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Monetary Policy Analysis and Forward-Looking Taylor Rule

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Statement of Authorship

I hereby declare that I am the sole author of this bachelor's thesis and that I have not used any sources other than those stated above. I further declare that I have not submitted this thesis at any other institution in order to obtain a degree.

May 6, 2019 in Prague ....

Signature

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Abstrakt

Cílem této práce je odhadnout vpřed hledící Taylorovo pravidlo pro Evropskou centrální banku (ECB) za účelem analýzy měnové politiky od doby založení eurozóny v roce 1999 do doby rozšířeného programu nákupu aktiv, jenž byl zahájen v roce 2015. Odhad je proveden metodou zobecněných momentů. Je formulován základní model a je testována jeho robustnost vůči změnám horizontů očekávané inflace a mezery produktu a také robustnost vůči změnám množiny instrumentů. Nakonec jsou do základního modelu přidány další vysvětlující proměnné. Na základě výsledků základního modelu se zdá, že Evropská centrální banka přizpůsobuje nominální úrokovou sazbu jak v závisloti na očekávané inflaci, tak v závisloti na odchylkách produktu od svého potenciálu. Odhad základního modelu, který předpokládá setrvačnost úrokových sazeb a který se zároveň zdá být robustnější než základní druhý model abstrahující od předpokladu setrvačnosti úrokových sazeb, naznačuje, že Evropská centrální banka reaguje na vyšší očekávanou inflaci dostatečným zvýšením nominální úrokové sazby tak, aby byla také zvýšena reálná úroková sazba. Dále byl odhadnut inflační cíl, oficiálně definovaný jako “pod ale blízko 2 %“. Za předpokladu setrvačnosti úrokových sazeb odhad inflačního cíle je 1.38 %, je-li inflace měřena indexem HICP a v případě použití proxy jádrového HICP k měření inflace je pak odhadnutý inflační cíl 1.85 %. Odchylka skutečného vývoje úrokových sazeb od vývoje určovaného odhadnutým Taylorovým pravidlem naznačuje, že zahájení rozšířeného programu nákupu aktiv v roce 2015 bylo spíše diskreční reakcí centrální banky.

Klíčová slova

Evropská centrální banka, vpřed hledící Taylorovo pravidlo, měnová politika, metoda zobecněných momentů

Abstract

The objective of this thesis is to report the estimation of the forward-looking Taylor rule for the European central bank (ECB) from the time of its establishment in 1999 to the time of Expanded Asset Purchase Programme, launched at the beginning of 2015, and use it for monetary policy analysis. The estimation is performed by the generalized method of moments. The baseline model is formulated and the robustness to changes in horizons of the inflation and output gap forecasts and changes of instrumental set is performed. As the last step additional regressors are added to the baseline model. According to the estimation of the baseline, the ECB changes nominal interest rate in reaction to both the expected inflation and the output gap deviations. The estimation of the baseline model with interest rate inertia assumption which seems to be more robust than the second one without interest rate inertia assumption, suggests that the ECB reacts to changes in the expected inflation by increasing the nominal interest rate sufficiently enough for the real rate to be increased as well. With interest rate inertia assumption the estimated inflation target, officially defined as "below but close to 2 %", is 1.38 % for inflation measured by HICP and 1.85 % when a proxy of core HICP is used to measure the inflation. The deviation of the actual path of the interest rate from the interest rate path resulting from the Taylor rule without the smoothing term suggests, that the launch of the Expanded Asset Purchase Programme in 2015 was rather a discretionary reaction of the central bank.

Keywords

European central bank, forward-looking Taylor rule, monetary policy, generalized method of moments

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List of abbreviations

CBPP Covered bond purchase programme
CPI Consumer price index
DSGE Dynamic stochastic general equilibrium model
ECB European central bank
FTOs Fine-tuning operations
GMM Generalized method of moments
GDP Gross domestic product
HICP Harmonised Index of consumer prices
LTROs Long-term refinancing operations
MROs Main refinancing operations
NAIRU Nonaccelerating Inflation Rate of Unemployment
OMT Outright Monetary Transactions
QE Quantitative easing
TFEU Treaty on the Functioning of the European Union

Introduction

From the establishment of the euro area in 1999 to 2009 the ECB conducted its monetary policy by decreasing and increasing nominal interest rates. The first set of unconventional measures — Enhanced Credit Support — was introduced in 2009. The conduct of unconventional monetary policy culminated in 2015 when the Expanded Asset Purchase Program was launched.

