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2019 Tetiana Kalinichenko

University of Economics, Prague

Faculty of Informatics and Statistics

MULTI CRITERIA DECISION ANALYSIS APPLIED ON SELECTED HIGHER EDUCATION INSTITUTIONS IN EUROPEAN UNION

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Author: Bc.Tetiana Kalinichenko
Supervisor: Ing. Martina Kuncová, Ph.D.

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Declaration

I hereby declare that I am the sole author of the thesis entitled “Multi criteria decision analysis applied on selected higher education institutions in European Union“. I duly marked out all quotations. The used literature and sources are stated in the attached list of references.

In Prague on ....

Tetiana Kalinichenko

Acknowledgment

I would like to express my sincere gratitude to my thesis supervisor Ing. Martina Kuncová, Ph.D. for providing valuable guidance throughout every step of my research. Her insightful comments and contributions helped me transform my knowledge into something meaningful and concrete.

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Abstract

The present increasing trend of people with higher education indicates undeniable demand for the highly qualified specialists on the labour market. However, the challenging task for an enrollee is to determine higher education institutions that value the qualitative aspect of education rather than quantification of enrolment rates. The ranking schemes may be seen as a practical tool to conclude the decision since they purposely account for criteria, which are most authoritative. While such education selection factors mostly remain unchanged predicated on gender and field of study, yet the importance of specific criteria differs among these subgroups. This research aims to facilitate Multi Criteria Decision Analysis within an educational framework to establish the rank among top 44 European Union universities. The main emphasis is placed on the dissimilarities of criteria importance order between genders as well as a field of study and their respective ranks.

The theoretical review addresses the decision making background encompassing, its history, process flow, and models. Decision analysis is considered within the scope of multi criteria input. Therefore, extensive multi criteria method review contributes to the selection of appropriate techniques for further analysis.

The ranking task is explicitly represented at the beginning of a practical part of the thesis. In order to enhance the accuracy of findings, information was gathered in the form of independent survey. The criteria weights concluded upon the results of the questionnaire finalize the model inputs. Aggregated data is incorporated into TOPSIS, VIKOR and WASPAS methods to achieve the final ranking of EU higher education institutions. Multi Criteria Decision Analysis results are introduced and explained in the closing part of practical implementation.

Keywords: Multi Criteria Decision Analysis, TOPSIS, VIKOR, WASPAS, Rank Ordering Method, university rankings, higher education institutions in European Union.

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List of Abbreviations

ARWU	Academic Ranking of World Universities
CU	Composite Unit
DE	Germany
DEA	Data Envelopment Analysis
DM	Decision Maker
DMU	Decision Making Unit
ELECTRE	Elimination and Choice Expressing Reality
EU	European Union
FR	France
IE	Ireland
MADM	Multi-Attribute Decision Making
MCDA	Multi Criteria Decision Analysis
MCDM	Multi Criteria Decision Making
MODM	Multi Objective Decision Making
PROMETHEE	Preference Ranking Organization Methods for Enrichment Evaluation
ROC	Rank Order Centroid
RR	Rank Reciprocal
RS	Rank Sum
SE	Sweden
SMARTER	Simple Multi Attribute Rating Technique Exploiting Ranks
TOPSIS	Technique for Order by Similarity to Ideal Solution
VIKOR	ViseKriterijumska Optimizacija i Kompromisno Resenje
WASPAS	Weighted Aggregated Sum Product Assessment

1 Introduction

Nowadays, education serves as a source of public progress as well as undeniably essential asset individual possess as a part of its fundamental rights. The important role of education lies in the indispensable condition for self-realization, contributing to the development of other vital aspects of life. In addition, the conception of education as such extends beyond the presence of a diploma but rather the quality of obtained knowledge to enhance mental capacity and to sustain the further development of a student. Existing employment practices also prove that institutions should focus primarily on the qualitative aspect of basic education, since the trends in science development are very rapid and often unpredictable. Thus, graduates with thorough and holistic knowledge of basic methods, concepts and algorithms will be of higher interest to employers, since they have better adaptability to new, changing environment and demonstrate superior learning ability, accounting for one's internal capability (Cheng, 2016). In such a manner it is uncomplicated to prove that the choice of a university often predetermines the success of a person in the future, although there are cases supporting the opposite (Bruni, 2016).

Higher education institutions are often regarded as the main origin of knowledge, however, in the present one has access to diverse scope of instruments (i.e. libraries, Internet, courses…) that initially provide common entity- knowledge, although the measurability of quality of such resources is questionable. Formal education, "*learning delivered by skilled and experienced teachers in structured and continuous manner*" (Young-adulllt.eu, 2019), is the most common and widely acceptable notion, which has more distinct measures that are employed to rank higher institutions based on number of various criteria. Given thesis aims to yield a ranking of the best universities, based on some pre-defined set of criteria such as employment rates, international connections, per capita academic achievements and others. Moreover, the research targets to classify possible dissimilarities between several science fields (natural, formal, applied and social).

The enrollee, seeking for a superlative education, faces a decision-making problem defined by certain criteria for evaluation in conjunction with a list of possible alternative universities. The greater the set of alternatives is, the higher the probability of obtaining the best possible outcome can be (Shi et al., 2011). In general, the decision-making procedure consists of two major components - emotional and rational (Kiddy & Partners, 2019). However, the hypothesis of rational, multi-criteria choice, when the emotional component is not taken into account or partially expressed by weight of each criterion, dominates in the decision-making theory.

1.1 Motivation

The essential purpose of this thesis is to establish the ranking of the best universities in selected European countries, taking into account general preferences of an enrollee when choosing a higher institution. There were approximately 3 300 higher institutions in European Union in 2006 (Eur-lex.europa.eu, 2019), however, this number is a subject to change but, indeed, with a large number of alternatives it becomes very difficult to make a rational choice. As part of the decision-making process, a sufficiently large number of methods have been developed for establishing a clear order, preference or division among options. The use of them does not require any significant efforts or set of professional knowledge from decision makers (experts). The modeling phase of the problem allows objective reflection on the existing problem, while MCDM methods such as TOPSIS, VIKOR and WASPAS aim to analyze given dataset and structure final findings, exploiting the calculated optimality vector with criteria weights for each research group. This research is implemented in two main sections: theoretical and practical. Literature review of the subject defines boundaries for each method of the decision-making analysis, whereas practical implementation of the problem is revised in the second part. The study is finalized by interpretation of differences between rankings, depending on science group and gender.

1.2 Objectives and Methodology

The fundamental goal of this work is to develop unique rank universities through the application of Multi Criteria Decision Analysis (MCDA) techniques. Main objectives for the scope of the study can be defined as follows:

1. Extensive literature review for the identification of available decision-making algorithms and the precise factors for their selection.
2. Definition of research problem, requirements and alternatives through empirical study.
3. Understanding and the ability to adjust to constraints faced during the implementation of methods to prove flexibility and the effectiveness of such.
4. Suggestion of the most suitable MCDA method for analogous types of studies based on findings.

The main goal of a discrete multi-criteria analysis is to identify the preferences of decision makers (DM) and since the subject expands on wide range of possible tasks, it is crucial to identify the correct methodology based on the unique property of a problem. This procedure may consist of determining the set of non-dominated alternatives; so called Pareto optimal. The simplest case targets to find one option, known as the best alternative or a winner (Miettinen, 2004). Transparent enough for the decision maker is an approach in which one is required to determine the best (non-dominated) solution by sequential, as a rule, pairwise comparison of alternatives. This can be done by means of a decision tree, or pairwise comparisons of alternatives in the matrix form. Consequently, the order of the set of alternatives is established either as a result of sequential pairwise comparison of alternatives, or by establishing order based on a comparison of the values of multi-criteria utility functions. Since the goal of the study is to order alternatives, the methodology is built upon well-recognized distance based (TOPSIS and VIKOR) and utility-based (WASPAS) methods (Raju and Kumar, 2010). A few different methods were applied to set ultimate order due to the fact that only diversified approaches guarantee reliability of results. Some MCDA models require determining weights of each criterion for ranking purposes. The author suggests SMARTER technique for quantification of weight significance (Edwards and Barron, 1994).

1.3 Structure of Thesis

The thesis begins by addressing the theoretical review of Decision Making in general followed by a detailed overview of Multi Criteria Decision Making methods and techniques. The second chapter focuses on the vital foundation of decision making theory, describing the history of process development along with classification of the main branches. The third chapter broadly reviews the notion of Multi Criteria Decision Making, subsequently revealing the breakdown structure within the subject with characteristics of particular class. In later chapters, the study focuses on the logic behind the ranking schemes, their creation and purposes. Chapter five starts with an explication of the study logic, aims and methods. After the problem description, follows an outline of the study, which takes a closer look at the data gathering, structuring, weight calculation technique and the software overview. Given the study environment of the thesis, the empirical part of the research presents the analysis along with an evaluation of its results. Chapter 6 also suggests the model correction to improve the outcome for particular alternative. The concluding part of the thesis revises the purpose of the research by means of critical result appraisement.

2 Theoretical Background

This chapter reveals the theoretical foundation of Multiple Criteria Decision Making (MCDM) with the main focus on fundamental notions and concepts in the field, enveloping the history and motivation behind its development. The section primarily seeks to explain a general overview of MCDA methods in order to shed some light on the enormous analysis possibilities in combination with the peculiar circumstances of their application. Alternatively stated, theoretical review has been performed in order to support the empirical research in the best way possible.

The first chapter aims to present a broad overview of decision-making by revealing notions of decision-making and closely related subjects, followed by its classification with peculiarities of kinds. In addition, section 2.2 refers to the early history of decision-making and its evolution. The conclusive phase of the theoretical introduction is finalized by definition of the decision process along with possible models.

2.1 An Overview of Decision Analysis

Conscious daily human activity is naturally linked to the decision-making processes. People have always made decisions based on their experience, intuition and common sense. In this case, as a rule, decision maker (DM) is unable to trace the exact path that led to the choice of the solution itself, although there is every reason to believe that one somehow weighted all alternatives according to possible criteria of the decision made. The ability to conclude the analysis with one particular option, providing the best solution in various difficult situations, was always regarded as art. However, attempts to systematically generate such decision were later generalized to a class of Multi-criteria decision making (MCDM) problems.

Decision making is the action of an analyst, who by means of explicit methods, aims to obtain a solution to the problem posed by a decision maker (DM), occasionally regarded as an analyst, in the decision making practice (Ehrgott, Figueira and Greco, 2005). Often, there is no conception of optimum within the framework of the model. From a traditional optimization standpoint, MCDA methods do not yield an optimal result (as in case of Operational Research field), hence, such techniques are mainly considered as an aid to the decision process. Subjectivity may also be argued, since construction of criteria set and their respective weights depend on subjective judgments of the DM. For this reason, MCDA strongly advocates for transparent and accurate suggestion of the optimal outcome versus the choice of an objective optimal solution. The core aim of MCDA is to provide better understanding of the difficulty faced by highlighting necessary trade-offs shared among alternatives. Such approach will assist the decision maker or a group of them to select the most ideal course of actions (Belton and Stewart, 2002).

Decision-making encompasses variety of branches, however, prevailing popularity is hands of Multi criteria decision making (MCDM), that, in turn, partitioned into multi-objective decision making (MODM) and multi-attribute decision making (MADM). It is worth mentioning, that very frequently the concepts of MADM and MCDM are very similar, thus, refer to the same group of problems (Triantaphyllou, 2000).

Multi objective decision-making (MODM) is focused on type of problems, where multiple objectives have to be satisfied simultaneously. MODM problems are defined by a set of (contradicting) objectives that are to be maximized or minimized along with set of constraints for optimization. This type of problems employs methods of mathematical programming, where solution set is large or infinitely large (Tzeng and Huang, 2011). MODM methods can be divided into Scalarization approach and Pareto approach. Vilfredo Pareto argued that "*optimal outcome does not exist, but rather set of solutions: non-inferior and inferior*". Each of the potential solutions from the set is referred to as non-inferior/efficient and is positioned on Pareto frontier, while inferior are enveloped by it (Gunantara, 2018). On the other hand, scalarization approach transforms multi objective problem into a string of single objective functions with apriori assigned weights or preferences (De Weck, 2004).

MADM/MCDM methods are performed in discrete decision space with relatively small number of possible outcomes. Due to this reason, such methods perform better under uncertainty condition as well as they are easier to calculate (Wallenius et al., 2008). MCDM class of problems encompasses wide range of problems grouped by distinctive features. Utility maximization problems seek such a result, which guarantee to maximize total utility (BusinessDictionary.com, n.d.). Moreover, some MCDM methods aim to minimize distance from ideal alternative (Fiala, 2013). Outranking methods form another class of problems that are based on preference relationship among the set of alternatives. DM evaluates his/her preference in pairwise comparison to state that *a* is at least as good as *b* (Bouyssou, 2001). Aforementioned classes will be explained in more detail.

2.2 History of Decision Making Theory

Decision-making practice falls far behind to ancient times, however, it becomes impossible to trace the first pioneers in the field (Köksalan, Wallenius and Zionts, 2011). One can argue that daily routine decisions are also classified as decision-making ones although they don't anticipate any mathematical modelling. There is no denial that ancient civilizations have incorporated decision-making in earlier stages of existence but with time, when problems have become more contradictory and complex, formal modelling enabled generalization of routine decisions into a class of such problems (Belton and Stewart, 2002).

Mathematicians in the early 18th century began to apply formal science in social fields to study individual preferences as well as utility theory that initiated the concept of the indifference curve. A little after that, the economist Vilfredo Pareto devoted his studies to notion of efficiency, currently known as Pareto-efficiency, which is recognized as a base ground in whole decision-making theory. It is worth mentioning the contribution of Frank P. Ramsey, who was a founding father of utility modelling, by proposing first class of axioms for alternatives with uncertain outcomes in the 20th century. Many of his followers contributed to a utility theory by exploring utility functions for gambling also incorporating rational preferences, based on the choices made. Ward Edwards, who focused on behavioral decision research, formulated questions of whether humans finalize decision on alternatives that maximize their utility functions. Contrary, Herbert A. Simon didn't support the idea of rationality, but instead claimed that people are "satisfiers", trying to reach certain aspiration level for satisfaction (Köksalan, Wallenius and Zionts, 2011). It goes without saying that there are many others, who contributed to the field of research, namely Bernard Roy for developing ELECTRE family of methods (Roy, 1991), George Dantzig famous for the simplex algorithm (Dantzig, 1987), Abraham Charnes and William Cooper, founding fathers of goal programming and Data Envelopment Analysis (DEA) (Charnes and Cooper, 2002), followed by others.

However, not only science itself provoked experts, social problems served as a main force to apply but also modify known techniques. One of the most progressive years for MCDA development occurred during World War II. The main objective for the USSR Government was to allocate uncountable natural resources in the most efficient way, in addition to it, the restriction was imposed by number of priorities as time was vitally important. Under such pressuring conditions, the government appointed Leonid Kantorovich to develop a method, which would find an optimal number of scare resources to achieve maximum return. An algebraic procedure was invented by Kantorovich, which was later known and recognized as linear programming (Munier, Hontoria and Jiménez-Sáez, 2019). Generally, all above-mentioned scientists worked under condition of such resources allocation that each alternative complied with all (sometimes contradictory) criteria requirements. Furthermore, the better an alternative is, the more efficiently it satisfies criteria set.

2.3 Decision Making Process and Model

It is common to believe that MCDM mainly implements established technique for data analysis, however, it is very unlikely for an analysts to receive a problem in a structured manner in order to deliver a solution. Practice proves that given a set of alternatives and criteria one has to go through multiple stages of data manipulation before any of the techniques can be applied. MCDM process is integrated into a much wider decision chain, in fact, it appears to represent one of the stages in this string (Belton and Steward, 2002). It goes without saying that every stage of the process is trivial for the final outcome. Figure 1 is provided for a better representation of decision-making process. The first phase in the process surely is identification phase. DM is required to distinguish the problem in place and provide explicit description to the analyst.

