirteen classes. In the view of theoretical knowledge, the division of assets into 28 classes is almost certainly exaggerated (Lam, 2016), since the inclusion of all these classes in one portfolio would mean their very small individual allocation (less than Swensen's (2009) recommended a minimum of 5%), and thus a negligible effect on the overall portfolio performance. The inclusion of ten to thirteen asset classes can be considered as a sufficient diversification, but it does not present a risk of the extinguishing impact of one class on the overall performance.

Looking at asset classes and their overlaps across various robo advisor portals, we see that all equally include US, international and emerging market stocks. Here, only the level of their more detailed division into subclasses is different.

A similar situation can be found in the field of bonds. All portals include corporate bonds and those originating from the emerging markets. In addition, US portals include municipal and government bonds in their portfolios. The Czech representative - Portu - is not naming these classes, which could be explained by their low interest rate, which could reduce the expected return of the entire portfolio while not significantly increasing its diversification. Taking into account the current strengthening of the Czech crown against the dollar, non-hedged investment in US government bonds could have a negative real return after being converted back into Czech crowns.

However, robo advisors approach the asset classes like real estate or commodities differently. While Wealthfron and Charles Schwab have included them in their portfolios, Betterment or Portu did not. Commodities can bring an interesting element of diversification into the portfolio, but it is redeemed with a relatively low long-term appreciation. Low returns can be well illustrated, for example, if we take a look at the performance of the price of gold compared to stocks and bonds. While gold averaged 0,7% between 1802-2012, bonds rose on average by 3,6% and shares even by 6,6%. (Siegel, 2014)

The question, then, is to what extent the benefit of greater diversification, or less riskiness of the portfolio, is offset by a decline in expected returns. If the portfolio risk does not fall significantly with the addition of classes of assets such as commodities and real estate, and, at the same time, the decrease in expected return is significant, it would not make much sense to include these classes into portfolios. It is possible that this is how Betterment or Portu approach the issue of commodities and real estate, and therefore, do not include them.

Using other than the risk-return point of view of portfolio composition, we can see other reasons why Wealthfront and Charles Schwab included asset classes, such as commodities and real estate, in their portfolios. For example, there may be some requirements of the portfolio managers or even clients for a portfolio exposure to a given real estate area or commodity. Indeed, there are some precious metal groups or selected real estate whose risk-return ratio is significantly better than the standard in these asset classes. Marketing can be

another reason. For example, broader diversification of portfolios and investment in real estate and commodities can have a positive impact on product attractiveness in clients' eyes.

Moving on to the bond components of the portfolios and their individual subclasses, here we can also argue about the appropriate classification. For example, American corporate bonds are questionable due to their relatively low returns and the high risks associated with them. First of all, corporate bonds, unlike government bonds (whose credit risk is considered to be negligible), carry credit risk - the risk that the issuer will not meet its obligations in the form of repayment of interest or principal. Low liquidity, or the ability to quickly exchange them for cash, can be considered as another risk factor. Corporate bond markets are much less liquid than, for example, government bonds markets (Swensen, 2005). Last but not least, there is a risk associated with the callability of corporate bonds. The issuer's ability to repay bonds before maturity prevents investors from, for example, in case of a fall in interest rates (and the associated increase in the present value of bonds), profiting from the sale of higher-value bonds. (Swensen, 2005). To bear all these risks, investors should be adequately compensated.

In the case of non-investment grade bonds - so-called high yield, the situation for investors may be even worse. First, these bonds, as their rating already indicates, carry higher credit risk. At the same time, according to the research, the liquidity risk of high yields of bonds compared to investment grade bonds is still noticeably higher, especially in times of high market turmoil. (Philips, 2012)

3.3. Allocation and the effective portfolio

NASDAQ (2018) online dictionary, for example, defines allocation as the distribution of each asset class in the portfolio. Allocation is the ratio of the individual classes in the investment portfolio. The choice of asset classes and their proportionality within the portfolio are key determinants of the expected return and risk. For each client, depending on his profile, investment objectives, and other factors, the allocation is different.

Historical research has shown that investment outcome success is predominately determined by the allocation decision process. (Brinson, Hood, Beebower, 1995)

- Investors are increasingly looking beyond their home market toward the full global equity opportunity set as the starting point for their investments
- Allocation decisions that start with the full opportunity set can be adjusted based on investor goals, expertise, philosophy and constraints
- Not considering the full opportunity set can introduce unintended bets and biases/risks, and can be an investment decision in itself

To determine the right allocation, several approaches can be used:

Probably the simplest option is to use Equally weighted or „1/n“ portfolio approach. Due to its simplicity, this approach is widely used in practice. (Bernartzi and Thaler, 2004) The

principle of this approach is that an investor simply takes the number of different asset classes (n) he has in his portfolio and attaches the weight to this asset class of 1/n. Despite its simplicity, some researches show that, when compared to more sophisticated approaches like mean variance optimisation, they provide equivalent results. This finding is explained by the fact that the gain from optimal allocation is offset by estimation error. (DeMiguel, Garlappi, Uppal, 2009)

Alternatively, mean-variance analysis and optimization can be used for portfolio composition. This model is based on the theoretical foundations made by Harry Markowitz in 1952 and further elaborated by William Sharpe in the sixties and seventies.

In essence, the model was the first mathematical description of the concept of risk diversification. Currently, it is a model that is the most widespread among robo advisors. (Robo-advice – a true innovation in asset management, 2017)

Markowitz (1952) based his theory on the idea that investors are looking for a portfolio with the lowest volatility (measured by standard deviation) for a given expected return, or looking for a highest expected return for a given volatility of the portfolio. Thus, the investors are looking for such portfolios whose volatility (standard deviation) is the lowest compared to all other available portfolios with the same expected return. Such portfolios are then called effective.

The graph of the effective portfolios is shown below. The range of effective portfolios composed only of risky instruments is shown by red and then the capital market line, which also contemplates the presence of a risk-free asset (i.e. the possibility to invest in the risk-free asset and the possibility to borrow and lend money for the given risk-free rate), is green.

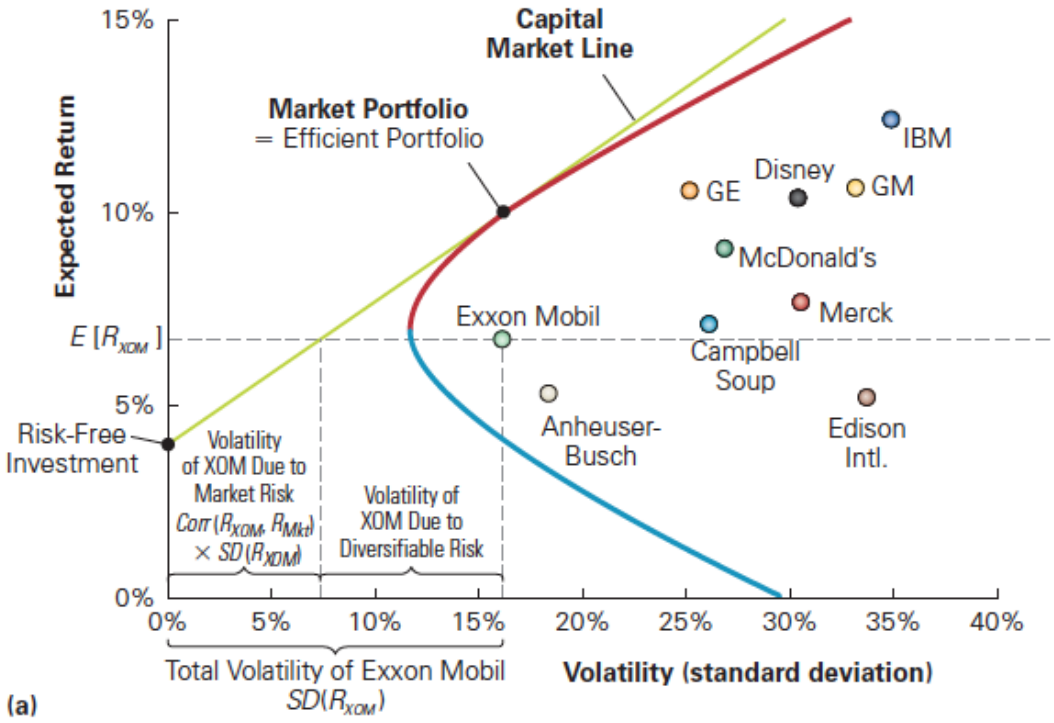

Figure 11 - Capital Market Line construction (Source: Berk, DeMarzo, 2014)

At the same time, the chart illustrates the Exxon (XOM) shares as well as the two main risk (volatility) components discussed in the previous chapter - systematic, market risk and specific, or diversifiable risk. From the chart, we can see that a rational investor could reach the same expected return as if holding only the Exxon stock with approximately half of the risk by investing partly into a fully diversified market portfolio and partly into a risk-free asset.

However, the theory of mean-variance optimization has several weaknesses in practice. As Haugh (2016) summarizes, there are, in particular, three problematic areas. Firstly, due to the use of estimates of expected returns, the model often builds extreme portfolios using a combination of extreme short and long positions. The second problem is the high sensitivity of the model to changes in the expected return of individual assets. Even a small swing can cause an extreme change to the recommended allocation. Last but not least, the model is also vulnerable to errors in covariant matrices that are used in the allocation calculation.

Markowitz's model is directly related to Sharpe's capital asset pricing model (CAPM), which allows determining the expected return on a given asset (share) using a beta - an asset risk measure. The key finding of the CAPM model is that in the equilibrium - determined as the tangent point of the effective portfolio boundary and a line rising from the return on a risk-free asset (the point where green and red curve meets in the above figure) - risk of an asset is not defined using standard deviations but its beta. At such point, the specific risk is fully diversified, and only systematic risk remains. The beta of the asset represents the sensitivity of the asset's return to the return of the entire market. Thus, the model assumes that there is

a linear dependence between the expected return on the asset and the expected return of the entire market. (Sharpe, 1964)

CAPM, however, is based on many assumptions about the markets, which are only very close to reality. These include, among others, the assumptions that all investors use this model to select and structure the portfolio, or that all investors have a homogeneous idea of the expected returns of individual assets, their variances, and correlations.

Besides, the Black-Litterman model, formulated in 1990 and further developed in 1992, can be used to build the portfolio. This model corrects certain shortcomings of the Markowitz and Sharpe models by including, in addition to a purely statistical view, the portfolio manager's view of the expected returns. As it is well known, historical developments - based on the calculations and analysis of variations in Markowitz's model - do not guarantee future development. Although the historical development of individual asset classes would suggest some future behavior, the portfolio manager may have a different opinion based on his fundamental analysis. This model, therefore, takes the human factor into account and incorporates it in the construction of the optimal allocation of the portfolio. This model may, to some extent, eliminate some extreme results obtained by the Markowitz model.

At the same time, the model also allows taking into account the level of certainty with which portfolio manager's expectations about future returns can occur. However, as Idzorek notes (2014), it can not generally be said that the Black-Litterman model would provide superior or better results and determine better portfolio composition than the traditional Markowitz model. Instead, this model will determine the best portfolio allocation for specific given expectations based on both the statistical calculations and the portfolio manager's views.

Thus, the Black-Litterman model is more advanced at the theoretical level than the Markowitz model. However, when it comes to its practical application, it reveals its complexity. It is a very advanced model that can be difficult to use even for many portfolio managers, mainly because of the need to incorporate own expectations of the future development of individual assets into the model. We can also mention a little younger model introduced by Qian (2005) called Equally weighted Risk Contribution Portfolio. The goal of this model is to distribute the weights to different asset classes in the portfolio in a way that each class contributes to total risk by the same portion. It means that the riskier the asset, the lower its weight in the portfolio. (Maillard, Roncalli and Teiletche, 2009)

Millard et. al. (2009) also published comparison of the three allocation approaches mentioned above – namely „1/n“, mean variance optimization and ERC. However, different results were obtained for different investment strategies. While all three models provide equivalent results in terms of portfolio volatility and return for agricultural commodity portfolio, ERC approach dominates others when it comes to globally diversified portfolio. Data from time period of 1995 to 2008 were examined with following results.

Statistics of three allocation strategies, globally diversified portfolio:

	1/n	MV	ERC
Return	7,17%	5,84%	7,58%
Volatility	10,87%	3,20%	4,92%
Sharpe ratio	0,27	0,49	0,67

Figure 12 - Performance of selected allocation strategies (Source: Millard et. al., 2009)

In addition to these models, several other things can affect the final allocation of portfolios. As stated in the KPMG study (The Robo Advising: Catching up and getting ahead, 2016), some of the robo advisors combine client assessment results with the knowledge of behavioral economics. Research, for example, shows that investors feel the pain of loss twice as intense as the enjoyment of profits made (Kahneman, 2011). So they may, for example, increase the proportion of more conservative assets (generally bonds) in the portfolio compared to more aggressive ones (stocks for example).

For many robo-advisors, the client himself can adjust the final allocation by his own decision. Either by altering, for example, the ratio of each asset class or by adding or removing the asset classes. Some portals, such as Motif Investing or M1 Finance, offer, for example, the ability to add sectorally or geographically targeted asset classes to pre-configured diversified portfolios. However, it is rather a marketing element designed to make the service more attractive to the end client, as the portfolio thus created can not be considered as effective from the point of view of the modern theory of the portfolio.

3.4. Passive investing

Since we will use a strategy of a globally diversified, index-following portfolios in the practical part of the thesis (which is considered as a passive investment strategy), the following chapter will present an introduction to this area and summarize the theoretical knowledge in this field.