The aim of this thesis is to estimate forward-looking Taylor rule to examine the ECB monetary policy from the time of the euro area establishment to the era of Expanded Asset Purchase Program. The estimation is performed by the generalized method of moments.

In the first chapter inflation targeting, at these days the most frequently adopted monetary policy regime, is described. Firstly, a brief history of inflation targeting is introduced and then basic pillars of the inflation targeting are mentioned. Following Svensson [32, 35, 33] a quadratic loss function is used to describe the inflation targeting.

The second chapter is focused on the ECB monetary policy, its objectives and strategy. After that, the monetary policy instruments are described. Finally, a history of the monetary policy conduct, with an accent on the unconventional measures, is explained.

In the third chapter an optimal interest rate rule, a generalization of the Taylor rule, is first derived in a simple three-equation (IS-PC-MR) macroeconomic model. Both backward-looking and forward-looking Taylor rule is then described with an emphasis on the Taylor principle and a potential implication, unless the Taylor principle holds.

The forth chapter focuses on the generalized method of moments which has become a popular method how to solve models with an assumption of rational expectations like the New Keynesian Phillips curve and the forward-looking Taylor rule or models of intertemporal optimization. The forward-looking Taylor rule is incorporated into the generalized method of moments framework at the end of the chapter.

In the chapter five, the data used for the estimation are described and other time series like the output gap are computed. In the last chapter, the estimation is performed. The baseline model is first formulated; the formulation is inspired by Clarida, Gali and Gertler [6, 7]. The estimated results are tested in terms of their robustness to changes in horizons of the inflation and output gap forecasts and changes of instrumental set. The results are interpreted and finally additional regressors are assumed.

1. Inflation targeting

Similar ideas like the inflation targeting are not from a historical point of view entirely new monetary policy concept. Knut Wicksell recommended in 1898 price level targeting. In Sweden in 1931 a monetary program of price stabilization was declared in order to mitigate concerns about rising prices. The declaration that gradually developed in a comprehensive monetary program was based on the idea that an institutional commitment to price stability could anchor inflation expectations. Important pillar of this monetary policy was it´s high level of transparency, which it has in common with modern inflation targeting. [3]

Nevertheless, the first adoption of inflation targeting as such was announced by the Reserve Bank of New Zealand in 1989 as a result of search for nominal anchor for monetary policy. Since then many central banks of both emerging and advanced countries have been forced to find adequate alternative for problematic exchange rate targeting like in the case of the Great Britain, the Czech Republic and Sweden or money supply growth targeting like in the case of the Great Britain, the Czech Republic and Spain. [24] As Mishkin and Posen states [23], one of the reasons of this monetary policy shift was weakening relationship between monetary aggregates growth and nominal income, thus monetary aggregate turned out to be insufficient as the only intermediate variable in the transmission mechanism.

At these days the central banks with inflation targeting regime are among others: Czech National Bank, Bank of Canada, European Central Bank, Bank of England and Reserve Bank of Australia. [24]

1.1 Pillars of inflation targeting

The inflation targeting is based, like other monetary policy regimes, on the long term neutrality of money. So higher Gross Domestic Product (GDP) growth and lower unemployment can be achieved in the short run throughout expansionary monetary policy but in the long term leads only to higher inflation. [23]

The inflation targeting can be characterised by three main pillars. The first pillar is an explicit quantitative inflation target that serves as a nominal anchor. Central bank aims to keep the rate of inflation within a specified target or target range over a certain time frame. The target is usually set in case of advanced economies at the level of 2 or 3 % For example the consumer price index CPI is often used to measure the quantified target. [34]

The second one is a framework for policy decisions. The central bank usually uses a short interest rate to adjust monetary conditions and the adjustment is performed in a forward-looking manner, because monetary policy actions impact the economy throughout the transmission mechanism with a lag.