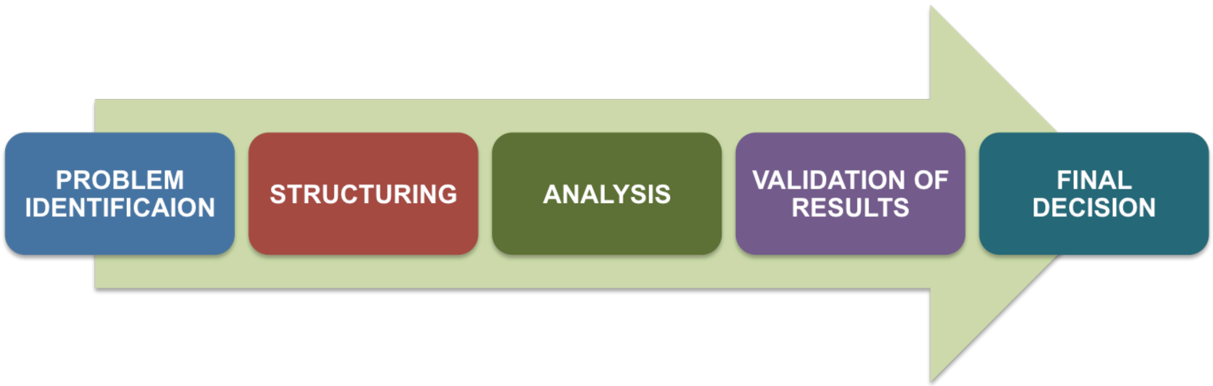

Figure 1 - Decision Making Process (Belton and Steward 2002)

Nevertheless, DM(s) themselves have a very vague idea of the problem perception with some raw input (alternatives, criteria, weights) in the beginning. Most likely such an approach will yield unsatisfactory results due to lack of structuring. Therefore, structuring phase is treated as a cognitive stage, requiring the clear formulation of key objective(s) in addition to the specification of inputs, surely after a problem was identified at the first stage. The next aspect to consider is the analysis. This may appear in many forms but mainly methodology of proceeding determines in what form criteria must be specified (preferences or weight vector), which alternatives (only non-dominated) remain in the analysis and how they are integrated and utilized (Hodgett, 2013). Last but not least, is the validation stage, which highlights potential calculation errors. Multiple feasibility tests or the DM(s) judgment is applied to assess the equality of results to either conclude the research or to return back to the previous stage for revaluation (Zardari et al., 2015).

As mentioned before, the decision-making process is rather complex, however, particularly modelling enables smooth communication between analyst and decision maker using "common language". Models represent explicit connections and problem schemes in simplified graphical form (Sanderson and Gruen, 2009). Typically, problem models are created during the structuring phase, when both parts (DM and the analyst) are encouraged to thorough communication to achieve maximal transparency during the negotiation process. Figure 2 represents the decision tree of a problem. Decision maker defines the main objective of the research in the first phase, namely the identification phase, of the decision-making process. The objective of a given study is to form rank of European universities according to different weight vectors.

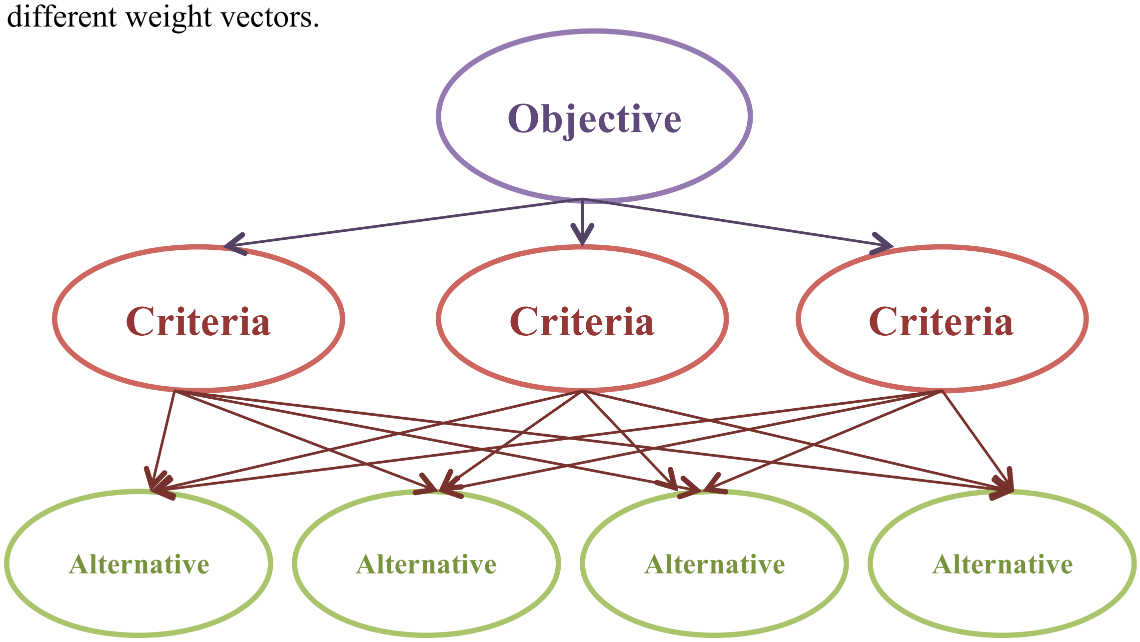

Figure 2 - Decision Making Tree (Pandey and Sharma, 2016)

According to the example, monocriterion methods are neglected in the thesis given that there are 8 criteria, which transform to multicriteria problem. Employment rates of graduates, per capita academic scores of professors, quality of teaching techniques and other criteria are considered in the course of study. Alternatives are typically selected depending on the availability of information.

3 Multi Criteria Decision Making

In literature many definitions of MCDM can be found, however, International Society of MCDM explains the notion as "*the study of methods and procedures by which concerns about multiple conflicting criteria can be formally incorporated into the management planning process*" (MCDM Society, 2019). The discipline stems from the field of Operations Research, which prepares scientific grounds for decision-making. More specifically, in terms of Operations Research, all decision-making problems are designed by utilizing mathematical models to form meeting ground for scientific (technological) and practical (managerial) approaches (Aristeias, 2019). While in Operations Research the main purpose of the subject is optimization (Mathworld.wolfram.com, 2019), MCDM delivers tools and methods for such optimization allowing for conflicting situations, which it intends to resolve (Zardari et al., 2015). Following optimization reasoning, DM identifies variables with the highest influence on the objective function and employs them to assess the relative success of alternatives. The branch of similar optimization sciences unfolds into system analysis, control theory, game theory, logic programming and even artificial intelligence (Mathworld.wolfram.com, 2019). Generally, decision-making is viewed from two main standpoints: single-criterion or multi-criteria class. Single-criterion group is characterized by a single decision criterion or measure that explicitly advocates for an alternative with the highest score. The objective function in such case equals the decision factor (Fülöp, n.d.). However, single-criterion methods prove to be insufficient to real life application due to their simplicity, which is hardly ever encountered in modern practices (Gade and Osuri, 2014).

Multi-criteria decision making focuses on problems with m criteria and n alternatives, while criteria set $C = \{c_1, \dots, c_m\}$ and set of alternatives $A = \{a_1, \dots, a_n\}$ are defined infinite space and clearly stated in structuring phase (Fülöp, n.d.). Alternatives correspond to objects at which a decision process is oriented towards. It is important to note that possible alternatives are not always qualified as feasible ones. The set A is a subject to alterations throughout the stages of the decision process. Nevertheless, it is required for the best alternative to be optimal and feasible. Criteria represent set C of decision factors that enable the evaluation of alternatives. Performance x_{ij} of each a_i is measured for all criteria $c(a_i)$. Frequently, results are scaled according to the objective function goal (min, max) of a criterion to reflect the general preference of a DM. Elements $c \in C$ are assigned with scores or degrees, represented by numbers for numerical scales and verbal valuation (e.g. good, better, best) if results belong to verbal scale (Ehrgott, Figueira and Greco, 2005). Criteria representation scales will be reviewed in a later section of the thesis.

Another crucial aspect of calculations is the weight of each criterion. From Table 1 it is visible that weights $W = \{w_1, \dots, w_m\}$ correspond to each criterion, reflecting the total importance of such in the evaluation process. On the one hand, weights $w_1, \dots, w_m$ express subjective importance since their values are calculated based on the personal valuation by DM(s). On the other hand, analyst may make use of weighting methods, if DM(s) failed to precisely quantify them, yet subjectivity is still present (Fülöp, n.d.).

	Criteria/Weights			
	w_1	w_2	-	w_m
Alternatives	c_1	c_2	-	c_m
a_1	x_{11}	x_{12}	-	x_{1m}
a_2	x_{21}	x_{22}	-	-
-	-	-	-	-
a_n	x_{n1}	x_{n2}	-	x_{nm}

Table 1 - Decision Matrix (Sbeity, Haidar and Dbouk, 2016)

Decision-making problem inputs are usually recorded in the decision matrix, which represents performance results in $X_{n \times m}$ matrix. It is assumed that all inputs are known beforehand. Even though, an analyst may be required to structure inputs or rescale criteria, if such manipulations are reasonable. The goal is then to apply relevant MCDM methods to evaluate overall scores with respect to each alternative and select such alternative a^* with the most desirable outcome.

3.1 Classification of MCDM Methods

In literature, there are many different MCDM methods. Each type has its unique and distinct characteristics, however, there are common features among them. Multiple authors define various methodologies for classification based upon the form of criteria entry, features of data and type of data processing. One way is to distinguish methods into a unique synthesis criterion approach, which excludes dissimilarities; outranking synthesis approach, dealing with weakness of previous; interactive local judgment approach (Roy, 2013).

1. **Unique synthesis criterion approach**

Methods of this types aggregate several viewpoints into one objective function, which is optimized in the course of evaluation. For instance, AHP (Analytical Hierarchy Process) (Saaty, 1980), TOPSIS (Technique for order by similarity to ideal solution) (Hwang and Yoon, 1981) and SMART (Simple Multi-Attribute Rating Techniques) (Triantaphyllou, 2000).

2. **Outranking synthesis approach**

Outranking methods utilize pairwise comparison of available alternatives along criteria by DM. Alternatives that are preferred in comparison receive a higher score. Following type of methods is frequently exercised for ranking (Zardari et al., 2015). It is necessary to note that outranking methods require less effort and are easily applicable. Yet, in comparison to the utility function these methods yield poorer models (Bozkurt, 2007). For instance, ELECTRE (Elimination and Choice Translating Algorithm) (Roy, 1968) and PROMETHEE (Preference Ranking Organization Methods for Enrichment Evaluation) (Brans, 1982).

3. **Interactive local judgment approach**

Following types of methods alternate stages, concluding each such step with a new compromise solution (Vincke, 1994). Such methods as Lexicographic, Conjunctive and Disjunctive are representative of the class.

Another way to determine the class of the method relies on the type of data used. Analyst may face problems with deterministic, stochastic (probabilistic) and fuzzy data inputs (Triantaphyllou, 2000). Deterministic data type assumes that data is known with certainty, meaning there exists deterministic relationship between each alternative and corresponding criteria. Situations, in which the condition of the environment and relationship is uncertain, generate probabilistic data. Fuzzy datasets are characterized by uncertainty along with inaccurate knowledge about alternatives (Malczewski, 1999). Views on criteria for methods classification range from data features to desirable outcomes, therefore numerous methodologies are available for grouping. Given study only mentions most common ones.

As displayed on Figure 3, all MCDM methods are also classified depending on the availability of preferences among criteria, generally, availability of information about preferences of the decision maker. Such classification appears to be the most common in MCDM environment. Thus, the following chapters of the research will rely on systematization of methods given the availability of information.

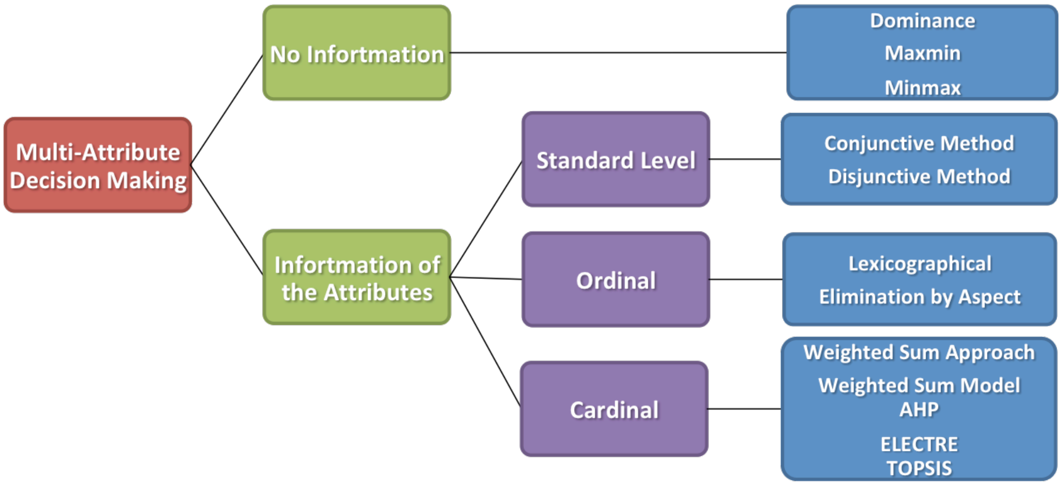

Figure 3 - Taxonomy of MCDM Methods (Chen and Hwang, 1992)

3.2 Methods with no Information

Occasionally, the decision maker fails to supply relative importance of each criterion or simply cannot define it. Common methods that allow working with alternatives constrained by the lack of information are Dominance, Max-min and Min-max methods (Wolny, 2016).

Dominance method is one of the simplest methods in optimization class. As name suggests, dominance relation is studied among all possibilities to identify the set or single alternative that satisfies dominance condition. The condition states that option A dominates B if it has better result on at least one criterion and simultaneously A is at least as good as dominated alternative B with respect to remaining criteria. The process continues until there exists an alternative that dominates all the others (Vallabhaneni, 2013). Dominance method results are considered to be highly reliable. Given that DM changes his/her criteria priorities, dominance relation is permanent (Hayes,1989).

Max-min method is another simple method, which frequently appears in the game theory, decision theory and artificial intelligence (En.wikipedia.org, 2019). Preliminary assumption states that DM is rather pessimistic towards the outcome, assuming minimal/worst results (Ravindran, 2009). The method aims to maximize one's minimum pay off according to worst performing criterion (En.wikipedia.org, 2019). According to Ravindran (2009) this technique "*maximizes the minimum normalized distance from the anti-ideal solution along each* *criterion*" (p.5.8). Considering calculation simplicity of the technique, nevertheless, method neglects a portion of available criteria information during analysis. In addition, Max-min is only favoured in cases when criteria are comparable, evaluated on the same scale (Linkov and Ramadan, 2004).

Another alternative method in decision-making analysis is **Min-max**, sometimes referred to as Regret, **method**. Similarly to Max-min technique it was originally formulated in game theory (En.wikipedia.org, 2019). General philosophy of the method is based on minimization of maximal possible loss. The loss is expressed by the total difference between the ideal (best) score of a criterion and the actual achieved value for an alternative. As opposed to Max-min, following method determines the optimal solution as the one closer to the ideal (Ravindran, 2009). Hence, the solution with a minimal relative deviation is treated as optimal.

