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Welfare Based Evaluation of Monetary Policy Conduct in the Czech Republic

Master Thesis

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I declare that this thesis was composed by myself, that the work contained herein is my own except where explicitly stated otherwise in the text, and that this work has not been submitted for any other degree or professional qualification.

Signed:

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Abstract

This thesis provides a welfare analysis of the inflation-targeting regime and its variations that take the nominal exchange rate stability into the consideration. The analysis done in this work aims to answer a question, whether there is a room for improvements in the current Czech monetary policy framework. In particular, this work utilizes a dynamic stochastic general equilibrium (DSGE) model of a small open economy and compares the current inflation-targeting regime to its hypothetical variations that incorporate nominal exchange rate stabilization. The results presented in this work suggest that the incorporation of the exchange rate stability could be beneficial in terms of welfare. However, as indicated by a robustness test, these results are sensitive to the calibration of the model and further research on this topic is advisable before making any conclusions. The other contribution of this thesis is showing how to solve a small open economy model without log-linearization. This allows solving the presented small open economy model and its variations using the higher-order approximation methods.

Key Words: Macroeconomics and Monetary Economics, Keynes, Monetary Policy, Comparative or Joint Analysis of Fiscal and Monetary Policy, Computable General Equilibrium Models, DSGE

JEL classification: E00, E12, E52, E63, C68

Abstrakt

Tato diplomová práce se zabývá analýzou režimu inflačního cílování a jejich alternativ, které berou v potaz stabilizační politiku měnových kurzů. Cílem práce bylo zkoumat dopady těchto alternativ na ekonomický blahobyt. Práce využívá DSGE modelu otevřené ekonomiky pro srovnání hypotetických alternativ s aktuálním režimem. Výsledky zprve ukazují pozitivní dopady na blahobyt při zapojení stability měnového kurzu vedle cenové stability. Tyto výsledky jsou ale citlivé na kalibraci modelu a je doporučeno další testování robustnosti těchto výsledků. Práce také ukazuje, jak vyřešit model otevřené ekonomiky bez použití log-linearizace, respektivě jak vyřešit prezentovaný model použitím odhadovacích metod druhého a vyšších řádů.

Klíčová slova: Makroekonomie a Monetární Politika, Keynes, Monetární Politika, Srovnávání Monetárních Politik, Modely obecné rovnováhy, DSGE

JEL klasifikace: E00, E12, E52, E63, C68

Contents

1 Introduction

The primary objective of the Czech National Bank is to maintain price stability. It does so by changing the monetary conditions using its available instruments. In the normal situation, it is primarily the policy rate that is used to affect market interest rates and therefore the behavior of market participants. The Bank conducts its monetary policy under the so-called inflation-targeting regime which works as the nominal anchor and therefore translates into price stability. The regime was introduced in 1998 after the failure of the central bank to maintain the fixed exchange rate regime. Arguably, since the shift to the inflation-targeting, the prices have to most extent stable. In addition, the Czech National Bank is considered to be the frontier of inflation-forecast targeting [Alichi et al., 2015]. The price stability is considered to be the major factor when it comes to the economy's performance. Stable prices, or in practice, the low uncertainty of its movements, help the public to plan their economic activities as consumption and investment. Unstable prices on, the other hand, mute the economic activity as people are uncertain about the future and the uncertainty makes it harder for them to make any long-term plans.

The scope of this work is to investigate whether the inflation-targeting regime can be improved by considering currency rate stability in addition to the price stability. Nowadays, including the Czech Republic, most countries have open borders and international trade plays an important role. The international trade is mostly affected by the nominal exchange rate and one could argue that its stability is also crucial for the public to make well-planned decisions. There is however a trade-off between the price and exchange rate stability as both are closely linked to the market interest rates. A full exchange rate stability, i.e. currency peg, is therefore not a feasible goal. But as is the case in many economic problems, in practice, the optimal solution usually resides between the two opposite extremes. The thesis conducts an analysis by incorporating the exchange rate stabilization, to some but not to the full degree, within the inflation-targeting regime.

The analysis is done using a dynamic stochastic general equilibrium (DSGE) model which provides a laboratory-like environment, where different policy regimes can be compared. The literature on this approach can be, for example, found in [Schmitt-Grohé and Uribe, 2004a] who try to find the optimal fiscal and monetary policy rules for a closed economy. The thesis focuses on the Czech Republic case and therefore the analysis is conducted on the small open economy model published by [Galí and Monacelli, 2005]. Their model is replicated in this thesis, however, the thesis contributes by solving the model in levels which allows solving the model using higher-order approximation methods as opposed to the authors who derived the model's equilibrium conditions using log-linearization (which allows only for linear approximation methods). The higher-order approximation solutions can be utilized in other research questions where the uncertainty and higher-order terms play a more significant role, i.e. welfare or risk premium studies.

The structure of the thesis is the following: The chapter 2 presents the relevant literature and describes the inflation-targeting and its transmission mechanism in more details. The chapter lays on the argument and the theory behind it and its linkage to the price stability goal. The chapter 3 then presents the DSGE model. The chapter introduces the underlying assumptions of the small open economy model and it then guides the reader how to derive the equilibrium conditions in levels. The third chapter 4 collects the equilibrium conditions of the model and takes the model to the data. The model is calibrated to reflect the Czech economy and then it concludes by presenting the welfare results of incorporating exchange rate stability as the additional goal within the inflation-targeting regime.

2 Literature review

2.1 Central banking

Central banks play undoubtedly an important role when it comes to a country's economy. There is a fair amount of consensus in the literature about the importance of monetary policy and its role in promoting sustainable economic growth that translates into economic welfare. Central banks can conduct their monetary policy in a way that stabilizes business cycles, avoiding unnecessarily deep and long recessions and at the same time avoiding excessive inflationary pressures. Poorly conducted monetary policy will on the other hand negatively affect welfare by destabilizing the economy, creating too inflationary or deflationary pressures and crippling the economy.

For example, an expansionary monetary policy might boost the economic growth in the short-term, but it might also end up generating too high inflationary pressures eventually resulting in welfare loss. The way how central banks specifically conduct their monetary policy will differ across countries based on their historical experience, specific environments, and beliefs of the monetary authority in charge. This thesis primarily focuses on the case of the Czech Republic and therefore the thesis pays most of its attention on inflation-targeting which is at present conducted by the Czech National Bank.

2.2 Price stability

The economic literature emphasizes the importance of stable prices and its role in supporting stable economic growth. Stable prices mitigate uncertainty among households and firms helping them in making the right economic decisions and long-term plans. Low uncertainty of the future price dynamics helps the public to make well-informed economic decisions about their consumption and investment choices, ensuring effective allocation of resources and maximizing welfare at the end. Arguably, the literature recommends the price stability to be the central goal of any sound monetary policy and advocates the notion that the price stability should be put at first when it comes to all economic variables relevant for economic welfare [Woodford, 2003], [Svensson, 2010].

One of the ways how to ensure stable prices is to set some form of a nominal anchor. There is an argument that without the nominal anchor, central banks will behave too opportunistically, i.e. they will focus only on policies with short-term gains. The opportunistic behavior will be reflected in too expansionary monetary policy that will achieve economic gains in terms of higher output in the short and medium run, but undesirable inflationary pressures in the long run leading to price instability. In addition, if the opportunistic behavior becomes frequent, the public will find the expansionary policy less credible and its effectiveness in stimulating output will diminish over time. Therefore, these central banks will have to eventually cause even more inflationary pressures in order to achieve a certain gain in output.

Nominal anchors can have different forms, one of them is, for example, a gold standard or a currency peg. Both are very similar in a way that a central bank commits itself to fix the exchange rate of its domestic currency, either to gold or to another currency. The rationale behind both of these is that the central bank will not be able to conduct expansionary policy without constraints as that would eventually lower domestic nominal interest rate which would cause capital (or gold) to outflow to countries with higher nominal interest rate and that would lead to currency depreciation.

Both of these forms however also introduce a set of other difficulties, especially a golden standard is unsuitable in modern economies. While the commitment to a fixed exchange rate prevents central banks from behaving irresponsibly, it also limits them in conducting expansionary monetary policy in situations in which it would be beneficial. Expansionary monetary policy does not have to be necessarily harmful as long as it is conducted at the right time and to the right extent. It can, for example, help the economy to deal with an on-going recession by stimulating a falling aggregate demand and it can prevent turning the recession into a depression. Not reacting to these situations due to an exchange rate commitment can be more costly than the inflationary pressures resulting from expansionary monetary policy[1].

2.3 Discretion, rules and inflation-targeting

Like any other economic problem, this is essentially a trade-off problem. One extreme solution is to leave the central bank left to do what is right (discretion), which brings the risk of having a policy-maker that acts irresponsibly by maximizing the short- and medium-run gains regardless of the long-run implications. The second extreme is to set a fixed rule for the central bank to follow, which limits the central bank's role in stabilizing the economy and reacting properly in unexpected situations. And as in any other economic problems, the solution resides somewhere between the possible two extreme options. Notably, this is eventually a problem caused by imperfect information and information asymmetry. Central banks are allowed to behave irresponsibly as households and firms do not have complete/sufficient information on how their policies affect future prices. And unless the central banks are motivated to collect information on the possible long-run cost of their policies, they will tend to focus on policies with immediate gains. The same applies to the rule situation. No one has perfect information about the future development and shocks

[1] This is known as the Mundell-Fleming trilemma put by [Mundell, 1963] and [Fleming, 1962]. A central bank in a modern open economy faces a trilemma - it can only choose two out of three options between having open borders to capital inflow/outflow, fixed exchange rate or be able to stabilize the economy using a monetary policy.

and no one can, therefore, construct a perfect rule that would capture all possible scenarios. Relying on a simple rule as an exchange peg or a gold standard can be quite unsuitable.

To sustain price stability and at the same time be able to act upon unexpected situations that affect the economy, the Czech National Bank functions under the inflation-forecast targeting framework. Under this approach, an official target for inflation rate is set as the ultimate goal of monetary policy. This introduces a constraint to prevent the central bank from systematically engaging in policies with too inflationary consequences but it also leaves the central bank some discretion for short- and medium-run stabilization objectives, i.e. stabilization of output or exchange rate. [Bernanke and Mishkin, 1997] call this regime a constrained discretion in the sense that the inflation-targeting regime increases the discipline and accountability of monetary policymakers as they have to explain not only the short- and medium-run but also the long-run implications of their policies. The inflation-targeting central banks are explicitly obliged to show the public the trade-off between the short-run (and the medium-run) gains and long-run costs. Similarly, a monetary policy that is costly in the short- and medium-run can be easier defended by its long-run benefits if the benefits are in line with the ultimate goal, price stability. Having a more complete information on the trade-offs, the public can make a better-informed decision whether they approve it or not. There is still uncertainty about the future implications of monetary policies but the inflation-targeting regime set the incentives for the bank to act to minimize this uncertainty.

The Czech National Bank implemented the inflation targeting framework in 1998. During the transformation period, the role of the central bank was to target monetary aggregate M2 and keep a fixed nominal exchange rate to the German mark and the US dollars [Clinton et al., 2017]. Both served as nominal anchors at that time. The central bank had to however later abandon the fixed exchange rate regime mainly due to a currency crisis in 1997. During 1995, the higher interest rate and the low currency risk became a source for higher foreign investment inflow. This caused pressures on the domestic currency appreciation and the Czech National Bank had to decide between keeping the inflation low or letting the Czech koruna to appreciate. The Czech National Bank utilized the sterilized currency intervention at first to deal with this issue, however, the effectiveness of these interventions had become weaker after some period. The speculative attacks on currency in 1997 then forced the Czech National Bank to leave the fixed exchange rate regime [Slaný et al., 2006]. After this event in 1998, the central bank decided to implement the inflation-targeting regime as a more suitable nominal anchor. The inflation-targeting regime underwent significant changes during the history of it. The most recent significant change is the shift to the inflation-forecast targeting regime in 2008 with the introduction of its medium/long-run forecast DSGE model [Clinton et al., 2017].

2.4 Inflation-targeting and price stability

There are technical reasons why inflation targeting is considered to be the same as ensuring price stability. Inflation is approximated using a price index that tracks prices of a selected set of goods. For example, the consumer price index is the weighted average of prices of goods consumed by an average consumer. This method of measurement cannot in time capture changes in weights of goods due to the substitution effect. Prices do not change simultaneously and when a good that is captured in the basket gets more expensive than the other included goods, a consumer might substitute the good with the others. A price index is also not able to capture changes in quality. An increase in quality of a good usually results in a higher price. This price increase should not be considered as inflation as the underlying good essentially becomes a different good by changing its quality. The consumer price index is not able to capture these changes of quality in time [CNB, 2018]. Price indexes are therefore often biased upwards and the true inflation is in many cases lower than the measured inflation. Central banks with low inflation targets are approximately targeting zero inflation which is in line with price stability [Bernanke and Mishkin, 1997]. Another argument that could be made is that the complete price stability is not feasible in reality and targeting low, but stable inflation-rate is sufficiently close to the idea of complete stable prices[2] [Woodford, 2002].

There are also practical justifications for targeting a low inflation rate. First of all, due to the imperfect measurement of inflation which is biased upwards, setting inflation target to the zero percent would result in higher deflationary risk and deflation can be a lot more harmful than a low and steady inflation rate. As [Bernanke and Mishkin, 1997] argue, inflation is harmful when its level and variability is high. High level of inflation results in a too fast diminishing purchasing power of the inflated currency. Higher variability, which is more costly than the level of inflation, increases the uncertainty of future prices and nominal interest rates which makes it harder for people to plan their consumption and investment paths. Targeting a low and consistent inflation rate reduces both of these issues. The second practical reason is based on the existence of wage rigidity [Tobin, 1972]. In practice, nominal wages are downward sticky, i.e. when the economy is hit by a negative productivity shock, firms are not willing to cut their wages even though it would be rational to do so. The firms rather fire their employees resulting in higher unemployment. A positive inflation is therefore helpful in decreasing real wage and helping the economy to deal with these type of shocks [Tobin, 1972].

2.5 Transmission mechanism

In the open economy under the inflation-targeting regime, the central bank can affect the inflation rate path via three different transmission channels: the aggregate demand channel, the expectation channel and the exchange rate channel.

The central bank can utilize its short-term policy rates as the main monetary policy tool to affect the inter-bank market interest rates and eventually the interest rate in the economy. The interest rate in the economy then affects the actual inflation rate via the

[2] A true price stability with zero inflation is the best optimal policy based on many standard macroeconomic models. However, this is non-achievable in the real world where we need to look after the second-best but realistic optimal policy.

aggregate demand transmission channel. Depending on the public's expectations about the future inflation path, a decrease in the nominal interest rate will lead to a decrease in the real interest rate. As the Theory of Interest predicts [Fisher, 1931], a decrease in the real interest rate lowers the opportunity costs of consuming today rather than saving and collecting the given interest in the future (which allows for higher future consumption), i.e. a decrease in the real interest rate makes current consumption cheaper relative to future consumption. Lower real interest rate therefore according to the theory increases consumption via an intertemporal substitution effect[3]. The lower real interest rate also positively affects investments as it lowers the investment costs (explicit costs in form of an interest for investors that have to take an investment loan or implicit costs of investing their own money instead of lending it for the market's interest rate) [Keynes, 1936].

The change in the nominal interest rate also affects the aggregate demand by affecting the availability of credit. By affecting the nominal interest rate, the monetary policy affects how much money people want to borrow and how much money banks are willing to lend. Notably, influencing the economy's nominal interest rate also affects how much money is created by the commercial banks which also plays a role in the evolution of inflation rate [Mcleay et al., 2014].

The second important transmission channel is the expectation channel. If the central bank has a strong credibility, announcing the inflation target can set the public's expectation about the future inflation and this alone can affect the actual inflation via wage and price setting behavior, i.e. if firms believe that the next year's inflation rate will be 2%, they will ex ante increase their prices and wages by 2%, confirming the targeted inflation rate. This expectation channel renders a strong argument why in general central banks should aim at being transparent. Transparency helps to gain and retain the credibility which then improves the effectiveness of monetary policy by lowering the cost of achieving its inflation goals [Alichi et al., 2015].

[3]There is also an income effect following a decrease in the real interest rate. The income effect is positive for debtors but negative for creditors. The overall income effect is however considered to be zero as the individual income effects counter themselves and the change in the real interest rate affects aggregate consumption only via the substitution effect.

In the case of an open economy, as is the case of the Czech Republic, there is an additional transmission channel that affects the inflation rate, the exchange rate channel. Influencing the exchange rate affects the relative price between domestic and foreign goods which directly affects the foreign demand for domestic goods. A depreciation of Czech koruna makes domestic goods cheaper for foreign countries leading to an increase in exported goods. Increase in the production of exporting firms increases employment which positively affects domestic demand. Higher aggregate demand (both from foreign and domestic households) then causes prices to increase (positive inflation). This channel is essentially the aggregate demand channel strengthened by the exchange rate. There is also a pure exchange rate channel which has a direct effect on inflation. This pure exchange rate channel is usually also faster than the aggregate demand one. A depreciation makes importing final goods used for direct domestic consumption more expensive. The increase in imported final goods then directly affects the inflation rate. A weaker currency also increases the cost of importing intermediate goods used for the production of domestic final goods. Finally, a higher inflation rate affects the nominal wage setting, the inflationary pressures caused by the change in the exchange rate, therefore, spreads to the whole of the economy[4] [Svensson, 1998].

Lastly, foreign shocks as for example changes in foreign inflation, foreign interest rates or foreign demand for domestic goods are transmitted to the domestic country through this exchange rate channel [Svensson, 1998].

2.6 Expectations, transparency and credibility

Mentioned above, the expectation channel emphasizes the role of the public's expectations on the future inflation rate path. Nonetheless, expectations have a strong role in the first aggregate demand channel as well [Svensson, 2007] [Alichi et al., 2015]. The Czech National Bank has a control over three short-term interest rates, while for the monetary

[4] In practice, to what degree the change in the nominal exchange rate affects the inflation rate depends on the pass-through effect [Hájek and Horváth, 2015].

policy, the most relevant rate is the 2-week repo rate. The repo rate is the rate at which commercial banks can lend their money to the central bank and so the central bank is one of the sources where commercial banks can allocate their excess liquidity. A change in the short-term repo rate alone is not enough to affect the real interest rate and the aggregate demand. What is important is to affect the expectations of the entire path of future short-term rates, which then effectively affects the long-run nominal interest rate. In combination with stable expectations about the future inflation rate path, a change in the nominal interest rate path then effectively affects the real interest rate path and therefore the aggregate demand [Clinton et al., 2017].