When the central bank changes the short term interest rate, it affects an aggregate demand with a certain lag. The aggregate demand has then an impact on inflation with another lag. Above aggregate demand channel, there are additional channels of transmission mechanism like expectations channel that in turn affect inflation with additional lag throughout wages and price-setting mechanism. In the open economy the transmission mechanism contains additional channel of exchange rate where the lags are considered to be shorter than those in the previous channels. [33]

The central bank adjusts monetary conditions based on an internal conditional forecast for inflation at the "monetary policy horizon" that is often one or two years ahead. The inflation forecast serves as an intermediate target variable.[34] Dynamic stochastic general equilibrium models (DSGE) are often used for a computation of the forecasts.

The third pillar and at the same time one of the most important features of inflation targeting is an emphasis on the central bank transparency and credibility, which means, that changes in the monetary policy are not a surprise for the public and the actions of policy makers are predictable. [18] Transparent central banks regularly release macroeconomic projections, notes and reports where undergone actions of the monetary policy makers are discussed and explained to the public. High transparency leads to higher credibility that is necessary for the inflation expectations to be successfully anchored. [33]

1.2 Inflation targeting in the context of targeting monetary policy rule

As already stated, the main goal of the inflation targeting is to stabilize inflation around the inflation target. If the inflation stabilization is the only goal, the inflation targeting is called strict. However when the real economy, represented by the output gap for example is also taken into account, the inflation targeting is flexible. As emphasised by Svensson [35], in practice the inflation targeting is the flexible one.

Inflation targeting can be exactly described by an intertemporal quadratic loss function, consisting of the expected sum of discounted current and future losses:

$$\mathcal{L}_t = (1-\delta)E_t \sum_{\tau=1}^{\infty} \delta^{\tau} L_{t+\tau}, \tag{1.1}$$

where δ means discount factor, whose value is between zero and one, E_t implies rational expectation conditional upon information of the central bank in the time t and L_t is quadratic social loss function defined as weighted sum of squared inflation and output gap:

$$L_t = \frac{1}{2}[(\pi_t - \pi^*)^2 + \lambda(y_t - y^*)^2], \tag{1.2}$$

where $\pi_t - \pi^*$ is a deviation of inflation from a target and $y_t - y^*$ is a deviation of output from the potential value - output gap[1] in the time t and λ is the weight on output gap stabilization relative to inflation stabilization, thus λ describes how much the central bank is output gap averse.

The inflation and the output gap are the target variables, whose appropriate values are π^* and zero, respectively. Zero output gap corresponds to the potential output.

For the flexible inflation targeting it holds that $\lambda > 0$, so both inflation ant output gap enter the loss function. For the strict inflation targeting λ equals zero. [36, 35]

1.3 Exemption from the inflation target

Consider now that the economy is struck by a negative supply shock. Under these circumstances higher inflation can occur and at the same time economic activity can slow down due to the rising costs.

When the central bank with flexible inflation targeting needs to cope with the supply shock, instead of sharp rise of interest rates deepening output contraction and volatility, the central bank might make an exception from achieving the inflation target. And when the central bank is transparent and credible enough, which means that the inflation expectations are low and well anchored, the central bank can afford slower inflation convergence to the inflation target as the general public will trust the central bank communique that higher inflationary pressures are just temporary and do not mirror true fundamentals of the domestic economy. Besides exemption from the inflation target, another possibility is to target core inflation adjusted for the variables affected by the negative supply shock the most such as energy. [24]

[1] For illustrative reasons the output gap will be somewhere noted as x_t

2. Eurosystem monetary policy

The Eurosystem consists of the ECB and 19 national central banks of the member states currently involved in the euro area. Conduct of the monetary policy follows the principle of decentralized implementation, that is to say the ECB is in the position of a coordinator and the national central banks execute the transactions. [8]

2.1 Eurosystem objectives

The primal goal of the Eurosystem monetary policy, according to Treaty on the Functioning of the European Union (TFEU), is to maintain price stability that helps to limit stress and uncertainty about the future. It in turn makes it easier to the individuals to distinguish changes in relative prices from changes in general prices which leads to more efficient allocation of resources.

Furthermore, besides the primal goal and without prejudice to it, the Eurosystem, as written in TFEU, is to support the general economic policies in the Union with a view to contributing to the achievement of the objectives including full employment and balanced economic growth.

However, the Eurosystem monetary policy framework is based on the principle of superior importance of maintaining the price stability. Since in the long run the central bank can affect only prices, the price stability appears to be the best contribution of the central bank to the economic welfare and long term economic growth. [40]

It would suggest that λ in equation 1.2 would be zero and thus it would correspond to the strict inflation targeting.