3.3 Methods with Aspirational Levels

Methods discussed in this chapter rely on level representation of DM preferences. Aspirational level depicts acceptable level of performance according to each criterion, represented in a vector form $G = \{g_1, \dots, g_m\}$. Selection process with aspirational levels involves interactive response from DM. If there doesn't exist a feasible solution satisfying standard level, g_i vector is adjusted accordingly and process continues till at least one alternative satisfies aspiration level $\left\{ X_i : x_{ij} \geq g_j \; for \; j = 1,2, \dots, m \right\}$ (Gal, Stewart and Hanne, 1999). Nowadays, methods employing levels have proven to be effective due to number of evident reasons, namely:

- adaptivity to changing DM's judgements,
- explicit definition of threshold,
- transparency of calculation and others.

Methods based on aspiration levels aim to find satisfactory alternatives instead of best performing once (Branke et al., 2008). Moreover, satisficing methods are more often applied to separate alternatives into subgroups of acceptable/unacceptable options as their filtration process in rather weak[1], generating multiple optimal solutions. Consequently, findings are rather fulfilling ("good enough") compared to most optimal results stemming from other MCDM methods (Hwang and Yoon, 1981). Representative, belonging to the group of

[1] Alternative is required to at least be equal or to exceed the cutoff threshold to be considered as satisfying.

satisficing methods, are Disjunctive, Conjunctive and PRIAM methods, which are explained in more detail in sections below.

Conjunctive method implies that performance of an alternative must exceed an aspirational level for all criteria to be selected, satisfying condition $\{X_i : x_{ij} > g_j \, for \, j = 1,2,\ldots,m\}$ (Talhofer, Hošková-Mayerová and Hofmann, 2019). Given method is considered as a screening method, typically used in a prephase for determination of acceptable alternatives, which are utilized for further optimization (Kahraman, 2008). However, analyst may apply method not only for elimination purposes but also for detection of best alternative. By increasing/decreasing the value of cutoff threshold, analyst may reduce or respectively increase number of selected alternatives. Aspiration level is modified in course of multiple iterations until result narrows down to a single choice satisfying the threshold (Hwang and Yoon, 1981). It is crucial to note that in order to receive set of any results, alternatives and aspiration thresholds must be defined in commensurate unit.

Disjunctive method is a counterpart to conjunctive method, in which condition is relax to satisfy at least one criteria from aspiration set. As mentioned by Kahraman (2008) "*an option must exceed the threshold for at least one criterion*" (p.4) to be considered as acceptable, satisfying condition$\{X_i : x_{ij} > g_j \, for \, at \, least \, one \, j = 1,2,\ldots,m\}$. This screening technique can also be applied to formulate accepted alternative set for more complex methods (Linkov and Ramadan, 2004). In such case, due to the fact that filtration condition is very mild, it is important to tighten the aspiration threshold, otherwise the set of alternatives will be too big (Talhofer, Hošková-Mayerová and Hofmann, 2019).

PRIAM (**PR**ogramme utilisant L'**I**ntelligence **A**rtificielle en **M**ulticritere) proposed by Levine and Pomerol (1986) as a type of unstructured and interactive method with relaxed mathematical assumptions. Calculation principles of the method rely on pairwise comparison of alternatives, where DM has an opportunity to modify his preference during an analysis. Therefore, adapting to such changes, PRIAM technique allows backward moves throughout the search. Class of such techniques is of high practical importance since they stimulate the dialogue between analyst and DM to avoid false results (Levine and Pomerol, 1986). Firstly, method requires the aspiration level corresponding to the minimum requirement for the best solution. Following step dichotomizes group of alternatives into acceptable $\{X_i: x_{ij} \geq g_j\}$ or redundant $\{X_i: x_{ij} \leq g_j\}$ options according to their performance in comparison with aspiration level. Procedure reiterates till algorithm is applied for each criteria present in the model (Pomerol and Barba-Romero, 2009). As a result, the exploration terminates on the alternative, which satisfies DM prerequisites on all levels. If following does not occur, DM is forced to modify aspiration set. Backtracking technique allows altering satisfaction levels in previous steps in order to explore a problem from a different perspective (Levine and Pomerol, 1986).

3.4 Methods with Ordinal Information

As previously mentioned, the information for the analysis is typically expressed in different units[2]: aspirational levels, ordinal and cardinal. Following chapter reveals notion of ordinal data, subsequently, describing existing MCDM methods that works with following data type. Since multi-criteria problems with ordinal data inputs are often encountered in practice, therefore, there are a significant number of interactive methods allowing for categorical variables. Ordinal data is often referred to as categorical data, values of which "*cannot be expressed in numerical units*" (Surbhi, 2016). Ordinal data provide more realistic research outcomes due to presence of qualitative inputs, which are inherent in wide spectrum of problems. Ordinal data characterized in a sense of order, however, frequently it is hard to determine the magnitude of difference between neighboring values. It must be noted that logical scaling is still present (Nic, 2013). Methods with ordinal information type assume that DM provides importance order of criteria $c_1 > c_2 > \cdots > c_m$ where c_1 is the most important criteria and c_m is least so (Fiala, 2013). Scientist distinguish between a non-strict preferences (this object is not worse than that) and strict ("more - less").

Lexicographic method assumes that the existing set of criteria is given in ordered by importance vector. For compared objects, the values of the most important criterion are measured first, so that A^1 is preferred according to the maximum value of $\mathbf{c_1}$. In the case when the values of the compared objects coincide according to the most important criterion, then procedure continues to the comparison based on the next criterion of importance. The

[2] There are also methods based on no information provided by DM.

procedure ends at that iteration, at which it is possible to order objects by preference, or when comparisons are made for all criteria. The notion of lexicographic ordering is associated with lexicographic structuring by the first letter (if not first, go on to the second) in vocabularies. The benefits of the method are clearly outlines by simplicity of the method and minimal calculation time (Pomerol and Barba-Romero, 2009). The disadvantage of the lexicographic method lies in its limited practical application since it is very rarely possible to clearly rank the criteria by importance. Furthermore, if $A^1 = 1$ for some $\mathbf{c_1}$, then for all other alternatives ordering loses its meaning because the winner is determined in a first step, unless DM requires ordered set of outputs (Sokolova and Solomatyn, 2002).

Elimination by aspects is similar to lexicographic method, where all alternatives are eliminated in the process of calculation. According to the method one should revise a set of alternatives according to the most important criteria, namely delete an option from set if $\{ X_i: x_{ij} < c_m^* \, for \, all \, i = 1,2, \dots, n\}$. Alternatives with weak performance according to the attribute are eliminated from the model. The algorithm continues to the second most important attribute to reduce existing set by the number of options that do not satisfy requirement. Method terminates at a single alternative, which yields best results corresponding to the importance vector (Tversky, 1972). It can be seen that such approach identifies the accurate result in a very efficient and effortless manner (Stevens and Pashler, 2002).

ORESTE, the name of the method is derived from "**O**rganisazion, **R**ang**E**ment ot **S**yn**TE**ze de donnecs relationnelles, in French". ORESTE method is recognized by a distinctive feature, which is a two-phase ordering technique. Firstly, an algorithm establishes the aggregated preorder of alternatives, known as weak order, after which analyst conducts indifference and incomparability tests to complete detailed preference order (Pomerol and Barba-Romero, 2009). As depicted on Figure 4, in steps 1 and 2 one computes global preference scores, specified by Dujmovic metrics $D = (d_{ij})$ (Fiala, 2013), after scores are structured in ascending order in Besson's ranking $R = (r_{ij})$. Based upon this ranking, one computes preference intensities c_{ij}, which are utilized for indifference and incomparability tests. Final order is derived from weak order and results of preference analysis (Tian et al., 2018).

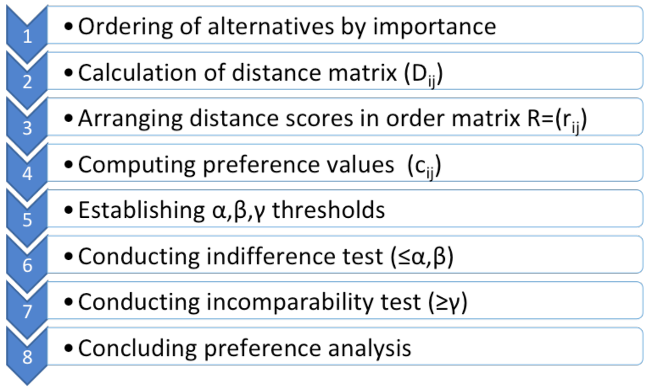

Figure 4 - ORESTE Method Algorithm (Fiala, 2013)

3.5 Methods with Cardinal Information

Further to earlier comments, preferences of decision maker can be expressed in various forms namely, in a form of aspiration levels, ordered criteria vector and quantitative criteria weights. Methods with cardinal information depict intensity of preference in quantitative measures by criteria weights. Weights are employed to quantify the dominance of particular criterion on the result (Vinogradova, Podvezko and Zavadskas, 2018). Weight vector $W = \{w_1, \dots, w_m\}$ denotes the importance of criteria, where each $0 < w_i < 1$ corresponds to the relative weight of a single criterion. MCDM methods apply normalized weights in following manner $\sum_{i=1}^{m} w_i = 1$ during the proceeding. Generally, the more influential criteria receive higher weight scores unless all are equally valued (Fiala, 2013). Weight assignment techniques play a crucial role in the decision making process particularly by enabling to incorporate the judgments of competent experts.

Weighting techniques rely on principles of subjective an objective evaluation. Subjective weights are derived from the information about preferences of decision maker or a group of such by employing mathematical techniques. Subjective weights encounter in practice more often due to ability to critically assess and quantify the competent opinion of experts/decision makers. Moreover, mentioned techniques are more representative of subjective choices and priorities of the decision makers(s). The noteworthy examples of such methods are the Analytical Hierarchy Process (AHP) (Saaty, 1980), the Factor Relationship (FARE) (Ginevičius, 2011) and others. On the contrary, according to Vinogradova, Podvezko and Zavadskas (2018) objective weighting techniques rely on "*the structure of the data array*" (p.2) to evaluate the influence of criteria. Therefore, alike methods are less common as they demonstrate the momentum influence of a criteria and not the overall preference. Objective methods favour correlation related methods such as Criteria Importance Through Intercriteria Correlation (CRITIC) (Diakoulaki, Mavrotas and Papayannakis, 1995), combined correlation algorithms with standard deviation - Correlation Coefficient and Standard Deviation (CCSD) (Wang and Luo, 2010) and similar. Accounting for the mathematical precision of results calculated by objective methods, it is hardy ever possible to conclude the problem without the subjective judgment. Therefore, it is advised to apply combination of subjective and objective algorithms. For the practical purposes of the research, subjective method SMARTER (Edwards and Barron, 1994) is applied for weight calculation in order to compute evident preferences of research groups. Detailed outline of the method is mentioned in the later section.

3.5.1 Utility Maximization

The concept of maximizing utility is formed on basis of optimization. Given one's utility function $U(a_i) = u\{u_1[c_1(a_i)],\ u_2[c_2(a_i)], \dots,\ u_m[c_m(a_i)]\}$, reflecting his/her individual system of preferences, method always tries to seek the optimum, i.e. best possible solution by maximizing $U(a_i)$ for $a_i \in A = \{a_1, a_2, \dots, a_n\}$. This function can be specified in the form of some analytical expression:

$$u(a_i) = \sum_{j=1}^{m} w_j\, u_j\left[c_j\,(a_i)\right] \qquad (1)$$

which is to be maximized. By evaluation of each decision, we explicitly or implicitly compare to it some value of utility function, which shows the degree of preference of this solution compared to the others. In order to determine such best/optimal solution various methods were developed that focus on construction and overall maximization of the utility function.

Weighted Sum Approach (WSA) is one of the simplest and most intuitive methods in decision-making. Algorithm scalarizes values of multiple objective functions to a unique equitation:

$$u(a_i) = \sum_{j=1}^{m} w_j \, x_{ij} \tag{2}$$

As name suggests, value of each objective function is weighted by pre-determined cardinal evaluation w_i, which provide a numerical impression of importance of an objective in terms of optimization task (Nedjah and Macedo Mourelle, 2005). However, the general case when criteria have different scales due to their “nature” may impose difficulties during calculation process. In this case, it becomes impossible to apply the optimization formula directly. At first, results are required to be scaled accordingly. Normalized results are dimensionless and their values lie within the same limits, typically from 0 to 1, and can be directly implemented into a formula (Deb, 2004). The optimal result is chosen based on the type of optimization function: minimization or maximization. In minimization problems outcome with a minimal value of objective function is considered to be optimal, while maximum value is treated as best for maximization type. **Weighted Product Approach (WPA)** is comparable to weighted sum approach, however, instead of summing alternative scores, methods performs multiplication:

$$u(a_i) = \prod_{j=1}^{m} [x_{ij}]^{w_i} \tag{3}$$

Similarly to WSA all alternative scores must be normalized and raised to the power of w_i. The best alternative yields the highest gain of $u(a_i)$ (Rao, 2007).

Weighted Aggregated Sum Product Assessment (WASPAS) is combination of above-mentioned methods for utility optimization class of problems. Method relies on normalised values of alternatives, which are calculated as:

$$\bar{x}_{ij} = \frac{x_{ij}}{max_i \, x_{ij}} \tag{4}$$

$$\bar{x}_{ij} = \frac{min_i x_{ij}}{x_{ij}} \tag{5}$$

(4) for beneficial criteria, (5) for non-beneficial/cost criteria. In subsequent steps algorithm determines the relative importance of an alternative based on WSA and WPA methods. WASPAS method adopts a joint optimality model:

$$U(a_i) = \lambda \sum_{j=1}^{m} w_j \, x_{ij} + (1-\lambda) \prod\nolimits_{j=1}^{m} [x_{ij}]^{w_i} \tag{6}$$

resulting from WSA and WPA combination. The interpretation of λ weight is case sensitive, although, optimum weight of it can be determined analytically by employing variances of "sub-functions". Nevertheless, common value of λ is considered to be 0.5. WASPAS method reduces to WPA with λ=0 and WSA with λ=1 (Chakraborty and Zavadskas, 2014). As argued by Zavadskas, Turskis, Antucheviciene and Zakarevicius (2012) such approach generates more robust and accurate ranking results being 1.3 times higher as opposed to WPA, moreover, the increase equals to 1.6 times in comparison with simple WSA. Mentioned method is very simplistic in terms of calculation, despite that it still produces quite in-depth analysis among possible options (Chakraborty et al., 2015). Under a scope of research, aimed to define the rank of European universities, WASPAS technique is applied for analysis of alternatives to structure results in following manner $A_1 > A_2 > \cdots > A_n$ according to the value of objective function. Results are present in the practical part of the thesis.

Further study of decision-making processes has led to the conclusion that in reality people rarely behave rationally. In fact, in most real-world situations, people accept satisfactory solutions, which are usually inferior to optimal solutions in theory, but are acceptable from the utility optimization point of view. There are number of reasons supporting the argument for development of more sophisticated techniques for problems without ranking requirements. Firstly, out of a large number of possibilities, a person sees only a few alternatives, and therefore it is unlikely that his choice will be optimal. Secondly, we often lack knowledge and our decisions are usually based on very rough and general ideas about reality. And last but not least, process is often guided by fuzzy, vague or even contradictory goals, which affects the quality of the decisions made.