In his paper, Barnawell (1987) was probably the first one to divide investors into two groups – active and passive. Active investors are those ones who are trying to pick a portfolio of those assets that will outperform some benchmark, or generally a market average. Passive investors, then, are those are not actively searching for such assets and their primary goal is to match the industry's or market's performance.

Since then, many researches have been made on this topic and the passive investing became more popular in recent years. Funds following passive strategy have grown significantly recently in most of the developed markets. Woolley and Bird (2003) see investors' dissatisfaction with active investment strategy as a main reason of this shift. And they are probably right because, as stated by the guru of passive investing, John Bogle (2015), active mutual funds have not over-performed market averages past decades and can claim no superiority over passive strategies.

Swensen (2005) also showed that most actively managed mutual funds fail in their goal of beating the market. Specifically, 78-95 % of mutual funds underperformed when compared

to market average. Such results, of course, make investors of actively managed strategies dissatisfied.

Active strategy – or stock-picking – is basically a discipline of identifying and monitoring companies with high growth potential. And, as empirical evidence proved, it is a very difficult task. (Swensen, 2005)

Malkiel (2016), further argues that most professional investors cannot outperform the market average because the markets price stocks so effectively. He further states it is almost impossible to beat the market using technical analysis due to the fact past prices do not contain enough information to be able to predict future performance. Fundamental analysis might provide better results but only very little investors are able to consistently beat the market.

Investors skills in this area seems to be determinant of success here. In the study, Sorensen, Miller, and Samak (1998) analysed the performance of investors with various skill levels of stock picking over the 12 year period between 1985 and 1997 and found that the optimal amount of allocation to indexing declines as stock picking skill increases.

Ellis (2017), however, states that as the markets are becoming increasingly dominated by institutions, individual investors have only little chance to outperform the market. Also, he notes that the costs associated with active portfolio management are generally higher than with passive investing making it even more difficult for investors to reach better results.

However, passive investing has its drawbacks, too. Woolley and Bird (2003), in their study on economics implications of passive investing, highlighted its negative impact. They found that "high level of passive investing is likely to contribute to excessive and wasteful investment which will result in lower economic growth and investors returns."

Due to the fact passive investors using index-based instruments (like for example Exchange Traded Funds or ETF's) without actually thinking about it, they are increasing demand and thus the price of the stocks of those companies that are part of the index. Such behaviour, when happening in a large scale, prevents markets to price stocks effectively. Some companies might get overvalued for no certain reason – or only for the reason they are part of some index.

So to conclude, while many authors including Malkiel and Ellis are in favour of passive investing claiming it is the best way for individual investors to invest, negative economic implications of this investing form have to be considered, too.

Robo advisors and passive investing

The reason passive investing was introduced in this chapter is mainly because most of the robo advisors use passive, index following strategies for their clients. ETF's are dominant type of asset for portfolio construction – at least for the world's biggest robo advisors like Betterment, Wealthfront, Wealthify or Scalable Capital.

As stated by Singh, and Kaur (2017), the "strategy followed by robo advisors is generally passive, 'buy and hold'. Investor with robo advisors buys stocks and holds them for a long period of time, regardless of fluctuations in the market."

Lam (2016) further notes that robo advisors could one day become the norm for passive investing. Individual investors might take robo advisors as a first choice when willing to invest their money but, at the same time, do not have the time or capacity to choose any active investing strategy.

3.5. Investor's risk profile

Selection of the right asset classes and their allocation within the portfolio of an individual investor should, among other things, be determined by **investors risk profile**. This chapter will briefly describe the issue of risk profile and its influence on portfolio construction. Moreover, it will present ways to how to asses an investor in this area.

Initial risk assessment and ongoing revalidation plays an important role in the whole portfolio management process. Good understanding of investors' risk profile is crucial because, as Merton (2014) states, every investor is unique and portfolio managers should avoid categorizing investors within certain investment strategies only based on their gender, initial wealth or age.

Risk capacity versus risk appetite

Firstly, it is important to distinguish between risk capacity and risk appetite. When talking about risk capacity, we refer to the actual ability of an investor to bear risk – ability to sustain potential losses, regardless to a willingness to undergo such risk. The risk capacity is based mainly on investors wealth and relative size of his investments. Having a low risk capacity should prevent an investor to choose aggressive investment strategies, or even invest at all. (Evensky, Horan, and Robinson, 2011)

On the other hand, risk appetite describes purely investor's willingness to hold risky assets. Willingness to undergo more volatile and uncertain results is crucial here. Based on Gai and Vause (2005), appetite depends on both the degree to which investors dislike uncertainty and the level of that uncertainty.

Therefore, we can conclude that when an investor has low risk capacity, he should stay within rather conservative investment strategies, despite having, for example, high-risk appetite. However, at the same time, an investor with high-risk capacity does not necessarily aim for aggressive strategy when his appetite is low. Both variables should always be considered.

Risk profile assessment

Finding the right way to assess investors on their risk appetite seems to be a challenge. For example, Choi, Fisman, Gale, and Kariv (2007) tried to asses investor's risk preferences using budget constraints through an innovative graphical interface. However, they conclude that "the behaviour of subjects is generally complex and we found it impossible to classify in a simple taxonomy."

As research shows, in general, we can state that "teens are more risk-seeking than adults when it pertains to losses; among investment professionals, women are more conservative than men when it pertains to gains; and even within these subgroups, every individual is unique and neither expected utility theory nor prospect theory appropriately captures the diversity in risk tolerance" (Muralidhar and Berlik 2017).

Portfolio managers might assess risk preferences for example during an interview with an investor. (Evensky, Horan, and Robinson, 2011) Many portfolio managers are also using questionnaires for risk preference assessment. Use of the questionnaire is simple and provides quick results. Set of questions is prepared, and based on the answers, investors are classified as a high or low-risk preference (appetite).

Robo advisors took questionnaires even further, making them automated. Usually, an onboarding process with a robo advisor starts by filling up couple of questions to determine investor's risk preference. However, as Muralidhar and Berlik (2017) states robo advisors often ask very naive questions to gauge risk appetite. For example, they only ask the question of whether an investor prefers to maximize gains or prevent losses or both equally to construct a portfolio.

There is, also, another issue with the risk profile assessment methods based on a questionnaire. Kahneman and Tversky (1979), in their Prospects Theory, found several anomalies and contradiction in human behavior. For example, they showed that the way the question is formulated influences the answers. Investors, when offered a choice formulated in one way, might display risk-aversion but when offered essentially the same choice formulated in a different way might display risk-seeking behaviour.

4. Portfolio rebalancing

In the previous chapters, the concept of wealth management with all its individual activities was described and, also, the whole investment process was described. This allows us now to deep dive into one of the specific wealth management activities – the portfolio rebalancing, which is the key focus area of this thesis. The previous chapters allowed us to set the issue of rebalancing into a broader context and explain various terms that will be used in the following chapter.

The **rebalancing and its tax impact** is the subject of the analysis in the practical part of this thesis. Therefore, the following chapter plays the crucial role in the right understanding of the whole topic and presents key empirical insights. The topic of rebalancing is connected with many terms and concepts explained in previous chapters – mainly asset classes, allocation, risk and return.

Unlike security selection and market timing – that are considered to be part of an investment strategy, **portfolio rebalancing is an act of investment policy**. And, as empirical evidence shows, investment policy dominates investment strategy, making it a key driver of portfolio performance. Despite the fact that choosing the right investment strategy can result in significant results, a study of Brinson, Hood and Beebower (1995) shows that these results are dwarfed by the return contribution from investment policy – the selection of asset classes and their normal weights (desired allocation). These findings are also supported by a more recent study from Ibbotson and Kaplan (2010) which states that 90 percent of the variability in returns of a typical fund across time is explained by policy.

At its core, rebalancing means realigning the portfolio structure into original or desired one. The **key goal of rebalancing is to maintain** certain portfolio allocation and, thus, **certain portfolio risk level**. Rebalancing is in fact a process of allocation management based on a predetermined set of rules. It is a process of constant monitoring of the portfolio and ensuring asset classes weights remain constant over the time. (Swensen, 2009)

The importance of rebalancing arises from the fact that, due to different returns produced by asset classes, actual portfolio structure tends to evolve over the time. Those asset classes which perform relatively better to others within the portfolio are de facto increasing their weight.

The deviation between the target weights of the investor's portfolio and the actual weights of the current investor's portfolio is called **drift**. Drift can either be expressed in an absolute way as a difference between actual and target weight or in a relative way as a ratio of actual to target weight. (Louw, 2018)

Since the goal of rebalancing is to keep the drift low to maintain target risk exposure, those **asset classes that have appreciated have to be sold** and, on the other hand, those that have declined in value have to be added. Basically, investors are selling the instrument which is going up and buying the ones that are going down.

4.1. Rebalancing triggers

Rebalancing can be triggered by two ways. Either be by time or by reaching some drift threshold. When using time as a trigger, the portfolio is rebalanced every day, month, quarter and so on, regardless of how much or how little the portfolio's asset allocation has drifted from its target. On the other hand, when using the drift threshold, time is ignored and rebalance is triggered when the portfolio's asset allocation has drifted from the target by a predetermined minimum rebalancing threshold such as 1 %, 5%, 10 %, regardless of the frequency. Rebalancing, then, can be as frequent as a day when the volatility is high or infrequent as every five years when the performance of asset classes is more or less the same.

Alternatively, a combination of both methods can be used. In such case, drifts would be calculated on a regular basis (monthly, yearly and so on) but only in case the drift exceeds the limit, the portfolio would be rebalanced. This implies that in case the drift is within the limit on the scheduled rebalancing day, the portfolio will not be rebalanced. Likewise, if the portfolio's asset allocation drifts by the minimum threshold or more at any intermediate time interval, the portfolio will not be rebalanced at that time either. (Jaconetti, Kinniry and Zilbering, 2010)

Rebalancing execution

The way rebalance is performed is a separate topic. If a portfolio is sufficiently out of balance, there are different ways investors can go. Donohue and Yip (2003) recommend rebalancing back to range edges. For example, if the target allocation is 60-40 stock-bonds and the relevant range for stocks is 55–65 % and the 65 % limit is surpassed, based on Donohue and Yip, transactions that would set the allocation back to 65 % of stocks should be executed.

In contrast, Masters (2003) recommends rebalancing half-way back from the range edge to the target allocation, 62.5 % from the preceding example. This conclusion arises from his cost-versus-benefit approach to rebalancing.

Opposed to all of the previous authors, Jaconetti, Kinniry, and Zilbering (2010) suggest that rebalance should restore a portfolio to its target allocation or to a close approximation of the target allocation, 60 % in our example.

The decision on what will actually trigger rebalancing and how rebalancing is done is generally made by investors itself or their portfolio managers. Threshold rebalancing can provide more accurate results since it is only triggered when needed but can be very costly during some high volatility periods. (Louw, 2018)

Apart from the buy/sell rebalancing, we can also use **cash flow** to restore the portfolio into target allocation. Basically, any type of cash flow whether it is money deposit, withdrawal or incoming dividends can be used for rebalancing. When there are some money inflows (deposits, dividends), those funds are distributed among the assets which are underweighted. By doing that, the obligation to sell the overweighted assets is minimalized. Likewise, any outflows, like withdrawals are used as to rebalance – those assets which are overweighted are

being sold first. Once the desired allocation is reached, all asset classes are being sold proportionally to keep investor in balance. (Louw, 2018)

4.2. Advantages of rebalancing

Empirical evidence supports the view that portfolio rebalancing improves the risk-return tradeoff of actual investment results. There are several papers dealing with this topic. For example, Chopra (1993) showed that using a two-asset framework, the rebalanced portfolio dominates the non-rebalanced portfolio in both risks and return. He used data from the years 1985 – 1990 and constructed a portfolio with the allocation of 60 % equities and 40 % bonds. Then, he ran a simulation of the portfolio performance for 72 consecutive months – once without any rebalance and once constraining the allocation not to deviate far from 60-40 stock-bond ratio. The rebalanced portfolio ended up with better results – experiencing both higher return and lower risk than the unrestrained portfolio.

Jaconetti, Kinniry, and Zilbering (2010) conducted another study based on a data set from 1926-2009. Using various rebalancing strategies, they found out that rebalancing generally decreased volatility by about 2 percent At the same time, the average annual return decreased too by about 1,5 percent compared to the non-rebalanced portfolio. Very similar results were reached using different rebalancing methods. The whole summary can be found in the table below.

Monitoring frequency	Minimum rebalancing threshold	Average equity allocation	Return	Risk
Monthly	0%	60,10%	8,50%	12,10%
Monthly	1%	60,10%	8,50%	12,10%
Monthly	5%	61,20%	8,60%	12,20%
Monthly	10%	61,60%	8,60%	12,20%
Quarterly	1%	60,20%	8,80%	12,20%
Quarterly	5%	60,90%	8,70%	12,10%
Quarterly	10%	62,60%	8,80%	12,30%
Annually	1%	60,50%	8,90%	11,90%
Annually	5%	60,70%	8,60%	11,80%
Annually	10%	63,00%	8,70%	12,10%
Never	None	84,10%	9,10%	14,40%

Figure 13 - Comparison of the portfolio performance for selected rebalancing strategies, 1926-2009 (Source: Jaconetti, Kinniry and Zilbering, 2010)

A separate study by Vanguard largely confirms these findings. The study, which is based on data from 1960 to 2013, compares two portfolios: a 60-40 stock-bond portfolio that is rebalanced annually and a 60-40 stock-bond portfolio that is not rebalanced. While the rebalanced portfolio provides a marginally lower return (9,12 % compared to 9,36 % for non-rebalanced portfolio), it does so with significantly lower risk (11,41 % compared to 14,15 %). (Kinniri, Jaconetti, DiJoseph and Zilbering, 2014)

An even more recent study conducted by Louw (2018) based on a data from 1998 to 2016, suggests, however, that rebalance does only slightly lower portfolio volatility. Again, Louw constructed a portfolio of 60-40 stock-bonds allocation ratio and tested it against historical

market data using different scenarios – no rebalance, time-triggered method and time/threshold triggered method. The simulations ended up with following results. The non-rebalanced portfolio ended up with an average annual return of 15,32 % and volatility of 11,77 %. Using any rebalance method lowered volatility by approximately 1 percent – no matter which one it was. At the same time, annual returns remained more or less unchanged. No transactional costs and taxes are considered. The data are summarized in the following table.