2.2 Eurosystem monetary policy strategy

The Eurosystem monetary policy strategy can be defined by a quantitative definition of price stability and a two-pillar analysis of the risks.

The inflation target is not precisely quantified in the treaties like TFEU or Maastricht treaty. To clarify this more precisely, the Governing Council of the ECB defined the target as a year-to-year increase in the Harmonised Index of Consumer Prices (HICP) below 2 %. In 2013 The Governing council clarified that it targets inflation below but close to 2 % over medium term. [8, 40]

The first pillar is a surprisingly considerable accent on a monetary aggregate, notably M3. The reference growth of M3 aggregate has been estimated at 4,5 % based on the growth of real GDP, inflation and money velocity. [18]

Nevertheless, the Eurosystem refused the monetary aggregate targeting due to an unstable relationship between prices and money growth. Instead, as Svensson [34] points out, the monetary aggregate growth was intended to be used as an indicator to the risk and stability, in a sense that a deviation from reference value indicates a certain risk to the price stability. Within the second pillar, additional variables that are expected to become a potential risk to price stability are assessed. The ECB regularly assesses output, fiscal policy, labour market conditions, price of financial assets, financial yields, balance of payments etc. Within the second pillar also the macroeconomic analyses are developed. The results have been monthly released in Economic Bulletin since 2015 when it replaced The Monthly Bulletin. [18, 40]

2.3 Eurosystem monetary instruments

The ECB uses a set of instruments and procedures to manage interest rates and to control liquidity in the interbank market. The set consists of open market operations, standing facilities and minimum reserves.

2.3.1 Open market operations

Dominant tool used to manage liquidity and short term interest rates are the open market operations, which are further divided into Main refinancing operations, Longer-term refinancing operations, Fine-tuning operations and Structural operations. Within the open market operations the financial assets are temporarily bought or sold to increase or decrease liquidity in the interbank market. They are conducted individually by the national central banks. [40]

The most important open market operations are the Main refinancing operations (MROs). The interest rate on MROs is set by Governing Council and it is called the main refinancing rate. MROs are reverse, liquidity-providing with regularity of one week and the same maturity. Performing these reverse transactions, the central bank buys assets from a commercial bank with a pledge of consecutive repurchase. [29]

Long-term refinancing operations (LTROs) are regular, open market operations with purpose to provide a long term liquidity, so that all the liquidity in the interbank market would not roll over every week. LTROs are executed monthly with maturity commonly three months.

In need of ad hoc open market operations to increase or decrease the liquidity when unexpected liquidity fluctuations occur, the central bank resorts to Fine-tuning operations (FTOs).

The structural operations are conducted to adjust a structural position of the Eurosystem with the financial market. They can be carried out by reverse transactions, outright operations or issuance of the ECB debt certificates. [40]

2.3.2 Standing facilities and minimum reserves

The ECB performs the monetary policy also by setting interest rates on standing facilities. In contrast with the open market operations, which are conducted by the ECB, the standing facilities are initiated by the commercial banks when they have an excessive or insufficient overnight liquidity. There are two standing facilities at disposal: the marginal landing facility and the deposit facility. The former enables banks to borrow overnight liquidity; collateral as a guarantee is required. The latter is used by the banks to deposit overnight liquidity at the central bank. The interests at the interbank market are normally more favorable than those at the standing facilities so under normal circumstances there is no big reason for the banks to use them.

It is required that all the credit institutions hold minimum deposits on the current account at the respective national central bank. In 2012 the minimal requirements were lowered from 2 % to 1 % of bank liabilities, primarily customers´ deposits. [40]

2.4 The Conduct of monetary policy and unconventional measures

From the establishment of Monetary Union in 1999 to the start of financial and banking crisis in the middle of 2007, the ECB conducted its monetary policy through increasing and decreasing main refinancing rate and rates at the standing facilities. At the dawn of financial crisis, the rate for the main refinancing operations was set at 4.5 %; 2.25 percent point higher than in January 2006. This hike was performed in order to weaken an inflationary pressure from robust economic growth and fast growth of money and credit supply.