Analytic Hierarchy Process (AHP) is the most simple, popular and effective method out of utility maximization class (Pomerol and Barba-Romero, 2009). AHP does not deliver to the decision maker (DM) any "optimal" decision, but allows him to interactively find an alternative that fulfils requirements on all levels. Analysis of the decision-making problem in the algorithm begins with the construction of a hierarchical structure[3] that includes the goal at the top of hierarchy, criteria that have the most influence on the choice and group of alternatives. This structure reflects the understanding of the problem by the DM and outlines the scope of the problem to the analyst. Evaluation accounts for both befit and cost criteria, measured by quantitative parameters and qualitative characteristics, moreover, objective data and subjective expert assessment is incorporated during all stages of the analysis (Mocenni, n.d.). After determining the elements of the hierarchy, factors and alternatives must be compared. Pairwise comparisons of criteria and alternatives are held in order to determine domination of one variable over the other. Following results are displayed in Saaty's matrix, where $S = \{s_{ij}\ for\ i, j = 1,2 \dots, n\}$ are scores of comparison. Pairwise comparison displays degree of preference of ith alternative over jth and vice versa in terms of a particular criterion. The preferred value s_{ij} is assigned an integer score on scale 1,2…9 (1= equally important; 9= extremely important) to reflect the extent of preference, while $s_{ji} = {}^{1}/_{s_{ij}}$ appoints a reciprocal value to inferior option. Once priority matrix is calculated, scores are transformed into a normalized matrix $V_{n \times m}$, where each cell:

$$v_i = {}^{[\prod_{j=1}^{m} s_{ij}]^{1/m}} \Big/ {}_{[\sum_{i=1}^{m} \prod_{j=1}^{m} s_{ij}]^{1/m}} \tag{7}$$

represents normalized pairwise scores of alternatives that enter the decision matrix. In a similar manner one can calculate the criteria weights. The concluding phase of the problem concerns the actual utility scores, utilized for final ranking. Aggregate utility score (8) is obtained via scalarization of normalized utilities and original criteria weights w_j (Fiala, 2013).

$$s_i = \sum_{j=1}^{m} v_j w_{ij}\,, \qquad i = 1,2,..,n \tag{8}$$

[3] Figure 2 graphically depicts the form of decision structure.

An alternative with the highest maximal aggregate utility score is considered as a compromise solution.

3.5.2 Minimization of Distance from an Ideal Alternative

The main idea of distance-based methods states that the most preferred alternative not only obtains the closest proximity to the ideal solution, but is also situated further than all other alternatives from the unacceptable solution. Hence, the optimal solution is a vector including the highest scores according to all criteria. On the other hand, the worst (unacceptable) solution is a vector containing the minimum values for each criterion. VIKOR and TOPSIS are well-recognized examples, which determine the ranking/ optimal solution based on measure of "closeness" to some theoretical ideal value. Despite the common characteristics of the methods, there are some major differences. While VIKOR method compares all results solemnly with the ideal score, TOPSIS method searches for an option, which will be in the closest distance to ideal, ensuring remoteness from the worst (Kapur, 2019). Nevertheless, above-mentioned methods are most often encountered for ranking problems.

TOPSIS (**T**echnique for **O**rder **P**reference by **S**imilarity to **I**deal **S**olution) is an approach suggested by Hwang and Yoon in 1981. Researches tried to prove that ideal solution can be found by examining the geometrical distance between an alternative and the positive ideal solution (PIS) along with negative ideal solution (NIS) (En.wikipedia.org, 2019). Definition of ideal solution is related to benefit and cost criteria. PIS intends to maximize the benefits criterion while simultaneously minimizing the loss. On the contrary, NIS will result in an option with maximal loss and minimal benefit (Kapur, 2019). TOPSIS algorithm calculates the normalized criteria matrix $R = (r_{ij})$, in which the values are determined by given formula:

$$r_{ij} = {y_{ij}} \Big/ {[\sum_{i=1}^{n}(y_{ij})^2]^{1/2}} \tag{9}$$

Normalization of inputs is required due to the incomparability problem that arises because some criteria are expressed on different scales. After receiving relative scores of alternative, one must proceed to weighted normalized matrix $W = (w_{ij})$. W matrix creates a product value of relative scores and corresponding weights of factors. The following step distinctively defines basis of the method, it finds a value H_j matching the best- achieved w_{ij}, along with D_j that stand for the worst w_{ij}, result. As mentioned previously, the analyst finds the Euclidean distance from the ideal, d^+, and anti-ideal, d^-, alternative integrating the weighted results and PIS and NIS values. The ranking is applied to the relative closeness, defined as:

$$c_i = {d_i^-}\Big/{d_i^+ + d_i^-} \tag{10}$$

Compromise alternative is the one with the highest relative closeness. TOPSIS method is widely used due to a number of evident advantages. Method does not loose the accuracy with increasing number of observations as well as criteria, while the algorithm remains simple and time efficient (Azadeh, Kor and Hatefi, 2011). Moreover, technique is superior to pairwise comparison methods for analysis of big datasets. It is worth mentioning, that with high number of criteria TOPSIS method fails to adjust for correlations among them. Therefore, such approach requires a prior screening of the problem to detect and eliminate existing dependencies (Xu et al., 2015). Compensatory character of mentioned technique approves the trade-offs among criteria, in particular an alternative with a poor score in one criteria can be compensated by another alternative with a greater score according to another criterion. Such interactive methods are more realistic due to flexibility of choice as compared to methods with strict cut-offs (En.wikipedia.org, 2019). This method is of a special importance for ranking problems because it contemporaneously takes into account the worst and best outcomes with the necessary quantifications. The feasibility of the method application may in some instances be distorted by atypical result, which is concurrently equidistant form ideal and anti-ideal solution (Li et al., 2011). Under such condition numerical representation of differences among results assist in the decision making process to determine the optimal rank. For the purposes of a given research, results from TOPSIS method assisted in setting the optimal rank of higher educational institutions.

VIKOR (**VI**se**K**riterijumska **O**ptimizacija I Kompromisno **R**esenje)[4] method is related to group of distance function methods that minimize the total regret (Yu, 1973). According to the

[4] In Serbian

original formulation of the problem by Opricovic (1998), VIKOR method was introduced to deal with problems under condition of conflicting criteria expressed in different units. As TOPSIS method VIKOR also recognizes an alternative, which lies closes to the ideal point, as the compromise solution. However, the difference between methods lies in the aggregation function and normalization techniques (Sayadi, Heydari and Shahanaghi, 2009). VIKOR method, on the one hand, searches for the optimal distance to PIS, disregarding the closeness to the negative solution. Therefore, method proposes such a solution that yields a maximum gain while the risk evaluation is omitted. According to VIKOR the decision maker is neutral to the risk evaluation (Sayadi, Heydari and Shahanaghi, 2009). VIKOR algorithm start with determination of the best and worst value according to all criteria, such that $f_i^* = \max f_{ij}$ represents the maximum achieved criterion value. In addition $f_i^- = \min f_{ij}$ corresponds to minimal score. Subsequently method searches to determine utility measure $minS_j$, which stands for maximum utility, and regret measure $minR_j$, denoting a minimum regret resulting from selection of an alternative. Both values are computed in form of L_p metric, namely

$$S_j = \sum\nolimits_{i=1}^{n} w_i \frac{\left(f_i^* - f_{ij}\right)}{\left(f_I^* - f_i^-\right)} \tag{11}$$

$$R_j = max\left[w_i \frac{\left(f_i^* - f_{ij}\right)}{\left(f_I^* - f_i^-\right)}\right] \tag{12}$$

where w_i indicates the weights of DM's preferences. The next phase focuses on the individual performance of alternatives VIKOR index expressed by Q_j scores:

$$Q_j = \frac{v\left(S_j - S^*\right)}{\left(S^- - S^*\right)} + \frac{(1 - v)\left(R_j - R^*\right)}{\left(R^- - R^*\right)} \tag{13}$$

calculated for all alternatives, where $S^* = minS_j$, $S^- = \max S_j$ same holds for $R^* = maxR_j$, $R^- = \max R_j$. Method introduces a second weighting factor v for the calculation of individual scores, which typically equals to $v = 0.5$, however could be compromised. Rank of alternatives is established by ordering VIKOR index and the option with a minimum Q_j score is considered as a compromise solution (Evangelides, Zormpa and Tzimopoulos, 2013). VIKOR method is widely applied for problems where the DM fails to provide the strict preferences in the begging of modelling. Moreover, method is especially beneficial when dealing with uncertainty since a compromise solution presents the total maximum utility revised for all the alternatives. However, the compromise solution stands more as the suggestion for the DM(s), the optimal solution remains the basis for negotiations. Because of the controversial nature of ranking schemes, the analyst has decided to facilitate VIKOR method for the main purpose of the research. The findings are presented in further sections.

3.5.3 Preference Relationship

Generally, the notion of preference relation describes human "*degree of dominance of one option over another*" (Rogers and Bruen, 1998). Bernard Roy has started to explore the topic of outranking techniques in 1968, which formed a group of European/ French methods in MCDM. Methods relating to this class are based on grounds of pairwise comparison of alternatives along all attributes. Therefore, such methods are not recommended for complete order because they don't provide the objective preferences of the DM(s) (Evans, 2017). Moreover, another distinctive difference concerns the utility function. In some situations it is not required to be specified in functional form or to be specified at all. However, the DM(s) has to present enough evidence to support the choice of one option over another (Kangas, Kangas and Pykäläinen, 2001). Outranking methods relate to non-compensatory class of methods. If an alternative yields poor scores with respect to an attribute, it may not be setoff by good results of corresponding alternative in other attribute. Due to this fact the DM has to meticulously evaluate the distance between alternatives on each criterion to conclude whether the distance is sufficiently large to assume strict preference according to the most important criteria (Kangas, Kangas and Pykäläinen, 2001). Preference relation in outranking classes is expressed by a dominance status, which at the same time rests on type of criterion used during analysis. Preference degree expresses extend to which one alternative relates to the other in pair. Table 2 displays the most common types of relationship.

Preference degree	**Interpretation**
$a\ I\ b$	Indifference between a and b
$a\ P\ b$	Strict preference of a to b
$a\ Q\ b$	Weak preference of a to b
$a\ R\ b$	Alternatives are incomparable

Table 2 - Preference relation between alternatives (Fiala,2013)

Outranking methods employ various types of preference criterion, which perform a function of decision principle during evaluation of alternatives a and b. The most dominant types criterion applied in "French school" of methods (ELECTRE, PROMETHEE) (Lootsma, 1990) are the "true criterion", "quasi" and "pseudo"-criterion.

- The "true criterion" is characterized by existence of preference without differentiation of its scale and magnitude. Any distance between alternatives implies the preference of one over another or indifference among compared, thus, forming linear preference function.

$$a\ I\ b \quad \rightarrow \quad f(a) = f(b)$$
$$a\ P\ b \quad \rightarrow \quad f(a) > f(b)$$

- The "pseudo-criterion" relates to an evaluation function, which is defined by thresholds (p, q) set by the DM(s). This criterion is invented to account for insensitiveness among criteria such, that thresholds introduce precision of sensitivity and distinguish between indifference and preference zones. Any small differences among alternatives may be neglected as long as they fall under the same threshold region. The first threshold q defines the border between an indifference and weak preference, while p threshold identifies the weak or strong preference.

$$a\ I\ b \quad \rightarrow \quad f(a) - f(b) \leq q$$
$$a\ Q\ b \quad \rightarrow q \ < \ f(a) - f(b) \leq p$$
$$a\ P\ b \quad \rightarrow \quad p < f(a) - f(b)$$

- The "quasi-criterion" the indifference is affiliated with a single threshold, where $p = q = c \geq 0$ represent the constant cut-off set by the DM(s). This criterion is typically imposed in problems dealing with uncertainty and vagueness (Colson and de Bruyn, 2014).

$$a\ I\ b \quad \rightarrow \quad |f(a) - f(b)| \ \leq q$$
$$a\ P\ b \quad \rightarrow \quad f(a) > f(b) > q$$

The main classes of problems representing, so called "French school", are the PROMETHEE and ELECTRE group of methods. The preference relationship approach is generally less recommended for ordering problems, as it establishes a weak order. Additionally, due to comparison nature of methods, it is suggested to apply mentioned techniques to finite sets.

Nevertheless, the computational mechanisms nowadays have been adjusted to perform the analysis with infinite sets (Bouyssou, 2019).

ELECTRE Family

ELECTRE (ELimination and Choice Expressing REality) family methods include ELECTRE I, ELECTRE II, ELECTRE III, ELECTRE IV, ELECTRE TRI and ELECTRE IS, which are the most widespread outranking methods (Fahmi, Kahraman and Bilen, 2016). The ELECTRE methods are aimed at solving problems with mainly finite set of alternatives $A = \{a_1, a_2, \dots, a_n\}$ evaluated according to the criteria set $C = \{c_1, c_2, \dots, c_m\}$. These methods do not quantify the difference among alternatives, but only establish the condition of superiority of one option over another. This key feature of the class allows the comparison of variants expressed in different units of measurement and / or with different scales. Each method of the family has its own individual characteristics, which makes their application most effective for various types of decision-making tasks. Each such problem may be handled is several ways, depending on the task and a specific ELECTRE method used.

The method ELECTRE I is the basic method of the family that was first explained by Bernard Roy in 1968. In the original formulation the author tried to obtain a set of all non-dominated solutions, from which one can choose the optimal compromise solution by application of preference threshold. For every pair of alternatives a and b the analyst must determine the concordance index (for each $a > b$) and discordance index (for each $b > a$), which can take on values from 0 to 1, and calculate the preference and dispreference degree accordingly. The outranking technique employs the thresholds to compare the calculated indices in order to determine preference relationship among the pair (Fiala, 2013). The main disadvantages of the basic methods are the inability to obtain a full ranking of alternatives. However, the procedure for solving problems using basic methods is quite fast and simple. Thus, the basic methods are most effective, if you need to make a quick ranking of alternatives, agreeing to accept low accuracy, or filter alternatives into groups according to a certain threshold.

As mentioned previously, ELECTRE I does not solve the problematic with sorting the alternatives from the best to worst, thus establishing the full rank of alternatives. Modifications of ELECTRE methods were proposed to correct for this and also develop new features of the following class. ELECTRE II was the first method to rank alternatives. The technique incorporated an embedded outranking relations divided into a strong and weak outranking enabled by two concordance levels. Following the ELECTRE II, ELECTRE III was developed to deal with inaccuracy of data along with imprecision of comparisons. Unlike on previous versions of the method, ELECTRE III introduces "pseudo-criterion" instead of "true-criterion" to form a fuzzy relation, characterized by credibility index. ELECTRE TRI was suggested as a correction for its predecessor to allocate alternatives into categories. Although, ELECTRE family consists of many diverse methods that are able to solve most of the existing real-life problems, the research in the field continues. Recent modifications focus on robustness caused by imperfect knowledge and elicitation parameter techniques (Figueira, Mousseau and Roy, 2005).

PROMETHEE Family

The PROMETHEE (Preference Ranking Organisation METHod for Enrichment Evaluations) class of methods relate to outranking class of methods, known for its simplicity and accuracy, developed by Brans, Vincke and Mareshal (1968). The PROMETHEE I, II, III and IV methods represent the class of "French school" that rely on pairwise comparison of alternatives to derive the rank. Unlike the ELECTRE methods, PROMETHEE extends the standard definition of a criterion (true, quasi or pseudo) to criterion with linear preference, linear preference with indifference area and level-criterion (Brans and Vincke, 1985). Moreover, PROMETHEE does not require a strict knowledge about the actual structure of preferences of the DM(s). When evaluating alternatives, the key task is to obtain information on whether some alternative is at least as attractive as another. The main stages of the algorithm can be separated into:

1. **The construction of a preference function**

The starting point begins with the formation a preference index, which reflects the preferences among alternatives for each of the criteria. Based on the information contained in the preference matrix, alternatives are compared in pairs according to existing criteria. Results are expressed by preference functions, which are calculated for each pair of options and can vary from 0 to 1, where 0 indicates indifference between options and 1 stands for a strict preference of one of the variants. The resulting preference relation can also be express in form of outranking graph

2. **The exploitation of the relation**

The total preference matrix or outranking graph is utilized to build a partial or a total order. According to PROMETHEE I the analyst must establish the outgoing, $\varphi^{+}(a)$, and incoming, $\varphi^{-}(a)$, flow for each node on the graph. Generally, the larger the outgoing flow, the more dominant a alternative is, while the smaller values of incoming flows indicate the slighter dominance over a. Evaluation of inflows and outflows presents the final partial preorder alternatives. Unfortunately, PROMETHEE I has a problematic of treating the incomparability, therefore, PROMETHEE II considers the net-flow, $\varphi(a) = \varphi^{+}(a) - \varphi^{-}(a)$, of nodes to establish complete order indicating indifference or outranking relation (Brans, Mareschal, 2005;Brans and Vincke, 1985). Moreover, GAIA (Geometrical Analysis for Interactive Aid) was introduced as a graphical complement to PROMETHEE class to enable graphical representation of the problem. Such approach allows the DM(s) to evaluate trade-offs among alternatives as well as visualize a force of criteria weights during PROMETHEE evaluation (Mareschal, 2014).