Rebalancing method	Statistics	
	Return	Risk
None	15,32%	11,77%
Time method - monthly	15,24%	10,66%
Time method – quarterly	15,29%	10,63%
Time method – yearly	15,32%	10,62%
Time/threshold - monthly	15,43%	10,86%
Time/threshold - quarterly	15,48%	10,95%
Time/threshold – yearly	15,12%	10,87%

Figure 14 – Comparison of the portfolio performance for selected rebalancing strategies, 1998-2016 (Source: Louw, 2018)

The study, therefore, confirms the findings given by Jaconetti, Kinniry, and Zilbering (2014) that the results are not meaningfully different when various rebalancing methods are chosen and the only drawback is that with increasing frequency of rebalancing, the costs are rising dramatically. The issue of costs related to rebalancing is discussed in more detail in the next chapter.

If we would combine these findings with a statement made by Chien, Y., Cole, H., & Lustig, H. (2012) that even small costs might suffice to deter investors from pursuing rebalance regularly, we can conclude that rational investors – if given a choice – would choose to rebalance portfolios yearly, or even less often, using the time/threshold method.

We can further conclude that rebalancing is extremely important because it helps investors maintain certain target risk level. However, the impact of rebalancing on returns still remains a debatable question.

4.3. Costs of rebalancing

On the other side, there are several drawbacks of rebalancing. Several **costs are associated with rebalancing**. Jaconetti, Kinniry and Zilbering (2010) provide a good summary of all the costs in their research paper on best practices for portfolio rebalancing.

- **Transaction costs** – these costs are related to the execution and processing of the trades. For individual stocks and exchange-traded funds (ETFs), the costs are likely to include brokerage commissions and bid-ask spreads (difference between the highest price of the buyer and the lowest price of the seller in the market. For mutual funds, costs may include also purchase (entry) or redemption fees.

- **Time and labour costs** – there costs are related to the time and labour it takes to compute the rebalancing amount. These costs are incurred either by the investor directly or by a professional investment manager. The costs may include administrative costs and/or management fees, if an investor hires a portfolio manager. With the boom of robo advisors, however, these costs are becoming marginal because the whole process is being automated by algorithms.
- **Tax costs** – Due to the fact that rebalance essentially means buying and selling some assets, **capital gains taxes may be due** upon the sale if the asset sold has appreciated in value (which is highly probable scenario since, as described earlier, asset classes that have appreciated are generally being sold). Tax impact of the rebalancing is the main topic for the practical part of this thesis.

As further stated by Jennings, Horan and Reichenstein (2010) high taxation on capital gains can substantially widen the no-trade zone where rebalancing should not occur. The fact that taxes are paid only on gains and not on losses can produce asymmetries in rebalancing ranges. Investors might want to wider the rebalancing threshold range for gains and tighten for losses. US investors, due to tax harvesting opportunity might experience even higher need for such actions due to the fact they can recapture at least part of the taxes then selling assets with loss.

Frequency of rebalance

In their paper, Jaconetti, Kinniry and Zilbering (2010), state that: „*there is no optimal frequency or threshold when selecting a rebalancing strategy. This paper demonstrates that the risk-adjusted returns are not meaningfully different whether a portfolio is rebalanced monthly, quarterly, or annually; however, the number of rebalancing events and resulting costs (taxes, time, and labour) increase significantly*."

Therefore, whichever approach one finds more persuasive, the investors always need to incorporate transactional costs, taxes and labour into their thinking – which will probably result in loosening the rebalance ranges or setting less frequent rebalance period.

Several papers were published on this topic, including above mentioned one by Jaconetti, Kinniry and Zilbering (2010) where they found that:

"monthly rebalancing with no threshold would require 1,008 rebalancing events, while annual rebalancing with a 10% threshold would require only 15 rebalancing events. As a result, we conclude that for most broadly diversified stock and bond fund portfolios (assuming reasonable expectations regarding return patterns, average returns, and risk), annual or semiannual monitoring, with rebalancing at 5% thresholds, is likely to produce a reasonable balance between risk control and cost minimization for most investors."

More detailed statistics can be found in the following table:

Monitoring frequency	Drift threshold	Number of rebalancing events	Average yearly rebalancing events
Monthly	1%	398	4,8
Monthly	5%	58	0,7
Monthly	10%	20	0,2
Quarterly	1%	210	2,5
Quarterly	5%	50	0,6
Quarterly	10%	21	0,3
Anually	1%	72	0,9
Anually	5%	28	0,3
Anually	10%	15	0,2

Figure 15 - Comparison of the rebalancing frequency for various strategies, 1926-2009 (Source: Jaconetti, Kinniry and Zilbering, 2010)

Almadi, Rapach and Suri (2014) conducted another cost-benefit analysis and found that monthly rebalance provides investors with the greatest rewards when unit transaction costs are less than approximately 50 basis points. Annual rebalance, then, is the most effective when the transaction costs are above 50 but below 400 basis points.

5. Czech tax legislative with regard to capital income

In the practical part, the thesis will analyze the tax costs of rebalancing. To provide a knowledge base for this issue, the following chapter will present an introduction into Czech tax legislative with relation to investing and performing portfolio rebalancing. Despite the fact that, for example, tax exemptions are not part the analysis presenten in the practipal part of the thesis, the following chapter will cover them to give a complete picture of the area.

Laws differ state by state in this area – not only in the rates but also in their form. Like other earnings, income from capital markets is subject to tax obligations. The tax framework is defined in the Czech Republic by Act No. 586/1992 Coll., On Income Taxes. The law in the current change declares the subject of taxation, tax rates for individuals and businesses as well as exceptions and possibilities of exemption from tax obligations.

5.1. Taxation on a capital gain

For individuals, Czech tax residents, whose securities are not included in business assets, income from the sale of securities (capital gain) is taxed according to the Income Tax Act at a valid rate – for 2018, it is 15 %.

This rate is applicable on all gains from the sale of securities, after deduction of the purchasing costs the other associated expenses related to holding those securities. In the tax return form, individuals take all the proceeds from the sale of securities over the last year (sale price*uantity) and deduct the purchasing costs (purchasing price*quantity) and the sum of expenses related to holding them. Brokerage or custody fees are, for example, part of those expenses. Asset managers and robo advisors often charge management fee which can also be deducted. In this case, the aggregate amount of the management fees associated with the holding of the security during the year in which it was sold may be used as an expense for the purposes of the tax return.

However, in certain cases, income from the sale of securities may be exempted from these taxes, discussed in the Chapter on Income Tax Exemption.

5.2. Taxation on an investment income from abroad

Investment portfolios can be constructed of assets or instruments issued by companies from foreign countries – especially when an investor is located in a small country, like for example Czech republic. These assets can generate some income in the form of dividends or interests. Similarly to capital gain, this income is also subject to taxation. Due to the fact that this income comes from abroad, the situation is a little more complicated.

Income - such as dividends or interest from abroad - are firstly taxed at a rate determined by the contractual relationship of the Czech Republic with the country from which the proceeds flow. As a rule, these taxes are paid in the form of withholding tax in the source state of income. For example, for income from the US or Germany is taxed at the same rate - 15%.

This rate is in, the case for the United States, determined in a document entitled "The convention between the United States of America and the Czech Republic for the avoidance of double taxation and the prevention of fiscal evasion with respect to taxes on income and capital" signed in 1993 by the presidents of both countries. In Article 10, for example, this document specifies that dividends paid by companies from one country to residents in another country may be taxed in the country of origin, but this tax rate will not exceed 15% (or 5% if the investor holds more than 10% of the voting shares of the company).

The dividends and interest are, as a rule, credited on the investor's account in the net amount (after deduction of withholding tax abroad). However, investors should be provided with a report stating both gross and net value, as well as the exact tax rate that has been applied. As the tax expert notes, although tariffs are clearly declared by the contractual relationship between the two states, it is common in practice that different rate is applied, whether higher or lower. (Jedlička, O., personal interview, April 20, 2018). If a higher tax rate is charged to the investor, it is possible to request a refund.

While filling the tax return in the Czech Republic, investors have to list all income from dividends and interest in the gross amount. If a double taxation treaty allows, the investor may then deduct the withholding tax already paid abroad. This reduces tax liability in the Czech Republic, and if the tax already paid abroad is equal to or even higher than the tax investor would have to pay in the Czech Republic (currently 15%), no additional tax liabilities arise.

Some countries – like for example US – require investors to sign a certificate confirmation if a tax residency in order to apply the right tax rate. In US, this form is called W-8BEN and investors have to submit signed form to their broker, asset manager or robo advisor. This institution then provides the document to designated legal authorities. In case US authority is for example not provided with such document, an increased rate of 30 % would be used.

Converting foreign currency income into the Czech crown

Some of the investor's foreign earnings, such as dividends, may arrive at investors' account in foreign currencies. Similarly, dividends and interest might be quoted in foreign currencies even on an account statement. It is because there are more methods to convert foreign currencies into the Czech crown for tax purposes and the wealth manager may not be familiar with the method the client is using.

If a taxpayer does not keep the accounting, he will most likely use the single rate for the conversion of the foreign currency determined in accordance with Section 38 (1) of Act No. 586/1992 Coll., On Income Taxes. This rate is being set once a year, always at the beginning of January, by the Financial Administration of the Czech Republic and published on its website.

The taxpayers who keep accounting may also use the corresponding daily exchange rates provided by the Czech National Bank. However, they must bear in mind that the chosen methods cannot be combined.

Purchasing price determination

Taxpayers have to deduct the costs of purchasing the security (purchasing price) from the income they receive by its sale (sale price). However, there are several valuation methods for determinig the purchasing price of the securities.

Investors who do not keep accounting can use the weighted average purchasing price. That means they only have to keep track of one purchasing price because whenever they buy additional piece of the same security, the weighted average purchasing price changes. Whenever then some sale of the security occurs, investor just uses the weighted average purchasing price in the tax declaration as the purchasing price.

Alternatively, investors who keep accounting can also use one of the accounting methods (i.e. FIFO). Using this method, taxpayers have to pair each piece sold with its original purchasing price.

Again, investors must bear in mind that the chosen methods cannot be combined.

5.3. Income tax exemptions

Provided that certain conditions are met, the income from the capital markets may be exempt from the tax.

The exemption (under the Section 4 (1) (a) of Act No. 586/1992 Coll., On Income Tax) is possible if the total income from the sale of securities for the taxable period does not exceed CZK 100 000. However, it is important to note that this amount relates to the total amount of income from the sale of securities - not just from one investment service provider - before any costs are deducted. If this amount is exceeded, all income from the sale of securities other than those that are exempt from income tax (for example, using the so-called "time test" described below) are taxed.

If the total income does not exceed CZK 100 000, it is still necessary to look at the amount of dividends received during the tax year. If the investor has only income from employment (only from one employer or gradually from more) and has no other income (according to the section 7-10 of the Income Tax Act) higher than CZK 6 000, he does not have to fill a tax declaration form. If the amount of dividends received is higher, it is necessary to fill in the tax declaration form and to pay the taxes.

Irrespective of the amount of income, all income from the sale of securities that have passed the so-called time test is exempt from tax and does not have to be declared. The time test is met when the time between the purchase and sale of securities exceeds three years. For more information on the time test, see Section 4 (1) (w) of Act No. 586/1992 Coll., on Income Tax.

If the investor meets the income tax exemptions listed above, there is no need to deal with the tax-related administration, since such income is not included in the tax return.

6. Analysis of portfolio rebalance on a historical data sets

The following chapter is dedicated to the analysis of portfolio rebalance using the historical data set from the years of 2006-2017. Firstly, the context of this problematics will be briefly explained concerning the specifics of the local Czech environment. Then, the exact goals of the analysis will be specified and hypothesis defined. Further, the way analysis will be done is described. Special attention is paid to the declaration of all the assumptions on which the whole simulation and analysis will be done. Finally, the results of the simulation will be presented and evaluated. The next chapter will then discuss the results and present a conclusion.

6.1. Context and problem description

As already briefly introduced in the previous part of this thesis, the main issue I am dealing with is the **application of the automated portfolio management method of rebalancing** – and its costs – namely tax impacts – for an investor located in the Czech Republic, being a Czech tax resident. Given the recent trend of growing popularity of using automated portfolio management services - offered by for example robo advisors – increasing amount of both private and institutional investors will face consequences this investment form brings along. At the same time, there are several restrictions for Czech investors to use the services of the automated portfolio management company from abroad. For example, foreign robo-advisors require investors to have a local tax number and confirm you are a local tax resident to even sign up.

Using a location perspective, this issue is state-specific due to different legislation and taxation in each country. For example, tax loss harvesting is possible in United States but not in the Czech Republic. Tax rates related to investing in the capital markets and tax exceptions differ, too.

Czech investors, while using the above-mentioned form of investing, might face higher than expected tax obligations due to the nature of the rebalance which lies in pursuing sales and purchases of instruments with the aim to return the portfolio into the desired allocation.