The US real estate bubble having burst, uncertainty about the health of balance sheets of many banks due to a possible exposure to the US housing market started spreading across the euro-area. The uncertainty was enhanced in September 2008 after the fall of investment bank Lehman Brothers. The banks stopped trusting each other when it came to the financial stability and solvency. As a result, banks with excessive liquidity hesitated to lend to the counterpart with a current lack of liquidity. The interbank market got frozen and considerable amount of financial segments collapsed. Without an intervention of the monetary authority, many banks would find it difficult to refinance its assets.

The crisis inevitably started spreading to the real economy as well with plummeting international trade and generally deteriorating economic prospects.

ECB in response to it cut main refinancing rate to historically low level of 1 % from 3.75 % within seven months between October 2008 and May 2009 and introduced a set of unconventional measures called Enhanced Credit Support [29]:

- Extension of LTROs maturity from three months to twelve months with an aim to reduce refinancing concerns. With longer liquidity planning horizon, banks were expected to continue providing credit to the economy.
- Since the euro-area banks were lacking the US dollar funding, ECB temporarily provided funding in the foreign currencies, especially in US dollars.
- Full allotment provision at fixed rate: For all refinancing operations (MROs, LTROs), the euro-area banks could have unlimited access to central bank liquidity against acceptable collateral.
- Extension of collateral list to include for instance asset-backed securities.
- First covered bond purchase programme (CBPP): In July 2009 the Eurosystem started buying covered bonds to resurrect the covered bond market, primary source of financing of the banks, which had become illiquid during the crisis. Amount of euro denominated covered bonds that the ECB was authorized to buy totalled 60 billion EUR.

In January 2010 concerns about the Greek sovereign debt sustainability and the state of predominantly South-European economies along with Ireland culminated. As a result, a secondary market with sovereign bonds of the troubled economies started drying up in terms of liquidity, which posed a threat to smooth functionality of the transmission mechanism, because the second market securities serves usually as a collateral for the commercial banks. [8]

Therefore, the governing council launched the Securities Markets Programme (SMP). The main objective, summarized by the then ECB president Trichet, was to “address tensions in certain market segments that hampered the monetary policy transmission mechanism”. The program was criticised for the lack of transparency, since neither the amount of acquired bonds nor the periodicity had been announced. [8, 46] Overall volume of the programme totalled

219 billion EUR. [18]

Ultimately at the end of 2011, however, SPM proved to be insufficient. Many euro-area sovereign bonds got downgraded, European economies were slowing down. Further uncertainty about the debt crisis arose.

In August 2012 the Governing Council announced a program of Outright Monetary Transactions (OMT). Under the program the ECB was prepared to purchase sovereign bonds on secondary market. OMT seemed to be similar to SMP. Mario Draghi in September clarified that the Board intentions would be more transparent. Another difference was a strict conditionality of the OMT. Unlike the SMP, the OMT was applicable only to those countries that still had an access to the market - Greece lost it in April 2010. The OMT managed to decrease market volatility as well as bond yields of Mediterranean countries. [46]

In the spring 2013 inflation in the euro-area slumped to 1.2 % from the peak of 3 % during the crisis. The slowing inflation along with the slow economic growth pushed the ECB to adopt a non-standard measure: Forward guidance - explicit declarations about probable development of policy interest rates under the conditional evolution of chosen macroeconomic variables. [8]

The press conference after the Governing Council meeting in July 2013 contained following expression [13]:

> “The Governing Council expects the key ECB interest rates to remain at present or lower levels for an extended period of time. This expectation is based on the overall subdued outlook for inflation extending into the medium term, given the broad-based weakness in the real economy and subdued monetary dynamics.“

The objectives of the forward guidance was to affect expectations about short term interest rates, which the central bank can directly control. Based on the expectations theory, this expectations about short term rates will affect longer term interest rates.

At the end of 2014 the Eurozone faced the negative inflation rate of 0.2 %. At the same time, however, the main refinancing interest rate was already near the zero lower bound and the rate of the deposit facility was at -0.2 %. Thus options of conventional monetary policy to further lower real interest rates by decreasing nominal interest rates were spent and powerless.

Trying to conduct further expansionary policy, ECB decided to launch Expanded Asset Purchase Program or so called Quantitative Easing (QE) in January 2015 to decrease longer term interest rates and ultimately bring the inflation back close to 2% target. Under the QE, ECB was buying long term government but also corporate debt, covered bonds and asset-backed securities. The intensity of the purchases peaked in 2016 when ECB monthly bought assets worth 80 billion EUR. Then the intensity faltered. In the last month of the QE in December 2018 the purchases totalled 15 billion EUR.