This is where the communication between the central bank and the public combined with the transparency of conducted monetary policy start to play an important role. The better is the central bank at communicating and explaining their intentions and goals, and the better is the central bank at justifying their policies, the more effectively will the central bank affect the entire paths of interest rates and inflation rates [Bernanke and Mishkin, 1997]. The Czech National Bank quarterly publishes Inflation Reports in detail since 2002 exactly for that reason, to affect the market expectations but also to increase its accountability and credibility which in turn lower the costs of achieving the inflation target. The forecasts done by the Czech National Bank is one of the primary inputs for the Bank Board's policy-making decisions. The inflation reports serve as the starting point for the Bank Board's members discussion about their policy steps and they also serve as the quantitative measurement of how the Czech National Bank is successful in achieving its goals. The Inflation Reports clarify the steps and intentions of the central bank, which as long as the central bank is believed to be credible, decrease the uncertainty about the future path of inflation and improve the consumption and investment planning of the private sector [Bernanke and Mishkin, 1997]. In addition, the transparency makes it easier for the "outsiders" to join the debate on how the monetary policy should be conducted. Conditional on the high transparency, the central bank is harder to be pressured by the government in situations where the government becomes opportunistic, i.e. in situations where it starts to pressure the central bank to conduct monetary policy in a way that has only short-run benefits (but costly long-run inflation). The Inflation Reports and higher transparency thus further reduce the short-run opportunism [Bernanke and Mishkin, 1997].

Apropos, the strong credibility and high transparency of the Czech National Bank arguably helped the central bank to be successful when implementing the exchange rate floor in 2013[5] as a mean to help the economy out of the recession and to avoid deflationary risks.

Lastly, since the exchange rate itself is a forward-looking variable and variable determined by expectations, the presence of the exchange rate channel in open economies furthermore emphasizes the importance of forward-looking expectations in monetary policy [Svensson, 1998].

The following example summarizes the described transmission mechanism. When the central bank expects that the future inflation path will get above its target (or its target range), the bank increases the short-term policy rate along with its future path. The increase in the policy rate path eventually affects the real interest rate which has a dampening effect on the economic activity. Higher opportunity costs linked with the higher real interest rate decreases consumption and investment, i.e. via the substitution effect and higher investment costs. The change in the policy rate also affects the nominal interest rate making loans more costly and reducing the availability of credit, furthermore supporting the negative effects of the higher policy rate on consumption and investments. Moreover, depending on the credibility of the central bank, the change in its policy rate path and forecasted inflation path published in the Inflation Report affect the inflation expectations. The households and firms then change their wage and prices depending on these expectations, eventually affecting the actual inflation. In case of an open economy, higher interest rates attract foreign capital which put pressure on the appreciation of the domestic currency. This decreases foreign demand and export creating further dampening

[5]Many central banks around the world were dealing with the zero lower bound, i.e. the central banks technically set their policy rates to zero which constrained them from further loosening their monetary policies. Further monetary expansions were arguably needed as the inflation rates and their expected paths were below the inflation targets. In the Czech Republic, the policy rate was reduced to zero in 2012 due to a newly started recession (one of the main cause was the euro zone's debt crisis and the subsequent fall in the aggregate demand). The Czech National Bank then depreciated the Czech koruna and implemented exchange rate floor as a mean to get out of the recession and stop the deflationary pressures [Alichi et al., 2015]

effects in the economy. The stronger domestic currency will also make it less costly to import goods, both final and intermediate, from abroad. All of these effects are at the end translated (with lags, which will vary according to the transmission channel) into deflationary pressures helping the central bank to meet its inflation goal.

2.7 The importance of forecasting in practice

It should be noted that the variables in the mentioned transmission mechanism affect each other with various lags. A change in the policy rate will have its highest effect on the aggregate demand and inflation usually after 4-6 quarters. The exchange rate channel will, on the other hand, affects the prices faster. A part of the Czech National Bank therefore strongly focuses on the forecast and structural modeling to estimate the expected effects within a given horizon.

For the medium-term horizon, the bank uses a DSGE model (called as QPM-g3 or Quarterly Projection Model g3) as the core forecasting model as it encompasses the forward-looking element, endogenous short-term interest rate[6] and the transmission mechanism described above. This DSGE model was introduced in 2008, the same year when the central bank started publishing expected path of market interest and policy rates conditional on the information present at the time of forecast modeling. The intention to start publishing these paths was to further increase the transparency and to improve the effects of monetary policies in shaping market expectations. The central bank also integrates other forecasting models in addition to the core model, e.g. it develops and uses satellite models in order to decompose the inflation rate into its components (core inflation, monetary-policy relevant inflation, etc.), to estimate the output gap evolution or to get better estimates of near-term forecasts [Clinton et al., 2017] [Alichi et al., 2015].

The Czech National Bank underwent a significant development under the inflation-targeting regime in order to improve the performance of the forecasting models but also

[6]The policy rate is endogenous as it, simply stated, reacts to the economic situation or its future outlook.

to improve the way the staff translates the forecast to the market and public. It started with near-term forecasting models in 1998 before implementing their first quarterly project model in 2002. The second and currently used quarterly project model is the mentioned DSGE model QPM-g3 was introduced in 2008 [Clinton et al., 2017].

The forecasts are periodically published to support the transparency of conducted monetary policy. The inflation reports with its forecasts serve as an important input for the policy-makers (the Bank Board members). Consequently, the market can build their predictions about the direction of monetary policy based on the same reports. The forecasts contain not only the expected path of the forecasted variables but also their confidence intervals which reflect the uncertainty about the future. The uncertainty is reflected in the market expectations about the possible future scenarios and their probabilities, affecting the behavior of the market. Hence, it is crucial to have the capacity to develop and maintain well-performing forecasting models to lower the uncertainty, not only of the policy-makers but also of the public.

2.8 Strict, flexible inflation-targeting and the Taylor rule

The literature on inflation targeting distinguishes between strict and flexible inflation targeting. The former focuses solely on the inflation while the latter allows the monetary authority to put weight on other relevant measures of economic activity such as the output gap or the real exchange rate. In fact, the Czech National Bank has implemented a flexible inflation targeting regime. [Vašíček and Musil, 2006] showed that the Czech National Bank reacts not only to changes in inflation but also to changes in the output gap. That is, the Czech National Bank sets the price stability as the main goal (and the only explicit goal), however, it also tries to stabilize the business cycles as long as it is in line with the main goal. The flexible inflation targeting constraints central banks from engaging in policies that have undesirable long-run consequences, but it is less rigid than the strict inflation targeting and allows the central bank to deal with unusual situations and to improve economic conditions through other channels.

It is also useful to mention the Taylor rule [Taylor, 1993] which prescribes a rule for central banks to follow. The Taylor rule sets exact coefficient how the central bank should change its policy rate in reaction to the current inflation and output gap. The flexible inflation targeting regime lies somewhere between this rule and a pure discretion. In practice, the current inflation and output gap is often not enough for a policymaker to make well-informed decisions and mechanically following the rule would be ignoring the varying importance of other relevant factors, i.e. stating that the current inflation and the current output is the only relevant information for inflation targeting is a strong assumption [Bernanke and Mishkin, 1997]. For example, if the prices increase due to changes in taxation or due to a temporary supply shock, it is more appropriate for the central bank not to react to this temporary higher inflation rate. The Taylor rule does not distinguish between the reasons for a higher inflation rate and would prescribe to increase the policy rates even in this case [Taylor, 1993]. The Taylor rule is, however, useful for modeling purposes as the approximation of the inflation targeting regime for its simplicity [Vašíček and Musil, 2006]. And even in practice, the Taylor rule can provide another reference point for policy-makers when making their policy decisions [Taylor, 1993].

2.9 Welfare implications

Essentially, the role of the central bank is to sustain economic conditions that are welfare-improving, which is to be achieved through monetary policy and through ensuring that economic conditions are aligned with the optimal welfare level. The researches have been therefore trying to evaluate monetary policies not only with regards to the dynamics of the basic and more easily measured macroeconomic variables as is output, inflation or nominal exchange rate (via an assumed loss function) but also directly with regards to their impact on welfare or utility (utility-based approach).

The basic premise is that the central bank can aggrandize welfare primarily by ensuring price stability. Workhorse New Keynesian DSGE models are in line with this view as the models create a rationale for stabilization policies (including policies that aim at price stability).

Specifically, introducing market imperfections as price rigidity and monopolistic markets into the basic DSGE framework creates the justification for the presence of a monetary policymaker [Woodford, 2002]. These market imperfections render monetary variables (nominal interest rate) to be non-neutral[7] in the short-run which in turn renders monetary policy to be effective in influencing output and employment. The existence of price rigidity creates a rationale for an active policy that would ensure more efficient allocation of resources[8]. In these workhorse models, the private agents (households) are assumed to care about their utility which in turns depends on the level of consumption and the level of leisure. The utility is not directly affected by prices, however since prices cannot change immediately due to assumed price rigidity, instability of the general price level can affect the relative prices of goods and therefore the equilibrium allocation of resources. The equilibrium allocation of resources in turns, under the price rigidity, can end up being not optimal and creating a dead-weight loss, i.e. welfare loss. That is, unstable prices have distortion effects which the central bank can mitigate by stabilizing the inflation rate [Woodford, 2002].

The standard New Keynesian models also support the notion that the monetary authority that focuses solely on inflation stabilization, also simultaneously stabilize other welfare-relevant economic variables, e.g. output gap. This result is known as the divine coincidence [Blanchard and Galí, 2007] and creates an argument for central bankers to primarily

[7]Mentioned nominal rigidities allow a DSGE model to produce real effects of monetary policy as captured by the empirical literature [Woodford, 2002]

[8]The basic DSGE models without market imperfections, on the other hand, implicated no rationale for a monetary or fiscal authority. In these models, prices always change immediately to clear the markets. The basic DSGE models, therefore, supported the argument that a policymaker should not try to smooth business cycles as the business cycles result from the optimal behavior, i.e. business cycles are in line with optimal allocation of resources. The transmission mechanism in these models also differ from New Keynesian ones and it relies on the capital accumulation.

focus on inflation stabilization. The divine coincidence, however does not have to hold in all New Keynesian models. [Blanchard and Galí, 2007], for example, showed that the divine coincidence does not hold under more nontrivial real imperfections and that the monetary authority has to face the trade-off between stabilizing inflation and stabilizing welfare-relevant output gap. Particularly in the case of [Blanchard and Galí, 2007], [**?**] implemented real wage rigidity into the model which creates a non-constant gap between natural and efficient output. Stabilizing inflation still leads to minimizing the gap between the natural and the actual output, i.e. the output gap, however, in the model, this output gap is different from the welfare-relevant output gap, i.e. the gap between the efficient and actual output. Other authors as [Debortoli et al., 2017] also argue that focusing on multiple goals rather than solely on inflation could be beneficial in terms of welfare.

The aim of the thesis is to test whether including exchange rate stabilization into the inflation targeting framework could in case of the Czech Republic promote any welfare gains. As mentioned in the previous paragraphs, central banks can affect aggregate demand and inflation by affecting the exchange rates. Intuitively, stabilizing exchange rate could help in stabilizing aggregate demand and inflation. However, the opposite can be true too. Focusing on exchange rate stabilization will limit the central bank on how it will be able to react to exogenous factors and shocks. In particular, if the central bank expected higher than desired inflationary pressures, it would in normal situations raise the policy rate in order to increase the expectations on interest rates in the economy. Higher interest rates, however, attract foreign capital which put pressure on appreciation. The Czech Republic experienced this trade-off during the transformation period when it aimed to target the monetary aggregate M2 but also at the same time wanted to hold a fixed exchange rate to a list of foreign currencies. This thesis includes exchange rate stabilization goal in a less extreme version, i.e. not in terms of fixed exchange rate regime. This work considers modeling the central bank that will put non-zero weight on the exchange rate stabilization without fixing the nominal exchange rate. That is, taking the mentioned example, under the expected inflationary pressures, the central bank will be allowed to raise its policy rate and let the domestic currency appreciate. The policy rate will be raised depending on how the exchange rate stabilization is important for the central bank. The more weight it will put on the nominal exchange rate, the less will be the central bank effective in reacting to inflation rate changes. In practice, this will simulate the central bankers that watch over the exchange rate for sudden changes, i.e. central bankers that allow the domestic currency to appreciate or to depreciate as long as the changes are marginal.

Similar welfare analysis is often performed under the DSGE framework which provides a ground to test and compare alternative monetary or fiscal policies. With the improvement in the computational power, DSGE models have become popular among economists for testing more complicated questions or simulating the outcomes of alternative policies. The DSGE models have the advantage of being immune to Lucas' critique [**?**] as it is built on the idea of forward-looking agents, i.e. the subjects in the economy makes their decision conditional on the past experience but also on the expectations of future paths of relevant variables. For example, [Schmitt-Grohé and Uribe, 2004a] tried to find the optimal fiscal and monetary policy rule using a DSGE model. Regarding the optimal monetary policy which was represented by a simple interest rate rule, their results suggested that the inflation coefficient in the interest rate rule does not play a major role as long as it high enough to ensure a determinacy of equilibrium[9], i.e. the welfare function around the optimal inflation coefficient is almost flat. The paper also suggested for the central banks to not put react too strongly to output changes as it could lead to significant welfare losses, especially if the central bank reacts to the output change in the opposite direction (increasing the policy rate when the output falls below its balanced growth path).

This thesis is built on [Galí and Monacelli, 2005] small open economy model. The solution of the model will differ from the authors' one in solving the model in levels. The authors of the model solved the model using log-linearization which is however not suitable for welfare analysis where one needs to solve the model using higher-order approximation methods. After deriving the model in levels, the work implements modified inflation-targeting by including interest rate rules that prescribe how the monetary authority reacts

[9]This is a standard requirement of workhorse New Keynesian DSGE models. For the model to render a unique equilibrium, the inflation-targeting central bank has to react strongly enough to inflation changes.

to changes in inflation, output and exchange rate. The work tries a different set of weights on exchange rate stabilization and compares it to the case of how the Czech National Bank behaves (approximated by the Taylor rule by [Vašíček and Musil, 2006]).

3 Model

3.1 Introduction to the model

The authors of the DSGE model presented here are [Galí and Monacelli, 2005]. It is a standard representative-agent RBC model with endogenous labor supply and stochastic productivity. The model includes two sources of inefficiency, monopolistic competition and price rigidity in a form of Calvo pricing [Calvo, 1983]. These inefficiencies also provide a reasoning for stabilization policies. There is no government presence in this model and the only authority that can conduct stabilization policy is the central bank using its monetary rules. Lastly, the model here consists of a continuum of small open economies. The continuum of small open economies is represented by the unit interval. Since each country is a part of an infinite amount of identically small open economies, any single country, including their policies, has no impact on the rest of the world, i.e. each country takes the rest of the world as exogenous to them.

This chapter lays down the assumptions and the behavior of households, firms and the monetary authority (the central bank). I will use the same notation as the authors [Galí and Monacelli, 2005]. The model is modeled from the viewpoint of a single small open economy - the domestic economy. Variables will refer to an economy $i \in [0, 1]$ with an index i. The domestic economy will have its variables denoted without the i-index.

The model is solved using a second-order perturbation method which departs from [Galí and Monacelli, 2005]. The conventional way of solving these type of models is to log-linearise them around its steady state (usually around the zero-inflation steady state) and use a first-order approximation method to find the policy functions that solve the equilibrium equations (around the steady state). The first-order approximation methods, however, omits the effects of uncertainty on the behavior of agents and thus on the level of welfare [Schmitt-Grohé and Uribe, 2004b]. It is therefore recommended to apply higher-order approximation methods when conducting welfare analysis using DSGE models. Omitting the log-linearisation results in a slightly different set of the equilibrium conditions as compared to the authors [Galí and Monacelli, 2005]. Log-linearization gets rid of some higher-order terms which are however needed if one wants to find the second-order effects on welfare. Log-linearization also allows for combining the first order conditions of the model into the New Keynesian Phillips curve and the dynamics IS curve, reducing the final set of equations and allowing for compact representation of the model's dynamics. Since the thesis wants to avoid log-linearization, these curves are not derived. That is, [Galí and Monacelli, 2005] derived the log-linearized set of conditions describing the modeled economy but thesis departs from their paper by showing the way how to derive the set of equilibrium conditions that can be approximated using higher-order methods[1].

3.2 Household

Every single economy is inhabited by an infinite amount of infinitely-lived households (also represented by the unity). It is assumed that these households are all identical in their preferences, i.e. households in the economy are homogeneous. This allows us to focus on a single (representative) household's optimization problem as the other households will behave identically. The representative household seeks to maximize its lifetime utility given the sequence of budget constraints. In each period, the household decides between consumption and leisure, both of which positively raise its utility. There is however a trade-off between those two variables. To increase consumption, the household has to

[1]Log-linearing the set of equilibrium conditions binds the solver to have only a linear approximation of the model, i.e. using higher-order approximation methods will still result in a linearized solution. If ones want to use higher-order approximation methods, he needs to keep the set of equilibrium conditions in its original non-linear form.

forfeit leisure and spend time working in order to pay for the higher consumption. At any time t, the household chooses optimal paths for its lifetime consumption and leisure given their rational expectations on wage and price evolution. These paths are conditional on the information known at time t and can change with new information. The discounted lifetime utility given the expectations at time $t = 0$ and the constant discount factor $\beta \in (0, 1)$ is

$$\mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \right\} \tag{3.1}$$

with households' preferences being described by the constant relative risk aversion utility function

$$U(C_t, N_t) \equiv \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \tag{3.2}$$

where σ measures the degree of relative risk aversion and the inverse of the inter-temporal elasticity of substitution, and φ is the inverse of the elasticity of labor supply.

Variable C_t denotes the composite consumption index summarizing the consumption of all domestic and foreign goods, and N_t denotes labor supplied by the household. The utility function is assumed to be strictly increasing in consumption, strictly decreasing in labor supply and strictly concave in both arguments. Households are therefore by definition better off, the more they consume, while worse off the more time they spend working, and marginal gains from higher consumption decrease as households consume more and more. Similarly, marginal gains from leisure decrease with increasing level of leisure.

The composite consumption index C_t is made up of a variety of domestic and imported goods[2]. In particular, the composite consumption is defined as

$$C_t \equiv \left[(1-\tau)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + \tau^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{3.3}$$

where $C_{H,t}$ and $C_{F,t}$ are consumption indexes for domestic and imported goods consumption, respectively. The indexes summarize the consumption of all the domestic and foreign

[2]The model assumes a range of differentiated goods that are imperfect substitutes to the consumers.

variety, also respectively, and from the viewpoint of domestic households, η measures the substitutability between domestic and foreign goods. The index $\tau \in [0,1]$ measures the openness of the domestic households to foreign goods. The lower the value of τ, the more are the domestic households biased towards goods produced in their country and the more they prefer domestic goods. Both indexes are given by the constant elasticity of substitution (CES) function. The index of domestic consumption is given by

$$C_{H,t} \equiv \left(\int_0^1 C_{H,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} \tag{3.4}$$

where $j \in [0,1]$ denotes the variety of domestic goods (home-produced goods) and ϵ denotes the elasticity of substitution between the varieties. Similarly, the index of imported (foreign) goods is defined as

$$C_{F,t} \equiv \left(\int_0^1 (C_{i,t})^{\frac{\gamma-1}{\gamma}} di \right)^{\frac{\gamma}{\gamma-1}} \tag{3.5}$$

where $C_{i,t}$ is the consumption index for consuming goods from country i and γ denotes the elasticity of substitution between the foreign consumption indexes. In particular, $C_{i,t}$ is the index of consumption for the variety of goods imported from a specific country i. The index of consumption for imported goods from a country i is given by the CES function as well.

$$C_{i,t} \equiv (\int_0^1 C_{i,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj)^{\frac{\epsilon}{\epsilon-1}} \tag{3.6}$$

where ϵ again denotes the elasticity of substitution between different j products produced within the country i[3].