Based on the size of their portfolio, risk – or volatility their portfolio – and frequency of rebalance, the tax implication might differ. And this multivariate issue is the primary focus of this thesis.

6.2. Goals and methodology

The primary aim of this thesis is to analyze the automated portfolio rebalancing with all its implications for a Czech investor. Tax impacts of the rebalancing are then one of the focus areas. The thesis specifically directs its attention on a recent period of the last 12 years, 1.1.2006 - 31.12.2017. There are several reasons for choosing this period. Firstly, the thesis aims to provide up-to-date information for current Czech investors on the topic of portfolio rebalancing. Secondly, some of the indexes I have chosen were created in the year 2005 – therefore, the longer price history does not exist. Lastly, the thesis wants to extend the

research done in this area by evaluating whether the findings reached on a global level and on more historical data sets are applicable also for the Cezch investor over the last 12 years.

The thesis therefore sets **two main goals**:

1. Analyse the effect of portfolio rebalancing on a globally diversified portfolio
2. Analyse the tax impacts of the rebalance for an investor located in the Czech republic

The **first goal** mainly focuses on the portfolio performance assessment. Portfolios with different risk levels will be tested against different rebalancing criteria – various drift thresholds and monitoring frequency. Within this part, the attention will mainly be paid to following areas and relations:

- Rebalancing as a way to maintain asset allocation,
- Impact of various rebalancing strategies on portfolio performance,
- Impact of rebalancing on portfolio volatility,
- Rebalancing frequencies caused by strategies with various drift threshold ,
- Portfolio volatility and its impact on a rebalancing frequency.

The **second goal** then assesses the impact of various rebalancing strategies on its tax impacts for the Czech investor. Currently applicable methodology of tax calculations based on the Czech law is used. In the simulation, only the 15 % tax rate will be used since it is assumed that there are no dividends and the exemptions are dependent on the absolute size of the portfolio and the numbers presented will be in a relative, not aboslute, form, as a percentage to portfolio size for example. Similarly to the first goal, the main focus of the analysis will be paid to the following areas and relations:

- Impact of various rebalancing strategies on taxable amount generation,
- Relation between frequency of rebalancing and tax costs,
- Relation between portfolio risk (volatility) and tax costs of rebalancing.

Apart from that, the thesis will assess the exact amounts of taxable gains/losses for each observed year. Taxes due will be calculated in both absolute and relative way (as a percentage to average portfolio value)

The following methodology part of this thesis will, therefore, have the following structure:

Firstly, the way the data will be collected and processed will be described. All the assumptions used in the simulation will be clearly stated.

Secondly, a globally diversified portfolio will be constructed. Due to the fact that portfolio management is not a primary topic for this thesis, a simple approach using benchmark portfolios will be used in the construction phase. Various instruments will be used in order to give exposition to different asset classes with the aim to approximate the market portfolio composed of stocks and bonds.

Thirdly, portfolio allocation will be set. The aim here is to create globally diversified portfolios with different risk levels. To be precise, three – conservative, neutral and aggressive – portfolios will be created. This will allow analysing the impact of portfolio risk on the need to rebalance the portfolio.

Moreover, the exact way how rebalancing is triggered in our simulation will be defined. Most importantly, the exact formula of how drifts are calculated will be set and allowed drift ranges will be described.

Finally, a simulation on real market data will be made. The simulation will be based on a hypothetical situation of an investor buying a globally diversified portfolio of the desired parameters defined above at the beginning of the year 2006 and holding it for 12 subsequent years. Then, portfolio performance will be simulated, and portfolio structure monitored. The drifts will be assessed and, when the allowed range is exceeded, the portfolio will be rebalanced.

The following chapter will then analyse the impact of rebalances on investor's capital income and its tax implications will be assessed. Results for different years, for different drift levels allowed and for different portfolio risk levels will be compared..

6.2.1. Data collection and simulation assumptions used

The whole simulation described in this thesis will be done using real historical market data. The way the data will be collected and adjusted is following:

- The market data will be obtained using Bloomberg Terminal platform. Daily **closing price** will be used to appraise the instruments' daily value.
- Whenever there is a price not available for a certain day (due to national holidays, market closes, etc.), previous available market price will be used.
- The daily market prices obtained in foreign currencies (USD, EUR) will be, for the purpose of the simulation, **converted into Czech crowns** using the Czech Central Bank's exchange rate fixing for that day. The exchange rate fixing is a rate declared once a day by Czech National Bank and corresponds to the trading in individual currencies in the foreign exchange market at 2:15 p.m. local time. This exchange rate should be used for accounting and legal purposes. (Czech National Bank, 2018)

The portfolio rebalancing is a complex issue and many other elements may influence both the rebalancing process and the overall portfolio performance in a real life. Therefore, in order to be able to assess only the impact of rebalancing on the portfolio performance, the effect of rebalancing has to be izolated from other variables. Therefore, the simulation in this thesis is based on several other **assumptions**:

- There are no fees (management, performance, entry nor exit)
- There are no options to hedge currency risk

- **There are no transaction costs associated with buying, selling and holding the instruments** (no trading costs, no custody costs, zero bid-ask spread, no labour costs associated with the rebalance preparation and execution). There are two reasons I am setting this assumption for my thesis. Firstly, determining the real costs might be very difficult due to no available historical data for particular bid-ask spreads at each time of the day. Secondly, the other costs (brokerage costs, custody etc) differ a lot provider by provider and setting an average is a debatable issue. And thirdly, most of the other relevant research papers use this assumption.
- Investors can only enter long positions (no short sales) and no leveraged positions exit
- **Dividends** and other proceeds obtained as a result of holding certain instrument **are not reinvested**
- Markets are liquid enough during the whole observed period so it is possible to buy and/or sell any desired amount of instrument at any point of time
- There are no settlement delays and other unexpected events

6.2.2. Globally diversified portfolio construction

In order to run a simulation, a globally diversified portfolio will be construnted first. The portfolio will be formed using stocks and bonds. The reason behind this formation is mentioned in the chapter 3 where the common way of portfolio selection used by robo advisors was described. Simple benchmark principles will be used in the composition and the main aim of the portfolio will provide broad diversification and therefore somehow approximate the market portfolio. In order to reach high diversification and exposure to various asset classes and subclasses, passive **index tracking instruments** will be considered as our investment universe. As it was mentioned in the chaper 3, it proved to be the most commonly used strategy by robo advisors.

Stock components

In the stock part of the portfolio, the goal is to approximate Markowitz's market portfolio. Therefore, as a benchmark, MSCI All Country World Index and its allocation is used. This index can be considered as a good proxy of such market portfolio because it covers more than 85 % of all investable equities. (MSCI, 2018) The allocation of the index – and thus our stock part too – is distributed between 2 main classes: developed and emerging markets. Based on their significance in terms of market capitalization, we can further subdivide among American, European and Pacific stocks.

American Stocks

Within the American stocks, the S&P 500 index (SPX Index) serves as a very good proxy for this asset class. This index consists of a portfolio of 500 U.S. large-cap stocks weighted by market capitalization. Investors can easily follow the performance of this index by purchasing

for example SPDR S&P 500 ETF Trust (SPY) which holds of all stocks included in the index. This ETF is being traded in dollars.

European Stocks

European stocks can be represented by Euro Stoxx 50 index (SX5E Index) which consists of 50 largest and most liquid European stocks. It is possible to buy for example iShares EURO STOXX 50 UCITS ETF (SX5EEX) which aims to track performance of this index. This ETF is traded in Euro and consists of companies like Siemens, Allianz, LVMH or Bayer.

Pacific Stocks

The dominant market in this region is Japan which also serves as a good proxy to the whole region. MSCI Japan Net Total Return Index can be taken as an index which covers performance of Japan. The index provides exposure to Japanese large and mid-cap equities. Investors can buy it using Xtrackers MSCI Japan UCITS ETF (XMK9) which physically replicates the performance of the index. Stocks of companies like Honda, Sony, Toyota or Nintendo are part of this ETF.

Emerging Market Stocks

Emerging market stocks are specific due to their low correlation with each other and with equities of companies from developed countries too. (DeFusco, Geppert and Tsetsekos, 1996) Therefore, they are good source of diversification. Emerging markets are often all put together and several indices exists to track their performance. Among others, FTSE Emerging Markets All Cap China A Inclusion Index (FQEACR Index) is a good representative. The index is comprised of more than 3 000 securities from 21 countries including Brazil, China, Taiwan or South Africa. Exposure to this index is possible through Vanguard FTSE Emerging Markets ETF (VWO).

Bond Components

Historically, bonds tended to be more conservative – offering lower risk and returns – than stocks. Moreover, their slightly negative long-run correlation proved even recently by Asgharian, Christiansen, and Hou (2015), have a positive effect on portfolio overall volatility. Similarly to stocks part of the portfolio, bond part also aims at reaching broad global diversification and minimalize the unique risk of eminent. Significance in terms of the eminent group, returns (interest rates) and risk-return ratio of various asset subclasses are considered when composing this portfolio part.

American Investment Grade Corporate Bonds

These bonds are emitted by American companies and have good rating meaning the risk of default is low. iBoxx USD Liquid Investment Grade Index (IBOXIG) is designed to track performance of these bonds. It consists of more than 1 000 bonds of companies like Apple, Goldman Sachs or Verizon. iShares iBoxx $ Investment Grade Corporate Bond ETF (LQD) then allows investors to replicate the performance of the fund.

European Investment Grade Corporate Bonds

There are European corporate bonds with high rating. The best index to track performance of these European corporate bonds is called The Bloomberg Barclays Euro Aggregate Corporate bond Index (LECPTREU). We can find bonds emitted by companies like Daimler, Volkswagen or Rabobank. Investors can buy iShares Core EUR Corp Bond UCITS ETF (EUN5) that aims to follow performance of the fund.

High Yield Bonds

To gain exposure to riskier bonds, high yield bonds are included. Bloomberg Barclays High Yield Very Liquid Bond Index (LHVLTRUU) tracks performance of such U.S. emitted non-investment grade bonds. These bonds have lower rating (BB+ and worse) meaning they are already being speculative but still offer moderate level of risk. It is possible to buy SPDR Bloomberg Barclays High Yield Bond ETF (JNK) which replicates performance of this index.

Emerging Market Bonds

An even riskier class is represented by bonds emitted by governments of developing countries. Thus, their rating is often even below High yield bonds emitted in developed countries. Similarly to Stocks from Emerging markets, inclusion of Emerging market bonds can have positive diversifying effect due to their low correlation with other asset classes. (Goetzmann, Li and Rouwenshorst, 2005) Index to track these bonds is JP Morgan Emerging Markets Bond Global Diversified (JPEICORE). It is possible to replicate the performance of this index by purchasing iShares JP Morgan USD Emerging Markets Bond ETF.

Government Bonds

Finally, bond part of the portfolio is completed with the least risky component of bonds emitted by governments of developed countries. FTSE World Government Bond Index (SBWGU) offers a good option to measure the global performance of these bonds.

Portfolio created using these asset classes give an investor exposure to the strong majority of the whole available investment opportunity set. Similar asset classes in terms of both type (equities and bonds) and geographical distribution (American, European Pacific) were also used for example in the paper from Black, Litterman (1992). Moreover, 9 different asset classes in a portfolio provide a good base for rebalancing which, at the same time, is not too broad.

6.2.3. Portfolio allocation with regard to various risk levels

To assess the impact of the portfolio risk on the frequency of the need to rebalance, portfolios with different risk levels are created. The portfolios are constructed using all the asset classes described in the previous chapter. However, **different allocation is used in order to reach different risk levels**. As already mentioned in the assumptions, only long positions will be used and no leverage will be considered.

To be more specific, portfolios of three different risk levels are created: **Conservative, Neutral and Aggressive**. The **ratio between** two main asset classes: **equities and bonds** is the main **determinant of the risk**. Moreover, the risk is also quantified using **standard deviation** and the final allocation within those two main asset group will be adjusted in order to reach certain quantified value of risk.

The allocation for conservative portfolio will be given by the ratio 20-80 stocks-bonds. Contrary, aggressive allocation will be characterized by 80-20 stock-bond ratio. Given the fact that many studies (see for example Chopra, 1993; Jaconetti, Kinniry and Zilbering, 2010; Kinniri, Jaconetti, DiJoseph and Zilbering, 2014; or Louw, 2018) were conducted on a portfolio with 60-40 stock-bond ratio allocation, our neutral portfolio will also be constructed using this ratio.

Annualized standard deviations are calculated using historical market data for the time period of 12 years (2006-2017). The exact allocations and other portfolio characteristics are summarized in the tables below.

Asset classes	**Portfolio**		
	Conservative	**Neutral**	**Aggressive**
Stocks/Bonds ratio	20-80	60-40	80-20
Stocks			
American stocks	11,90%	34,43%	45,32%
European stocks	4,35%	14,97%	20,19%
Pacific Stocks	2,34%	5,99%	8,28%
Emerging Market Stocks	1,41%	4,61%	6,21%
Bonds			
American Investment Grade Corporate Bonds	34,30%	13,06%	7,62%
European Investment Grade Corporate Bonds	14,86%	5,81%	2,52%
High Yield Bonds	10,29%	7,60%	2,68%
Emerging Market Bonds	8,26%	5,81%	2,52%
Government Bonds	10,29%	5,72%	2,66%
Cash	2,00%	2,00%	2,00%
Total (Stocks, bonds and cash)	**100%**	**100%**	**100%**

Figure 16 - Portfolio asset classes and their weights, different risk strategies

Based on the daily historical market data from the years 2006- 2017, we can determine two key characteristics of our portfolios – their volatility measured by annual standard deviation and their average annual return (in %). The data are summarized in the following table:

Portfolio characteristics (2006-2017)	**Portfolio**		
	Conservative	**Neutral**	**Aggressive**
Pricing frequency	Daily	Daily	Daily
Annualized standard deviation	9,30%	11,50%	13,66%
Average historical annual return	5,08%	5,24%	4,97%

Figure 17 - Portfolio performance, various risk strategies, 2006-2017 (Source: author, Bloomberg data)

We can observe that, even for aggressive portfolio, the volatility is not that high compared to volatilities of single stocks which can be even well above 30 %. The reason for such results, of course, can be found in diversification. Because the portfolios created are widely diversified across various asset classes, even for high stocks/bonds ratio, the volatility is still quite moderate. Through passive exchange-traded funds, the investor gains exposure to hundreds of different equities and thousands of different bonds.