For the research purposes of a study, outranking methods yield poor results due to number of reasons. A rank reversal problem concerns with a change in the ranking order after introduction of new alternatives or method application changes. Following problem appears more often in the course of PROMETHEE proceeding, which produces unreliable ranking results as a consequence of their increased sensibility. In addition, outranking methods fail to structure the problem during the structuring phase, such matter imposes a higher risk of the problem misunderstanding by the DM(s) and complications for the analyst. Until recent days, such methods missed clear guidelines for the weight calculation techniques. The DM(s) was obliged to present estimated weights of the criteria. This may be treated by application of scoring techniques, however, there are no existing methods from outranking nature to adequately account for it (Figueira et al., 2012). One of the possible suggested solutions is to incorporate AHP weighting approach in a PROMETHEE environment (Marcharis et al., 2004). On the other hand, methods are able to deal with uncertain and fuzzy information. As well as application of outranking methods is justified when simultaneously handling the qualitative and quantitative criteria units (Liaise-kit.eu, n.d.).

4 Data Envelopment Analysis

Data Envelopment Analysis (DEA) identifies and assesses alternatives with low efficiency scores, in order to enhance the performance of such by identification and transformation of weak performance criteria. DEA analysis makes use of mathematical linear programming technique to determine inefficient and efficient decision making units (DMU), which are referred to as alternatives within the DEA framework. In addition, inefficient DMUs are compared to role-model (efficient) alternatives to quantify the lack in order to achieve Pareto-frontier. For comparison of results method generates relative efficiency scores on scale from 0, completely inefficient, to 1, efficient alternative, which lie on Pareto-frontier.

$$Efficiency = \frac{u_1 O_{1j} + \cdots + u_p O_{pj}}{v_1 I_{1j} + \cdots + v_p I_{pj}} \tag{14}$$

where u stands for the weight of output, v is the weight of input and O and I are the values of outputs and inputs respectively. In terms of mentioned approach, efficiency is regarded as generation of maximum output with the set of inputs (Mendes et al., 2015).

DEA models are applied in MCDM analysis to increase the performance of inferior options. Evaluation relies on the input and outputs, which are selected to adequately express the performance of each DMU. The selection of variables can follow from a specific theory or an expert knowledge. Depending on the research goal, the DM(s) can choose the set of key indicators to evaluate the behaviour of DMU, thus forming the set of outputs. Once objectives are in place, inputs are collected to describe the above-mentioned set. It is important to mention that DEA technique allows for inclusion of generally incomparable inputs and outputs. Within the bounds of research the analyst enables the controllable set of inputs, however, there are model modifications that integrate uncontrollable inputs (Mendes et al., 2015). For the purpose of university ranking outputs are treated as measures of university success, which are employment reputation, institution's recognition on an international level expressed by international connections as well as willingness of experienced and highly-qualified academic staff to work for an institution representing the teaching quality. The list of potential inputs must sufficiently describe the success drivers, outputs. The set of inputs may be described by per capita academic performance, access rates, citation frequency and awards rewarded to staff and alumni. It is important to mention that there is a simplified technique to filter the input/output groups in case the analyst is unable to clearly state the success measures. Outputs are perceived as desirable outcomes while inputs are less preferential (Gillen and Lall, 1997).

Once the set of inputs and outputs are identified, DEA models must be defined in terms of optimization method as well as the analyst is supposed to specify relationship among output and input groups. Optimization character of the problem is formulated either by minimization of input or maximization of output. Minimization of input approach guarantees to maintain the equal level of output under condition that inputs are minimized. This strategy is employed for cost reduction problems. However, for the purpose of university rank improvement one must review the output maximization approach, which aims to maximize the total output for known set of inputs. Relative efficiency of university has to be maximized in order to attain the higher scores according to some criteria. Such option is particularly appropriate for universities, which are ranked low according to established final rank, thus the criteria with the highest weight must be increased. With respect to scaling mode, determining the relationship between groups, DEA model differentiates the constant and variable return to scale. Constant return to scale anticipates direct linear proportion of outputs to inputs, with increasing inputs succeeds the increase in outputs. It is rarely the case within the research definition. Increasing access rates (input) do not prompt alike change in employment rates or teaching quality. As one can conclude, variable return to scale is more realistic. Such approach presumes that alteration of inputs may increase as well as decrease the efficiency (Avkiran, 2006).

5 University Ranking Schemes

5.1 Ranking as a Mathematical Concept

The ranking reflects the type of relatedness for two or more items, where the relationship between neighbouring alternatives is defined in terms of "rated higher/lower or equal" to the other (En.wikipedia.org, 2019). In mathematics, ranking belongs to the class of weak orders that allows ties (equal ranks) among options. Concurrently, a generalization of weak orders results in totally ordered class, which relies on strict preferences among adjacent alternatives. A weak order is well-defined transitive, binary relation between options. Weak ordering is sometimes recognized as a strict weak ordering, however, these notions prove to have minor distinctions. Regenwetter et al. (2006) describe the strict weak order as "the asymmetric part of a weak order" meaning ties have a negative perception as they fail to define strict preference of alternatives. One may argue that the expected outcome of a ranking problem must be defined in the form of the total order. However, it is irrational to demand the total order in the decision-making, moreover, sometimes such doesn't exist due to imperfect knowledge. As one of the disadvantages, the following condition produces artificial and spurious results. Likewise, total orders do not explicate the compensatory nature of all MCDM problems. The DM(s) has an opportunity to change the importance of criteria when working with interactive models, and such modifications may result in rank reversal problems in total order. Thus weak orders are regarded as an acceptable generalization of order with no ties. Interpretation of such rankings is very straightforward, intuitive and realistic (Wang et al., 2008).

In MCDM analyses are typically complex involving a big amount of data to be examined, thus it leads to more complicated decision-making models. As mentioned previously, all MCDM problems require to either determine the rank of alternatives in order to detect a "winner" or structure the result in the form of rankings to obtain an overview of the complex problem. The different kinds of sophisticated methods allow transforming the initial complex problem into a list of ordered options (the rankings) according to the criteria of interest. Corresponding integration of the raw data and sound approaches generate the comprehensive output, which may be easily understood by the decision maker(s). Given research aims to rank EU universities by utilizing Multi Criteria Decision Making methods to identify the total order of the top 5 universities. The results of the procedure establish an ordered sequence of universities, in which their location corresponds to their preferences (i.e. in the first place there is the most preferred choice, in the second - the less preferred, etc) according to all criteria specified. Thus, the following ranking records the names of universities in the order of their preference, forming a unique permutation subjective to the input material. The output of the study represents a single list of universities.

This analysis is aimed to determine the ranking of universities by assigning the rank to each alternative based on its performance. In such a situation it is reasonable to review multiple ranking strategies for order definition. *Standard competition ranking (1224)* assigns the same rank to alternatives that yield the same results. The order of following alternatives remains unaffected as the gap is filled out with the same ranks and order continues. If $a > b = c > d$, standard competition ranking appoints a with the first place, b and c with the second and d with the forth (Spoj.com, n.d). In a similar manner, one can define *modified competition ranking (1334)* where equal alternatives are awarded a lower rank among two possible (Community.tableau.com, 2018). Given the same example, a is ordered the first, b and c are the third and d is the fourth in order. The later types of ranking are not recommended for the purposes of the study, as they do not yield fully accurate results. The distance between equal alternatives is not taken into account, thus, the reliability of the order might be questionable. *Dense ranking (1223)* is treated in a similar manner although it does not leave gaps while ordering. The next alternative, given that previous two are equally valued, instantly receives the following rank. Referring to the example above, the order is a - the first, b and c - the second and d - the third. Dense ranking produces results that are simple for understanding since there are no gaps in the order. Such comprehensive nature of results definition is generally more preferable for the DM(s) because they do not have to dig deeper to interpret the missing rank value. The most common and preferred type is the *ordinal ranking (1234)*. This type of output is characterized by assigning a definite ordinal number to each alternative neglecting the equivalency of results. The appointment of rank for options that are equal is done arbitrarily or randomly. A more general and accepted way is to apply an arbitrary method, which may embody other stratification attributes. Techniques like alphabetical ordering, frequency of occurrence and others guarantee a more consistent order of preference, as any piece of additional information allows achieving advance individualizing. Thus, an alternative a is awarded the first place, b - the second (alphabetical ordering applied), c- the third, d- the forth. *Fractional ranking (1 2.5 2.5 4)* awards equal alternatives with the rank equivalent to the mean of ranks they would obtain under the ordinal rank. Following ranking scheme is the most common and applied ranking in the statistics field, particularly the order statistics. If an ordinal rank cannot be achieved, the fractional technique expresses the ranking closest to the ordinal (Cichosz, 2015). Both ordinal and dense approaches are highly effective when defining the university ranking because the results are structured in a very transparent manner without any gaps or reverse orderings (modified standard ranking). Therefore, a combination of stated techniques provides the highest accuracy. The top 5 universities are required to be structured in an ordinal way to detect absolute leaders by digging into a smallest detail, specifying the minor differences, while the dense ranking is applied to the rest.

5.2 University Rankings

For the future perspective of financial and social stability in life, teenagers are asked to make the choice, influencing the direction of development to a certain extent. Nowadays, given a great range of available higher institutions, colleges, and online schools, many applicants seek for guidance and transparency during the decision-making process. Published university rankings appear to be a key tool in decision aiding because they evaluate a wide range of institutions based on a combination of various attributes that might appear essential to many (Baty, 2018). The aim of university ranking may appear obvious to many. The framework of rankings is explicated by critical evaluation on how institutions *"transform inputs to outputs"*. University rankings employ the set of inputs (funding, academic knowledge, etc.) to transform them into a measure of goal accomplishments, such as outputs (graduation rates, employment rates, student academic achievements, etc.). As it may be suggested, the higher investment into inputs, thus generates more significant results in terms of quantity and quality. Many leading universities employ this strategy to achieve an admired outcome and to improve their rating positions (Shin, Toutkoushian and Teichler, 2011).

It goes without saying that objective evaluation is hardly ever possible due to a large amount of unorganized data, subjective judgments and lack of clearly identified selection criteria. The most well recognized global university rankings are QS “World University Ranking”, Times Higher Education World University Ranking and Academic Ranking of World Universities. These rankings assess the performance of universities based on extensive and explicit educational indicators, encompassing the teaching approach, general level of knowledge, academic and research achievements, international ties and many other influential factors for most excellent institutions. Each of the global rankings rely on its own explicit methodology: some are more performance oriented, while others account for student satisfaction levels and international diversity. Thus, there is no single authoritative rank, however, the “best” universities are included in all ratings, varying by position on the list. All universities are somehow unique in its strengths, thus it is crucial for an enrollee to orient the choice of the ranking adapting to its own needs. The best tactics lie in the choice of such an institution that possesses a strong reputation and prestige in the field of interest. Rankings are particularly helpful to students, who desire to study abroad. Very rarely those students have a chance to receive feedback without visiting universities of interest (Shin, Toutkoushian and Teichler, 2011). Academically oriented rankings provide that kind of detailed discipline based overview. Yet, the universities including many branches/ faculties receive a high overall rank, which may diminish the differences in quality among available programs (Angerilli, 2013). While noting many beneficial sides of university rankings, this topic has been highly debated over the past years. With an increasing number of rankings available, there is less and less opportunity to reach consensus and indicate absolute measures of “educational success”. The following arguments best describe the opinion of opponents.

- Firstly, the failure to objectively characterize and measure the performance of the university by a single quantitative indicator is the main argument against the rankings (En.wikipedia.org, 2019).
- Secondly, according to some professors, the rankings are biased by availability of information as well as research-related measures. Mainly medical and scientific fields determine the academic achievements of universities. Often academic publications in these areas are translated to English, while social sciences and humanities have lower citations rates because they are mostly available in native languages (Nilsson, 2016).
- Thirdly, some ratings include manipulative characteristics such as “reputation” or “prestige”, which do not demonstrate educational efficiency (En.wikipedia.org, 2019).

- Fourthly, following the input/output evaluation model some rankings tend to underestimate the importance of either inputs or outputs. Thus the reliability of rankings decreases due to the omission of fundamentals. Results of objective function maximization lose its sense, as they appear contrary to the assumption of availability of inputs and outputs (Shin, Toutkoushian and Teichler, 2011).
- The last but not least matter concerns the immeasurable scale of some criteria. When selecting a higher institution an enrollee frequently refers to indicators that are hard to measure. Academic performance and reputation play a role, however, the experience of student life is a very personal and important aspect to acknowledge. The part of any educational success is to be driven and inspired by colleagues, enjoying the campus life and other related measures. These results shape a unique combination, which captures the substantial share of a final decision although are not directly measured by ratings (Pop, 2018).

Undeniably, university rankings have had an influence on the tactics and behaviour of higher institutions. The race for the higher rank forces institutions to adapt to the main evaluation criteria, thus increasing investments in academic research. Universities devote a lot of attention to a number of publications and citation rates. Moreover, new environment promotes changes within the organizational structure of higher institutions by mainly promoting professors with a required number of recognized international publications. Another aspect partially driven by the world's globalization refers to increased interest in international ties. Universities in English-speaking countries benefit more compared to some others. Consequently, institutions open more English programs to remain competitive as well as they require regular students to mandatorily pass few courses in English. For better or worse, the traditional conservative culture aimed at academic performance of professors is slowly but firmly shifting towards the students. Student satisfaction with an overall "student life" forces universities to expand beyond the academia. Many foresee the essential changes to future rankings, which encompass transparency, comparability and suitability issues. Existing unique ranks should be stratified into multiple rankings considering the discipline, size and overall mission. Furthermore, global rankings must be revised to reflect the local difference, which may heavily affect the student's choice. More importantly, university ranks must be disciplebased. This correction will avoid exclusion of some universities oriented towards humanities and social sciences from the top ranks. The fundamental goal of rankings is to assist the students and their parents to choose a high-quality institution (Shin, Toutkoushian and Teichler, 2011), accordingly, availability of reliable and transparent information is the main aid to facilitate the choice.

6 Problem Analysis: EU University Ranking

The empirical study of this chapter aims to perform MCDM analysis by employing mentioned above methods to evaluate the ranking of European Union's universities. Analysis was applied to 44 universities located in the European Union within the framework of 8 criteria discussed in the later part. The data was acquired from the widely acknowledged databases of rankings such as QS "World University Ranking", Times Higher Education World University Ranking, and Academic Ranking of World Universities (ARWU). Through the application of TOPSIS, VIKOR and WASPAS methods in the RStudio software the analyst intends to build the rank of top 5 universities among available 44 based on the combined output of suggested methods. For this purpose, the questionnaire was constructed to conduct the poll in order to determine differences and quantify weights of criteria depending on gender as well as science field. Concluding part of the chapter observes the differences among obtained rankings of universities according to genders along with a comparison of the rankings defined by a field of study. The results are presented in the form of 5 distinct rankings respectively.