During the QE, the ECB purchased assets worth 2,6 trillion EUR. The ECB plans to reinvest the maturated, under QE accumulated bonds.

To quantify the effect of unconventional measures during the time of zero lower bound, Wu and Xia [43] constructed the so called shadow interest rates which can go to the negative territory. Wu and Zhang [45] then in case of FED plugged the shadow interest rate to New Keynesian DSGE model and they concluded that the shadow interest rate was a good substitute to the funds rate.

3. Monetary policy rules: Taylor rule

Following Svensson´s [33] definition, the monetary policy rule is a prescribed guide for monetary policy conduct. It is possible to divide the monetary policy rules into instrument rules and targeting rules.

In general, the targeting monetary policy rule means an assignment to minimize certain loss function as, for example, the one introduced in 1.2. In this respect, conduct of monetary policy is to set the instrument rate in a way to reach target criterion of the target variables.

On the other hand, instrument rules are prescribed functions of predetermined or forward-looking variables, or possibly both. Well known instrumental rules are for example McCallum´s [22] rule for monetary base or Taylor´s [39] rule for short term nominal interest rate. Since the Taylor rule will be the subject of this chapter and it will be used for the monetary policy analysis, it seems convenient now, as has already been shadowed in chapter 1.1, to expect existence of short term imperfect process of price and wage adjustment causing the short term trade–off between the output and inflation to exist.

3.1 Optimal interest rate rule in IS-PC-MR model

IS-PC-MR model consists of three equations: backward-looking Phillips curve on the side of supply, IS curve on the side of aggregate demand and monetary rule. It is possible to derive from these equations monetary policy rule proposed by Taylor. The model, initially formulated by Svenson [32], is taken from Carlin and Soskice [5].

To derive monetary rule consider inflation targeting central bank minimising output and inflation fluctuations around its targets throughout the loss function:

$$L_t = \frac{1}{2}[\lambda(\pi_t - \pi^*)^2 + (y_t - y^*)^2], \tag{3.1}$$

where λ is the weight of inflation loss relative to output gap describing how much averse the central bank is to the inflation deviation.

Backward–looking Phillips curve is defined as

$$\pi_t = \pi_{t-1} + \zeta(y_t - y^*) \tag{3.2}$$

where the current inflation is a function of lagged inflation and output gap. Phillips curve serves here as a constraint of the central bank optimizing problem to minimize its loss function. To find out the monetary policy rule of the central bank, the best combinations of inflation and output need to be found within the loss function with respect to the Phillips curve.

Since in this model the central bank can have an effect only on the output by manipulating its interest rate; inflation is affected after that via Phillips curve, the monetary rule can be derived by finding the value of y_t which minimizes the L_t at any π_{t-1}.

It can be done by substituting the Phillips curve 3.2 into 3.1 and taking derivative with respect to y_t which yields:

$$(y_t - y^*) + \zeta\lambda(\pi_{t-1} + \zeta(y_t - y^*) - \pi^*) = 0. \tag{3.3}$$

As $\pi_{t-1} + \zeta(y_t - y^*) = \pi_t$ is the Phillips curve, it can be substituted back to 3.3:

$$(y_t - y^*) = -\zeta\lambda(\pi_t - \pi^*). \tag{3.4}$$

Equation 3.4 is the derived monetary policy rule and it shows the output and inflation combinations that the central bank will opt for. An inverse relation between output and inflation is determined by the slope of the Phillips curve and central bank inflation loss aversion.

The Phillips curve has been defined and the monetary policy rule has been derived. The last part of the model that has not yet been presented is the IS curve that in the output gap form is defined as:

$$(y_t - y^*) = -\gamma(r_{t-1} - r^*), \tag{3.5}$$

where the r_{t-1} denotes a real interest rate, r^* is the equilibrium real rate and γ denotes the sensitivity of the aggregate demand to changes in the real interest rate. According to the IS curve the output gap will deviate depending on the deviation of real interest rate from equilibrium real rate.