Speaking about the average annual return, it might seem strange on the first sight that the aggressive portfolio has the lowest return. However, we have to consider the timeframe of our data set. 2006-2017 is a period consisting of both crisis (2008-09) and a very strong bull market (2013-15). Despite the fact the aggressive portfolio demonstrates much higher returns during the bull market period, it plummeted dramatically during the crisis. Therefore, as an average, the returns are similar. To give a comparison, you can find characteristics of the same portfolios for a time period of only 2012-2017 below:

Portfolio characteristics (2012-2017 only)	Portfolio		
	Conservative	Neutral	Aggressive
Pricing frequency	Daily	Daily	Daily
Annualized standard deviation	8,06%	10,20%	12,26%
Average historical annual return	3,95%	6,59%	8,42%

Figure 18 - Portfolio performance, various risk strategies, 2012-2017 (Source: author, Bloomberg data)

The situation can be even better demonstrated by the following graph presenting the comparison of the total performance of three investment strategies. We can see that during the crisis, the aggressive portfolio dropped significantly while the conservative strategy led to only moderate losses.

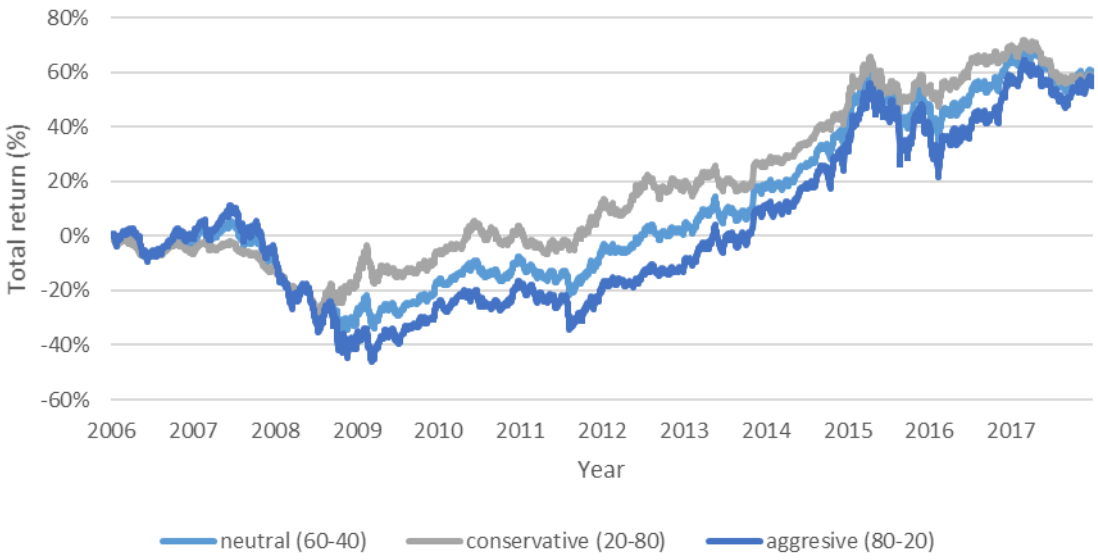

Figure 19 - Historical portfolio performance - buy and hold, no rebalancing, 2006-2017 (Source: author)

6.2.4. The rebalancing algorithm

It is important to explicitly declare under which conditions the rebalance is triggered and how drift is calculated. There are several methods to both handle the rebalancing process and calculate drift. In this thesis, following procedure will be used.

Time-threshold method

Rebalancing will be performed using the time-threshold strategy. Portfolio would be monitored on a regular basis but only if the drift threshold is exceeded, the portfolio will be rebalanced.

Monitoring frequency

Monitoring frequency means how often the portfolio will be examined whether drift thresholds are exceeded. In our simulation, I am testing three scenarios – weekly, monthly and quarterly monitoring.

Drift threshold

If the drift threshold is exceeded during the monitoring event, rebalance is executed. Similarly to monitoring frequency, three different drift thresholds are being tested – 1 %, 5 % and 10 %.

Drift calculation

The way drifts are calculated might influence the drift size. In our simulation, drifts will be calculated as a relative deviation of actuall asset class asslocation from its target allocation. The formula then is as follows:

$$\frac{Actual\ allocation\ (\%) - Target\ alocation(\%)}{Target\ alocation\ (\%)}$$

To illustrate this on an example, let's assume American stocks do curently represent 33 % of the value of our portfolio. Their target weight is just 30 %. The drift is therefore:

$$\frac{33\% - 30\%}{30\%} = 0{,}1\ or\ 10\%$$

Further, same drift thresholds are used for both positive and negative drift.

6.3. Simulation results

The following chapter will present results of the simulation. Firstly, results regarding the portfolio performance and other characteristics will be presented. Secondly, the attention will be paid to tax implications of the rebalancing throughout the years.

6.3.1. Portfolio performance analysis

The portfolio performance will be assessed under the following criteria:

- Portfolio volatility (measured as an annualised standard deviation)
- Average annual return
- Risk-adjusted return (measured by the Sharpe ratio[1])
- Average stock allocation (measured as average percentage of daily weights of stocks in the portfolio
- Average number of rebalancing events per year (division of total number of rebalancing events by the number of years observed)

The following part will compare portfolio performances of different rebalancing strategies. Moreover, the non-rebalanced portfolio (following just buy and hold strategy) will always be used as a benchmark performance. Firstly, we will take a look at results provided by rebalancing with different monitoring frequencies. The results of weekly, monthly and quarterly monitoring will be presented consecutively.

WEEKLY monitoring	**Drift**	**Average stock allocation**	**Volatility**	**Average annual return**	**Sharpe ratio**
conservative 20-80	non-rebalanced	17,20%	9,30%	5,08%	0,28
	1%	20,00%	8,81%	5,47%	0,34
	5%	20,04%	9,09%	5,47%	0,32
	10%	20,19%	9,15%	5,48%	0,32
Neutral 60-40	non-rebalanced	54,94%	11,50%	5,22%	0,24
	1%	60,00%	11,26%	6,43%	0,35
	5%	60,14%	11,68%	5,67%	0,27
	10%	60,30%	11,95%	5,98%	0,26
Agressive 80-20	non-rebalanced	76,52%	13,66%	4,97%	0,18
	1%	80,00%	13,20%	6,25%	0,28
	5%	80,09%	13,56%	6,04%	0,25
	10%	80,15%	14,00%	5,93%	0,24

Figure 20 - Portfolio statistics for various strategies with weekly monitoring, 2006-2017 (Source: author)

From the table above, we can see that **rebalancing helps maintaining the desired allocation significantly**. If we take a look at the average stock allocation column, we can see that the average stock allocation of non-rebalanced portfolios deviates from their target by about 3-5 %[2]. Rebalanced portfolios, on the other hand, keep their stock allocation close

[1] Sharpe ratio uses a **risk free rate** in its calculation of risk-adjusted return. The rate used in my calculations – 2,52 % - is based on data sets provided by the Czech National bank (2018) and is calculated as the average coupon rate of all the medium- and long-term bonds issued by the Czech government during the time period of 2006-2017 (the same period as used for the portfolio simmulaiton).

[2] One might wonder why the average stock allocation for non-rebalanced portfolio is below its target when generally equities tend to outperform bonds and therefore tend to gain more portfolio weight

to the target (20, 60 and 80 % for conservative, neutral and aggressive portfolios respectively). On average, the stock allocation never deviated by more than 0,3 % from the target, no matter what the drifts are. Interesting to say, rebalancing with 1 % drift and weekly monitoring provides perfect results in this manner – the actual average allocation is completely in line with the target providing an investor with maximum control over the portfolio allocation.

Not only rebalance helps to maintain portfolio allocation. From the table, we can further see that the risk-adjusted return (measured as a Sharpe ratio) improved for all the rebalancing strategies and also for portfolios with various risk levels. That means the investor is better compensated for the risk he bears. **Rebalancing with weekly monitoring slightly decreased volatility for the conservative portfolio but increased volatility for riskier ones** (by not more than 0,5%)**. However, it also improved average returns** (by 0,9-1,6 %) **resulting in a higher Sharpe ratio**. The same conclusion can be made for all the risk strategies and drift thresholds. However, the findings are the most evident for the rebalancing strategy with only 1% drift threshold. The Sharpe ratio improved on average by 17 % for the conservative portfolio, 25 % for neutral portfolio and 42 % for the aggressive strategy.

MONTHLY monitoring	Drift	Average stock allocation	Volatility	Average annual return	Sharpe ratio
conservative 20-80	non-rebalanced	17,20%	9,30%	5,08%	0,28
	1%	20,01%	9,13%	6,64%	0,43
	5%	20,06%	9,14%	5,89%	0,37
	10%	20,11%	9,15%	5,93%	0,37
Neutral 60-40	non-rebalanced	54,94%	11,50%	5,22%	0,24
	1%	59,97%	11,85%	6,66%	0,35
	5%	60,07%	11,86%	6,46%	0,33
	10%	60,20%	11,91%	6,20%	0,31
Agressive 80-20	non-rebalanced	76,52%	13,66%	4,97%	0,18
	1%	80,16%	14,02%	6,21%	0,26
	5%	80,04%	14,06%	5,89%	0,24
	10%	79,98%	14,09%	6,35%	0,27

Figure 21 - Portfolio statistics for various strategies with monthly monitoring, 2006-2017 (Source: author)

Taking a look at rebalancing strategy with monthly monitoring, we can see that the results are very similar to the strategy with weekly monitoring. Again, rebalance with monthly monitoring provides significant improvements in the average stock allocation – making it

over the time. The reason here is that we are considering data set from time period with recession. In 2008 and 2009 the stocks dropped so significantly that their weight decreased for example to only 68 % for aggressive strategy (80 % is the target). Contrary, if we would consider only data from bull market period, the average stock allocation for buy and hold portfolio would be above its target.

very close to target for all the drift and risk strategies. The results are almost the same as for weekly monitoring.

Apart from the conservative portfolio, rebalancing increased portfolio volatility compared to benchmark non-rebalanced portfolio. This is, however, caused by the fact that average stock allocation of the non-rebalanced portfolios is below its target and especially for neutral and aggressive strategies, rebalancing meant increasing equity part of the portfolio. Addition of more volatile assets goes, naturally, hand in hand with the increase of overall volatility.

What is more important is the fact that the average portfolio return was increased by even more than half to one percent for all the risk strategies. An investor is therefore compensated for bearing the additional risk.

Therefore, it can be concluded that not only does **rebalance with monthly monitoring** maintain target asset allocation but also **improves the risk-adjusted return**. The Sharpe ratio increased for all the drift strategies and all the levels of risk. The improvements are very similar for all the forms of rebalancing except for rebalancing strategy with 1% drift which provides slightly better results. The Sharpe ratio improved on average by 37 % for the conservative portfolio, 37 % for neutral portfolio and 42 % for the aggressive strategy.

QUARTERLY monitoring	**Drift**	**Average stock allocation**	**Volatility**	**Average annual return**	**Sharpe ratio**
conservative 20-80	non-rebalanced	17,20%	9,30%	5,08%	0,28
	1%	20,01%	9,17%	5,58%	0,33
	5%	20,04%	9,18%	5,61%	0,34
	10%	20,15%	9,17%	5,49%	0,32
Neutral 60-40	non-rebalanced	54,94%	11,50%	5,22%	0,24
	1%	59,91%	11,84%	5,53%	0,25
	5%	59,96%	11,82%	5,49%	0,25
	10%	60,21%	11,86%	5,33%	0,24
Agressive 80-20	non-rebalanced	76,52%	13,66%	4,97%	0,18
	1%	79,94%	14,01%	4,94%	0,17
	5%	80,03%	14,00%	4,98%	0,18
	10%	79,92%	14,06%	4,73%	0,16

Figure 22 - Portfolio statistics for various strategies with quarterly monitoring, 2006-2017 (Source: author)

Finally, we take a look at the results given by rebalancing strategy with quarterly monitoring. Similar to previously mentioned strategies, the asset allocation is well maintained. The rebalanced portfolios never deviate by more than 0,2% from their target allocation.

The situation is however different with the portfolio performance. Compared to weekly and monthly monitoring strategies, **rebalancing with quarterly monitoring does not really affect volatility and return**. The differences compared to the benchmark portfolio are very small. Therefore, even the Sharpe ratio remains very close to the original one.

To conclude, we can say that **rebalancing provides very good level of the target allocation control** – no matter whether the portfolio is being monitored the portfolio on a

weekly, monthly or quarterly basis. However, **weekly and monthly monitoring provides** – contrary to quarterly monitoring – **improvements in portfolio risk-adjusted returns**. This conclusion can be made for all the drift threshold levels and various portfolio risk strategies.