[3]The model assumes that the substitutability between varieties is the same within any country, whether it is a domestic or a foreign country.

Each household maximizes its utility subject to the time-varying budget constraint with time subscript t

$$\int_0^1 P_{H,t}(j)C_{H,t}(j)dj + \int_0^1 \int_0^1 P_{i,t}(j)C_{i,t}(j)djdi + \mathrm{E}_t\left\{Q_{t,t+1}D_{t+1}\right\} \leq D_t + W_tN_t \quad (3.7)$$

where $P_{H,t}(j)$ denotes the price of domestic good j, $P_{i,t}(j)$ denotes the price of good j imported from a country i. D_{t+1} denotes a nominal pay-off in the period $t+1$ of a risk-less bond held at the end of period t. $E_t\left\{Q_{t,t+1}\right\}$ represents the stochastic discount factor for one-period ahead nominal pay-off. It is assumed that the bond markets are complete and that the households have access to a complete set of contingent claims, traded internationally. W_t represents nominal wage and $w_t \equiv \frac{W_t}{P_t}$ denotes real wage.

We can simplify the budget constraint by aggregating the consumption indexes. To find the aggregates, we have to find the expressions for the optimal allocation of any expenditure on the variety of goods within the domestic country and within any country i, the optimal allocation of expenditure between all foreign good indexes and the optimal allocation of expenditure between the domestic and imported goods indexes. We have to find the expression for the optimal bundle consisting of all domestic and foreign goods which maximize the composite consumption given any expenditure constraint.

Let M be an arbitrary expenditure level that a consumer can spend on the variety of domestic goods $j \in [0,1]$. The optimal allocation of that expenditure level between the domestic varieties is the one that maximizes the domestic consumption index given the expenditure constraint M. That is

$$\max_{C_{H,t}(j)} \left[\int_0^1 C_{H,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}} \quad (3.8)$$

subject to

$$\int_0^1 P_{H,t}(j)C_{H,t}(j)dj \leq M_t \quad (3.9)$$

The first order condition is of this problem is

$$\frac{\epsilon}{\epsilon-1}\left[\int_0^1 C_{H,t}(j)^{\frac{\epsilon-1}{\epsilon}}dj\right]^{\frac{1}{\epsilon-1}}\frac{\epsilon-1}{\epsilon}C_{H,t}(j)^{-\frac{1}{\epsilon}}-\lambda_t P_{H,t}(j)=0$$
$$C_{H,t}^{\frac{1}{\epsilon}}C_{H,t}(j)^{-\frac{1}{\epsilon}}=\lambda_t P_{H,t}(j) \tag{3.10}$$

The condition implies that the relation between any two different domestic goods j and $z \neq j$ is

$$C_{H,t}(j)=C_{H,t}(z)\left[\frac{P_{H,t}(j)}{P_{H,t}(z)}\right]^{-\epsilon} \tag{3.11}$$

which after plugging into the expenditure constraint gives

$$M_t=\int_0^1 P_{H,t}(j)C_{H,t}(z)\left[\frac{P_{H,t}(j)}{P_{H,t}(z)}\right]^{-\epsilon}dj$$
$$=P_{H,t}C_{H,t}(z)\left[\frac{P_{H,t}(z)}{P_{H,t}}\right]^{\epsilon} \tag{3.12}$$

where $P_{H,t}\equiv\left[\int_0^1 P_{H,t}^{1-\epsilon}(j)dj\right]^{\frac{1}{1-\epsilon}}$ is the domestic price index. It then follows that the optimal level of a domestic good j, $C_{H,t}(j)$, is given by

$$C_{H,t}(j)=\left[\frac{P_{H,t}(j)}{P_{H,t}}\right]^{-\epsilon}\frac{M_t}{P_{H,t}} \tag{3.13}$$

Substituting this expression into the index of consumption for domestic goods will yield the optimal allocation within the variety of domestic goods given M.

$$C_{H,t}=\left\{\int_0^1\left[\frac{P_{H,t}(j)}{P_{H,t}}\right]^{-\epsilon\frac{\epsilon-1}{\epsilon}}\left[\frac{M_t}{P_{H,t}}\right]^{\frac{\epsilon-1}{\epsilon}}dj\right\}^{\frac{\epsilon}{\epsilon-1}}$$
$$=\left[\int_0^1 P_{H,t}(j)^{1-\epsilon}dj\right]^{\frac{\epsilon}{\epsilon-1}}\frac{M_t}{P_{H,t}}P_{H,t}^{\epsilon} \tag{3.14}$$

which finally implies the expression we need

$$P_{H,t}C_{H,t} = M_t = \int_0^1 P_{H,t}(j)C_{H,t}(j)dj \tag{3.15}$$

In addition, by combining (3.13) and (3.15), we can get the demand schedule for domestic goods

$$C_{H,t}(j) = \left[\frac{P_{H,t}(j)}{P_{H,t}}\right]^{-\epsilon} C_{H,t} \tag{3.16}$$

which states that the demand for a domestic good j depends on its relative price, the overall domestic consumption and on the elasticity of substitution between varieties. Lower elasticity gives producers the power to raise their prices without fully affecting the demand for their products. Given our assumption, these conclusions on the optimal set of varieties within domestic country hold also within any foreign country i. That is

$$P_{i,t}C_{i,t} = \int_0^1 P_{i,t}(j)C_{i,t}(j)dj \tag{3.17}$$

and

$$C_{i,t}(j) = \left[\frac{P_{i,t}(j)}{P_{i,t}}\right]^{-\epsilon} C_{i,t} \tag{3.18}$$

where it follows that $P_{i,t} \equiv \left[\int_0^1 P_{i,t}^{1-\epsilon}(j)dj\right]^{\frac{1}{1-\epsilon}}$ is the country i price index.

Following the same steps and assuming that $P_{F,t} \equiv \left[\int_0^1 P_{i,t}^{1-\gamma}di\right]^{\frac{1}{1-\gamma}}$ is the price index for imported goods, we can find the optimal allocation of expenditure between all foreign countries[4].

$$P_{F,t}C_{F,t} = \int_0^1 P_{i,t}C_{i,t}di \tag{3.19}$$

$$C_{i,t} = \left(\frac{P_{i,t}}{P_{F,t}}\right)^{-\gamma} C_{F,t} \tag{3.20}$$

Lastly, we have to find the optimal allocation of expenditures between domestic and imported good indexes. That is, we need to find the desired allocation of an arbitrary

[4]The equations (3.15) and (3.17) give the optimal allocation within each country.

expenditure M between $C_{H,t}$ and $C_{F,t}$ that maximizes the composite consumption C_t.

$$\max_{C_{H,t},C_{F,t}} \left[(1-\tau)^{\frac{1}{\eta}}(C_{H,t})^{\frac{\eta-1}{\eta}} + \tau^{\frac{1}{\eta}}(C_{F,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{3.21}$$

subject to

$$P_{H,t}C_{H,t} + P_{F,t}C_{F,t} \leq M_t \tag{3.22}$$

The first order conditions to this problem are

$$\begin{aligned}
\left[C_{H,t}\right] : &\left[(1-\tau)^{\frac{1}{\eta}}(C_{H,t})^{\frac{\eta-1}{\eta}} + \tau^{\frac{1}{\eta}}(C_{F,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{1}{\eta-1}}(1-\tau)^{\frac{1}{\eta}}C_{H,t}^{-\frac{1}{\eta}} = \lambda_t P_{H,t} \\
&C_t^{\frac{1}{\eta}}(1-\tau)^{\frac{1}{\eta}}C_{H,t}^{-\frac{1}{\eta}} = \lambda_t P_{H,t} \\
\left[C_{F,t}\right] : &\, C_t^{\frac{1}{\eta}}\tau^{\frac{1}{\eta}}C_{F,t}^{-\frac{1}{\eta}} = \lambda_t P_{F,t}
\end{aligned} \tag{3.23}$$

which gives the optimal ratio between the domestic and the foreign consumption indexes.

$$\begin{aligned}
\left(\frac{1-\tau}{\tau}\right)^{\frac{1}{\eta}}C_{H,t}^{-\frac{1}{\eta}} &= C_{F,t}^{-\frac{1}{\eta}}\frac{P_{H,t}}{P_{F,t}} \\
C_{H,t} &= \frac{1-\tau}{\tau}C_{F,t}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}
\end{aligned} \tag{3.24}$$

Substituting the expression above into the expenditure term gives

$$\begin{aligned}
M_t = P_{H,t}\left(\frac{1-\tau}{\tau}\right)C_{F,t}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta} + P_{F,t}C_{F,t} \\
M_t\tau P_{F,t}^{-\eta} = C_{F,t}\left[(1-\tau)P_{H,t}^{1-\eta} + \tau P_{F,t}^{1-\eta}\right] \\
M_t\tau P_{F,t}^{-\eta} = C_{F,t}P_t^{1-\eta} \\
C_{F,t} = \tau\left(\frac{P_{F,t}}{P_t}\right)^{-\eta}\frac{M_t}{P_t}
\end{aligned} \tag{3.25}$$

where the third line uses the CPI definition $P_t \equiv \left[(1-\tau)P_{H,t}^{1-\eta} + \tau P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}$. Similarly, it also holds that the optimal domestic consumption index given the expenditure constraint M is

$$C_{H,t} = (1-\tau)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \frac{M_t}{P_t} \tag{3.26}$$

Finally, substituting both terms into the composite consumption index yields

$$\begin{aligned}
C_t &= \left[(1-\tau)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + \tau^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \\
&= \left[(1-\tau)^{\frac{1}{\eta}} (1-\tau)^{\frac{\eta-1}{\eta}} \left(\frac{P_{H,t}}{P_t}\right)^{1-\eta} + \tau^{\frac{1}{\eta}} (\tau)^{\frac{\eta-1}{\eta}} \left(\frac{P_{F,t}}{P_t}\right)^{1-\eta}\right]^{\frac{\eta}{\eta-1}} \frac{M_t}{P_t} \\
&= \left[(1-\tau)P_{H,t}^{1-\eta} + \tau P_{F,t}^{1-\eta}\right]^{\frac{\eta}{\eta-1}} \frac{M_t}{P_t} P_t^{\eta}
\end{aligned} \tag{3.27}$$

and therefore

$$P_t C_t = M_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t} \tag{3.28}$$

The demand schedules for the domestic and the foreign goods are then

$$C_{H,t} = (1-\tau)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \tag{3.29}$$

$$C_{F,t} = \tau\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{3.30}$$

The expressions (3.15), (3.17), (3.19) and (3.28) give the optimal allocations of resources within each group of goods. We can use these equations to rewrite the original budget constraint. Each household then solves the following optimization problem

$$\max_{C_t,N_t} \mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \right\} \tag{3.31}$$

subject to

$$P_t C_t + \mathrm{E}_t \left\{ Q_{t,t+1} D_{t+1} \right\} \le D_t + W_t N_t \tag{3.32}$$

The first-order conditions of the household's problem are

$$\left[C_t\right]: \qquad C_t^{-\sigma} = \lambda_t P_t \tag{3.33}$$

$$\left[N_t\right]: \qquad N_t^{-\varphi} = \lambda_t W_t \tag{3.34}$$

$$\left[D_{t+1}\right]: \qquad \beta^t \lambda_t \mathrm{E}_t \left\{ Q_{t,t+1} \right\} = \beta^{t+1} \mathrm{E}_t \left\{ \lambda_{t+1} \right\} \tag{3.35}$$

Combining the consumption and the labour first-order conditions gives the intra-temporal optimality condition describing the optimal amount of consumption and labor given the real wage at time t.

$$C_t^{\sigma} N_t^{\varphi} = w_t \tag{3.36}$$

Combining the first and the third condition gives the Euler equation for the risk-less bonds

$$\mathrm{E}_t \left[Q_{t,t+1} \right] = \beta \mathrm{E}_t \left\{ \left(\frac{C_{t+1}}{C_t}^{-\sigma} \Pi_{t+1}^{-1} \right) \right\} \tag{3.37}$$

where $\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$ is the CPI inflation. No-arbitrage condition implies $E_t \left\{ Q_{t,t+1} \right\} = \frac{1}{R_t}$, where $R_t \equiv 1 + i_t$ is the nominal gross return from the risk-less one-period discount bond and i_t denotes the nominal interest rate. This is one of the important transmission mechanism features of the New Keynesian models as it allows the central bank to affect the consumption-saving decision via affecting the nominal interest rate (in combination with the later-introduced price stickiness that renders the nominal variables to be non-neutral in the short-run).

3.3 Final goods firms

This section lays down the assumptions about firms in this model. The domestic goods supply side consists of two sectors, the final good sector and the intermediate goods sector. The final good sector is fully competitive and the final good firms are therefore price-takers. The final good firms buy intermediate goods from their producer and sell them in bundles to the consumers. The intermediate goods $Y_t(j)$ are bundled by the Dixit-Stiglitz aggregator into Y_t

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \tag{3.38}$$

and sold to consumer for the domestic price index, $P_{H,t}$. The final good firms therefore face the profit-maximization problem

$$\max_{Y_t(j)} P_{H,t} \left[\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}} - \int_0^1 P_{H,t}(j) Y_t(j) dj \tag{3.39}$$

with its first-order condition

$$[Y_t(j)] : \quad P_{H,t} \left[\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{1}{\epsilon-1}} Y_t(j)^{-\frac{1}{\epsilon}} = P_{H,t}(j) \tag{3.40}$$

which implies a downward-sloping demand for each variety

$$Y_t(j) = \left[\frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\epsilon} Y_t \tag{3.41}$$

As stated in the consumer section, the presence of differentiated goods implies a market power for intermediate producers and that the producers can charge to some degree for a positive mark-up without losing all their customers. The demand for the j is a function of relative domestic price and the price elasticity of demand ϵ.

3.4 Intermediate goods firms

The sector with intermediate goods producers is a monopolistic competition. It consists of a large amount of producers that have power to change their price, i.e. they are price-makers as opposed to the final good firms. Each producer $j \in [0,1]$ produces a differentiated intermediate good, $Y_t(j)$ under the same CRS production technology using the hired labour. The production function is homogeneous of degree one. Specifically, the firms produce its intermediate products using the Cobb-Douglas production function

$$Y_t(j) = A_t N_t(j)^{1-\alpha} \tag{3.42}$$

where A_t is the total factor productivity (TFP) which is stochastic and follows the AR(1) process

$$\ln(A_t) = \rho_A \ln(A_{t-1}) + e_t^A \tag{3.43}$$

with the productivity shock e_t^A and the auto-regressive parameter ρ_A. The productivity shock is a normally distributed random variable $e_t^A \sim \mathcal{N}(0, \sigma_A^2)$.

The model introduces price rigidity via [Calvo, 1983] pricing. Each period, there is a chance θ that the producer is unable to change its price. Each producer thus faces two problems. It needs to minimize its production cost given the demand for their good ((3.41)) and it also needs to optimally set its price given the knowledge that they might be stuck with that price in the next following periods.

The first optimization problem focuses on the minimization of the production factor cost given the production technology and the demand that each firm $j \in [0,1]$ faces.

$$\min_{N_t(j)} W_t N_t(j) \tag{3.44}$$

subject to

$$A_t N_t(j)^{1-\alpha} \geq \left[\frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\epsilon} Y_t \tag{3.45}$$

Note that a Lagrangian to this problem is

$$\mathcal{L} = W_t N_t(j) + MC_t(j)\left[\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} Y_t - A_t N_t(j)^{1-\alpha}\right] \tag{3.46}$$

with the Lagrangian multiplier $MC_t(j)$ being the nominal marginal cost of increasing the output by a single unit. The first-order condition to this problem is

$$\begin{aligned}[N_t(j)]:\ W_t &= MC_t(j)(1-\alpha)A_t N_t(j)^{-\alpha} \\ &= MC_t(1-\alpha)A_t N_t^{-\alpha}\end{aligned} \tag{3.47}$$

Since all firms face the same labour price and have the access to the same production technology, all producers will want to hire the same amount of labour for a given wage and thus all producers face the same nominal marginal cost. Real marginal cost[5] is defined as $mc_t \equiv \frac{MC_t}{P_{H,t}}$. This implies that

$$mc_t = \frac{w_t}{(1-\alpha)A_t N_t^{-\alpha}} \frac{P_t}{P_{H,t}} \tag{3.48}$$

Next, the producers face the pricing problem. The real profit of a producer j at time t is

$$\begin{aligned}\text{RealProfit}_t(j) &= \frac{P_{H,t}(j)}{P_{H,t}} Y_t(j) - \frac{W_t}{P_{H,t}} N_t(j) \\ &= \frac{P_{H,t}(j)}{P_{H,t}} Y_t(j) - (1-\alpha) mc_t Y_t(j)\end{aligned} \tag{3.49}$$

The introduction of Calvo pricing [Calvo, 1983] makes the pricing-problem dynamic, the second problem of the producers therefore becomes

$$\max_{P_{H,t}(j)} \sum_{k=0}^{\infty} \theta^k\ \mathrm{E}_t \left\{ Q_{t,t+k} Y_{t+k}(j) \left(\frac{P_{H,t}(j)}{P_{H,t+k}} - (1-\alpha) mc_{t+k} \right) \right\} \tag{3.50}$$

[5]The relevant price level for the consumer is the CPI level, while for the domestic producers, it is the domestic price level that is relevant. The relationship between those two price levels is described in a section below.

subject to the sequence of demands for their variety

$$Y_{t+k}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t+k}}\right)^{-\epsilon} Y_{t+k} \tag{3.51}$$