6.1 Data Collection

The dataset of universities for further analysis was combined based on the multiple ranking sources. Initially, 44 EU universities were collected from a list of 100 best universities in Academic Ranking of World Universities for 2018 (Shanghairanking.com, 2019). Figure 5 displays the share of participating countries. It is important to note that not all EU countries are present in the top 100 universities in ARWU (which includes a total of 1500 university participants).

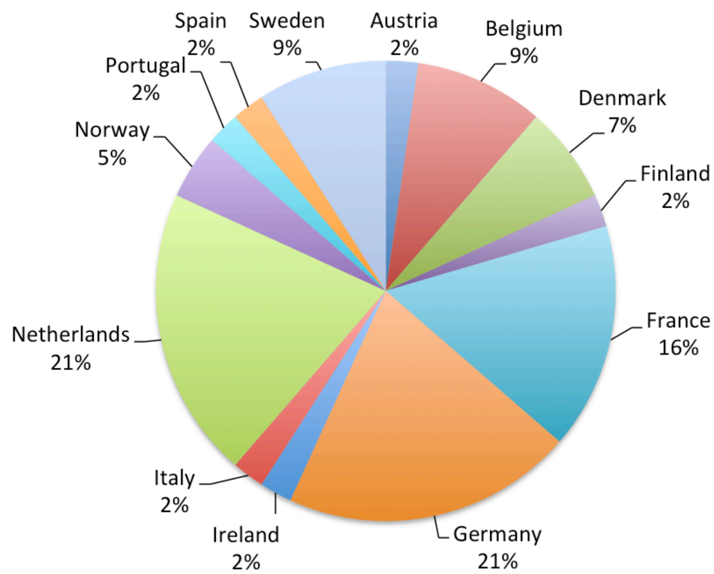

Figure 5 - Representation of Participating Countries
(own calculation based on data from Shanghairanking.com, 2019)

An institution must first satisfy the selection criteria and only excellent 500 institutions are ranked as the world's leading higher education institutions according to ARWU. Universities are filtered by multiple selection criteria, representing academic performance such as graduates and staff with Nobel Prizes and medals, citation frequency, and per capita academic achievements of a university (Shanghairanking.com, 2019). Another valuable source is The Times Higher Education Ranking Survey, which gathered information from 10 162 respondents across 138 countries, ensuring equality of response distribution among countries and disciplines (Timeshighereducation.com, 2019). In addition, data came from QS "World University Ranking" that covers 85 countries across the world with 1 011 considered institutions. QS "World University Ranking" ranks institutions according to the academic and employer reputation, international ratio and others (Topuniversities.com, 2019) The combination of criteria drawn from multiple rankings guarantees the reliable and effective data to describe a university performance, avoiding rank specific bias. The section A.1 in Appendix provides the score dataset of criteria used for the purposes of the analysis. Throughout the study, the author is assumed to perform as the decision maker.

6.2 Criteria Selection

The main question under ranking debate focuses on measures of institutional effectiveness and quality, thus criteria of effectiveness. There are many theoretical assumptions what may be a measure of such, however, there is only a very small amount of practical research and evident data to support the assumption. Even with the considerable amount of investigation and research, the lack of empirical data evidence to define the effectiveness of universities is still apparent and controversial (Shin, Toutkoushian and Teichler, 2011). The criteria selection within the framework of the study is chosen to sufficiently well meet desired prerequisites according to the DM's vision. Unfortunately, no unique ranking scheme evaluates the research and academic quality to a fair extent. Employing a combined criteria model based on multiple rankings may enable more sound analysis.

1. *Alumni*

The alumni criterion stands for the number of university graduates, who won Nobel Prizes as well as Field Medals. Any student, who successfully obtained a bachelor's, master's or doctoral degrees in the higher institution is recognized as graduate. In case a graduate has more than one degree from the same institution, the person is only counted once. The criterion is seasonally weighted to better reflect nearest past without neglecting previous accomplishments. The highest weight of 100% receives a graduate from 2001-2010, 90% is awarded to those graduated between 1991-2000, 80% for period of 1981-1990, following the linear decrease 10% remains within 1911-1920 (Shanghairanking.com, 2019).

2. *Award*

The award criterion assumes the similar to alumni definition. It focuses on the staff members, who obtained Nobel Prizes in the field of Physics, Economics and Medicine as well as Field Medals in Mathematics. Staff member is the one who was employed by an institution in the moment of winning the prize. If a professor is associated with more than one university, each of them receives the reciprocal of the total number of institutions. Nobel prizes are also weighted based on the proportion of an effort, in case it is shared. The weight of 100% receive prizes/medals after 2011, 90% for period between 2001-2010, 80% for winners between 1991-2000 and so on, until the 10% is remains in a period of 1921-1930 (Shanghairanking.com, 2019).

3. HiCi

The criterion represents the amount of Highly Cited Researchers chosen by Clarivate Analytics. Publications of 2017 were provided to ARWU for calculation of the 2018 index. Only the primary authors are considered during the calculation. (Shanghairanking.com, 2019).

4. Per Capita Performance (PCP)

Per Capita Performance combines the three above mentioned indicators plus the nature and science publications along with the total number of cited articles based on Social Science Citation Index. The Nature and Science indicator represents the number of papers published in related fields, while the total citation frequency is expanded citation index accounting for all the primary author references. PCP indicator is a weighted average of above indicators divided by the number of the full-time employed staff in an institution (Shanghairanking.com, 2019).

5. Employment Reputation (EMPL)

Employment reputation of an institution estimates the academic performance, namely the skills graduates possess and their direct work applicability. Employability is the one of key indicators to assess the relevance and quality of university education. This metric is based on the QS Employer Survey, which collects over 40 000 responses from potential employers. They are asked to score institutions on the basis of competences, effectiveness and quality of skills graduates show at the work place (Topuniversities.com, 2019). Criteria scores were transformed from original view due distinct clustering. Values are grouped within a certain interval, thus clusters need to account even for minor differences since the criterion (in most cases) is considered to the most important. Based on the interval width the analyst defined number of clusters and the scale, which is from 1 to 10 in terms of employment reputation criteria.

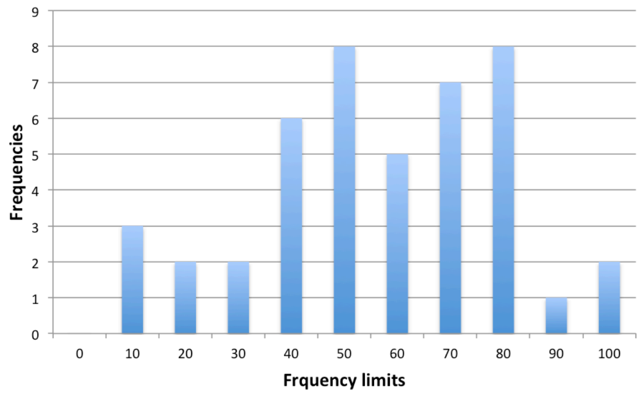

Figure 6 - Frequency of Original EMPL Scores
(own calculation based on data from Topuniversities.com, 2019)

6. *International Connections (INTRN)*

Universities with strong international relations demonstrate high levels of competitiveness and thus attract ambitious enrollee from all around the globe. Higher institutions benefit from an acquired international brand not only by increased number of interested students, but also promote global outlook within an institution. Multinational environment encourages exchange of knowledge, practices and opinions. Nowadays, international outlook is well regarded and appears to be valuable even to future employers.

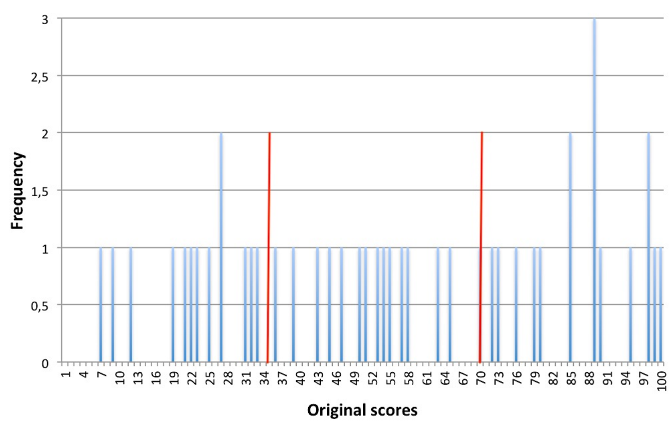

Figure 7 - Frequency of Original International Scores
(own calculation based on data from Topuniversities.com, 2019)

As in previous example, the criterion used during analysis is rescaled because original scores are evenly spread within few intervals as it can be seen on Figure 7. According to frequency distribution all values falling within the first interval receive score – 1, the second – 2 and the third – 3.

7. *Teaching (TCHNG)*

From the student's perspective teaching quality is the crucial component of learning success. More than 20 000 Academic Reputation Survey respondents underpin this criterion by explicitly evaluating the level of teaching skills as well as their quality. This criterion does not solemnly relies on survey, also staff-to-student ratio is representative of the commitment. Another important aspect to consider is an institutional income, which demonstrates the indoor financial investment opportunities (research investments, infrastructure and facility support…). Teaching can also be perceived by number of students who are willing to continue studying to the highest level, doctorates level. Thus, doctorate-to-bachelor's ratio is another component to review (Topuniversities.com, 2019).

8. *Access*

Nowadays, access may be one of the important indicators to check. As more and more universities expand their access on the global level, the competition among enrolee enables the chance to select only smart, eager and prudent students. The scores are calculated as a proportion of the number of available positions (the total amount of full-time equivalent places) divided by an index of population size, namely the square root of population. This indicator is utilized to demonstrate the chances of the country's residents to be enrolled (Topuniversities.com, 2019).

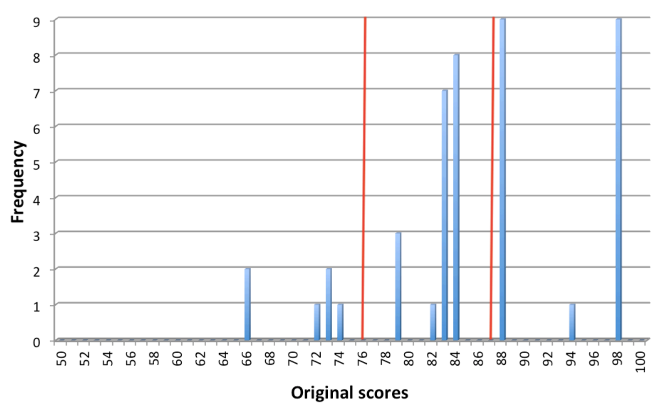

Figure 8 – Frequency of Original Access Scores
(own calculation based on data from Topuniversities.com, 2019)

Figure 8 depicts original access score, which indicate the high chances of acceptance with values varying from 66 to 100 points. Thus, results were split into 3 equal intervals to represent cluster similarities. The values in lowest interval (60-76) were awarded with one point, the second received two points, while the third – three.

It is important to note that data was utilized with minor modifications to scale as well as application of proxy values. On rare instances when an institution failed to provide a particular metric within 2018, the rankings handled such problem by altering the score with a conservative estimate (proxy). The omission or "zero" penalization will harshly underestimate an alternative. Scale modification of some criteria was necessary to depict similarities of groups, the values of which didn't vary sufficiently within the cluster itself. However, if values were widely spread out, one must leave the original scores to encompass and quantify the differences. Within the scope of the research all criteria are to be maximized to reach the highest value of the objective function for each particular alternative.

6.3 Survey

The preliminary purpose of the survey is to collect data in order to explore an extend of factor significance with regard to the gender and profession preferences. Interviewees were asked to assign the relative scores on the scale from 0, if they disagree with a statement, to 4, indicating full agreement. The summation of scores from individual surveys through all subgroups of representatives enabled the final evaluation of the criteria importance, thus the one with maximum score proved to be the most influential to relevant class. Such scoring approach allows avoiding order bias associated with the position of a statement in the sequence. The criteria appearing higher in the order have a higher probability to be selected as best. Moreover, given a long list of criteria, respondents may loose concentration (Finch, 2017). There were total of 80 interviewed individuals out if which the age distribution is mainly scatter around the age of 22-25 with 34 respondents, followed by 30 representatives aged 18-21 and only 16 respondent aged above 25. The biggest proportion of respondents corresponds to the age category that recently dealt with the university selection challenge, accordingly assuming they possess a strong prior knowledge.

Selection bias is another crucial milestone to consider during surveying as it may result in undercoverage, caused by inadequate representation of the groups. Also, a nonresponse bias, stemming from considerable differences between groups of respondents and those who are unwilling to participate, hence capturing biased responses with similar trend (Stattrek.com, n.d.). The equal representation of genders (50% of males and 50% females) verified the compliance with the balance prerequisites to avoid biased results according to sex. The next four target groups were associated with field of study encompassing social sciences, concerned with human interaction within a society, natural sciences, studying the natural phenomena. Followed by formal sciences that aim to explain the formal systems in particular computer science, information theory, logic, statistics, etc. and applied sciences, which implement existing scientific inventions for the purposes of practical use (business, law, medicine…) (Definitions.net, n.d.). An overall total of 80 respondents is broken down into equal groups according to each field, where total of 20 respondents (10 males, 10 females) is regarded as the sample group for the class. Based on the survey results the following order was obtained in Table 3. The table of criteria scores is enclosed in Appendix A4.

Field of Science				Gender	
Social	**Natural**	**Applied**	**Formal**	**Female**	**Male**
1.Empl/Tchng	1. Tchng	1. Empl	1. Empl	1. Tchng	1. Empl
2. Empl/Tchng	2. Empl	2. Tchng	2. Tchng	2. Empl	2. Tchng
3. Intrn	3. Award	3.Intrn	3. Intrn	3. Intrn	3. Award
4. PCP/Award	4. HiCi/ PCP	4. Award	4. PCP/ Award	4. Award	4. Intrn
5. PCP/Award	5. HiCi/ PCP	5. PCP	5. PCP/ Award	5. PCP	5. PCP
6. Alumni	6. Alumni	6. HiCi/ Alumni	6. HiCi	6. HiCi	6. HiCi
7. HiCi	7. Access	7. HiCi/ Alumni	7. Access	7.Alumni/Access	7. Alumni
8. Access	8. INTRN	8. Access	8. Alumni	7.Alumni/Access	8. Access

Table 3 - Criteria Order of Importance (own findings)

According to survey findings the top most important criteria when selecting a higher institution are employment reputation, teaching quality and international ties of a university. Nowadays, international connections and environment play an important role because potentially international level requirements anticipate higher quality of an institution. It may appear contradictive but research related criteria are of lower significance, mainly because the percentage of students interested in continuing the academic path is lower, than those who intend to apply the knowledge in practice after the completion. The order of the importance specifies the strict preference between alternatives, however, there is no assumption about the magnitude of such relationship. For the direct method application preferences must be transformed by application of mathematical weighing methods to a single number – the weight. Generally, the more important an attribute is, the higher the weight is.

6.4 Weight Calculation

Score determination stage is followed by the criteria weighting in which the preferences of the DM are implemented to quantify the degree of preference of one criterion over another. The aim of the thesis is focused on the empirical study to research the priorities and weight differences between attributes, depending on the gender of enrolee. As well as practical part of the study intends to explore peculiarities influencing the choice of higher institution depending on the field of interest. The data for the weight determination is collected through the series of short interviews, were respondents were requested to fill out a questioner, attached in Appendix A3. The criteria previously chosen by the DM are embedded in the survey in form of multiple-choice questions. Consequently, obtained results are extracted to further evaluate the overall weight of the criterion through application of technique encountered in Simple Multi Attribute Rating Technique Exploiting Ranks (SMARTER) method (Edwards and Barron, 1994). The technique is closely revised in the later section.