Comparison of the crisis and post-crisis period is also an interesting thing to mention. When analysing the results, I found out that the performance results differ a little for the first half of the time period (what we can consider being a crisis period) and the second half (2012 onwards, what we can consider as only a booming period). While we can't say that rebalancing generally decreases volatility for the whole period of 2006-2017, we can say that it does decrease volatility during the booming period. It can be explained by the fact that, during the bull market, rebalancing is mainly about selling equities which outperform bonds and buying more conservative assets (like those bonds). Therefore, the overall volatility decreases compared to non-rebalanced portfolio. Moreover, during the booming period, the average equity allocation is always above the target – again, for the same reason of better performance of the equities. The results for Sharpe ratio improvements are similar for both crisis and post-crisis period. The increase in Sharpe ratio caused by rebalancing is however less evident for the booming period.

Rebalancing frequency

The fact that rebalancing frequency would increase with more frequent monitoring and lower drifts would be generally expected. The question is, however, how significant the increase is for various drift levels and monitoring periods. The results of how often rebalance would happened for our observed time period are summarized in following table.

Portfolio		Drift	Number of rebalancing events	Average yearly rebalancing events	Sharpe ratio
Weekly	Conservative 20-80	1%	547	46	0,34
		5%	155	13	0,32
		10%	52	4	0,32
	Neutral 60-40	1%	536	45	0,35
		5%	124	10	0,27
		10%	41	3	0,26
	Agressive 80-20	1%	545	45	0,28
		5%	137	11	0,25
		10%	45	4	0,24
Monthly	Conservative 20-80	1%	130	11	0,43
		5%	75	6	0,37
		10%	33	3	0,37
	Neutral 60-40	1%	130	11	0,35
		5%	71	6	0,33
		10%	30	3	0,31
	Agressive 80-20	1%	130	11	0,26
		5%	71	6	0,24
		10%	29	2	0,27
Quarterly	Conservative 20-80	1%	43	4	0,33
		5%	38	3	0,34
		10%	25	2	0,32
	Neutral 60-40	1%	43	4	0,25
		5%	37	3	0,25
		10%	21	2	0,24
	Agressive 80-20	1%	43	4	0,17
		5%	36	3	0,18
		10%	20	2	0,16

The results provide us with following two key insights:

Firstly, we can see that the **size of the drift is a crucial determinant of rebalancing frequency**. One percent drift threshold triggers rebalance almost every time monitoring happened – for example, it triggered rebalancing 45 times (out 52 weeks) for weekly monitoring, 11 times (out of 12) for monthly monitoring and 4 times (out of 4) for quarterly monitoring. This means **one percent drift threshold triggered rebalancing in approximately 90 % of monitoring events**.

If we would take a look at five percent drift threshold, rebalance is triggered significantly less frequently. Precisely, in about 25 % of monitoring events with weekly frequency (about 12 rebalancing events in a year), 50 % of monitoring events with monthly frequency (6 rebalancing events) and 75 % of monitoring events with quarterly frequency (3 rebalancing events). The increasing percentage can be explained by the fact that drifts generally increase

with the longer time period between monitoring. Though, the absolute numbers of rebalancing events are significantly lower compared numbers given by one percent drift.

Even more significant are the results for the ten percent drift threshold. Using this strategy, rebalance would be triggered in only 7% cases of weekly monitoring (4 times), 25 % cases of monthly monitoring (3 times) and 50 % cases of quarterly monitoring (2 times).

If we would then compare the risk-adjusted returns provided by different rebalancing strategies, we would find that the Sharpe ratio remains more or less the same for all the strategies (only very slight increase can be observed for one-percent drift threshold rebalancing strategy). Combining these findings, we can conclude that it is not rational choosing the rebalancing strategy with one percent drift threshold while it doesn't provide any significant improvement in risk-adjusted return or allocation control and, at the same time, triggers rebalancing during almost every monitoring. **Using five- and ten-percent drift threshold rebalancing strategy provides very similar results in terms of risk-adjusted return while the number of yearly rebalancing events remain moderate**.

Interesting to mention, the combination of one percent drift threshold and quarterly monitoring led to rebalancing being triggered every single time. Using such small drift for quarterly monitoring is irrelevant. Even five percent drift threshold triggers rebalancing in 3 out of 4 times.

Secondly, we can see that **rebalancing is triggered with a very similar frequency for all the risk strategies**. For example, using one percent drift threshold strategy with weekly monitoring (the one that triggers rebalancing most often) would lead to on average 46 rebalancing events for a conservative portfolio, 45 for neutral portfolio and 45 for an aggressive portfolio. Similarly, five percent drift threshold strategy with monthly monitoring would lead to the equal number of 6 rebalancing events for all risk strategies. Therefore, it can be concluded that **higher portfolio risk (higher volatility) does not result in more often rebalancing, no matter which rebalancing strategy is used**.

Average numbers of rebalancing events for various strategies (2006-2011 and 2012-2017)			
Monitoring	Drift	2006-2011	2012-2017
Weekly	1%	48	40
	5%	14	8
	10%	5	3
Monthly	1%	13	9
	5%	7	5
	10%	3	2
Quarterly	1%	4	4
	5%	3	3
	10%	2	2

Figure 23 - Average numbers of rebalancing events for various strategies during and post-crisis period (Source: own calculation)

If we would again compare the crisis and booming period, we would find that **rebalancing is more often triggered during the period of high volatility in the market**. For example,

five-percent drift threshold rebalancing strategy with monthly monitoring had triggered rebalancing on average 7 times per year during the period of 2007-2011 but only on average 5 times during the years 2012-2017. The situation is very similar even for strategies with other drift triggers. Rebalancing frequency generally increased during the crisis period. The only expection is the rebalancing strategies with quarterly monitoring. There, the number of rebalancing events remain unchanged.

6.3.2. Analysis of tax impacts of rebalancing

In the following part, the tax costs of rebalancing will be analyzed. For each year of our observed period, **the gain/loss on the portfolio resulting from only rebalancing events was calculated**. No additional sales of the assets are considered. This gain/loss is not an actual gain/loss because the investor does not get any proceeds since they are immediately reinvested to other asset classes to maintain the target allocation (which is the principle of rebalancing). It will be therefore called "**taxable amount**". This term will refer to the taxable gain produced by rebalancing only.

Further, whenever the thesis talks about average values of taxable amount, only the years when profits were made on sales are included in the calculations of average. Since the goal of this analysis is to examine taxable amounts and liabilities (which are only calculated when profit is made), including losses in the calculations would distort the results.

In the simulation, I assume an investor bought the portfolio at the beginning of the year 2006 and held it for 12 consecutive years. There were no deposits or withdrawals and transactions only happened as a consequence of the need to rebalance. The results would, of course, change for different period examined. Time of the initial purchase plays an important role in calculating the taxable amount. The more the portfolio appreciates from its purchasing value, the higher are the absolute gains (and thus tax liabilities) on assets sold.

I have used the standard approach to the gain/loss calculation for tax purposes, as described by the Czech legislative. Whenever an asset was being sold, gain/loss was calculated as a difference between sell and purchase price. I have used **the average purchasing price** of the asset to calculate the taxable amount.[3] Taxes are then to be paid only in those years where the investor ended up with gain (positive taxable amount). No taxes are paid on losses.

For each rebalancing strategy and year, figures for gains/losses resulting from rebalancing and related tax liabilities will be presented.

The **figures will be shown in a relative way – as a percentage of the average value of the portfolio** during the year. This allows us to apply results to various portfolio sizes. However, we then have to keep in mind that the absolute amount of tax liabilities might

[3] Other methods like for example FIFO can be also used in the Czech environment. However, average purchasing price was used due to the fact that rebalancing can result in small amounts of assets being sold in various days throughout the whole year and matching the assets sold with their actual purchasing price might get very chaotic.

grow every year when the portfolio increases its value while the percentage of the tax amount to portfolio value remains similar.

Firstly, let's take a look at the results for the neutral portfolio (60-40 stocks-bonds ratio):

Monitoring	Drift		2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	Average
Weekly	1%	rebalancing events	45	49	52	52	50	52	51	49	48	50	46	38	**49**
		gain/loss	-0,3%	0,2%	-11,8%	-6,8%	1,0%	0,7%	2,3%	4,2%	5,5%	7,5%	5,3%	3,1%	**3,3%**
		tax liabilities		*0,0%*			*0,1%*	*0,1%*	*0,3%*	*0,6%*	*0,8%*	*1,1%*	*0,8%*	*0,5%*	***0,5%***
	5%	rebalancing events	9	12	22	17	12	13	10	8	6	10	10	5	**11**
		gain/loss	0,0%	0,3%	-9,0%	-4,6%	0,0%	-0,3%	1,1%	1,6%	2,3%	3,6%	3,8%	1,4%	**1,6%**
		tax liabilities	*0,0%*	*0,0%*			*0,0%*		*0,2%*	*0,2%*	*0,4%*	*0,5%*	*0,6%*	*0,2%*	***0,2%***
	10%	rebalancing events	3	3	8	7	3	6	2	3	2	3	2	1	**4**
		gain/loss	0,1%	0,6%	-6,1%	-3,5%	-0,3%	-0,6%	0,3%	0,5%	1,0%	2,4%	2,3%	0,4%	**0,9%**
		tax liabilities	*0,0%*	*0,1%*					*0,0%*	*0,1%*	*0,1%*	*0,4%*	*0,3%*	*0,1%*	***0,1%***
Monthly	1%	rebalancing events	10	12	12	12	12	12	12	12	12	12	11	12	**12**
		gain/loss	0,0%	0,5%	-4,5%	-3,7%	-0,8%	-0,6%	0,4%	1,8%	2,4%	4,1%	3,0%	2,0%	**1,8%**
		tax liabilities	*0,0%*	*0,1%*					*0,1%*	*0,3%*	*0,4%*	*0,6%*	*0,4%*	*0,3%*	***0,3%***
	5%	rebalancing events	5	7	9	7	9	6	8	5	5	6	5	4	**6**
		gain/loss	0,1%	0,6%	-4,4%	-2,9%	-0,8%	-0,6%	0,1%	1,1%	1,5%	2,5%	2,7%	1,1%	**1,2%**
		tax liabilities	*0,0%*	*0,1%*					*0,0%*	*0,2%*	*0,2%*	*0,4%*	*0,4%*	*0,2%*	***0,2%***
	10%	rebalancing events	2	3	3	4	3	3	3	3	2	2	1	2	**3**
		gain/loss	0,2%	0,6%	-2,2%	-1,3%	-0,5%	-0,3%	0,0%	0,1%	1,5%	0,8%	2,2%	0,3%	**0,7%**
		tax liabilities	*0,0%*	*0,1%*					*0,0%*	*0,0%*	*0,2%*	*0,1%*	*0,3%*	*0,0%*	***0,1%***
Quarterly	1%	rebalancing events	3	4	4	4	4	4	4	4	4	4	4	4	**4**
		gain/loss	0,1%	0,4%	-2,5%	-2,1%	-0,6%	-0,3%	0,1%	0,8%	1,6%	2,4%	1,9%	0,6%	**1,0%**
		tax liabilities	*0,0%*	*0,1%*					*0,0%*	*0,1%*	*0,2%*	*0,4%*	*0,3%*	*0,1%*	***0,2%***
	5%	rebalancing events	2	3	4	4	4	3	4	4	3	3	1	3	**3**
		gain/loss	0,1%	0,4%	-2,5%	-2,1%	-0,6%	-0,2%	0,1%	0,8%	1,6%	1,1%	2,0%	0,6%	**0,8%**
		tax liabilities	*0,0%*	*0,1%*					*0,0%*	*0,1%*	*0,2%*	*0,2%*	*0,3%*	*0,1%*	***0,1%***
	10%	rebalancing events	0	2	3	3	2	2	2	2	1	3	1	1	**2**
		gain/loss	0,2%	0,4%	-2,0%	-1,9%	0,0%	0,4%	0,1%	0,9%	1,6%	1,1%	2,0%	0,0%	**0,8%**
		tax liabilities	*0,0%*	*0,1%*				*0,1%*	*0,0%*	*0,1%*	*0,2%*	*0,2%*	*0,3%*		***0,1%***

Figure 24 – Yearly tax amounts caused by various rebalancing strategies, as a percentage of the portfolio value, Neutral portfolio, 2006-2017 (Source: own calculation)

From the table, we can see that during the first half of the observed period – due to the crisis – the rebalance ended up generating losses or only very small gains or taxable amounts (less than 1 % of the portfolio value). The reason behind this fact is straightforward. During the crisis, rebalancing resulted in selling assets of lower value than what was their purchasing value, therefore generating loss.

The situation changed in the year 2012 which was the first year when prices of the assets got above their average purchasing power, mainly influenced by the initial 2006 value and therefore some taxable amount was generated. From the tax perspective, the situation is therefore very similar to the situation when the initial investment would be done at the beginning of the year 2012, where the prices were on the same level as in the year 2006. During the years of 2009-2011, when markets were already growing, no taxable amount was generated by rebalancing because the average purchasing price was still above current price.

By that, it can be concluded that tax impacts of the rebalancing are highly influenced by the purchasing price of the investment and the market performance in the following years. Situation would, of course, be different if the investor would buy the portfolio for example in the year 2009 – rebalancing would result in generating taxable gain/amount straight from the first year. Moreover, the faster the market grows, the greater the tax costs are.

For example, we can see from the table that rebalancing generated biggest amounts in the year 2015 – up to 7,5 % of the portfolio value for one percent drift rebalancing strategy with weekly monitoring. The year 2015 has outperformed all the other observed years. Our neutral portfolio grew by 26 % in that year compared to its annual average of 5,4 % over the last 12 years.