In other words, producers maximize the present value of real profit flow given the price stickiness. The flow of real profits is discounted by the stochastic discount factor $\mathrm{E}_t\left\{Q_{t,t+k}\right\}$ and by the probability of being stuck with the set price for the following k periods, θ^k. The first-order condition to this problem is

$$\begin{aligned}
&\sum_{k=0}^{\infty}\theta^k\,\mathrm{E}_t\left\{Q_{t,t+k}Y_{t+k}\left[\frac{1}{P_{H,t+k}}\left(\frac{P_{H,t}(j)}{P_{H,t+k}}\right)^{-\epsilon}-\epsilon\left(\frac{P_{H,t}(j)}{P_{H,t+k}}\right)^{-\epsilon}\frac{1}{P_{H,t+k}}+\right.\right.\\
&\qquad\left.\left.+\,\epsilon(1-\alpha)mc_{t+k}\left(\frac{P_{H,t}(j)}{P_{H,t+k}}\right)^{-\epsilon-1}\frac{1}{P_{H,t+k}}\right]\right\}=0\\
&\sum_{k=0}^{\infty}\theta^k\,\mathrm{E}_t\left\{Q_{t,t+k}Y_{t+k}\left(P_{H,t}(j)^{-\epsilon}P_{H,t+k}^{\epsilon-1}-\epsilon P_{H,t}(j)^{-\epsilon}P_{H,t+k}^{\epsilon-1}+\right.\right.\\
&\qquad\left.\left.+\,\epsilon(1-\alpha)mc_{t+k}P_{H,t}(j)^{-\epsilon-1}P_{H,t+k}^{\epsilon}\right)\right\}=0\\
&\sum_{k=0}^{\infty}\theta^k\,\mathrm{E}_t\left\{Q_{t,t+k}Y_{t+k}(1-\epsilon)P_{H,t}(j)^{-\epsilon}P_{H,t+k}^{\epsilon-1}\right\}=\\
&\qquad=-\sum_{k=0}^{\infty}\theta^k\,\mathrm{E}_t\left\{Q_{t,t+k}Y_{t+k}\epsilon(1-\alpha)mc_{t+k}P_{H,t}(j)^{-\epsilon-1}P_{H,t+k}^{\epsilon}\right\}
\end{aligned}$$

$$P_{H,t}(j) = \frac{\epsilon}{\epsilon-1}\frac{\sum_{k=0}^{\infty}\theta^k\,\mathrm{E}_t\left\{Q_{t,t+k}Y_{t+k}P_{H,t+k}^{\epsilon}(1-\alpha)mc_{t+k}\right\}}{\sum_{k=0}^{\infty}\theta^k\,\mathrm{E}_t\left\{Q_{t,t+k}Y_{t+k}P_{H,t+k}^{\epsilon-1}\right\}} \tag{3.52}$$

We can substitute for $\mathrm{E}_t\left\{Q_{t,t+k}\right\}$ with the Euler equation (3.37), i.e. $\mathrm{E}_t\left\{Q_{t,t+k}\right\} =$ $= \beta^k\,\mathrm{E}_t\left\{\left(\frac{C_{t+k}}{C_t}\right)^{-\sigma}\frac{P_t}{P_{t+k}}\right\}$. Moreover, since nothing on the right hand side depends on j, we can get rid of j reference and replace it with the general notation of the optimal updating price $P^{\#}_{H,t}$ (i.e., all updating firms update to the same price).

$$P^{\#}_{H,t} = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{k=0}^{\infty}(\beta\theta)^k \,\mathrm{E}_t \left\{ C_{t+k}^{-\sigma} P_{t+k}^{-1} Y_{t+k} P_{H,t+k}^{\epsilon} (1-\alpha) mc_{t+k} \right\}}{\sum_{k=0}^{\infty}(\beta\theta)^k \,\mathrm{E}_t \left\{ C_{t+k}^{-\sigma} P_{t+k}^{-1} Y_{t+k} P_{H,t+k}^{\epsilon-1} \right\}} = \frac{\epsilon}{\epsilon - 1} \frac{X_{1,t}}{X_{2,t}} \tag{3.53}$$

Expressions $X_{1,t}$ and $X_{2,t}$ are auxiliary variables for the infinite sums. The optimal resetting-price equation describes how the optimal resetting-price depends on the weighted average of expected future real marginal costs mc_{t+k}. Producers choose price that minimizes the expected future changes between marginal costs and marginal revenues. In the case of flexible prices ($\theta = 0$), the expression collapses to $P^{\#}_{H,t} = \frac{\epsilon}{\epsilon-1}(1-\alpha)mc_t$. Since $\epsilon > 1$, firms prefer to charge a constant mark-up $\frac{\epsilon}{\epsilon-1} > 1$ over their real marginal costs[6].

Before proceeding to the next section, we need to get rid of the infinite sums by rewriting the expression recursively and replace the price levels with inflation rates to make the expression stationary[7]. This can be done by dividing the whole expression by $P_{H,t}P_t^{-1}$. Defining $x_{1,t} \equiv \frac{X_{1,t}}{P_{H,t}^{\epsilon} P_t^{-1}}$ and $x_{2,t} \equiv \frac{X_{2,t}}{P_{H,t}^{1-\epsilon} P_t^{-1}}$, the auxiliary variables become

$$\begin{aligned}
x_{1,t} &= C_t^{-\sigma} Y_t (1-\alpha) mc_t + \sum_{k=1}^{\infty} (\beta\theta)^k \,\mathrm{E}_t \left\{ C_{t+k}^{-\sigma} \left(\frac{P_{t+k}}{P_t} \right)^{-1} Y_{t+k} \left(\frac{P_{H,t+k}}{P_{H,t}} \right)^{\epsilon} (1-\alpha) mc_{t+k} \right\} \\
&= C_t^{-\sigma} Y_t (1-\alpha) mc_t + \\
&\quad + \frac{P_{t+1}^{-1}}{P_t^{-1}} \frac{P_{H,t+1}^{\epsilon}}{P_{H,t}^{\epsilon}} \sum_{k=0}^{\infty} (\beta\theta)^{k+1} \,\mathrm{E}_t \left\{ C_{t+k+1}^{-\sigma} \left(\frac{P_{t+k+1}}{P_{t+1}} \right)^{-1} Y_{t+k+1} \left(\frac{P_{H,t+k+1}}{P_{H,t+1}} \right)^{\epsilon} (1-\alpha) mc_{t+k+1} \right\} \\
&= C_t^{-\sigma} Y_t (1-\alpha) mc_t + \beta\theta \,\mathrm{E}_t \left\{ \Pi_{t+1}^{-1} \Pi_{H,t+1}^{\epsilon} x_{1,t+1} \right\}
\end{aligned} \tag{3.54}$$

[6]Conditional on $(1-\alpha)$ which is the share of labor in production. Usually, α denotes the capital share in production, however this model omits the presence of capital.

[7]This is required in order to solve the model.

$$x_{2,t} = C_t^{-\sigma} Y_t + \left(\frac{P_{t+1}}{P_t}\right)^{-1} \left(\frac{P_{H,t+1}}{P_{H,t}}\right)^{\epsilon} \sum_{k=0}^{\infty} (\beta\theta)^{k+1} \mathrm{E}_t \left\{ C_{t+k+1}^{-\sigma} \left(\frac{P_{t+k+1}}{P_{t+1}}\right)^{-1} Y_{t+k+1} \left(\frac{P_{H,t+k+1}}{P_{H,t+1}}\right)^{\epsilon-1} \right\}$$
$$= C_t^{-\sigma} Y_t + \beta\theta \, \mathrm{E}_t \left\{ \Pi_{t+1}^{-1} \Pi_{H,t+1}^{\epsilon-1} x_{2,t+1} \right\} \tag{3.55}$$

and the updating price then becomes

$$\Pi_{H,t}^{\#} = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{k=0}^{\infty} (\beta\theta)^k \, \mathrm{E}_t \left\{ C_{t,t+k}^{-\sigma} \Pi_{t+k}^{-1} Y_{t+k} \Pi_{H,t+k}^{\epsilon} (1-\alpha) mc_{t+k} \right\}}{\sum_{k=0}^{\infty} (\beta\theta)^k \, \mathrm{E}_t \left\{ C_{t,t+k}^{-\sigma} \Pi_{t+k}^{-1} Y_{t+k} \Pi_{H,t+k}^{\epsilon-1} \right\}} = \frac{\epsilon}{\epsilon - 1} \frac{x_{1,t}}{x_{2,t}} \tag{3.56}$$

where $\Pi_{H,t}^{\#} \equiv \frac{P_{H,t}^{\#}}{P_{H,t}}$, $\Pi_{H,t+k} \equiv \frac{P_{H,t+k}}{P_{H,t}}$ and $\Pi_{t+k} \equiv \frac{P_{t+k}}{P_t}$.

The expression of the overall domestic price level evolution can be derived from the Calvo assumptions [Calvo, 1983]. In each period t, the domestic price level consists of the fraction $(1-\theta)$ of firms that were able to change their prices in the last period $(t-1)$, and of the fraction θ that had to keep the previous $t-1$ price level. The price level at time t is then

$$P_{H,t}^{1-\epsilon} = \int_0^{1-\theta} (P_{H,t}^{\#})^{1-\epsilon}(j) dj + \int_{1-\theta}^{1} P_{t-1}(j) dj \tag{3.57}$$

Since there is a large number of firms (an infinite amount) and the price-updating firms are chosen randomly, the sum or integral of individual prices over some subset of the unit interval is proportional to the integral over the entire unit interval [Sims, 2014], i.e. $\int_{1-\theta}^{1} P_{t-1}(j) dj = \theta \int_0^1 P_{t-1}(j) dj = \theta P_{t-1}$. The expression above can be then written as

$$P_{H,t}^{1-\epsilon} = (1-\theta)(P_{H,t}^{\#})^{1-\epsilon} + \theta P_{H,t-1}^{1-\epsilon} \tag{3.58}$$

or

$$1 = (1-\theta)(\Pi_{H,t}^{\#})^{1-\epsilon} + \theta \Pi_{H,t}^{\epsilon-1} \tag{3.59}$$

which describe the evolution of the domestic price level and domestic inflation rate recursively.

3.5 Aggregation of the supply side

To get the expression for the aggregate output, use the intermediate-good demand (3.41), the production function (3.42) and integrate it over all producers $j \in [0, 1]$

$$\begin{aligned} \int_0^1 A_t N_t(j)^{1-\alpha} dj &= \int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} Y_t dj \\ Y_t &= A_t N_t^{1-\alpha} \frac{1}{Z_t} \end{aligned} \tag{3.60}$$

where $Z_t \equiv \int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon}$ is a price dispersion. As mentioned, the price rigidity introduced via the Calvo pricing [Calvo, 1983] becomes the source of one market inefficiency[8]. Since $Z \geq 1$, the presence of the nominal rigidity is always costly. In particular, the price dispersion leads to lower aggregate output entailing output loss. If firms are not allowed to charge the optimal price, they will as the result not produce the optimal amount of output.

$$\begin{aligned} Z_t &= \int_0^1 \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} dj \\ &= \int_0^{1-\theta} \left(\frac{P^{\#}_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} dj + \int_{1-\theta}^1 \left(\frac{P_{H,t-1}(j)}{P_{H,t}}\right)^{-\epsilon} dj \\ &= (1-\theta)\left(\frac{P^{\#}_{H,t}}{P_{H,t}}\right)^{-\epsilon} + \int_{1-\theta}^1 \left(\frac{P_{H,t-1}(j)}{P_{H,t-1}}\right)^{-\epsilon} \left(\frac{P_{H,t-1}}{P_{H,t}}\right)^{-\epsilon} dj \end{aligned}$$

or

$$Z_t = (1-\theta)(\Pi^{\#}_{H,t})^{-\epsilon} + \theta \Pi^{\epsilon}_{H,t} Z_{t-1} \tag{3.61}$$

[8]The other source of inefficiency is the presence of the monopolistic competition implying non-zero profits.

3.6 Small open economy identities

Terms of Trade This section describes the assumptions and identities that link our small open economy with the rest of the modeled world. The bilateral terms of trade is defined as $S_{i,t} \equiv \frac{P_{i,t}}{P_{H,t}}$ and it expresses the price of country i's goods in terms of domestic goods. The effective terms of trade can be derived by integrating the bilateral terms of trade over $i \in [0, 1]$ countries

$$S_t = \int_0^1 \frac{P_{i,t}}{P_{H,t}} di = \frac{P_{F,t}}{P_{H,t}} \tag{3.62}$$

Using the CPI definition $P_t \equiv [(1-\tau)P_{H,t}^{1-\eta} + \tau P_{F,t}^{1-\eta}]^{\frac{1}{1-\eta}}$, we can derive the relationship between the CPI and the domestic price levels

$$h_t \equiv \frac{P_t}{P_{H,t}} = \left(1 - \tau + \tau S_t^{1-\eta}\right)^{\frac{1}{1-\eta}} \tag{3.63}$$

or

$$\frac{\Pi_t}{\Pi_{H,t}} = \left(\frac{1 - \tau + \tau S_t^{1-\eta}}{1 - \tau + \tau S_{t-1}^{1-\eta}}\right)^{\frac{1}{1-\eta}} \tag{3.64}$$

The gap between the CPI and domestic inflation is therefore proportional to the change in terms of trade. Lower domestic prices make the home country more competitive compared to the rest of the world which then, depending on the elasticity η, improve terms of trade.

The Law of One Price The model assumes that the law of one price holds every period, i.e. $P_{i,t}(j) = \varepsilon_{i,t} P_{i,t}^i(j)$ must hold at all time and for all $i, j \in [0, 1]$ with $\varepsilon_{i,t}$ being the bilateral nominal exchange rate (the price of country i's currency in terms of the domestic currency) and $P_{i,t}^i(j)$ being the price of country i's good j in terms of country i's currency.

Integrating over j goods and i countries yields

$$\int_0^1 \int_0^1 P_{i,t}(j)djdi = \int_0^1 \int_0^1 \varepsilon_{i,t} P^i_{i,t}(j)djdi$$
$$P_{F,t} = \varepsilon_t P^*_t \tag{3.65}$$

where ε_t is the effective nominal exchange rate and P^*_t is the world price index. It is useful to note that for the world, there is no distinction between the world's CPI and the world's domestic price levels, i.e. $P^*_t = P^*_{H,t} = P^*_{F,t}$. Combining the last expression with terms of trade implies $S_t = \frac{\varepsilon_t P^*_t}{P_{H,t}}$ which results in

$$\Delta\varepsilon_t \equiv \frac{\varepsilon_t}{\varepsilon_{t-1}} = \frac{S_t}{S_{t-1}} \frac{\Pi_{H,t}}{\Pi^*_t} \tag{3.66}$$

An increase in the domestic inflation or a decrease in the world inflation leads to a nominal depreciation, ceteris paribus. This is consistent with the relative purchasing power parity, i.e. an expected increase in domestic prices has to be compensated with the currency depreciation in order to keep the purchasing powers of foreign currencies constant (and vice versa, an increase in foreign prices will, ceteris paribus, lead to the domestic currency appreciation).

The Real Exchange Rate The bilateral real exchange rate is defined as the ratio of the two countries' CPI in terms of the domestic currency, $\Omega_{i,t} \equiv \frac{\varepsilon_{i,t} P^i_t}{P_t}$. The effective real exchange rate is then

$$\int_0^1 \Omega_{i,t} di = \int_0^1 \frac{\varepsilon_{i,t} P^i_t}{P_t} di$$
$$\Omega_t = \frac{\varepsilon_t P^*_t}{P_t} = S_t \frac{P_{H,t}}{P_t} \tag{3.67}$$

where the last line depicts the positive relationship between real exchange rate and terms of trade (note, that the CPI/domestic price ratio also positively depends on the terms of trade as shown in the previous paragraph).

3.7 International risk sharing

The model presented here assumes complete bond markets implying that the same Euler equation (3.37) must hold in any other country, i.e. the prices of bonds must equal across the world. This implies

$$\mathrm{E}_t\left\{Q_{t,t+1}\right\} = \beta \mathrm{E}_t\left\{\left(\frac{C^i_{t+1}}{C^i_t}\right)^{-\sigma}\left(\frac{P^i_t}{P^i_{t+1}}\right)\frac{\varepsilon^i_t}{\varepsilon^i_{t+1}}\right\} \tag{3.68}$$

which is the Euler equation of country i expressed in terms of the domestic currency (of the modeled small economy). Since $\mathrm{E}_t\left\{Q_{t,t+1}\right\}$ must hold across the world, it must also hold that

$$\beta \mathrm{E}_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{P_t}{P_{t+1}}\right\} = \beta \mathrm{E}_t\left\{\left(\frac{C^i_{t+1}}{C^i_t}\right)^{-\sigma}\frac{P^i_t}{P^i_{t+1}}\frac{\varepsilon_t}{\varepsilon_{t+1}}\right\} \tag{3.69}$$

implying

$$C_t = \Omega_{i,t}^{\frac{1}{\sigma}} C^i_t \upsilon_i \tag{3.70}$$

where $\upsilon_i \equiv \left(\frac{C_{t+1}}{C^i_{t+1}}\Omega_{i,t+1}^{-\frac{1}{\sigma}}\right)$ is constant and in case of symmetric initial conditions it holds that $\upsilon_i = 1$. Integrating over i gives the relationship between the domestic consumption and the world's consumption conditional on the effective real exchange rate (and given the initial conditions which are constant over time).

$$C_t = \Omega_t^{\frac{1}{\sigma}} C^*_t \tag{3.71}$$

3.8 The trade balance

The net export is defined in terms of domestic output as a fraction of steady state output Y.

$$nx_t \equiv \frac{1}{Y}\left(Y_t - \frac{P_t}{P_{H,t}}C_t\right) \tag{3.72}$$

The effect of price increase on the net export depends on the elasticity of substitution.

3.9 Equilibrium

Each producer sell its products to domestic and foreign consumers.

$$\begin{aligned} Y_t(j) &= C_{H,t}(j) + \int_0^1 C^i_{H,t}(j)di \\ &= \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} \left(C_{H,t} + \int_0^1 C^i_{H,t}di\right) \end{aligned} \tag{3.73}$$

The last term $C^i_{H,t}(j)$ denotes the country i's demand for goods produced by the domestic economy and the second line comes from the assumption of symmetric preferences across countries. This assumption also implies that the amount of consumption goods imported into the modeled economy (foreign goods consumption $C_{F,t}$) must be equal to the amount of domestic goods exported into the world (the rest of the $i \in [0, 1]$ countries).