Weight assignment stage is the decisive step in the decision-making process because the final rank of universities is largely defined by the values of the criteria with the highest weights. However, the allocation and quantification of such is a challenging MCDM task, unless all criteria possess same significance level and one can apply the Equal Weights Method (EW). The idea behind the method relies on the concept of equality among factors and assigns the same weight to each criterion. Being the easiest methods, equal weighting also performs poorly since it does not capture the differences. The vast majority of weighting methods can be classified into subjective, objective and combined methods. The subjective methods tend to reflect only the judgment of the DM concerning a criterion. The pairwise comparisons, Deplhi method, AHP, SMART and others are representative of the group. On the other hand, methods that employ mathematical models and algorithms without accounting for the DM's judgment are known to be objective methods. The typical examples of such are Entropy method, TOPSIS and Variation coefficient (Zardari et al., 2015). An integrated approach combines assumption for both subjective and objective approach and evaluates the weight based on the DM's judgment as well as objective decision matrix. Within the framework of the study the direct assignment of weights seems unreasonable, as there is not a single DM but rather group of voters, who provide the order of criteria importance. The group of the decision makers are not likely to reach the consensus on the definition of exact weights. Even if such exists, availability and willingness to specify particular weights still remains a problem. Supporting arguments clearly suggest that application of the weighting techniques to ordered criteria tend to produce more reliable and accurate weights, avoiding the weight judgment biases. Barron and Barret (1996) argued that generated weights possess higher precision as compared to assigned weights.

Edwards and Barron employed SMART Exploiting Ranks (SMARTER) technique to increase the accuracy of weight calculation by sorting the criteria by the importance $c_1 \geq c_2 \geq \cdots \geq c_4$. Consequently, method calculates surrogate weights with the help of Rank Ordering Methods, namely Rank Order Centroid, proposed by Barron and Barrett (1996), or the equivalent measures mentioned below. The surrogate weights stand for an approximation of the "true" criteria weights, thus must fulfil the same requirements: $w > 0$ as well as $\sum_{j=1}^{m} w_j = 1$ (Barfod and Leleur, 2014). Rank Ordering method is perceived as one of the simplest but effective approaches to translate the list of ranks $(r_1, \dots, r_k)$ into a numerical scale $(w_1, \dots, w_m)$. The general idea behind the method infers that if no prior information about the criteria weights is available (except non-negativity and sum assumptions), then any vector satisfying the condition is acceptable (Edwards and Barron, 1994). Stillwell et al. (1981) proposed several suggestions, which make use of the order information, for weight determination techniques: rank sum (RS), reciprocal of the ranks (RR) and rank exponent (RE). The Rank Exponent method assumes a prior knowledge of the most important factor either evaluated by the DM(s) or through interactive procedures. Since the estimate of such parameter defining the weight is impossible due to the original nature of the ordering approach used, it will not be reviewed within the scope of the study. Instead this study is extended by Rank Order Centroid approximation.

- **Rank Sum Weight Approach**

The rank sum approach generates weight, which represents "*the individual ranks of criteria normalized by dividing by the sum of the ranks*" (Barfod and Leleur, 2014). The name of the technique demonstrates the normalization procedure during the weight determination. Mentioned method assumes linear relationship among criteria in which weights are determined according to formula:

$$w_j(RS) = \frac{K - r_j + 1}{\sum_{j=1}^{K} K - r_j + 1} \tag{15}$$

where r_j represents the rank of j-th attribute for all $j = 1,2, \dots, K$.

- **Rank Reciprocal Weight Approach**

The rank reciprocal calculates the weight of a criterion by using "*the reciprocal of the ranks which are normalized by dividing each term be the sum of the reciprocals*" (Barfod and Leleur, 2014). As the name may suggest the relationship among attributes is inverse. Similarly to RS technique, the DM has to first assign the ordinal scores to each item under review after which the weight is defined as stated:

$$w_j(RR) = \frac{1/_i}{\sum_{j=1}^{K} 1/r_j} \tag{16}$$

where r_j represents the rank of j-th attribute for all $j = 1,2, \dots, K$.

- **Rank Order Centroid Weight Approach**

Rank order centroid approach aims to minimize the total error corresponding to each weight by employing the centroid of all weight vector variations, since there is no evidence to assume the superiority of particular vector. Thus, the estimate produced maintains the order of criteria importance while the centroid guarantees the error-minimization. It is important to mention, that under the assumption of Rank Ordering method the vectors of weights are uniformly distributed, which does not imply the uniform distribution of specific weights. Consequently, individual weights differ and can be determined using following equation:

$$w_j(ROC) = {}^{1}/_{K} \sum_{j=1}^{K} {}^{1}/_{r_j} \tag{17}$$

where r_j represents the rank of j-th attribute for all $j = 1,2, \dots, m$.

Any of the mentioned methods for weight determination yield reasonable and satisfactory results. However, as proven by Barron and Barrett (1996) during the original research ROC weight outperformed its ranking counterparts. Authors simulated 100 random data matrixes as well as weight vectors (referred to as true vector) and approximated the weights by ROC, RR, RS and EW. In all 100 instances there was an evident superiority of ROC weights, namely ROC>RR>RS>EW.

In a similar manner, above-mentioned technique can be implemented for the purposes of the university rank determination. Given the preference order of the criteria defined in a previous section, it is possible to evaluate and quantify the magnitude of importance of each particular criterion. The final female weight vector is presented in Table 4.

Female	ROC	RS	RR
Tchng	0,340	0,222	0,368
Empl	0,215	0,194	0,184
Intrn	0,152	0,167	0,123
Award	0,111	0,139	0,092
PCP	0,079	0,111	0,074
HiCi	0,054	0,083	0,061
Alumni	0,033	0,056	0,053
Access	0,016	0,028	0,046

Table 4 - ROC, RS and RR weight vectors (own calculation)

The section only reviews one example for weight calculation based on the female gender in order not to overload the thesis but at the same time shed some light on calculation technique and on quantitates of results. The remaining weight vectors for field of study and male gender are computed in a similar fashion.

The three candidate metrics may be seen as reasonable estimates despite the fact that RS and RR weight vector rely on more heuristic procedure, while ROC weights have more reliable statistical premises. The choice of the satisfying vector reckons upon the DM views and vision of the "true" weights. In some cases, generated results might be to discriminative by placing a larger weight on the more important criteria, similarly to ROC weight, whereas some techniques produce "flatter" results as RS.

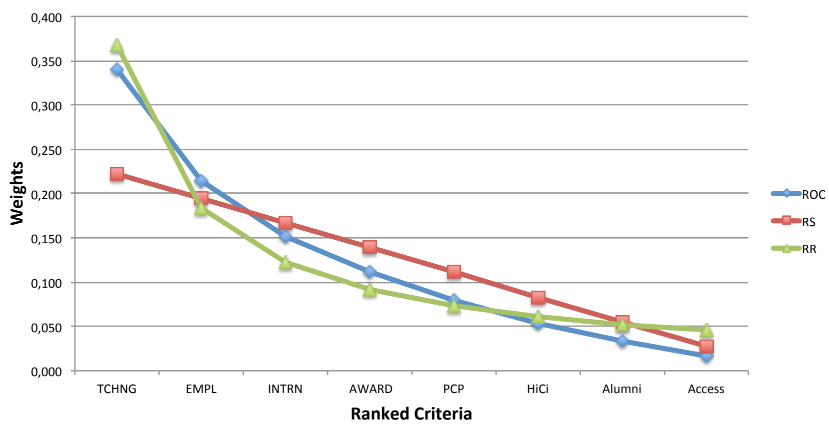

Figure 9 - Surrogate Weights (own calculation)

The comparison of weight vectors presented on Figure 9 displays substantial differences. In considered example ROC and RR weights demonstrate similarities mainly due to resembling nature of generating functions. RR emphasizes the highest and the lowest ranked criteria, at the expense of criteria in the middle, thus over/underestimating end-points in the criteria order. Indeed RR results depart notably form ROC as well as RS weights. Putting into comparison ROC and RS vectors with less dramatic differences, it may be observed that middle ranked criteria by RS technique are compensated by the lower weight in the highest ranked one, while ROC is less accommodating to criteria in the middle of the rank. It may be concluded that ROC as well as RR approaches predominate when searching for the winner, while RS reduces the influence of extreme points (Danielson and Ekenberg, 2017). Rank order centroid weights seem to capture the drastic differences among other alternative techniques. Moreover, ROC weights provide similar accuracy of the findings with smaller input and computational effort when compared to AHP approach (Olson and Dorai, 1992). For the purposes of considered analysis ROC weight vector is exploited on the grounds of supporting arguments mentioned before.

6.5 RStudio Software

RStudio is an "*open-source statistical tool initially introduced for statistical computing and analysis"*, nevertheless, applied in many other fields except statistics, for general purposes of data exploration (analytical and graphical) (RStudio.com, 2019). Despite the fact of being an open-source software, RStudio is a very powerful tool for data analysis with expanding number of features developed by employed staff together with volunteers, who represent the a bigger share. It is important to note that R is a software environment yet RStudio stands for an integrated development environment (IDE) to use R. To address an enormous number of problems all users develop so called packages, which consist of user–defined codes written in R programming language. The following resides at the repository system (CRAN) where all add-on packages with detailed function description are stored for a public use (Verzani, 2011). Users serving as volunteers are encouraged to share their knowledge about new analysis features or techniques via sharing the pre-set collection (package) of build-in functions for certain purpose.

Within the scope of MCDM analysis the widely encountered R packages are MCDM (TOPSIS, VIKOR, WASPAS, MetaRanking), topsis (TOPSIS method), OutrankingTools (ELECTRE), MCDS (AHP,ELECTRETRI…) and others (Gonz'alez-Arteaga and de Andr'es Calle, 2017). Suggested method TOPSIS, VIKOR and WASPAS in context of research are incorporated in MCDM package created by Blanca A. Ceballos Martin (2016). The package includes codes for implementation of specified MCDM methods as well as RIM and Multi-MOORA techniques. The result output depends on the method of interest: scores of alternatives (VIKOR) and the defined rank (TOPSIS, WASPAS).

7 Practical Application on Selected Universities

The purpose of the section is to explicitly review the university-ranking task within the framework of proposed MCDM methods. The conclusions drawn in the end of the section rely solemnly on the results of empirical study followed by the final rank of top 5 universities based on the academic field of study as well as gender. The analyst provides the comments to define the differences and reasons for such to happen. Firstly, the methods are executed in the statistical software after which the program output in structured and presented to the DM for evaluation. Secondly, results are manipulated with the help of Data Envelopment Analysis to identify the criteria, which reduce the overall score for lower rated universities and to suggest how to enhance the existing condition. Moreover, the analyst expresses the critiques and proposals for future modifications of university rankings.

7.1 Implementation of Proposed Methods

Due to availability of various software programs implementation of the methods becomes less demanding, thus the analyst can focus more on the development of improved techniques or result interpretation. The vital part of any analysis is embedded in the ability to utilize existing tools to avoid calculation mistakes (usually encountered during manual or semi-automated computations) and ensure higher accuracy the produced findings. The outputs attached in this section concern the female ranking example mentioned in earlier section. The full list of results is enclosed in Appendix A5.

TOPSIS

User-defined function `TOPSISLinear` introduced in `MCDM` package in RStudio executes the Technique for Order of Preference by Similarity to Ideal Solution approach. The normalization procedure in this case is accomplished through linear transformation of maximum (Ceballos Martin, 2016). The function `TOPSISLinear(d,wf,cb)` requires:

- `d=decision`, which stands for the decision matrix $X_{n\times m}$.
- `wf=weights`, symbolizing the vector of weights.
- `cb`, representing the vector of benefit `cb(i)="max"` or cost criteria `cb(i)="min"`.

The output of the method found on Figure 10 delivers the c_i (R values within the package context) values along with the ranking of the first ten alternatives out of 44 available. (Ceballos Martin, 2016).

```
> TOPSISLinear(d,wf,cb) # results for females
   Alternatives          R Ranking
1              1 0.4772774      16
2              2 0.4513267      22
3              3 0.6264247       6
4              4 0.4987669      15
5              5 0.4651859      20
6              6 0.5358534      11
7              7 0.4715951      18
8              8 0.3765404      32
9              9 0.5072994      14
10            10 0.5665344       8
```

Figure 10 - TOPSIS Output (own findings)

VIKOR

The method is based on the distance relation to the ideal solution, therefore, is also widely employed for the ranking purposes. `VIKOR` function attached to the package `MCDM` calls for the same type of input `VIKOR(d,wf,cb,v)`, although there is a new introduced parameter:

- `v`, which stands for the weighting factor ranged from 0 to 1 used for Q scores calculation[5].

For the purpose of the study the v coefficient is set to 0.5 to balance the maximum group utility (S) coupled with individual utility (R). Generally, if $v = 1$, the analyst favors maximization of group utility, on the other hand $v = 0$ implies minimization of individual utility (Chatterjee and Chakraborty, 2016). VIKOR method output is more explicit compared to other methods, it offers S, R and Q scores. However, the final ranking is applied only to the Q values of alternatives. Figure 11 displays the common type of RStudio output when implementing VIKOR function. Only ten first alternatives are presented on the figure as an example.

[5] The detailed explanation of the factor is provided in theoretical section.

```
> VIKOR(d,wf,cb,v)
   Alternatives         S          R          Q Ranking
1             1 0.5366012 0.18724638 0.43541714      18
2             2 0.5546091 0.17246377 0.42266121      15
3             3 0.3698153 0.11100000 0.16029617       6
4             4 0.5075316 0.21763285 0.46804363      22
5             5 0.5440682 0.27101449 0.59598204      31
6             6 0.4377611 0.17574879 0.33452622       8
7             7 0.5182912 0.23077295 0.50095978      26
8             8 0.6158490 0.24227053 0.60081692      32
9             9 0.5246507 0.16178744 0.37881370      12
10           10 0.5038886 0.15200000 0.34401947      10
```

Figure 11 - VIKOR Output (own findings)

WASPAS

Weighted Aggregated Sum Product Assessment (WASPAS) method defined in the package `MCDM` combines the Product and Weighted Sum Approach. Thereby, one function enables calculation of 3 methods either separately, with scores for particular method, or in combination, resulting in WASPAS values. The function `WASPAS(d, wf, cb, lambda)` relies on initially same inputs, however does require:

- `lambda` represented by the weight factor [0,1] for the $U(a_i)$ index calculation.

During the analysis $\lambda = 0.5$ to achieve the highest accuracy of estimation, otherwise $\lambda = 0$ method is transformed to WPA or if $\lambda = 1$, it reverses to WSA (Chakraborty et al., 2015). The function returns the scores of WPA, WSA and the U index used for the ranking of alternatives.

```
> WASPAS(d, wf, cb, lambda) #Female results
   Alternatives       WSM       WPM         Q Ranking
1             1 0.5661581 0.0000000 0.2830790      31
2             2 0.5636016 0.5265695 0.5450855      16
3             3 0.6879108 0.0000000 0.3439554      25
4             4 0.5998501 0.5497497 0.5747999      12
5             5 0.5815217 0.0000000 0.2907608      29
6             6 0.6521849 0.6271885 0.6396867       5
7             7 0.5945254 0.5655565 0.5800409       9
8             8 0.5084455 0.4521182 0.4802819      20
9             9 0.5920412 0.5772143 0.5846277       8
10           10 0.6206813 0.5861909 0.6034361       6
```

Figure 12 - WASPAS Output (own findings)

In the similar manner methods were applied with the help of RStudio to other classification groups for all 44 alternatives. The aggregation of mentioned TOPSIS, VIKOR and WASPAS approaches guarantee an integrative solution to produce accurate final rankings due to increase of precision and scrupulosity. Integrated approach eliminates and solves disadvantages of methods when each is exercised alone. Among these methods, all suffer from deficient weight elicitation methodology, hence Rank Ordering Method potentially handled existing problem. However, TOPSIS method does not justify the group utility as well as individual regret from selecting an option, but instead purely measures the distance. Incorporation of VIKOR results makes a judgment about possible losses and gains from selecting an alternative. Therefore, it accounts for the disadvantage of TOPSIS and enhances the accuracy of the final ranking. Similarly, robust WASPAS method guarantees maximization of total utility function for each university effectively raising the probability of choosing the best university.