Looking further at the results, we can see that during the market growth years, **the biggest taxable amounts were generated by those strategies which resulted in highest number of rebalancing events** (one and five percent drift threshold strategies with weekly or monthly monitoring. However, no direct relation can be seen. The strategy of one percent drift threshold with weekly monitoring, for example, resulted in about 5 times more frequent rebalancing but only doubled the taxable amounts compared to five percent drift strategy with the same monitoring. The trades triggered by rebalancing were therefore more frequent but smaller.

Next, the results with other portfolio strategies – conservative and aggressive – will be compared:

Monitoring	Drift		2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	Average
Weekly	1%	rebalancing events	45	49	52	52	50	52	51	49	48	50	46	38	**49**
		gain/loss	-0,4%	-0,5%	-7,6%	-2,5%	2,0%	1,8%	2,8%	3,4%	4,2%	6,0%	4,8%	2,4%	**3,4%**
		tax liabilities					*0,3%*	*0,3%*	*0,4%*	*0,5%*	*0,6%*	*0,9%*	*0,7%*	*0,4%*	***0,5%***
	5%	rebalancing events	9	12	22	17	12	13	10	8	6	10	10	5	**11**
		gain/loss	-0,2%	-0,3%	-5,6%	-2,4%	0,9%	1,1%	1,6%	2,3%	2,5%	2,9%	3,5%	1,0%	**2,0%**
		tax liabilities					*0,1%*	*0,2%*	*0,2%*	*0,3%*	*0,4%*	*0,4%*	*0,5%*	*0,2%*	***0,3%***
	10%	rebalancing events	3	3	8	7	3	6	2	3	2	3	2	1	**4**
		gain/loss	0,0%	0,2%	-3,7%	-2,0%	0,5%	0,3%	0,9%	1,2%	0,9%	1,7%	2,6%	1,6%	**1,1%**
		tax liabilities		*0,0%*			*0,1%*	*0,0%*	*0,1%*	*0,2%*	*0,1%*	*0,3%*	*0,4%*	*0,2%*	***0,2%***
Monthly	1%	rebalancing events	10	12	12	12	12	12	12	12	12	12	11	12	**12**
		gain/loss	-0,1%	-0,1%	-3,1%	-1,9%	0,4%	0,5%	1,7%	1,9%	2,0%	3,6%	2,5%	1,3%	**1,7%**
		tax liabilities					*0,1%*	*0,1%*	*0,3%*	*0,3%*	*0,3%*	*0,5%*	*0,4%*	*0,2%*	***0,3%***
	5%	rebalancing events	5	7	9	7	9	6	8	5	5	6	5	4	**6**
		gain/loss	0,0%	0,0%	-2,8%	-1,9%	0,3%	0,4%	1,1%	1,2%	1,5%	2,0%	2,6%	1,2%	**1,3%**
		tax liabilities					*0,0%*	*0,1%*	*0,2%*	*0,2%*	*0,2%*	*0,3%*	*0,4%*	*0,2%*	***0,2%***
	10%	rebalancing events	2	3	3	4	3	3	3	3	2	2	1	2	**3**
		gain/loss	0,0%	0,1%	-1,7%	-1,0%	0,2%	0,2%	0,8%	0,7%	1,0%	1,7%	1,2%	1,9%	**0,8%**
		tax liabilities	*0,0%*	*0,0%*			*0,0%*	*0,0%*	*0,1%*	*0,1%*	*0,2%*	*0,2%*	*0,2%*	*0,3%*	***0,1%***
Quarterly	1%	rebalancing events	3	4	4	4	4	4	4	4	4	4	4	4	**4**
		gain/loss	0,0%	0,0%	-1,3%	-1,4%	0,3%	0,6%	0,8%	1,0%	1,1%	2,4%	2,0%	0,6%	**1,0%**
		tax liabilities		*0,0%*			*0,0%*	*0,1%*	*0,1%*	*0,2%*	*0,2%*	*0,4%*	*0,3%*	*0,1%*	***0,1%***
	5%	rebalancing events	2	3	4	4	4	3	4	4	3	3	1	3	**3**
		gain/loss	0,0%	0,0%	-1,3%	-1,4%	0,3%	0,6%	0,8%	1,0%	1,1%	1,8%	2,0%	0,6%	**0,9%**
		tax liabilities		*0,0%*			*0,0%*	*0,1%*	*0,1%*	*0,2%*	*0,2%*	*0,3%*	*0,3%*	*0,1%*	***0,1%***
	10%	rebalancing events	0	2	3	3	2	2	2	2	1	3	1	1	**2**
		gain/loss	0,0%	0,0%	-1,1%	-1,2%	0,2%	0,3%	0,9%	0,6%	1,0%	1,8%	1,9%	0,5%	**0,7%**
		tax liabilities	*0,0%*	*0,0%*			*0,0%*	*0,0%*	*0,1%*	*0,1%*	*0,1%*	*0,3%*	*0,3%*	*0,1%*	***0,1%***

Figure 25 – Yearly taxable amounts caused by various rebalancing strategies, as a percentage of the portfolio value, Conservative portfolio, 2006-2017 (Source: own calculation)

Monitoring	Drift		2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	Average
Weekly	1%	rebalancing events	46	49	52	52	51	52	52	49	50	50	48	42	**49**
		gain/loss	-0,1%	0,7%	-11,9%	-8,6%	-0,6%	-1,1%	1,1%	3,5%	5,1%	7,0%	5,0%	3,2%	**3,7%**
		tax liabilities		*0,1%*					*0,2%*	*0,5%*	*0,8%*	*1,1%*	*0,7%*	*0,5%*	***0,5%***
	5%	rebalancing events	10	14	24	19	16	14	10	9	8	9	9	4	**12**
		gain/loss	0,0%	0,6%	-9,3%	-5,6%	-1,1%	-1,4%	0,2%	1,5%	2,6%	2,9%	2,9%	0,9%	**1,5%**
		tax liabilities	*0,0%*	*0,1%*					*0,0%*	*0,2%*	*0,4%*	*0,4%*	*0,4%*	*0,1%*	***0,2%***
	10%	rebalancing events	3	4	8	7	5	5	3	4	2	3	4	1	**4**
		gain/loss	0,2%	0,8%	-5,1%	-4,2%	-1,0%	-1,4%	0,1%	0,6%	1,0%	2,7%	2,5%	0,4%	**1,0%**
		tax liabilities	*0,0%*	*0,1%*					*0,0%*	*0,1%*	*0,1%*	*0,4%*	*0,4%*	*0,1%*	***0,2%***
Monthly	1%	rebalancing events	10	12	12	12	12	12	12	12	12	12	12	12	**12**
		gain/loss	0,1%	0,8%	-4,7%	-3,9%	-2,0%	-1,5%	-0,6%	1,3%	2,3%	3,6%	3,0%	2,1%	**1,9%**
		tax liabilities	*0,0%*	*0,1%*						*0,2%*	*0,3%*	*0,5%*	*0,4%*	*0,3%*	***0,3%***
	5%	rebalancing events	5	6	9	10	8	7	6	6	5	6	5	3	**6**
		gain/loss	0,1%	0,8%	-4,7%	-3,5%	-2,0%	-1,2%	-0,5%	0,8%	1,6%	2,0%	2,6%	0,4%	**1,2%**
		tax liabilities	*0,0%*	*0,1%*						*0,1%*	*0,2%*	*0,3%*	*0,4%*	*0,1%*	***0,2%***
	10%	rebalancing events	2	3	4	3	2	4	3	2	2	2	1	2	**3**
		gain/loss	0,2%	0,9%	-2,1%	-1,4%	-0,9%	-0,8%	-0,7%	0,1%	1,5%	1,0%	1,7%	0,3%	**0,8%**
		tax liabilities	*0,0%*	*0,1%*						*0,0%*	*0,2%*	*0,2%*	*0,3%*	*0,1%*	***0,1%***
Quarterly	1%	rebalancing events	3	4	4	4	4	4	4	4	4	4	4	4	**4**
		gain/loss	0,2%	0,6%	-2,8%	-2,4%	-1,3%	-1,0%	-0,5%	0,5%	1,6%	2,1%	1,6%	0,4%	**1,0%**
		tax liabilities	*0,0%*	*0,1%*						*0,1%*	*0,2%*	*0,3%*	*0,2%*	*0,1%*	***0,2%***
	5%	rebalancing events	2	3	4	4	4	3	4	4	3	3	3	2	**3**
		gain/loss	0,2%	0,7%	-2,8%	-2,4%	-1,3%	-0,9%	-0,4%	0,5%	1,6%	1,5%	1,4%	0,4%	**0,9%**
		tax liabilities	*0,0%*	*0,1%*						*0,1%*	*0,2%*	*0,2%*	*0,2%*	*0,1%*	***0,1%***
	10%	rebalancing events	0	3	2	3	2	2	2	2	0	3	1	1	**2**
		gain/loss	0,3%	0,6%	-2,5%	-2,0%	-0,4%	0,2%	-0,2%	-0,3%	2,2%	1,0%	1,6%	0,0%	**1,0%**
		tax liabilities	*0,0%*	*0,1%*				*0,0%*			*0,3%*	*0,2%*	*0,2%*		***0,1%***

Figure 26 - Taxable amounts caused by various rebalancing strategies, as a percentage of the portfolio value, Aggressive portfolio, 2006-2017 (Source: own calculation)

We can see that the results do not significantly differ for various risk strategies. Despite slight differences in the values of taxable amounts in specific years, the overall trend and the averages are very similar for all risk strategies. For both conservative and aggressive strategy, the **values of taxable amounts generated by rebalancing lay mostly between the range of 1-5 % of the portfolio value**.

To compare the overall results throughout various rebalancing strategies and risk profiles, average figures are summarized in the following table. Moreover, the table also provides comparative figures generated by running the simulation only over the period of 2012-2017 (only the growing period).

Average yearly taxable amounts							
		2006-2017			**2012-2017**		
Monitoring	Drift	Coservative	Neutral	Aggressive	Coservative	Neutral	Aggressive
Weekly	1%	3,4%	3,3%	3,7%	3,3%	5,1%	5,7%
	5%	2,0%	1,6%	1,5%	2,3%	2,8%	3,3%
	10%	1,1%	0,9%	1,0%	1,1%	1,6%	1,7%
Monthly	1%	1,7%	1,8%	1,9%	2,1%	3,0%	3,1%
	5%	1,3%	1,2%	1,2%	1,6%	2,1%	2,2%
	10%	0,8%	0,7%	0,8%	1,2%	1,5%	1,5%
Quarterly	1%	1,0%	1,0%	1,0%	1,3%	1,7%	1,8%
	5%	0,9%	0,8%	0,9%	1,3%	1,6%	1,7%
	10%	0,7%	0,9%	1,0%	1,1%	1,5%	1,8%

Figure 27 - Average yearly taxable amounts for various rebalancing strategies, as % of the portfolio value (Source: author)

Several insights can be found in the table. Firstly, the average taxable amounts might seem to be higher in the only-growing-period. However, this might only be caused by the fact that the averages for the period of 2006-2017 are lowered by the very low taxable amounts generated in the first half (crisis period) of our observed period.

Secondly, **the risk level of the portfolio did not really matter in the longer run, but mattered in the only-growing-period**. To be precise, portfolio volatility did not affect taxable amounts but the portfolio return did. The reason behind is, as we found in the previous part of the analysis, the greater volatility did not increase the rebalancing frequency.

While the figures for various rebalancing strategies did not significantly differ for conservative, neutral and aggressive portfolio during the whole period of 2006-2017, the figures differ if the simulation runs only for the time period of 2012-2017. In the later period, the more aggressive strategy generated higher taxable amounts for all rebalancing strategies. The difference is most significant in the weekly monitoring strategy. The reason to explain this can be found in the different performance of the portfolio over the two periods. While the average annual returns were very similar for different risk levels looking at the whole period, the neutral and aggressive portfolio performed better than the conservative one in the period of 2012-2017. By that, it can be concluded that **higher average return on the portfolio results in higher taxable amounts.**

Thirdly, taxable amounts from the rebalancing on average did not exceed 6 % of the portfolio value in a given year. On the other hand, none of the tested strategies resulted in generating taxable amounts lower than the average of 0,5 %. For **most of the rebalancing strategies, the taxable amounts were between 1-4 % of the portfolio value, yearly**. In the years of record returns, the rebalancing produced increased taxable amount – up to 7 % of the portfolio value.

Tax liabilities can be then simply counted as the taxable amount multiplied by the current Czech tax rate – 15 %. In our case, it would mean the **average yearly tax liabilities caused by rebalancing of about 0,2-0,6 % of the portfolio value**. To illustrate it, holding a rebalanced portfolio with the average value of 1 million Czech crowns would result in tax liabilities of 2 000-6 000 Czech crowns.

At this point, the topic of tax exemption comes to play, too. We know that investors do not have to pay taxes if their taxable gains (tax amounts) are lower than 100 000 CZK within one year. If we would assume there were no other sales of assets made by the investor within that year (not only on this portfolio but on all his portfolios together) and no dividends or interests obtained, the portfolio of average value of 1,5 million Czech crowns (or lower) would most likely not exceed the 100 thousand taxable limit.

Furthermore, if an investor holds certain asset for a time period of 3 year or longer, there are no tax impacts related to the profit generated by the sale of such asset. In our simulation, this rule would apply to all of those instruments bought in the year 2006 and then, to all instruments bought during rebalancing events that were held for 3 and more years. Investor

has to however keep a precise record of all instruments bought with exact date and quantity to be able to match sold and bought prices.