Using the expression for the optimal allocation on imported goods by country of origin (3.20) and the demand for the foreign goods (3.30) gives

$$\begin{aligned} C^i_{H,t} &= \left(\frac{P_{H,t}}{\varepsilon_{i,t} P^i_{F,t}}\right)^{-\gamma} C^i_{F,t} \\ &= \tau \left(\frac{P_{H,t}}{\varepsilon_{i,t} P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t \end{aligned}$$

which if we insert together with the demand for domestic good (3.29) into the (3.73) gives

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} \left((1-\tau)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \tau \int_0^1 \left(\frac{P_{H,t}}{\varepsilon_{i,t} P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t di\right) \tag{3.74}$$

Finally, using the demand function of the final good firms (3.41) results in

$$
\begin{aligned}
Y_t &= (1-\tau)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \tau \int_0^1 \left(\frac{P_{H,t}}{\varepsilon_{i,t} P^i_{F,t}}\right)^{-\gamma} \left(\frac{P^i_{F,t}}{P^i_t}\right)^{-\eta} C^i_t di \\
&= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\tau)C_t + \tau \int_0^1 \left(\frac{P_{H,t}}{\varepsilon_{i,t} P^i_{F,t}}\right)^{-\gamma} \left(\frac{\varepsilon_{i,t} P^i_{F,t}}{\varepsilon_{i,t} P^i_t}\right)^{-\eta} \left(\frac{P_{H,t}}{P_t}\right)^{\eta} C^i_t di\right] \\
&= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\tau)C_t + \tau \int_0^1 \left(\frac{\varepsilon_{i,t} P^i_{F,t}}{P_{H,t}}\right)^{\gamma-\eta} \Omega^{\eta}_{i,t} C^i_t di\right] \\
&= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\tau)C_t + \tau \int_0^1 \left(\frac{\varepsilon_{i,t} P^i_{F,t} P_{i,t}}{\varepsilon_{i,t} P^i_{i,t} P_{H,t}}\right)^{\gamma-\eta} \Omega^{\eta-\frac{1}{\sigma}}_{i,t} C_t di\right] \\
&= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\tau)C_t + \tau \int_0^1 \left(S^i_t S_{i,t}\right)^{\gamma-\eta} \Omega^{\eta-\frac{1}{\sigma}}_{i,t} C_t di\right]
\end{aligned}
$$

$$
Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[1-\tau+\tau S^{\gamma-\eta}_t \Omega^{\eta-\frac{1}{\sigma}}_t\right] C_t \tag{3.75}
$$

which is the domestic market clearing condition. The third line uses the fact that $\Omega_t = \frac{\varepsilon_{i,t} P^i_t}{P_t}$, the fourth line uses the international risk sharing equation $C_t = C^i_t \Omega^{\frac{1}{\sigma}}_{i,t}$ and the fact that $P_{i,t} = \varepsilon_{i,t} P^i_{i,t}$. For the last line, note that the effective terms of trade of country i implies $\int_0^1 S^i_t di = \int_0^1 \frac{P^i_{F,t}}{P^i_{i,t}} di = \frac{P^*_{F,t}}{P^*_t} = 1$.

Lastly, the world market clearing condition is

$$
Y^*_t = C^*_t \tag{3.76}
$$

3.10 Monetary policy and interest rate rules

The model is closed with the setting of a monetary policy rule. Monetary policy is portrayed by a simple interest rate rule (a feedback rule).

$$
R_t = \phi_R R_{t-1} + (1-\phi_R)\left[\frac{1}{\beta} + \phi_\Pi(\Pi_t - \Pi) + \phi_Y(Y_t - Y_{f,t}) + \phi_\varepsilon(\Delta\varepsilon_t - 1)\right] + \nu_t \tag{3.77}
$$

where the variables without the time subscripts denote their steady state values, and $Y_{f,t}$ denotes output in the case of fully flexible prices, i.e. the potential output. Coefficients ϕ_π, ϕ_y and ϕ_ε measure the magnitude of policy rate responses to each variable's deviations from its target (potential output in case of the output and steady state values in case of the other variables). The smoothing parameter ϕ_R reflects the importance of not making sudden changes in the policy rate. Note, that in the model, there is no distinction between the policy rate and the nominal interest rate. Also, the real interest rate is not present in this model and the transmission link between the nominal interest rate, inflation and consumption are built in the Euler equation (3.37). Monetary policy shocks can be propagated into the interest rate rule via $v_t \sim \mathcal{N}(0, \sigma_v^2)$.

$$v_t = \phi_v \, v_{t-1} + e_{vt} \tag{3.78}$$

In general, the policy rate should increase when there are rising inflationary pressures and the economy starts to overheat, and similarly, the policy rate should decrease when the economy starts falling into a recession and the economy experiences deflationary pressures. This rule-like behavior is believed to stabilize and sustain economic growth (in normal situations and in most countries, where the evaluation of monetary policies have been done). This is portrayed in the right-hand side of the equation when the coefficients ϕ_π and ϕ_y are both positive. When the economy deviates from its inflation target, which is in this thesis the zero-inflation rate, i.e. price stability[9], the policy rate goes up to slow down the economy and to mitigate the inflationary pressures. Similarly, when the actual output is below its potential output, i.e. negative output gap, the central bank reacts by lowering the policy rate to close the output gap (or get close to closing it).

This mechanical rule is a form of the Taylor rule [Taylor, 1993] that prescribes how the

[9]In practice, a slightly positive inflation is still considered to be consistent with price stability. The price indexes are up-ward biased as they do not capture quality improvements in time and the substitution towards cheaper goods as prices go up. Another argument of targeting a positive and low inflation rate is based on wage rigidities. For example, when the economy is in recession and the nominal wages are down-ward rigid, a slightly positive inflation helps get the real wage into its new lower equilibrium, helping the labor market to clear out.

policy rate should change in response to movements in inflation and real output. As mentioned at the beginning of this thesis, the Taylor rule is a simplification of how central banks react to these variables and it can sufficiently proxy the inflation-rate targeting regime in models. In practice, however, the inflation-rate targeting regime demands more information, especially the forward-looking outlooks, when making decisions about changing the policy rate. [Taylor, 1993] was aware of the difficulty to follow some sort of a mechanical rule in the real world and recommended to at most use it as one of the information inputs among others.

There are two parts of the interest rate rule that deviates from the seminal Taylor rule [Taylor, 1993], the first part incorporate the interest rate smoothing into the rule. Depending on the calibration of the smoothing parameter ϕ_R, the monetary authority will not change the policy rate abruptly. The parameter ϕ_R measures the importance of staying close to the previous policy rate. In practice, policy-makers also do not want to change the policy rate abruptly as that could cause unnecessary shocks to the economy.

The second part includes the exchange rate stability as another goal. The stable exchange rate is also arguably relevant for the welfare as it can affect the relative prices and thus the equilibrium allocation of resources and price levels. There however exists a trade-off between the price stability and the stability of the nominal exchange rates. In the extreme case of pure exchange rate stability, i.e. exchange rate peg / fixed exchange rate regime, the central bank is not allowed to change its policy rate at all. In that case, the monetary authority cannot react to inflationary pressures or output gap. This trade-off is portrayed in the model and is also supported by the literature for countries as the US, Germany, France, Italy, Japan, and the UK. In these countries, the fixed-exchange-rate periods were accompanied with higher fluctuations in real output and worse inflation performance [Taylor, 1993], and as mentioned at the beginning, in the Czech Republic, the conflict between the two goals was a reason to leave the currency peg by the end of 1997.

This thesis aims to study the potential benefits of a less extreme case, where the exchange rate stability is still important for the central bank but not in a rigid manner as the currency peg. The coefficient ϕ_ε measures the importance of the exchange rate stability, i.e. how much weight the central bank puts on the stability of the nominal exchange rate. Note, that what is important for the central bank are the changes in the nominal exchange rate, not its level. I assume, that the central bank allows the nominal exchange rate to departs from its initial level as long as the exchange rate stabilizes at the new level. This is another distinction from the fixed exchange rate regime, where the central bank does not allow the exchange rate to change its level (unless it devaluates or revaluates the exchange rate). The goal of the thesis is to find out whether there can be some welfare gains from partly trading-off price stability for exchange rate stability.

In the case of the Czech Republic, [Vašíček and Musil, 2006] shows that the inflation targeting regime conducted in the Czech Republic can be represented by a modified Taylor rule with estimated coefficients

$$R_t = 0.65 R_{t-1} + (1 - 0.65)\left[1.27(\Pi_t - \Pi) + 0.47(Y_t - Y_{f,t})\right] + v_t \tag{3.79}$$

A similar estimation of a Taylor rule is used in [Slaný et al., 2006]. The 65% of the current policy rate is affected by the previous rate. The rest 35% is then affected by inflation and the output gap. As expected, the central bank put 2.7 times more weight on inflation gap than on the output gap. A 1% increase in the inflation gap leads on average to increase in the policy rate by 0.44%, while 1%-increase in the output gap leads to an increase in the policy rate by only 0.16%. This reflects the fact that the fundamental goal of the Czech National Bank is the price stability, but that the central bank also reacts to output dynamics as long as the reaction is aligned with the fundamental goal. I will use this estimated Taylor rule as the benchmark case against which I will compare the hypothetical central banks that put different weight on the exchange rate stability as well.

3.11 Marginal cost and output gap

Monetary policy reacts to the deviation of the output from its potential, i.e. it reacts to the output gap. The potential output is defined as the flexible-price output or the output that could be achieved under no nominal rigidities. The potential output changes with real factors, e.g. productivity shocks. To find the output gap, it is convenient to first go through the supply side and the (domestic) marginal costs dynamics. The consumer's intra-temporal condition (3.36), the aggregate output (3.60), the international risk-sharing condition (3.71), the world clearing condition (3.76) and $\Omega_t = S_t h_t^{-1}$ imply[10]

$$w_t = C_t^{\sigma} N_t^{\varphi} = Y_t^{*,\sigma} S_t h_t^{-1} Y_t^{\frac{\varphi}{1-\alpha}} Z_t^{\frac{\varphi}{1-\alpha}} A_t^{-\frac{\varphi}{1-\alpha}}$$

and since $mc_t = \frac{w_t}{(1-\alpha)A_t N_t^{-\alpha}} h_t$, it holds that the equilibrium real marginal costs must be[11]

$$\begin{aligned} mc_t &= Y_t^{*,\sigma} S_t Y_t^{\frac{\varphi}{1-\alpha}} Z_t^{\frac{\varphi}{1-\alpha}} A_t^{-\frac{\varphi}{1-\alpha}-1-\frac{\alpha}{1-\alpha}} Y_t^{\frac{\alpha}{1-\alpha}} Z_t^{\frac{\alpha}{1-\alpha}} \frac{1}{1-\alpha} \\ &= Y_t^{*,\sigma} S_t Y_t^{\frac{\varphi+\alpha}{1-\alpha}} Z_t^{\frac{\varphi+\alpha}{1-\alpha}} A_t^{-\frac{\varphi+\alpha}{1-\alpha}-1} \frac{1}{1-\alpha} \end{aligned} \tag{3.80}$$

We can see that the real marginal costs are increasing in the term of trades and domestic output. Both variables affect real wage through the income effect on the labor supply resulting from their impact on domestic consumption [Galí and Monacelli, 2005]. Technology, through its direct effect on labor productivity and through its effect on employment and real wage for a given output affect marginal costs as in the closed economy case [Galí and Monacelli, 2005]. Lastly, the world output affects the real marginal costs as well.

[10]Combining the latter three leads to $C_t^{\sigma} = Y_t^{*,\sigma}\Omega_t = Y_t^{*,\sigma} S_t h_t^{-1}$, and the aggregate output can be written as $N_t = Y_t^{\frac{1}{1-\alpha}} Z_t^{\frac{1}{1-\alpha}} A_t^{-\frac{1}{1-\alpha}}$.

[11]After substituting for the real wage and the labour.

The output gap is defined as the difference between the actual and the potential output.

$$Y_{gap,t} \equiv Y_t - Y_{f,t} \tag{3.81}$$

Where the potential output is consistent with the flexible-price case output. Under flexible-price situation, the real marginal costs is always constant and equals to the inverse of constant markup, $mc_t = \frac{\epsilon-1}{\epsilon}(1-\alpha)^{-1}$. The flexible-price output is therefore

$$Y_{f,t} = \left(Y_t^{*,-\sigma} A_t^{\frac{\varphi+1}{1-\alpha}} \frac{\epsilon - 1}{\epsilon} \right)^{\frac{1-\alpha}{\varphi+\alpha}} \tag{3.82}$$

4 Solution of the model and welfare results

4.1 Equilibrium conditions and the solution method

The previous sections introduced the model's assumptions and identities and showed the derivation of the equilibrium conditions of the economy expressed in levels (as opposed in log-deviations which would be the result of log-linearization - the case of [Galí and Monacelli, 2005]). This section summarizes the equilibrium conditions and describes the solution method.

[Utility function]

$$\mathbf{U}_t = \frac{{C_t}^{1-\sigma}}{1-\sigma} + \frac{{N_t}^{1+\varphi}}{1+\varphi} \tag{3.2}$$

[Intra-temporal condition]

$$w_t = {C_t}^{\sigma} {N_t}^{\varphi} \tag{3.36}$$

[Euler equation]

$$Q_t = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \Pi_{t+1}^{-1} \tag{3.37}$$

[No-arbitrage condition]

$$R_t = \frac{1}{Q_t}$$

[Productivity Process]

$$\ln(A_t) = \rho_A \ln(A_{t-1}) + e_t^A \tag{3.43}$$

[Real marginal cost]

$$mc_t = h_t \frac{w_t}{(1-\alpha)\, {N_t}^{(-\alpha)}\, A_t} \tag{3.48}$$

[Auxiliary variable 1]

$$x_{1t} = (1-\alpha) mc_t\, {C_t}^{(-\sigma)}\, Y_t + \beta\,\theta\, {\Pi_{Ht+1}}^{\epsilon}\, {\Pi_{t+1}}^{(-1)}\, x_{1t+1} \tag{3.54}$$

[Auxiliary variable 2]

$$x_{2t} = {C_t}^{(-\sigma)}\, Y_t + {\Pi_{t+1}}^{(-1)}\, \beta\,\theta\, {\Pi_{Ht+1}}^{\epsilon-1}\, x_{2t+1} \tag{3.55}$$

[Calvo pricing]

$$\Pi^{\#}_{Ht} = \frac{\epsilon}{\epsilon-1}\frac{x_{1t}}{x_2} \tag{3.56}$$

[Price evolution]

$$1 = (1-\theta)\, {\Pi^{\#}_{Ht}}^{1-\epsilon} + \theta\, {\Pi_{Ht}}^{\epsilon-1} \tag{3.59}$$

[Agregate Output]

$$Y_t = \frac{A_t\, {N_t}^{1-\alpha}}{Z_t} \tag{3.60}$$

[Price dispersion]

$$Z_t = (1-\theta)\, {\Pi^{\#}_{Ht}}^{(-\epsilon)} + \theta\, {\Pi_{Ht}}^{\epsilon}\, Z_{t-1} \tag{3.61}$$

[Auxilary variable CPI / Domestic price]

$$h_t = \left(1-\tau+\tau\, {S_t}^{1-\eta}\right)^{\frac{1}{1-\eta}} \tag{3.63}$$

[CPI / Domestic inflation]

$$\Pi_t = \frac{\left(1-\tau+\tau\, {S_t}^{1-\eta}\right)^{\frac{1}{1-\eta}}\, \Pi_{Ht}}{\left(1-\tau+\tau\, {S_{t-1}}^{1-\eta}\right)^{\frac{1}{1-\eta}}} \tag{3.64}$$

[Nominal interest rate]

$$\Delta\varepsilon_t = \frac{S_t}{S_{t-1}}\frac{\Pi_{H,t}}{\Pi^*_t} \tag{3.66}$$

[Real interest rate]

$$\Omega_t = \frac{S_t}{\left(1 - \tau + \tau\, {S_t}^{1-\eta}\right)^{\frac{1}{1-\eta}}} \tag{3.67}$$

[Risk-sharing condition]

$$C_t = C_t^*\, {\Omega_t}^{\frac{1}{\sigma}} \tag{3.71}$$

[Net export]

$$nx_t = \frac{1}{Y}\left(Y_t - \frac{P_t}{P_{H,t}} C_t\right) \tag{3.72}$$

[Clearing condition / Equilibrium]

$$Y_t = C_t\, {h_t}^{\eta} \left(1 - \tau + \tau\, {S_t}^{\gamma-\eta}\, {\Omega_t}^{\eta - \frac{1}{\sigma}}\right) \tag{3.75}$$

[Monetary policy rule]

$$R_t = \phi_R R_{t-1} + (1-\phi_R)\left[\frac{1}{\beta} + \phi_\Pi(\Pi_t - \Pi) + \phi_Y(Y_t - Y_{f,t}) + \phi_\varepsilon(\Delta\varepsilon_t - 1)\right] + \nu_t \tag{3.77}$$

[Monetary policy shock]

$$\nu_t = \phi_\nu\, \nu_{t-1} + e_{\nu t} \tag{3.78}$$

[Output gap]

$$Y_{gap,t} = Y_t - Y_{f,t} \tag{3.81}$$

[Flexible-price output]

$$Y_{f,t} = \left(Y_t^{*,-\sigma} A_t^{\frac{\varphi+1}{1-\alpha}} \frac{\epsilon - 1}{\epsilon}\right)^{\frac{1-\alpha}{\varphi+\alpha}} \tag{3.82}$$

[World market clearing condition]

$$C_t^* = Y_t^*$$

[World's outpout]

$$Y_t^* = Y$$

[World's inflation]

$$\Pi_t^* = \Pi$$

This system of stochastic difference equations represents the general equilibrium of the model and the solution to it is a set of policy and transition functions, usually written in the state-space form. The policy functions map the dynamics of the control variables across the state variables[1]. DSGE models usually do not have a closed-form analytical solution and one cannot solve it by hand. A standard-way way of analyzing these model is by applying a perturbation method which provides an approximate solution around some given point, i.e. around the model's deterministic steady state or any other chosen point. Specifically, the perturbation method applied to DSGE models means finding Taylor expansions for the non-linear set of equations. The Taylor expansion provides a set of equations that approximate the equilibrium conditions. A solution algorithm is then applied to solve for the policy functions of the approximated set of equations. These policy functions will usually sufficiently describe the dynamics of the variables around the chosen point (deterministic steady state) [Flotho, 2009] [Judd, 1998][2][3].

The current standard in the literature is to use a first-order approximation method as it is sufficient for most purposes, i.e. a model is linearised (in logs or in levels) and the non-linear set of equations are approximated by linear equations. First-order approximation methods are however not sufficient for welfare comparison of various policies as most policy rules, including the Taylor-like rules, have no first-order effects on the deterministic steady state, i.e. using the first-order approximation will give the same welfare results for all of the interest-rate rule variations. This is called the certainty equivalence principle [Schmitt-Grohé and Uribe, 2004b] [Kim and Kim, 2003a]. On the other hand, higher-order approximation methods take into account the volatility of shocks. For the higher-order approximation methods, there exists a stochastic steady state which is different from the deterministic steady state. The stochastic steady state corrects the deterministic steady

[1]Mapping of consumption given price etc.

[2]A more straight-forward way would be to solve the model using the value function iteration known from dynamic programming. This provides a global solution, i.e. solution that is sufficiently accurate at any given point as opposed to a local solution that is sufficiently accurate only around the steady state, or around any other chosen point where the local approximation method is applied.

[3]The approximated conditions can be solved using the eigenvector-eigenvalue decomposition or the method of undetermined coefficients [Flotho, 2009].

state by accounting for the pre-cautionary behavior of agents. The agents move to a different steady state with a rationale that shocks might happen in the future, i.e. they take the uncertainty measured by the shock variance into their behavior. This behavior is not captured up to a first-order approximation. For the second-order approximation, uncertainty affects the decision rules by a constant term [Schmitt-Grohé and Uribe, 2009] [Adjemian et al., 2018].