7.2 Evaluation of Results

According to the original formulation of the problem, the final EU university ranking was constructed depending on the gender, the top 5 universities according to the female and male weight vector, and academic field, resulting in the ranking of institution with respect to social, natural, formal and applied field of sciences. The joint table of TOPSIS, VIKOR, WASPAS results are provided for each category. To achieve the final rank the analyst employed the Average ranks ranking. Each alternative a_i received an average rank $\overline{r_j}$ of defined by formula:

$$\overline{r_j} = \frac{\sum_i^n r_j^i}{n} \tag{18}$$

where r_j stands for the rank of an alternative according to i^{th} method (Brazdil and Soares, n.d.). After the average rank is calculated for all 44 alternatives, the averages are ranked to achieve the final university rank in each category.

Female

The primer criteria for females when choosing a higher institution concerns teaching quality, employment rates and international ties of university. Consequently, the values of these factors to the large extend define the choice of university. Karolinska Institute in Sweden, being the best national institution and occupying 44th position in the original ranking ARWU, is placed first (Shanghairanking.com, 2019). Interestingly, TOPSIS method ranked Technical

University in Munich as the compromise alternative (PIS) not only because original scores are higher along the teaching and employment criteria, moreover, it is furthest away from the worst alternative (NIS). VIKOR and WASPAS methods neglect the distance from the worst solution but Karolinska Institute indeed achieves maximum value of objective function according to these two approaches.

Institution	**Country**	**TOPSIS**	**VIKOR**	**WASPAS**	$\overline{r_j}$	**Overall rank**
Karolinska Institute	SE	2	1	1	1,33	1
Technical University Munich	DE	1	3	2	2	2
University of Munich	DE	3	2	3	2,67	3
Heidelberg University	DE	4	4	4	4	4
Sorbonne University	FR	8	10	6	8	5-6
University of Copenhagen	DK	11	8	5	8	5-6

Table 5 - The Final Rank: Female (own findings)

Male

The importance criteria vector for males regarded employment rates, teaching quality and academic staff awards with the most interest for the higher institute selection. Technical University in Munich, ranked the second best university on the national level and 48th on the global rank, is the most attractive higher institution from male viewpoint (Shanghairanking.com, 2019). Given alternative has better scores according to all criteria except the awards where Karolinska Istitute has more privilege. This fact gives precedence to the Swedish institution according to VIKOR approach results.

Institution	**Country**	**TOPSIS**	**VIKOR**	**WASPAS**	$\overline{r_j}$	**Overall rank**
Technical University Munich	DE	1	2	1	1,33	1
Karolinska Institute	SE	3	1	2	2,00	2
University of Munich	DE	2	4	3	3,00	3
Heidelberg University	DE	4	3	4	3,67	4
Sorbonne University	FR	7	7	5	6,33	5

Table 6 - The Final Rank: Male (own findings)

Social Sciences

Social sciences related to studies of relationships among members of society give preference to practical skills (employment reputation) and teaching quality mainly because the branch itself stems from day-to-day human activities rather than knowledge "explored" in laboratories. The leading institutions are shared among same candidates. It is important to mention that Heidelberg University, which is the best university on the national level, has relatively low performance according to chosen methods. It could be explained by high employment reputation of technical universities due to rapid technological progress and increasing profession need. Such formulation suggests that universities with a higher share of technical programs have market-driven and "artificial" gain in employment reputation.

Institution	Country	TOPSIS	VIKOR	WASPAS	$\overline{r_j}$	Overall rank
Technical University Munich	DE	1	2	1	1,33	1
Karolinska Institute	SE	2	1	2	1,67	2
University of Munich	DE	3	3	3	3,00	3
Heidelberg University	DE	4	4	4	4,00	4
Sorbonne University	FR	8	12	6	8,67	5

Table 7 - The Final Rank: Social Sciences (own findings)

Natural Sciences

Applicants interested in professions related to natural science tend to consider teaching quality, employment reputation and academic staff awards more. Heidelberg University has remarkable achievements in natural sciences by being in top 50 global universities by subject fields, namely physics and chemistry according to ARWU (Shanghairanking.com, 2019). Under these circumstances Heidelberg University moved up in the rank, thus forming the trio of top 3 best German universities for enrolee interested in studying natural sciences.

Institution	Country	TOPSIS	VIKOR	WASPAS	$\overline{r_j}$	Overall rank
Technical University Munich	DE	1	2	1	1,33	1
Heidelberg University	DE	2	1	3	2,00	2
University of Munich	DE	3	3	2	2,67	3
Karolinska Institute	SE	4	4	4	4	4
Sorbonne University	FR	5	5	5	5,00	5

Table 8 - The Final Rank: Natural Sciences (own findings)

Applied Sciences

Applied sciences form the field where known theory is adapted in practice for instance business (economic theory is utilized for capital generation). The top ranked universities are similar to previous results, however, there is a new institution entering the rank - Trinity College Dublin. Trinity College is the best national higher education institution in Ireland offering courses in business, law, health sciences, engineering and mathematics.

Institution	Country	TOPSIS	VIKOR	WASPAS	$\overline{r_j}$	Overall rank
Technical University Munich	DE	1	1	1	1,00	1
Karolinska Institute	SE	3	2	2	2,33	2
University of Munich	DE	2	3	3	2,67	3
Heidelberg University	DE	5	4	4	4,33	4
Trinity College Dublin	IE	11	8	7	8,67	5

Table 9 - The Final Rank: Applied Sciences (own findings)

Formal Sciences

Formal sciences form a class of sciences (computer science, statistics, mathematics, decision theory and etc.) that aim to create tools to characterize and depict other branches of science. The final rank of universities according to the importance criteria vector yield alike the applied science field results in the view of the fact that preference orders of criteria are equivalent, thus method performance is almost identical in both cases.

Institution	Country	TOPSIS	VIKOR	WASPAS	$\overline{r_j}$	Overall rank
Technical University Munich	DE	1	1	1	1,00	1
Karolinska Institute	SE	3	2	2	2,33	2
University of Munich	DE	2	3	3	2,67	3
Heidelberg University	DE	5	4	4	4,33	4
Trinity College Dublin	IE	11	11	8	9,00	5

Table 10 - The Final Rank: Formal Sciences (own findings)

7.3 Efficiency analysis of universities

As a conclusion of a MCDM analysis, it was proved that given the set of alternatives one can easily form a rank, satisfying his/her pre-defined order of criteria importance along with the relative weight of each such criterion. It is essential to note that such analysis does not suggest any improvements but rather orders the available alternatives. The decision may be finalized upon results, nevertheless, the analysis may be extended by supplementing the suggestion how results can be manipulated to improve current ranking position. DEA analysis can be employed with the help of RStudio software to identify the efficient and inefficient alternatives.

According to DEA output, Technical University Munich, Karolinska Institute, University of Munich and other mentioned earlier universities appear to be efficient, thus, have higher chances to appear on the top of the rank. Efficiency testing pinpointed results of TOPSIS, VIKOR and WASPAS by indicating that universities at the bottom of the ranks are inefficient alternatives. Based on RStudio efficiency testing output (attached in Appendix A6), University of Frankfurt with the lowest efficiency score equal to 0.608 has been elected for further analysis in order to quantify the increase required to reach the efficiency frontier. University of Frankfurt is compared to the closest efficiency reference units, which are Catholic University of Leuven (U_1) and Delft University of Technology (U_2). The combination of such units forms a composite unit (CU), where shadow prices (λ) serve as weights. Shadow prices are linked to the constraint, which limit the efficiency score to 1. Thus to measure the distance from an alternative to the efficiency convex hull, one must solve the equation:

$$CU = 0.066 * U_1 + 0.934 * U_2 \quad (19)$$

Total sum of CU for all criteria lead to the formation of the minimal requirement to achieve efficiency (Bharskarjit, 2018). Table 11 depicts the calculation for University of Frankfurt in order to arrive at the Pareto-frontier. Required amount quantifies the solution of equation (19) according to particular criteria, while current scores represent existing ranking scores of an institution. The difference displays the discrepancy between the alternative and composite unit, situated on the efficiency border:

- $= 0$, appears on the frontier,
- < 0, change required to reach the frontier,
- > 0, excess from the minimal requirement.

Criteria	Required amount	Current score	Difference
Access	2,93	3,00	0,07
Alumni	10,65	30,00	19,35
Award	0,00	0,00	0,00
Empl	9,87	6,00	-3,87
HiCi	14,80	16,60	1,80
Intrn	3,00	1,00	-2,00
PCP	25,22	25,70	0,48
Tchng	53,45	32,50	-20,95

Table 11 - Calculation of DEA Improvements for University of Frankfurt (own findings)

To reach the efficiency frontier University of Frankfurt has to increase its performance indicators according to three criteria, namely employment reputation, international ties and teaching quality. These criteria emerge as the most important factors during the ranking application regardless of classification group. Thus, employment reputation criterion needs to reach the total score of 9.87, increasing its current result by 3.87 units. International relations scores on the level of 3 units are considered satisfactory, therefore, current result must be raised by 2 units in order to reach the frontier. Moreover, teaching quality momentarily falls short of 21 units to please efficiency requirements. Needless to say that after suggested improvements an alternative would become efficient. DEA is widely applied powerful technique to assist the analyst in concluding suggestions for improvements.

7.4 Research Summary

Thought the analysis the order of the best universities changed among Sorbonne, Heidelberg, Munich Technical Universities and Karolinska Insitute manly due to the fact that they have the highest scores according to the most important criteria such as employer reputation, teaching quality and international connections paired with awards received by members of academic staff. Referring to the statement earlier, ROC weights tend to emphasize on the most influential criteria more, as a result, Munich Technical University is the best choice in five ranks because of the highest score in employment criteria. Karolinska Institute has average scores in all the criteria, therefore, occupies the second place in most of the ranks. In comparison to Swedish institution, the University of Munich generally has better values according to most important factors, however, very poor results in the other four criteria. The trade-off among high scores in important criteria and very low result in less significant factors does not compensate in the overall score. Remaining universities essentially perform weaker, thus receive lower rank. Interestingly, ranking results may vary depending on methods and weight vector employed during analysis so maybe given a different weight calculation technique, the top 5 universities could have been different.

The universities, which appear higher in original ARWU ranking, yield high scores during the performed study as well. Considering the fact that for the purpose of research ARWU ranking was modified with new criteria taken from QS "World University Ranking" and Times Higher Education World University Ranking, the leading universities remain unchanged. Top five universities reviewed in the previous sections are also the leading European Universities in the above-specified rankings, suggesting superior quality of institution regardless of criteria.

Institution		Alumni	Award	HiCi	PCP	Empl	Intrn	Tchng	Access
Sorbonne University	FR	33,3	27,1	25,3	26,2	8	1	49,3	2
Heidelberg University	DE	19,7	25	28,7	35,6	8	2	63,6	3
Technical University Munich	DE	36,6	21,3	25,3	35,5	10	2	60,3	3
University of Munich	DE	28,3	18,9	23,5	34,9	9	2	65,4	3
Karolinska Institute	SE	25,4	26,3	25,3	48,6	8	3	57	2

Table 12 - Data on Top 5 Best Universities

It is important to note that TOPSIS, VIKOR, and WASPAS methods identify practically same best universities, which potentially suggests that the weight calculation technique is more important for the rank establishment. Referring to section 6.4 some techniques over or underestimate criteria importance by employing hash penalization on less influential criteria. For the purposes of ranking analysis, one should utilize methods that are capable of establishing an order of criteria. More importantly, the analyst should pay close attention to the data processing (weight calculation) and aim to improve the technique to represent the importance of factors in such a way so that no criterion is neglected or under/overestimated.

7.5 Topic Critiques

Despite advantages of university ranking visible to many, there are opponents of such approach. In most cases, there are evident downsides to the ranking schemes irrespective of how qualitative the analysis is.

1. An enrolee can find a higher institution attractive even though it has low rank on the global scale. Small universities do not meet the participation requirements for the rankings, however, can still demonstrate outstanding academic achievements.
2. Even if university receives a rank, the significance of the differences between neighbouring universities is not always obvious.
3. Measurability of some important criteria is questionable. The rankings do not capture criteria that appear influential to students such as campus life, job/internship offers during studies, extra curriculum activities and others.

Mentioned arguments affect the impression of objectiveness and reliability of the rankings, however, rank assessment along with personal evaluation aid students to make informed and sound decisions.

8 Conclusion

Within the study framework, university rankings are incorporated in the context of Multi Criteria Decision Making problem. In addition, the research addresses an overview of existing methods, providing the general picture of subject systematics. Coincidently, the prime aim of the study is to shed some light on the application of existing MCDM theories in the ranking environment. An empirical example intended to construct the ranking of top 44 universities in the European Union predicated upon the vector of criteria importance given gender and field of study. The structuring phase of the problem states the alternatives, criteria coupled with weights, explaining the preference relationship. However, one of the most important steps in decision-making flow is the actual data processing/analysis phase. For purposes of such analysis, it is crucial to identify factors that are adequate, measurable and relevant in terms of problem environment. Unfortunately, the criteria for the rank definition differ depending on the ranking methodology. Since no absolute best and definitive set of criteria for university rankings exist, the establishment of factors employed during the study is based solemnly on the DM's point of view. After such have been identified, the weight calculation techniques seek to transparently reflect the preferences of the DM(s) in order to receive a reliable outcome. Due to a wide range of weight determination techniques, the calculation is performed by incorporation of various approaches to compare their performance. The ROC technique seems to produce accurate weights from a more reliable statistical perspective. TOPSIS, VIKOR, and WASPAS methods are exercised within the scope of the thesis so that the evaluation is done in a transparent and gradual manner. The final university ranking is derived by means of Average Rank Ranking realized by a combination of method findings. The result of the analysis is delivered in the form of the rank of top 5 EU universities taking into account the gender or the field of study.

The higher reliability of the research can be achieved by an increased sample representing the gender and field of study. However, it may be argued since peculiarities, which distinguish each group, are very pronounce and evident and with the increasing sample may only become more distinct. Yet, this matter can be addressed from a different standpoint. Aggregation or transformation of existing weight techniques, alternatively application of new ones allow changing weight vector. While the preferences are preserved, the weights may influence the rankings results, consequently decreasing the reliability of existing findings. These topics might appear interesting for the purpose of further research.

Prior synthesis of the result led to suggestions for the modification. The university ranking can be practically extended to propose the solution for lower ranked universities. Data Envelopment Analysis demonstrated lack of efficiency, extrapolating further this means that some alternatives are dominated, which potentially stems as a cause of the low general performance. Hence, the emerging possibilities for improvement were put into practice and evaluated through DEA model to increase the potential of lagging alternatives.

All in all, the purpose of the study is fulfilled and closely revised during the given research. The final university ranking can be constructed by means of mentioned methods, although, does not imply that other formulations should be neglected. The suggestion for future work relates to an increase in calculation accuracy. The following may be achieved by selection or construction of new weighting techniques, which provide a less extreme difference between the most and least important criteria. Moreover, the employment of different method combinations might be explored to derive the optimal approach for ranking problems.

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