Important to mention, the goal for our simulation was to quantify taxable amounts generated by rebalancing and subsequent tax liabilities. Since similar results were obtained by running the simulation from the year 2012 (where taxable gains were generated by rebalancing straight from the first year), we can conclude that the results of average taxable amounts generated by rebalancing can be applied in a more general way.

The relation between average taxable amount and number of rebalancing events

Looking at the numbers of rebalancing events given by various strategies and the corresponding average taxable amounts, we can see that there might be some dependence between those two variables. See the following table:

Average yearly taxable amounts, numbers of rebalancing events for various strategies (2006-2017)

		Coservative		Neutral		Aggressive	
Monitoring	Drift	#	avg. taxable amount	#	avg. taxable amount	#	avg. taxable amount
Weekly	1%	46	3,4%	45	3,3%	45	3,7%
	5%	13	2,0%	10	1,6%	11	1,5%
	10%	4	1,1%	3	0,9%	4	1,0%
Monthly	1%	11	1,7%	11	1,8%	11	1,9%
	5%	6	1,3%	6	1,2%	6	1,2%
	10%	3	0,8%	3	0,7%	2	0,8%
Quarterly	1%	4	1,0%	4	1,0%	4	1,0%
	5%	3	0,9%	3	0,8%	3	0,9%
	10%	2	0,7%	2	0,9%	2	1,0%

Figure 28 - Average yearly taxable amounts and numbers of rebalancing events for various strategies, 2006-2017 (Source: author)

To measure the relation between the average taxable amount and number of rebalancing events I used linear regression from the tools of statistics. In general, a linear regression model estimates the relation between the dependent variable (in this case the taxable amount) and the explanatory variable(s), which is the number of rebalancing events in my model. For this exact purpose, the measure is the so-called R squared (R^2), and its value can be between 0 and 1, depending on the tightness of the relation. In the model, we examine a sample of 27 based on available data, which we find sufficient in this case (generally although the statistically relevant samples should contain at least 30 items, however, based on the modelling limitations it can be different). The function of the line that best fits our dataset is the following, with an R^2 of 0,94:

$$\text{Average taxable amount} = 0.81\ \% + 0.06\ \% * \text{Number of rebalancing events}$$

It means that due to the high rate of R^2, the relation is statistically strong between the variables and ceteris paribus increasing the number of rebalancing events by 1 unit would result in 0,06 % change in average regarding the taxable gain. The F-test of the regression model (value of 460,97) suggests that the built model is statistically relevant at a significance level of 5 %.

6.3.3. Results summary

Based on the analysis of the portfolio performance over the time period of 2006-2017 using various risk strategies and rebalancing approaches, the following conclusions can be drawn:

- **Rebalancing generally significantly helped maintaining the target portfolio allocation.** Over the period of 2006-2017, the actual allocation does not deviate by more than 0,3 % from its target, no matter which rebalancing strategy was used.

- **Rebalancing with weekly and monthly monitoring increased risk adjusted return**. The Sharpe ratio on average improved by on average 30 % for weekly monitoring and by 40 % using monthly monitoring. (average improvement for all risk strategies)
  By contrast, the same conclusion can't be reached with rebalancing with quarterly monitoring where the Sharpe ratio remained more or less unchanged. The increase in Sharpe ratio was either zero or less than 3 %.

- **Rebalancing generally did not decrease portfolio volatility**. To be precise, rebalancing did decrease volatility only during bull market period, when equities outperform bonds. During the recession, however, rebalancing increases volatility because more stable bonds are being sold and falling equities bought to the portfolio to maintain the target allocation.

- **Lower drift thresholds trigger rebalancing more often.** One-percent drift threshold triggered rebalancing in approximately 90 % of monitoring events. Five-percent drift thresholds resulted in rebalancing being triggered 12, 6 and 3 times using weekly, monthly and quarterly monitoring, respectively. Ten-percent drift threshold led to on average only 3 rebalancing events per year, for all monitoring strategies.

- **Rebalancing was triggered with very similar frequency for all the risk strategies.** Higher portfolio risk (higher volatility) does not result in more often rebalancing, no matter which rebalancing strategy is used**.** Conservative, Neutral and Aggressive portfolio strategies triggered similar numbers of rebalancing events.

- **Rebalancing was more often triggered during the period of high volatility in the market, like during the crisis for example.** With the exception of quarterly monitoring strategy, rebalancing was triggered significantly more often during the years 2006-2011 compared to 2012-2017.

- **The tax impacts of rebalancing are highly influenced by the purchasing price of the investment and the market performance in the following years**. Of course, whenever the current price of the asset gets bellow its average purchasing price, no taxable amounts are generated by rebalancing.

- The **portfolio volatility does not affect taxable amounts** but portfolio return did - **higher average return of the portfolio results in higher taxable amounts.**

Highest taxable amounts by rebalancing (up to 7 % of the total portfolio value) were generated in the year 2015 in which portfolios grew by record number of 26 %.

- **A strong statistical relation can be found between the number of yearly rebalancing events (frequency) and the average taxable amounts.** With an R^2 of 0,94, following function of the line that best fits my data set can be found: Average taxable amount = 0.81 % + 0.06 % * Number of rebalancing events

- For **most of the rebalancing strategies, the taxable amounts were between 1-4 % of the portfolio value, yearly.** The average values of the taxable amounts over several years never deviated from the range of 0,5-7 %, using various rebalancing strategies.

- **The amount of the taxes to be paid were therefore on average between 0,2-0,6 % of the portfolio value, yearly.** This amount of taxes paid have to be considered while calculating real return of the portfolio. We can conclude that **rebalancing would have decreased annual portfolio returns by on average 0,2-0,6 %.**

6.3.4. Rebalancing strategy evaluation

We have seen that all the rebalancing strategies provided an excellent level of maintaining the target allocation which is the primary purpose of rebalancing. Moreover, apart from rebalancing strategy with quarterly monitoring, both monthly and weekly monitoring strategies increase the portfolio risk-adjusted return. That means investors are either earning more for the risk they bear or are earning the same returns with lower risk. Either way, they are better compensated for the risk they bear. Going for the strategy which would provide the best improvement of risk-adjusted return would seem like the best option. Rebalancing strategies with one percent drift threshold and either monthly or weekly monitoring would then be the best option. However, we have to consider costs, too.

We have seen that more frequent rebalancing generated higher taxable amounts – and therefore higher tax liabilities. So, the question is what strategy provides the best combination

of rebalancing frequency and subsequent costs and benefits in the form of improvements of the risk-adjusted returns. The following table presents the summary of our analysis results:

Average yearly taxable amounts, numbers of rebalancing events and Sharpe ratios for various strategies (2006-2017)

		Coservative			Neutral			Aggressive		
Monitoring	Drift	#	Sharpe ratio	avg. taxable amount	#	Sharpe ratio	avg. taxable amount	#	Sharpe ratio	avg. taxable amount
Never	-	-	0,28	-	-	0,24	-	-	0,18	
Weekly	1%	46	0,34	3,4%	45	0,35	3,3%	45	0,28	3,7%
	5%	13	0,32	2,0%	10	0,27	1,6%	11	0,25	1,5%
	10%	4	0,32	1,1%	3	0,26	0,9%	4	0,24	1,0%
Monthly	1%	11	0,43	1,7%	11	0,35	1,8%	11	0,26	1,9%
	5%	6	0,37	1,3%	6	0,33	1,2%	6	0,24	1,2%
	10%	3	0,37	0,8%	3	0,31	0,7%	2	0,27	0,8%
Quarterly	1%	4	0,33	1,0%	4	0,25	1,0%	4	0,17	1,0%
	5%	3	0,34	0,9%	3	0,25	0,8%	3	0,18	0,9%
	10%	2	0,32	0,7%	2	0,24	0,9%	2	0,16	1,0%

Figure 29 - Comparisson of various rebalancing strategies and portfolios - average numbers of rebalancing events, sharpe ratios, taxable amounts, 2006-2017 (Source: author)

We can see that while rebalancing strategies with one percent drift threshold generally provide the highest improvement in the risk-adjusted return, they result in a significantly higher number of rebalancing events and taxable amounts compared to five and ten percent drift strategies.

The difference in risk-adjusted return between five and ten-percent drift threshold rebalancing strategies with both weekly and monthly monitoring is marginal. At the same time, ten percent drift threshold strategies resulted in a significantly lower number of rebalancing events (3 times less for weekly monitoring and two times less for monthly monitoring). Also, as it was found out in the analysis, rebalancing quantity influenced the taxable amounts.

The **strategies of ten-percent drift threshold and either weekly and monthly monitoring** which provided very similar results in terms of cost/benefit ratio **turned out to be the best rebalancing strategies** in our observed period and for our portfolios. They both provided a slight increase in risk-adjusted return and ended up generating taxable costs of about 1 % of the total portfolio value.

One percent taxable amount of the portfolio value means decreasing the portfolio return by about 0,15 % annually by the rebalancing strategies which turned out to be the best. The positive effect of rebalancing on the risk-adjusted returns is therefore lower.

Moreover, those strategies, on average, triggered rebalancing only 2-4 times in the year for all the risk strategies. It is an essential finding for applicability of the simulation results in real life situations. In our simulation, I have set an assumption that there are no transactional costs. In real life, however, there are transactional costs which question the effect of risk-adjusted return benefits of rebalancing. The improvements of risk-adjusted returns observed in the simulation are not that significant, and the existence of transactional costs might minimize them to zero (or even turn them to negative numbers) – especially for rebalancing strategies which resulted in the high number of rebalancing events like the one percent drift

threshold strategies. Similarly, for rebalancing strategies with quarterly monitoring which itself did not bring any improvements in risk-adjusted returns, the total real-life effects of rebalancing on performance might be negative.

The findings therefore suggest that for the wealth managers or robo advisors, the best rebalancing strategy to choose is the one with ten-percent drift threshold and either weekly or monthly monitoring based on how accurate they want to be with keeping the actuall allocation in line with the target one. While weekly monitoring will make sure the weights of different asset classes are in almost perfect order all the time, it might result in more frequent rebalancing, especially during the high-volatility period, like for example the crisis. If the wealth managers or robo advisors anticipate times of recession, it might be good idea to turn to monthly monitoring to prevent excess rebalancing which could result in high tax costs while not compensating investors adequately in terms of risk-adjusted return. On the other hand, if a bull market is anticipated by the wealth managers, monitoring the portfolio on a weekly basis might be good option mainly to keep the target portfolio risk level while generate only moderate tax costs.

7. Conclusion

Although the topic of portfolio rebalancing has been researched in several papers, the aspect of its tax impacts for an investor located in the Czech Republic remained yet uncovered. This thesis analysed the effect of portfolio rebalancing on a globally diversified portfolio and assessed what the tax costs of various rebalancing strategies for a Czech investor are.

The analysis is based on the simulation of rebalancing on a globally diversified portfolio over the period of the last 12 years (2006-2017). The results obtained by the analysis, therefore, provide up-to-date insights into the topic of portfolio rebalancing with its benefits and costs which can be practically used by both investors and investment firms providing investment services in the Czech Republic when deciding what rebalancing strategy, if any, to pursue on their investments.

Based on the analysis of various portfolio risk strategies and rebalancing parameters, the following can be concluded. The findings generally support the results of other studies conducted on the topic of the effect of rebalancing on portfolio performance. While all rebalancing strategies, in general, help maintain the target portfolio allocation, the impact of rebalancing on the risk-return trade-off remains disputable and depends on many factors. Among the main determinants, we have to mention the rate of transactional costs related to trading due to the need to rebalance, the amount of tax costs related to the sale of assets, the length of observed time period or the current phase of the economic cycle (if only the short time frame is considered).

Especially when using a rebalancing strategy with low drift thresholds (e.g., 1 or 5 %) and frequent monitoring (e.g. weekly), high amount of rebalancing events occurred, and following costs are raising dramatically, offsetting the otherwise slightly positive effect of rebalancing on the risk-adjusted returns.

The findings presented in this thesis also provide a unique insight into the relation of portfolio risk level and rebalancing frequency, which often remained unaddressed in academic papers. In my simulation, three different portfolio allocations with various risk (volatility) levels were tested. The findings suggest that despite higher volatility – and therefore higher fluctuation of the value of the assets – the portfolios did not require to be rebalanced more often in the observed period. Whether an investor would have chosen to go for rather a conservative or aggressive strategy, his choice would not have affected the number of rebalancing events.

Moving further into the area of costs related to rebalancing, the resultsof my simmulation suggest that there is a strong relation between the number of rebalancing events (or rebalancing frequency) and the tax costs of rebalancing. Not only the transactional costs are rising with increasing number of rebalancing events, but tax costs are rising dramatically, too. By that, we can say that with the increasing number of rebalancing events happening in a year, the costs outweigh benefits and rebalancing becomes somewhat harmful than helpful.

Rebalancing strategies with ten-percent drift threshold and either weekly or monthly monitoring turned out as providing the best overall results in the observed period. These strategies resulted in triggering on average three rebalancing events yearly and generating taxable amount worth of about 1 percent of the total portfolio value. Moreover, they maintained the target allocation in a range of deviation lower than 0,3 percent and improved the risk-adjusted return (measured by the Sharpe ratio) by about 30 % compared to the non-rebalanced portfolio, if not considering the transaction and tax costs.

As a result, we can conclude that rebalancing strategies with reasonable drift threshold (such as ten percent) and either monthly or weekly monitoring frequencies are likely to provide investors in the Czech Republic with sufficient risk control while keeping the tax costs on a moderate level for most portfolios with globally diversified stock and bond holdings.

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