Intuitively, when a linear approximation method is applied, only the first moments of the random variables are taken into account and the agents' best guess about any future shock in the steady state is zero (its mean). Using higher order approximation takes into account additional moments of the random variables, for example, the variance of the productivity shocks. This then results in a stochastic steady state that is different from the deterministic one (where the random variables/shocks are set to their means).

The equations are solved using perturbation in MATLAB and Dynare. Dynare is a platform added to MATLAB that was developed for DSGE and OLG (overlapping generations) models. The advantage of using Dynare is its wide applicability on most of the DSGE models[4]. The MATLAB and Dynare codes are attached in the appendices C and D, respectively.

4.2 Steady state

The model is solved around the zero-inflation deterministic steady state, i.e. $\Pi = \Pi_H = \Pi^* = 1$. The zero-inflation steady state implies $\Omega = 1$, $Z = 1$, $h = 1$ and $\Delta\varepsilon = 1$. The Euler Equation (3.37) implies $Q = \beta$.

The Calvo pricing with the auxiliary variables in steady state imply $mc = (1-\alpha)\frac{\epsilon-1}{\epsilon}$.

[4]For example, [Krusell and A. Smith, 1998] that have heterogeneous agents and productivity shock modeled using Markov chain transition matrices cannot be solved using Dynare unless making some changes in its underlying assumptions.

Combining the intra-temporal condition (3.36) with the market clearing condition (3.76) and the production function leads (3.42) to

$$
\begin{aligned}
w &= C^{\sigma} N^{\varphi} \\
&= N^{(1-\alpha)\sigma+\varphi} \\
mc(1-\alpha)N^{-\alpha} &= N^{(1-\alpha)\sigma+\varphi} \\
N &= [mc(1-\alpha)]^{\frac{1}{(1-\alpha)\sigma+\varphi+\alpha}} \qquad (4.1)
\end{aligned}
$$

where the third line uses $mc = \frac{w}{(1-\alpha)N^{-\alpha}}$.

The rest then follows that

$$
\begin{aligned}
C = N^{1-\alpha} &= C^{*} = Y = Y^{*} && (4.2) \\
w &= N^{\sigma(1-\alpha)+\varphi} && (4.3) \\
x_1 &= \frac{(1-\alpha)mcC^{-\sigma}Y}{1-\beta\theta} && (4.4) \\
x_2 &= \frac{C^{-\sigma}Y}{1-\beta\theta} && (4.5)
\end{aligned}
$$

4.3 Calibration

The model presented in this thesis will be evaluated quantitatively using the calibration approach. The calibration maps the theoretical model into the data. The parameters describing the preferences of agents[5] and the environment (underlying structure of the model) are set according to microeconomics studies.

The calibration with respect to the Czech Republic is primarily taken from [Vašíček and Musil, 2006] who calibrated a similar small open economy model. First, the parameter τ is set to 0.60 which roughly reflects the average import/GDP ratio in the Czech Republic in between the

[5] Deep parameters

years 1998-2017 [CSO, 2017]. [Vašíček and Musil, 2006] set that value to 0.40 to reflect the imported consumption to the total consumption ratio in between 1999 and 2003.

One period in the model represents a single quarter in the real world. The discount factor β that reflects the patience of the households is set to 0.99. This discount behavior is associated with a 4% annual real interest rate. This calibration is widely used in the literature.

Parameter σ is set to 0.82, which implies an intertemporal elasticity of 1.22 ($\frac{1}{\sigma}$). This parameter affects the interest rate effect on consumption. The lower the value of σ, the slower the marginal utility from consumption falls as consumption rises and the more are households willing to allow their consumption vary over time (i.e., the less is the consumption smoothing important for the household and the more they react to changes in the interest rate).

Parameter η is set to 0.38 which reflects a low substitutability between domestic and foreign goods. This value is taken from the [Vašíček and Musil, 2006] who argue that the elasticity was very low in the first half of the 90s and although it has become higher across time, the substitutability between domestic and foreign good still remains relatively low. The parameter is arguably hard to estimate but as [Vašíček and Musil, 2006] argue, any value lower than 1 is reasonable. [Slaný et al., 2006] for example estimated the elasticity of substitution between domestic and foreign goods to be 0.28. The estimation is lower than of [Vašíček and Musil, 2006] since [Slaný et al., 2006] used data from 1995 covering the transformation period. The low substitutability can be also explained by significant heterogeneity of goods outside of the country.

Assuming a low substitutability among foreign countries, I apply the same value for γ as well.

Parameter φ is set to 1.08 implying Frisch elasticity of labor supply to be 0.93 rendering the labor supply somewhat inelastic. This reflects the Czech conditions of low geographical labor mobility, long-run unemployment and that a rise in real wage is usually associated with an increase in labor productivity rather than with a higher employment [Vašíček and Musil, 2006].

The Calvo parameter is set to 0.63, implying that 37% of firms update their price every three months. The parameter reflects the average duration of price contracts to be set to roughly 2.7 months [Vašíček and Musil, 2006].

The technological process is also set according to [Slaný et al., 2006] and [Vašíček and Musil, 2006]. The auto-regressive parameter is set to $\rho_A = 0.97$ which reflects the persistent impact of each technological improvement. The parameter is close to one, implying that the auto-regressive productivity process is almost a random walk process. A single shock has persistent effects on the modeled economy. This reflect time periods used for estimation [Slaný et al., 2006], [Vašíček and Musil, 2006]. The standard deviation of productivity e_a is set to 0.95%.

Lastly, the production function parameter α is set to 0.5 [Štork et al., 2009]. The table 4.1 summarizes the calibration of the model.

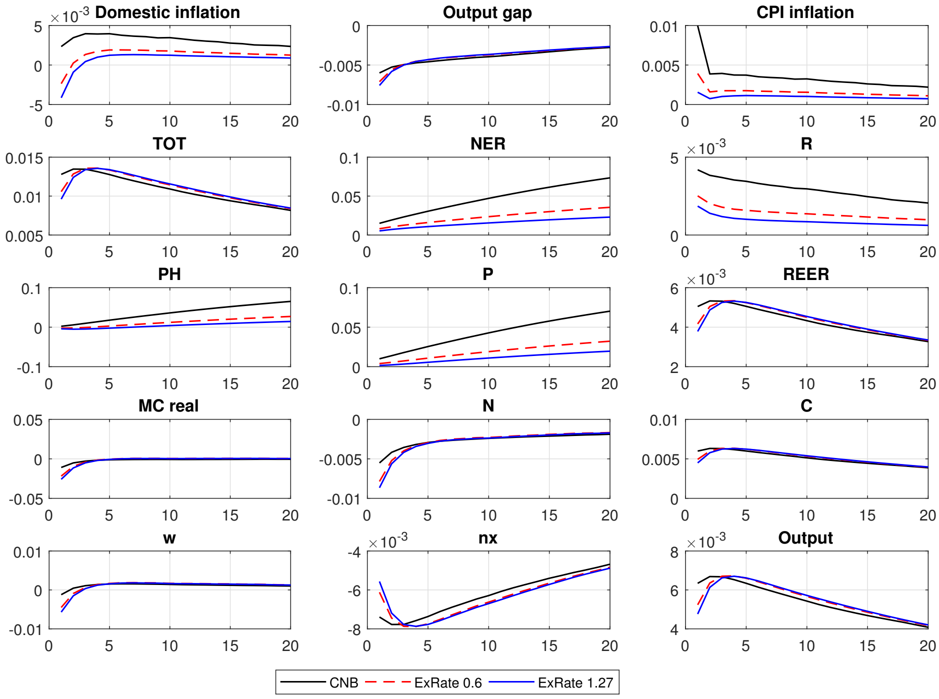
TABLE 4.1: Calibration

Parameter	Value	Description
β	0.99	Discount factor
σ	0.82	Coefficient of relative risk aversion
φ	1.08	Labor parameter (implies a labour supply elasticity of 0.93)
α	0.5	Production function parameter
ρ_A	0.97	Persistence of TFP
e_A	0.0095	Std. dev. of TFP
θ	0.63	Price stickiness parameter
τ	0.6	Openness to imported goods
η	0.38	Elasticity of substitution between domestic and foreign goods
γ	0.38	Elasticity of substitution among foreign countries
ϵ	9	Elasticity of substitution inside a single country
ϕ_Π	1.27	Interest rate rule parameter - Feedback to inflation rate
ϕ_y	0.47	Interest rate rule parameter - Feedback to output deviation
ϕ_r	0.65	Interest rate rule parameter - Interest rate smoothing
ϕ_ε	-	Interest rate rule parameter - Exchange rate

4.4 Impulse response analysis

This part analyses the dynamics of the model and the response of the monetary authority to productivity shock. The impulse response functions will differ from [Galí and Monacelli, 2005] due to the different calibration. The appendix A contains the impulse response functions generated for [Galí and Monacelli, 2005] calibration.

FIGURE 4.1: Impulse Response Functions to a positive productivity shock $\rho_A = 0.97$ and the volatility $\sigma_A^2 = 0.0095^2$



Abbreviations: TOT = Term of Trades, NER = Nominal Exchange Rate, PH = Domestic price, P = CPI price, REER = Real Exchange Rate, MC real = Real marginal cost

The figure 4.1 shows the variable responses to a productivity shock with the horizontal axis denoting time in quarters and the vertical axis measuring deviations from the steady state (in levels). The figure contains three cases of the interest rate rule. The black line represents the interest rate rule estimated for the Czech Republic [Vašíček and Musil, 2006]. The red dashed line represents the interest rate rule with exchange rate stabilization parameter set to $\phi_\varepsilon = 0.6$. The last blue line represents the highest considered weight on the exchange rate stabilization, $\phi_\varepsilon = 1.27$.

Higher productivity helps to decrease marginal costs of domestic production. Since firms desire a constant markup, firms react by gradually[6] cutting their prices to hit that desired

[6]Gradually as the Calvo mechanism let the firm change their prices.

markup, i.e. a positive shock to domestic productivity drops inflation in the beginning and it then follows an upward-sloping path. Looking at the Czech National Bank case, these deflationary pressures are however countered by higher demand for domestic goods.

In particular, an increase in the total factor productivity increases the supply of domestic goods, further supporting the decrease in domestic prices. On the other hand, the boost in production and productivity increases the wealth of domestic households. Higher income of households raises consumption and demand for both domestic and foreign goods, creating inflationary pressures on domestic and CPI price levels.

In the case of exchange rate stabilization, the former effect initially dominates the wealth effect, causing the domestic prices to decline relative to the foreign prices. This decrease in domestic prices relative to foreign ones has a positive effect on the terms of trades which supports export and a further increase in domestic production and wealth. In the case of the Czech Republic, the domestic prices increase at the beginning but the CPI prices rise more, resulting in a larger initial response in the terms of trade.

Net export decreases in all three cases, reflecting that domestic demand for imported goods relatively increases more than the domestic production exported to foreign countries. Increase in demand for foreign goods has inflationary effects on foreign prices. In addition, the law of one price implies that higher the increase in foreign prices, the higher the CPI inflation in the modeled economy and therefore the further the domestic currency depreciates. And the other way around, the weaker the domestic currency gets, the more expensive it is to import foreign goods, implying higher CPI inflation.

Overall, this leads to a positive CPI inflation in the domestic economy in all of the three cases, and the monetary authority contracts the economy by increasing the nominal interest rate at the beginning and then following a downward-sloping path.

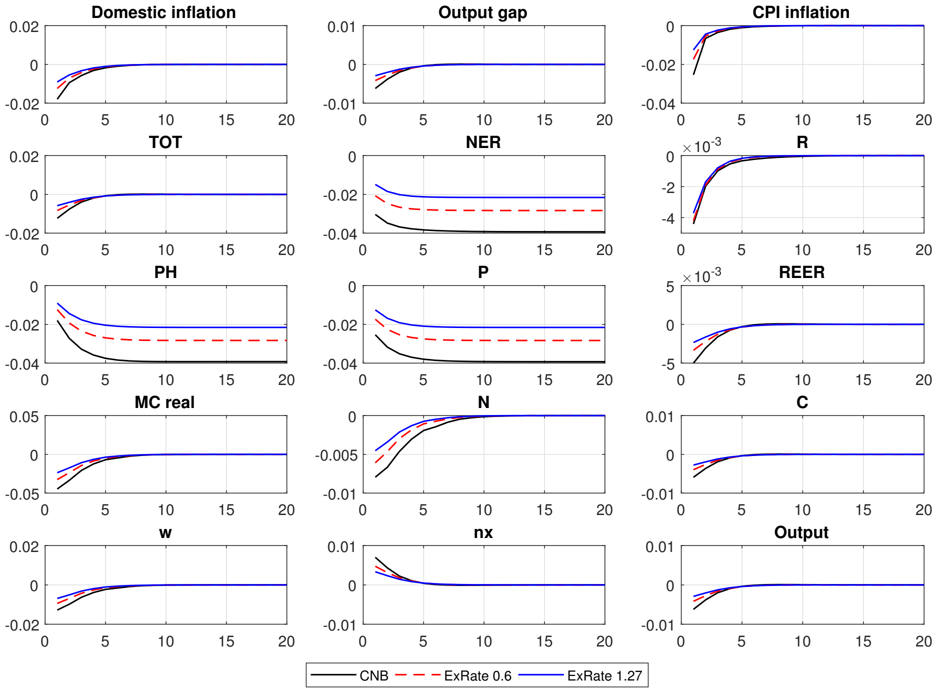
The higher are the inflationary pressures on the CPI level, the stronger is the reaction of the policy-maker by increasing its policy rate.

Under the exchange rate stabilization, the nominal depreciation and the reaction of the terms of trade are more muted. This dampens the positive effect of the productivity shock on the export which causes a more moderate increase in demand. A muted response in demand then generates less inflationary pressures as opposed to the Czech Republic case and the central bank does not have to react so strongly when increasing its policy rate. The more muted change in aggregate demand and thus the more muted the wealth effect then let the productivity shock to cause more deflationary pressures in the economy. The more pronounced stabilization of actual output and CPI inflation is traded with the more amplified reaction of the output gap, i.e. under the exchange rate stabilization, the economy is further away from its improved potential.

Although, the model does not explicitly contain real interest rate we can utilize the Fisher equation to find its dynamics (ad-hoc). Following the dynamic version of the Fisher equation[7], the paths of CPI and nominal interest rate imply a drop in real interest rate with an upward-sloping path. A path, according to the theory, consistent with a jump in consumption with a downward-sloping path.

[7] $\mathbb{E}_0 \sum_{t=0}^{\infty} R_t = \mathbb{E}_0 \sum_{t=0}^{\infty} r_t + \mathbb{E}_0 \sum_{t=0}^{\infty} \Pi_t$ where r_t denotes the real interest rate. The expected path of nominal interest rate must equal to the sum of expected paths of the real interest rate and inflation. If the CPI inflation path is persistently above the nominal interest rate path, the real interest rate path must be negative in order to dampen the higher CPI inflation (assuming the Fisher equation holds at all time).

FIGURE 4.2: Impulse Response Functions to a positive monetary policy shock $\rho_\nu = 0.50$ and the volatility $\sigma_\nu^2 = 0.01^2$



Abbreviations: TOT = Term of Trades, NER = Nominal Exchange Rate, PH = Domestic price, P = CPI price, REER = Real Exchange Rate, MC real = Real marginal cost

The figure 4.2 shows variable responses to a positive policy rate shock. The positive policy rate shock contracts the economy as described in the theoretical part of the thesis. The unexpected jump in the nominal interest rate induces an increase in the real interest rate resulting in the fall of aggregate demand. Lower aggregate demand mutes both domestic and foreign production and results in deflationary pressures. Foreign prices fall faster than domestic prices, resulting in negative terms of trade and the appreciation of domestic currency. The appreciation further supports deflationary pressures of the consumer prices. The net export, on the other hand, jumps up at the beginning and stays positive until the policy rate shock dies out. The demand for imported goods thus falls faster than the production of exported goods.

The monetary authority counter-reacts to the contraction by lowering its interest rate. We can see that in this case, the reaction of the output gap and CPI inflation is less amplified for the interest rate rules with a positive weight on the exchange rate stabilization.

4.5 Welfare analysis

This section compares the welfare implications of various interest rate rule settings, i.e. we are comparing the expected values of welfare associated with different interest rate rules. An important advantage of using a DSGE model, a model with microeconomic foundations of private-agent optimization, is that there is a natural welfare criterion. The preferences of private agents are built in the model and the natural welfare criterion in the context of the model is, therefore, the level of expected utility of the representative household (utility-based approach) [Woodford, 2002].

The criterion used in this thesis is welfare conditional on starting in the deterministic steady state while factoring for future shocks, i.e. conditional welfare. At time t, the conditional welfare W_t of a private agent is given by the expected sum of contemporaneous utilities over his lifetime.

$$W_t = \mathrm{E}_t \left\{ \sum_{k=0}^{\infty} \beta^k U(C_{t+k}, N_{t+k}) \right\}$$

That is, the lifetime utility conditional on information known at time t. This can be written recursively as

$$W_t = U_t + \beta \mathrm{E}_t \{W_{t+1}\}$$

Which we can solve using Dynare and perturbation method, the same way we have solved for other policy functions. To make the different interest rate rules and their associated conditional welfare results comparable, the same starting point t must be set for all the variations of the interest rate rule. I, therefore, assume that the agents are starting in the non-stochastic steady state of the economy, which is identical across all interest rate rules considered in this thesis. In general, different interest rate rules will be associated with a different stochastic steady state and using the conditional welfare criterion accounts for the transitional dynamics of the economy towards the stochastic steady state [Schmitt-Grohé and Uribe, 2004a]. Note, that only the productivity shocks are considered in this analysis. The monetary policy shocks were implemented only to show the response of the economy to changes in the policy rate.

The unconditional expectations of lifetime utility (unconditional welfare) which integrate out initial conditions and future shocks are also calculated in this thesis. The welfare criterion is, however, the conditional welfare and the unconditional one is calculated only for the comparison.

The conditional welfare represents the steady state welfare that takes into account that shocks might happen in the future. For the second order approximation, the conditional mean is calculated using the steady state plus the uncertainty condition. Following the Dynare Reference Manual [Adjemian et al., 2018], the conditional mean of welfare starting at the non-stochastic steady state is equal to

$$W_t = W + 0.5\Delta^2 \tag{4.6}$$

For the second-order approximation, the conditional welfare is, therefore, a sum of its non-stochastic steady state and the uncertainty correction. The expression Δ^2 denotes the shift effect of the variance of future shocks and comes from the second partial derivative with respect to the perturbation parameter, see section 4.13.4 in Dynare Reference Manual for more details [Adjemian et al., 2018].

It is important to solve the model using at least second-order approximation for purpose of this thesis. A log-linearized or linearized solution is in general accurate enough when one wants to estimate the business cycles statistics, getting the covariance structure or fitting the model to data. It is however inaccurate for calculating welfare effects of various monetary and fiscal policies that have the same deterministic steady state, i.e. when the policies have no first-order effects on the model's non-stochastic steady state [Sims, 2000]. Intuitively, the first-order approach is especially inaccurate in open-economy models as shown by [Kim and Kim, 2003a]. The first-order approximation omits higher-order terms which matters for measuring risks and welfare analysis [Kim and Kim, 2003b].

One could use unconditional welfare criterion instead of the conditional one. [Kim and Kim, 2003a] and [Schmitt-Grohé and Uribe, 2004b] however opposed to this practice as unconditional expectations of welfare do not take into account the welfare effects of transitional dynamics from the deterministic to stochastic steady state.

The welfare results of our model are in the table 4.2. The table shows both the conditional and unconditional welfare values. As mentioned, the relevant criterion is the conditional welfare but in our case, the unconditional welfare gives similar results. The first line refers to the interest rate rule that represents the Czech Republic case as estimated by [Vašíček and Musil, 2006]. The values below then refers to the welfare under the monetary authority that reacts to exchange rate fluctuations. The greater the parameter ϕ_ϵ, the more the monetary authority puts weight on stabilizing the exchange rate. The results show positive welfare gains which are increasing for the moderate values of the parameter ϕ_ϵ. Specifically, the welfare gains increase until the parameter ϕ_ϵ hits 0.6. After reaching this weight, the welfare gains start to decrease but stays positive for the whole range of considered weights[8].

The last line in the table shows the expected lifetime utility for a fixed exchange rate regime (currency peg), where the central bank does not follow an interest rate rule but holds $\Delta\epsilon_t = 0$ at all times. The currency peg still indicates welfare gain as opposed to the benchmark situation, however, the gain is lower compared to some of the inflation-targeting regimes that put weight on the exchange rate stabilization. This suggests that

[8]Other then in the currency peg case, I did not consider the case where the policy-maker puts more weight on the exchange rate stability than on the price stability.

there is probably a point where more exchange rate stabilization does not lead to higher welfare.

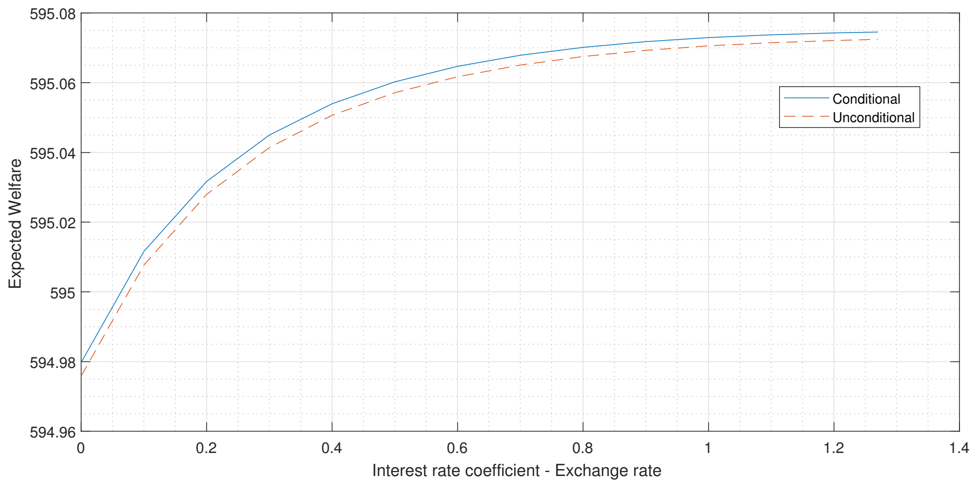
TABLE 4.2: Welfare results for $\phi_r = 0.65$, $\phi_\Pi = 1.27$, $\phi_y = 0.47$ and $\rho_A = 0.97$

ϕ_ϵ	**Conditional**	**Welfare gains in %**	**Unconditional**	**Welfare gains in %**
0	594.9797	-	594.9758128	-
0.1	595.0116	0.0054%	595.0076999	0.0054%
0.2	595.0317	0.0087%	595.0279625	0.0088%
0.3	595.0450	0.0110%	595.0414111	0.0110%
0.4	595.0540	0.0125%	595.0506288	0.0126%
0.5	595.0602	0.0135%	595.0570986	0.0137%
0.6	595.0647	0.0143%	595.0617181	0.0144%
0.7	595.0679	0.0148%	595.0650552	0.0150%
0.8	595.0701	0.0152%	595.0674819	0.0154%
0.9	595.0718	0.0155%	595.0692497	0.0157%
1	595.0729	0.0157%	595.0705325	0.0159%
1.1	595.0737	0.0158%	595.0714537	0.0161%
1.2	595.0743	0.0159%	595.0721021	0.0162%
1.27	595.0745	0.0159%	595.0724293	0.0162%
PEG	595.0554	0.0127%	595.0547	0.0133%

Overall, the results indicate that the public could benefit from the exchange rate stabilization under the inflation-targeting regime. Interestingly, the model suggests that the currency peg might be in terms of welfare be more appropriate than the "pure" inflation-targeting.

The figure 4.3 below portrays the concave concave relationship between the expected lifetime utility and the weight on exchange rate stabilization.

FIGURE 4.3: Expected welfare conditional on how the central bank reacts to exchange rate movements



4.6 Robustness of the results

To test the robustness of the results, this section considers a different calibration. In particular, I argue that the persistence of the productivity process might be overestimated. The value $\rho_A = 0.97$ was estimated by [Vašíček and Musil, 2006] and on the period that still reflects the transformation period of the Czech economy. The auto-regressive process with this coefficient behaves almost as the random walk and the convergence of the process to its steady state is quite slow. Each technological shock has, therefore, a long-lasting impact on the economy (almost permanent).

In the transformation period, the Czech economy had a lot of investment opportunities that were not fully utilized at that time. Improvements in technology, education etc. had arguably more prone impacts on the productivity than I would assume it would have now. I set the persistence parameter to $\rho_A = 0.90$ too see how the persistence of the shock plays a role in conducting monetary policy.

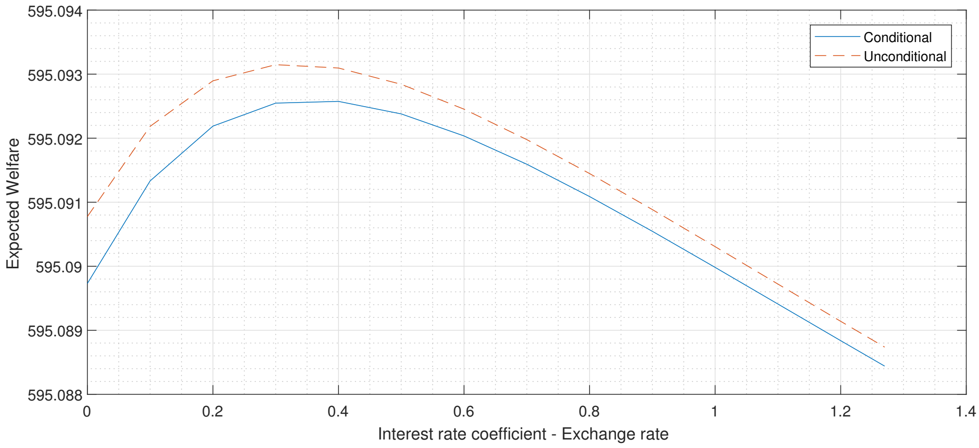
This section tests the robustness by setting the auto-regressive parameter to $\rho_A = 0.90$ that reflects a less persistent impact of a productivity shock. The results of the different calibration of the AR(1) productivity process is shown in the table 4.3 and plotted in the figure 4.4.

$\phi_e ps$	**Conditional**	**Welfare gains in %**	**Unconditional**	Welfare gains in %
0	595.0897	-	595.0908	-
0.1	595.0913	0.0003%	595.0922	0.0002%
0.2	595.0922	0.0004%	595.0929	0.0004%
0.3	595.0925	0.0005%	595.0931	0.0004%
0.4	595.0926	0.0005%	595.0931	0.0004%
0.5	595.0924	0.0004%	595.0928	0.0003%
0.6	595.0920	0.0004%	595.0925	0.0003%
0.7	595.0916	0.0003%	595.0920	0.0002%
0.8	595.0911	0.0002%	595.0914	0.0001%
0.9	595.0905	0.0001%	595.0909	0.0000%
1	595.0900	0.0000%	595.0903	-0.0001%
1.1	595.0894	-0.0001%	595.0897	-0.0002%
1.2	595.0888	-0.0002%	595.0891	-0.0003%
1.27	595.0884	-0.0002%	595.0887	-0.0003%
PEG	595.0656	-0.0040%	595.0658	-0.0042%

TABLE 4.3: Welfare results for $\phi_r = 0.65$, $\phi_\Pi = 1.27$, $\phi_y = 0.47$ and $\rho_A = 0.90$

Under the different calibration of the model, the welfare implications of exchange rate stabilization are now different. A slight nominal exchange rate stabilization still entails some welfare gains. But now there is a cut-off point where putting more weight on the exchange rate yields a welfare loss. In addition, the fixed exchange rate regime is now not a better alternative to the inflation-targeting regime.

FIGURE 4.4: Expected welfare given how the central bank reacts to exchange rate movements

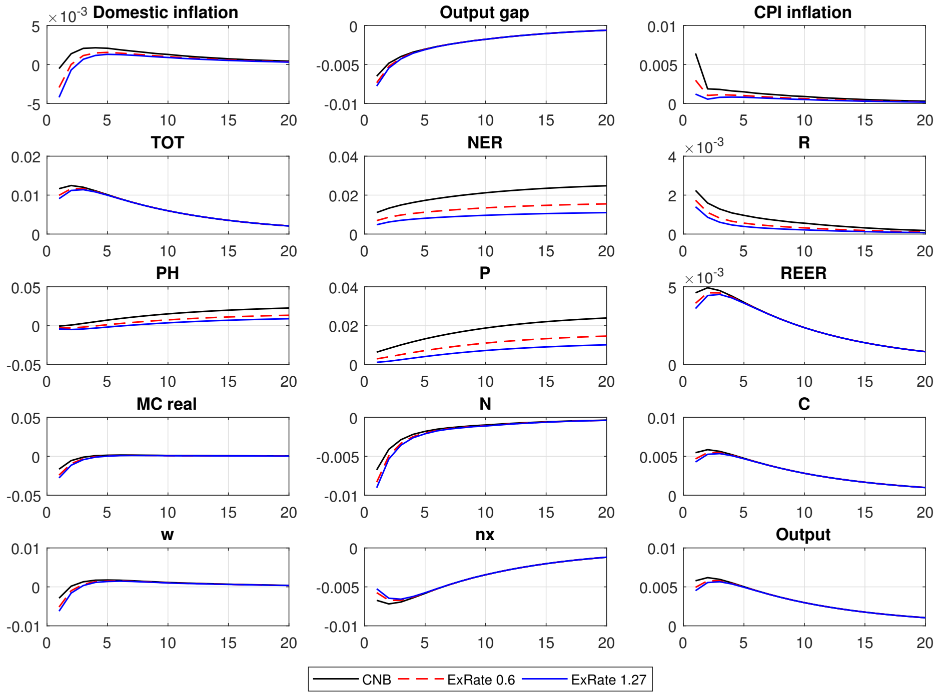


The figure 4.5 shows the impulse response functions given the lower productivity shock persistence. The different results under this calibration come from the different impact of the productivity shock on the terms of trades and inflation.

In the case of the lower shock persistence, a productivity shock has a lower impact on the terms of trade at the beginning as compared to the original calibration. The terms of trades also converge faster to its steady state and the impact of the productivity shock has, therefore, a more mitigated effect on the nominal exchange rate which quickly stabilizes after the shock occurrence.

On the other hand, in the previous productivity process calibration, the shock generated long-lasting deprecatory pressure on the domestic currency and it took more than 50 quarters before the nominal exchange rate became stable at its new level (see B). The long-lasting depreciation of the currency also amplified the inflationary pressures (The depreciation caused an increase in export resulting in higher domestic production and wealth. Higher wealth then increased the aggregate demand which in the combination with more expensive import due to the depreciation generated inflationary pressures in the economy.). In that case, the productivity has more noticeable implications on the economy and the central bank can make the public better off by mitigating these long-lasting destabilizing effects.

FIGURE 4.5: Impulse Response Functions to a positive productivity shock $\rho_A = 0.90$ and the volatility $\sigma_A^2 = 0.0095^2$



In the second case, with $\rho_A = 0.90$, the deprecatory pressures last only briefly before the nominal exchange rate stabilize at its new level and thus there is a less need of the central bank to intervene. The central bank can still mitigate the level at which the new nominal exchange rate will end up but the welfare results indicate that more relevant for the private agents is how the nominal exchange rate changes rather than its level.

Overall, this section with robustness check shows that the welfare implications of exchange rate stabilization are sensitive to the calibration of the model. It would be advisable to do more robustness checks in the future before making any general conclusions. For example, one could implement capital or wage rigidity in the model which also usually result in different implications [Schmitt-Grohé and Uribe, 2004a]. One could also consider a different type of shocks relevant for our economy (change in foreign countries' demand, change in foreign/imported inflation etc.).

5 Conclusion

This thesis investigated whether it would be beneficial to consider, to some degree, nominal exchange rate stability along with the price level stability. Taking into account the Czech economy, the results are inconclusive. The thesis conducts the welfare analysis using the DSGE model of a small open economy as it provides an environment where different policy rules can be tested and compared. Thanks to its microeconomics foundations, the DSGE models are immune to Lucas' critique and offer a utility-based comparison.

The DSGE model used in this work is taken from [Galí and Monacelli, 2005] but differs in the solution approach. The authors used a standard approach of log-linearizing the equilibrium system of equations which is not sufficiently accurate for welfare analysis as argued in this thesis. The first contribution of this work is therefore in showing how to replicate the [Galí and Monacelli, 2005] model without using log-linearization. The model can be therefore approximated by higher-order approximation methods and used for questions related not only to welfare but for example to risk premiums (where the higher-order terms play an important role and should not be omitted by linearization).

The primary scope of this work was to see if there is a room for improvement under the inflation-targeting regime. Specifically, if the exchange rate stability could improve economic welfare. The literature emphasizes the importance of price stability as one of the main factors improving the economy and arguably, the Czech Republic has been so far successful in maintaining the price stability since the introduction of the inflation-targeting framework in 1998. The thesis tests alternative versions of inflation-targeting regimes that also put some importance on the stability of exchange rates.

The exchange rate stabilization is arguably also welfare-improving, however, there is often a trade-off between having stable exchange rates and targeting a desirable inflation rate as both of them are linked to the main policy instrument, policy rate. In the extreme case, under the fixed exchange rates, the central bank is restricted to use the policy rate to target the inflation rate. Having completely stable exchange rates is not therefore feasible under the inflation-targeting regime. This thesis considers less extreme variations of exchange rate stabilization by implementing the exchange rate stabilization into the inflation-targeting regime (instead of replacing the inflation-targeting regime completely). The results show that putting some weight on keeping the nominal exchange rate stability is beneficial but the robustness test also shows that these welfare gains are conditional on the calibration of the model. Specifically, after assuming a less persistent productivity shock, the welfare gains of the exchange rate stabilization policy are more muted and the results also show that putting too much weight on the exchange rate could result in welfare loss.

It should be emphasized that the calibration used in this thesis is mostly taken from [Vašíček and Musil, 2006] and [Slaný et al., 2006] to reflect the Czech Republic case. These papers were published a long time ago and their calibration was based on the transformation period of the Czech Republic. Estimating the parameters for a more recent period might result in different calibration and thus different results. I would, therefore, recommend estimating the model's parameters to see how the parameters stayed stable across the time.

Other improvements or future course regarding this topic would be to try different assumptions of the model. One should start by considering wage rigidity or capital to see how are the results immune to more realistic assumptions. Omitting the presence of the government and fiscal policy could also be crucial for considering an alternative to the current inflation-targeting regime. This thesis considered the introductory version of a small open economy model published by[Galí and Monacelli, 2005]. Since part of the thesis' scope was to find a way how to use a higher-order approximation method, for the tractability reasons any additional variation were not considered.

Finally, one should also consider how it would be practical to have an additional goal besides the price stability. The arguable advantage of having a single goal is that the public clearly understands the role of the central bank. Trying to add an additional goal, especially one as is the nominal exchange rate, might create confusion and negatively affect the transparency and the perception of the public on monetary policy (which are both important for the conduction of monetary policy itself). In practice, one would have to consider whether the potential benefits of altering the inflation-targeting regime could beat the costs of changing the monetary policy regime. Further work on this topic should be therefore done before making any stronger conclusions.

List of Figures

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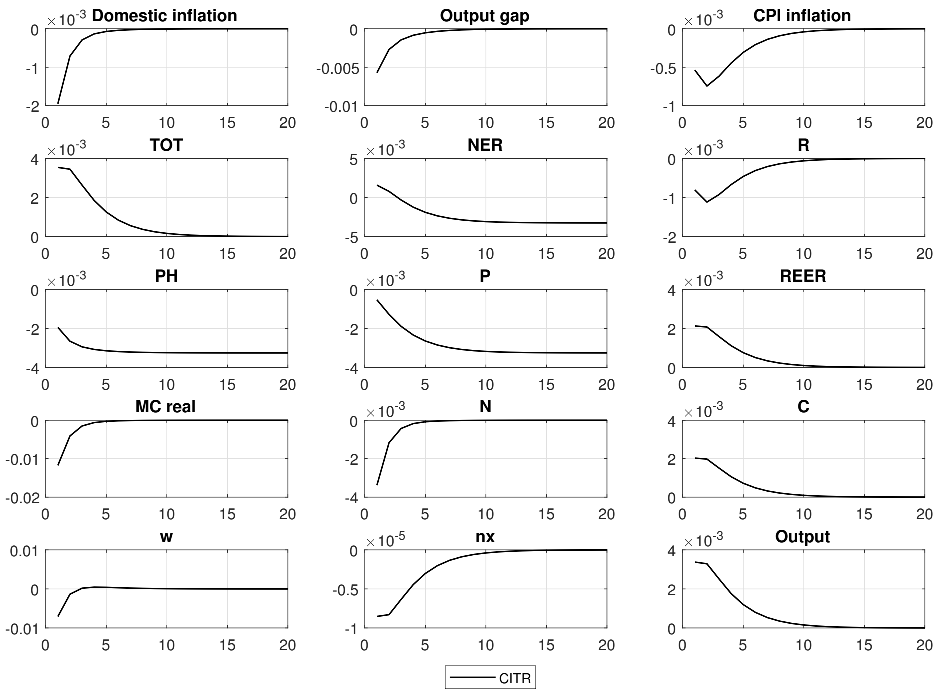
Appendices

A Gali and Monacelli (2005) replication

The figure below replicates the CPI inflation targeting regime under the [Galí and Monacelli, 2005][1].

Figure A.1: Impulse Response Functions to a positive productivity shock under CPI inflation targeting (CITR) - calibrated according to [Galí and Monacelli, 2005]

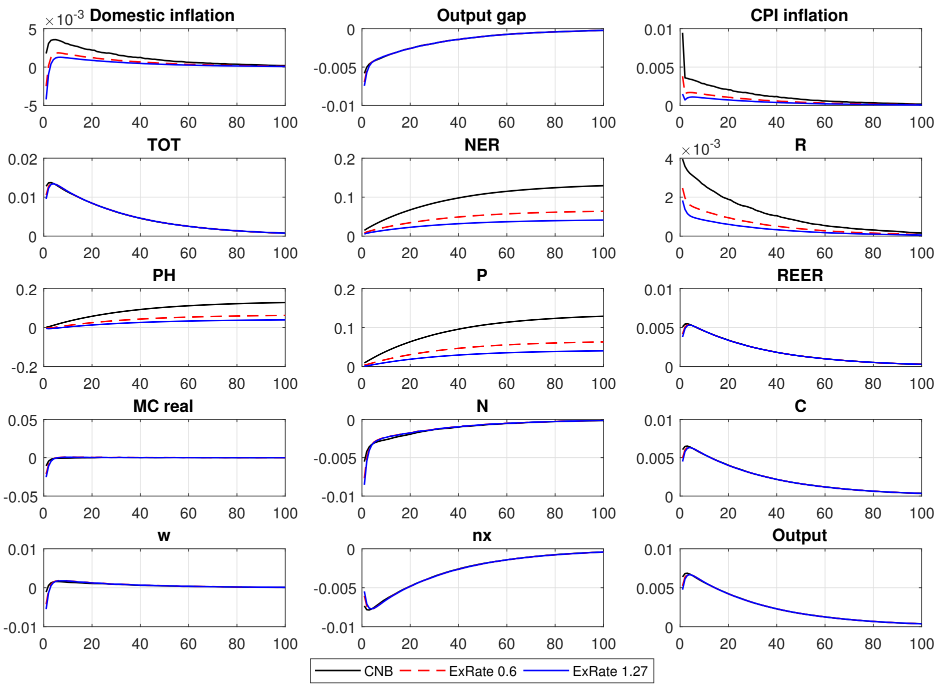


[1]Slight deviations were made when replicating the [Galí and Monacelli, 2005] dynamics, e.g. the parameter η was set to 0.99 instead of 1 in order to solve the model without rewriting the equations into its limit versions. Also, the impulse response functions were generated in levels, i.e. without log-linearization.

B Impulse response functions to the productivity shock - 100 quarters

These are the same impulse response functions to the productivity shock as in the figure 4.1 and differ in the length of considered period.

FIGURE B.1: Impulse Response Functions to a positive productivity shock $\rho_A = 0.97$ and the volatility $\sigma_A^2 = 0.0095^2$



Abbreviations: TOT = Term of Trades, NER = Nominal Exchange Rate, PH = Domestic price, P = CPI price, REER = Real Exchange Rate, MC real = Real marginal cost

C MATLAB code

This the main (MATLAB) file (*welfare.m*) that is used for running the Dynare code and for extracting the results (impulse response functions, conditional and unconditional welfare).

```
clc, clear all, close all;
%% Cell arrays for IRFs and Welfare measure
interestrules=14; %Number of interest rate rules (+1 for PEG)
calibrations=14; %Number of different calibrations
irfperiods=20; %Impulse Response Function periods

IRF_ALL=cell(calibrations,interestrules+1);
U=table(zeros(interestrules,1), zeros(interestrules,1), 'VariableNames'
    ,...
     {'Conditional' 'Unconditional'}); %Table for saving welfare results
      (of each iteration / different MP)
Welfare=table(zeros(interestrules,1), zeros(interestrules,1), ...
     'VariableNames', {'Conditional' 'Unconditional'}); %Table for
    saving welfare results (of each iteration / different MP)

%% Welfare calculations
i=1; %Policy regimes
j=1; %Calibrations
phi_pis=repelem(1.27,14); %Different weights on inflation (Interest
    rate rule for MP)
phi_ys=repelem(0.47,14); %Different weights on output gap (How strongly
     monetary authority reacts)
```

```
phi_epss=(0:0.1:1.30); %Different weights on output gap (How strongly monetary authority reacts)
phi_epss(14)=1.27;

for i=1:(calibrations)
    phi_pi=phi_pis(i); %Taking different weights from phi_pis vector
    phi_y=phi_ys(i);
    phi_eps=phi_epss(i);
    save MP_weights.mat phi_pi phi_y phi_eps %Saving the weights into a file that will be loaded in dynare
    dynare SOE_CITR.mod;

    % Non-stationary variables
    PH=ones(1,21); %Domestic price level
    P = ones(1,21); %CPI level
    NER = ones(1,21); %Nominal exchange rate level

    % Stationary variables
    PIH = 1+oo_.irfs.Pi_H_eps_A; %Domestic inflation
    PI = 1+oo_.irfs.Pi_eps_A; % CPI inflation
    Delta_NER = 1+oo_.irfs.delta_eps_eps_A; %Change in NER

for ii=(2:(irfperiods+1)) %Evolution of the non-stationary variables (given inflation, changes in NER)
    PH(ii) =  PH(ii-1)*PIH(ii-1);
    P(ii) = P(ii-1)*PI(ii-1);
    NER(ii) = NER(ii-1)*Delta_NER(ii-1);
end

    %To start from zero
    PH=PH-1;
    P=P-1;
    NER=NER-1;

    %Extracting IRF from Dynare results
```

```
irf = array2table(zeros(irfperiods,16), 'VariableNames', {'PIH' 'Y_gap'
    'PI' 'ToT' ...
    'NER' 'IR' 'PH' 'CPI' 'REER' 'MC' 'N' 'C'...
    'w' 'nx', 'Y', 'Y_flex'});
irf{:,1} = transpose(oo_.irfs.Pi_H_eps_A);
irf{:,2} = transpose(oo_.irfs.y_gap_eps_A);
irf{:,3} = transpose(oo_.irfs.Pi_eps_A);
irf{:,4} = transpose(oo_.irfs.S_eps_A);
irf{:,5} = transpose(NER(2:end));
irf{:,6} = transpose(oo_.irfs.R_eps_A);
irf{:,7} = transpose(PH(2:end));
irf{:,8} = transpose(P(2:end));
irf{:,9} = transpose(oo_.irfs.q_eps_A);
irf{:,10} = transpose(oo_.irfs.mc_eps_A);
irf{:,11} = transpose(oo_.irfs.N_eps_A);
irf{:,12} = transpose(oo_.irfs.C_eps_A);
irf{:,13} = transpose(oo_.irfs.w_eps_A);
irf{:,14} = transpose(oo_.irfs.nx_eps_A);
irf{:,15} = transpose(oo_.irfs.Y_eps_A);
irf{:,16} = transpose(oo_.irfs.Y_flex_eps_A);

    %Saving IRFs (j~different calibration, i~different MP rule)
IRF_ALL{j,i}=irf;

%% Utility and Welfare
U_DR= oo_.dr.inv_order_var(1); % Declaration order -> Decision Rule (DR
   ) order // Utility is declared 1st -> oo_.dr.inv_order_var(1) gives
    the DR order of the utility
U{i,1}=oo_.dr.ys(1) + 0.5*oo_.dr.ghs2(U_DR); % oo_.dr.ys(Declaration
   Order of Utility) + 0.5*oo_.dr.ghs2(DR Order of Utility) // The
   stochastic steady state (Dynare Manual 4.13.4)
U{i,2}=oo_.mean(1); % Unconditional mean

```

```
Welfare_DR= oo_.dr.inv_order_var(2); % Declaration order -> Decision
    Rule (DR) order // Utility is declared 1st -> oo_.dr.inv_order_var
    (1) gives the DR order of the utility
Welfare{i,1}=oo_.dr.ys(2) + 0.5*oo_.dr.ghs2(Welfare_DR); % oo_.dr.ys(
    Declaration Order of Utility) + 0.5*oo_.dr.ghs2(DR Order of Utility
    ) // The stochastic steady state (Dynare Manual 4.13.4)
Welfare{i,2}=oo_.mean(2); % Unconditional mean
end%%

%% Plot IRFs
CITR=IRF_ALL{1,1};
CITR_EPS1=IRF_ALL{1,7};
CITR_EPS2=IRF_ALL{1,14};

figure
for ii=1:15
subplot(5,3,ii)
plot(CITR{:,ii}, 'black', 'LineWidth', 1, 'LineStyle', '-')
hold on
plot(CITR_EPS1{:,ii}, 'red', 'LineWidth', 1, 'LineStyle', '--')
plot(CITR_EPS2{:,ii}, 'blue', 'LineWidth', 1, 'LineStyle', '-')
hold off
grid on

if ii==1
    title('Domestic inflation')
elseif ii==2
    title('Output gap')
elseif ii==3
    title('CPI inflation')
elseif ii==4
    title('TOT')
elseif ii==5
    title('NER')
elseif ii==6
```

```
    title('R')
elseif ii==7
    title('PH')
elseif ii==8
    title('P')
elseif ii==9
    title('REER')
elseif ii==10
    title('MC real')
elseif ii==11
    title('N')
elseif ii==12
    title('C')
elseif ii==13
    title('w')
elseif ii==14
    title('nx')
elseif ii==15
    title('Output')
end
end
legend({'CNB', 'ExRate 0.6', 'ExRate 1.27'}, 'orientation', 'horizontal
    ')

%% Welfare table
figure
plot(phi_epss, Welfare.Conditional(1:14),'DisplayName','U.Conditional
    (1:14)')
hold on
plot(phi_epss, Welfare.Unconditional(1:14),'DisplayName','U.
    Unconditional(1:14)', 'LineStyle', '--')
grid on
grid minor
hold off;
ylabel('Expected Welfare')
```

```
xlabel('Interest rate coefficient - Exchange rate')
legend('Conditional', 'Unconditional')
```

D Dynare code

This the Dynare file (*SOE_CITR.mod*) that contains the model. The Dynare files has to be saved as .mod files in order to be able to run it with the attached matlab code C.

```
var
    U           ${\mathbf{U}}$ (long_name='Utility')
    Welfare     ${W}$ (long_name='Welfare')
    Pi          ${\Pi}$ (long_name='CPI inflation')
    Pi_H        ${\Pi_H}$ (long_name='Domestic inflation')
    S           ${S}$ (long_name='Terms of Trade')
    delta_eps   ${\delta \varepsilon}$ (long_name='Change in NER')
    R           ${R}$ (long_name='Gross nominal interest rate')
    q           ${\Omega}$ (long_name='REER')
    mc          ${mc}$ (long_name='Real MC')
    N           ${N}$ (long_name='Labour')
    C           ${C}$ (long_name='Consumption')
    w           ${w}$ (long_name='Real wage')
    nx          ${nx}$ (long_name='Net export')
    C_star      ${C^*}$ (long_name='World consumption')
    Q           ${Q}$ (long_name='Bonds')
    Pi_H_bar    ${\Pi_H^\#}$ (long_name='Domestic inflation')
    x1          ${x_1}$ (long_name='Auxiliary variable 1')
    x2          ${x_2}$ (long_name='Auxiliary variable 2')
    Z           ${Z}$ (long_name='Z')
    Pi_star     ${\Pi^*}$ (long_name='World inflation')
    Y_star      ${Y^*}$ (long_name='World output')
    Y           ${Y}$ (long_name='Output')
    A           ${A}$ (long_name='TFP')
    y_gap       ${Y_{gap}}$ (long_name='Output gap')
```

```
    h           ${h}$ (long_name='P-PH auxiliary variable')
    Y_flex      ${Y_{flex}}$ (long_name='Flexible-price Output')
    nu          ${\nu}$ (long_name='MP shock')
    ;

varexo eps_A ${{\varepsilon}_A}$ (long_name='TFP Shocks')
    eps_nu ${\varepsilon_{\nu}}$ (long_name='MP shocks');

parameters
     phi_pi ${\phi_{\pi}}$
     phi_eps ${\phi_{\varepsilon}}$
     sigma ${\sigma}$
     varphi ${\varphi}$
     betta ${\beta}$
     epsilon ${\epsilon}$
     alppha ${\alpha}$
     thetta ${\theta}$
     tau ${\tau}$
     eta ${\eta}$
     rho_A ${\rho_{A}}$
     gamma ${\gamma}$
     phi_y ${\phi_y}$
     phi_r ${\phi_r}$
     rho_nu ${\phi_{\nu}}$;

    betta = 0.99;       //Discount factor
    sigma =  0.82;      // Intertemporal Elasticity of Substitution (1/
   sigma)
    eta = 0.38;        // Elasticity of Substitution (EoS) between good
    bundles produced in different foreign countries
    gamma=0.38;         // EoS between domestic and foreign goods (from
    the viewpoint of domestic country)
    varphi = 1.08;      // Frish elasticity of labor supply (1/varphi)
    alppha = 0.5;   // Labor intensity of production function
    rho_A = 0.97;       // AR parameter (for TFP AR process) 0.97
```

```
    rho_nu = 0.5;        // AR parameter (for MP-shocks AR process)
    thetta = 0.63;    // Probability of being stuck with current price (Calvo parameter)
    tau=0.6; //0.75      // Degree of openness / Weight on imported goods
    epsilon = 9;         // EoS between varieties produced within country (within any given country)
    phi_r = 0.65;         // Taylor rule - smoothing parameter for policy rate

    load InitParams;
    set_param_value('phi_pi',phi_pi);
    set_param_value('phi_y',phi_y);
    set_param_value('phi_eps',phi_eps);


// 1 for interest rate rule (0 for currency peg)
@#define MP1 = 1

model;
    // Household problem
        [name = 'Utility function']
        U =  C^(1-sigma)/(1-sigma) + N^(1+varphi)/(1+varphi);
        [name = 'Welfare']
        Welfare =  U + betta*Welfare(+1);
        [name = 'Labor supply FOC']
        w=C^sigma*N^varphi;
        [name = 'Euler equation for bonds']
        Q=betta*(C(+1)/C)^(-sigma)/Pi(+1);
        [name = 'Gross nominal interest ratae']
        R=1/Q; // Bond's price and Gross nominal interest rate (R)

    // SOE identities
        [name = 'Auxiliary variable P/PH']
        h=(1-tau+tau*S^(1-eta))^(1/(1-eta));
```

```
        [name = 'CPI / DPI ratio']
        Pi=Pi_H*((1-tau+tau*S^(1-eta))^(1/(1-eta)))/((1-tau+tau*S(-1)
    ^(1-eta))^(1/(1-eta)));
        [name = 'Change in the nominal exchange rate']
        delta_eps=S/S(-1)*Pi_H/Pi_star;
        [name = 'REER']
        q=S/(1-tau+tau*S^(1-eta))^(1/(1-eta));
        [name = 'Risk sharing condition']
        C=C_star*q^(1/sigma); // Risk sharing

    // Intermediate good producers
        [name = 'Real marginal cost']
        mc=w/((1-alppha)*N^(-alppha)*A)*h;
        [name = 'Optimal domestic price']
        Pi_H_bar=epsilon/(epsilon-1)*(x1/x2);
        [name = 'Auxiliary variable 1 for Calvo']
        x1=C^(-sigma)*Y*(1-alppha)*mc+betta*thetta*Pi_H(+1)^(epsilon)*
    Pi(+1)^(-1)*x1(+1);
        [name = 'Auxiliary variable 2 for Calvo']
        x2=C^(-sigma)*Y+betta*thetta*Pi_H(+1)^(epsilon-1)*Pi(+1)^(-1)*
    x2(+1);

    // Aggregate variables
        [name = 'Aggregate output']
        Y=A*N^(1-alppha)/Z;  // Aggregate Output
        [name = 'Home inflation evolution']
        1=(1-thetta)*(Pi_H_bar)^(1-epsilon) + thetta*Pi_H^(epsilon-1);
        [name = 'Home PD evolution']
        Z=(1-thetta)*Pi_H_bar^(-epsilon) + thetta*Pi_H^epsilon*Z(-1);

    // Equilibrium
        [name = 'Equilibrium condition / Market clearing']
        Y=h^eta*(1-tau+tau*S^(gamma-eta)*q^(eta-1/sigma))*C;

```

```
    // World
        [name = 'World consumption']
        C_star=STEADY_STATE(Y);
        [name = 'World output']
        Y_star=STEADY_STATE(Y);
        [name = 'World inflation']
        Pi_star=STEADY_STATE(Pi_star);
        [name = 'Net export']
        nx=(1/STEADY_STATE(Y))*(Y-C*h);

    // Exogenous AR processes
        [name = 'TFP process']
        log(A)=rho_A*log(A(-1))+eps_A;
        [name = 'Shocks to MP']
        nu = rho_nu*nu(-1)+eps_nu;

    // Output gap and shocks
        [name = 'Output gap']
        y_gap=Y-Y_flex;
        [name = 'Output under flexible prices']
        Y_flex=(Y_star^(-sigma)*A^(1+(varphi+alppha)/(1-alppha))*(epsilon-1)/epsilon)^((1-alppha)/(varphi+alppha));

    // MP rule
    @#if MP1==1
        [name = 'Interest rate rule']
        R = (1-phi_r)*(1/betta) + phi_r*R(-1) + (1-phi_r)*(phi_pi*(Pi-STEADY_STATE(Pi)) + phi_y*y_gap + phi_eps*(delta_eps-1)) + nu;
    @#else
        [name = 'Currency PEG']
        delta_eps=1;
    @#endif

end;
```

```
write_latex_dynamic_model (write_equation_tags);
write_latex_definitions;

steady_state_model;
    h=1;
    nu=0;
    A=1;
    Z=1;
    S=1;
    q=1;
    delta_eps=1;
    Pi=1;
    Pi_star=1;
    Pi_H=1;
    Pi_H_bar=1;
    Q=betta;
    R=1/Q;
    mc=(epsilon-1)/epsilon*(1-alppha)^(-1);
    N=((1-alppha)*mc)^(1/(sigma*(1-alppha)+varphi+alppha));
    C=N^(1-alppha);
    C_star=C;
    w=N^(sigma*(1-alppha)+varphi);
    Y=C;
    Y_star=Y;
    x1=C^(-sigma)*Y*(1-alppha)*mc/(1-betta*thetta);
    x2=C^(-sigma)*Y/(1-betta*thetta);
    U = C^(1-sigma)/(1-sigma) + N^(1+varphi)/(1+varphi);
    nx=(1/Y)*(Y-C);
    Y_flex=Y;
    y_gap = 0;
    Welfare=U/(1-betta);


end;
```

```
resid(1);

steady;

check;

shocks;
var eps_A = 0.0095^2; // 0.0095^2 ;
var eps_nu = 0; //0.01^2;
end;

stoch_simul(order = 2,irf=20, periods=0, nograph, noprint);
```