To derive the interest rule for r_{t-1}, the Phillips curve 3.2 is substituted into monetary rule 3.4:

$$\pi_{t-1} - \pi^* = -\left(\zeta + \frac{1}{\zeta\lambda}\right)(y_t - y^*). \tag{3.6}$$

And finally the substitution of the IS curve 3.5 into 3.6 yields after slight rearrangement interest rate rule:

$$r_{t-1} - r^* = \frac{1}{\gamma\left(\zeta + \frac{1}{\zeta\lambda}\right)}(\pi_{t-1} - \pi^*). \tag{3.7}$$

Unlike the Taylor rule, the real interest rate r_{t-1} is a function of only deviation of inflation. The output gap does not enter the policy rule. The degree of central bank reaction is determined by the parameter of all three equations.

The presented model has assumed as is seen from the IS curve, that r_{t-1} has an effect on output with one period lag, affecting y_t. To make the model more realistic, other lags can be added - now it takes one period for the output to affect the inflation. This assumption leads to a change of y_t in Phillips curve to y_{t-1} and π_t in the monetary rule changes to π_{t+1} because the inflation term in loss function of the central bank shifts one period forward as well. The resulting modified equations are:

$$\begin{aligned} \pi_t &= \pi_{t-1} + \zeta(y_{t-1} - y^*) \\ (y_t - y^*) &= -\zeta\lambda(\pi_{t+1} - \pi^*) \\ (y_t - y^*) &= -\gamma(r_{t-1} - r^*) \end{aligned}$$

Repeating the same substitutions [1] as before but with the modified baseline equations finally yields:

$$r_{t-1} - r^* = \frac{1}{\gamma\left(\zeta + \frac{1}{\zeta\lambda}\right)}[(\pi_{t-1} - \pi*) + \zeta(y_{t-1} - y^*]. \tag{3.8}$$

In 3.8 the real interest rate is a function of both inflation and output.

3.2 Basic Taylor rule

Claiming that for most of the central banks it is preferable to change monetary policy conditions based on both inflation and output, John Taylor [39] proposed in 1993 a monetary policy rule in which federal funds rate is a function of deviation of real GDP from a target and deviation of inflation from a target. Since then the rule has attracted considerable attention and numerous modifications has been proposed.

The rule can be described in general as:

$$i_t = r^* + \pi_t + \alpha_\pi(\pi_t - \pi^*) + \alpha_y(y_t - y^*), \tag{3.9}$$

where the i_t is the federal funds rate recommended by the Taylor rule.

When an economy is in its long term equilibrium, the output is on its potential and the inflation equals the inflation target. The suggested nominal interest rate then equals the equilibrium real rate plus the inflation target.

Taylor without an econometric procedure suggested representative values of parameters α_π and α_y to be 0.5 and both inflation target and equilibrium real rate to be 2 % and found out that this reaction function well described

[1] To perform the substitutions once more successfully, it is necessary to shift time in the Phillips curve one period forward, for details see appendix A

monetary policy of Federal Reserve (FED) during the 1987–1992 period. It is worthwhile to mention that 3.9 with parameters α_π, α_y equal to 0.5 is equivalent with the derived interest rate rule 3.8 where slopes of Phillips curve, IS curve and monetary rule all equal 1.

Even though it seems that FED followed monetary policy rule similar to the 3.9, Taylor does not recommend that the central bank should follow it mechanically because such a simple rule cannot involve all important information. But he does not reject it either as a possible additional indicator assessed by the Federal Open Market Committee during the decision process about a future development of federal funds rate.

The coefficient α_π is required to be consistent with the Taylor principle demanding α_π to be higher than zero. Assuming the Taylor principle, the central bank reacts to an increase of 1 percent point in the inflation by rising nominal interest rate by $1 + \alpha_\pi$ percent points. If the central bank does not increase the nominal interest rate enough, then the increase in output and inflation will cause the real interest rate to fall, which will have another expansionary impact on the aggregate demand.

The necessity of this assumption is clear from equation of the aggregate demand [1]: $y_t = y^* - \gamma(r_t - r^*)$ where no lag on the real interest is assumed and where the exogenous real interest rate was replaced by the Fischer equation: $r_t = i_t - E_t[\pi_{t+1}]$:

$$y_t = y^* - \gamma(i_t - E_t[\pi_{t+1}] - r^*),$$

where the nominal interest rate was further replaced by the Taylor rule 3.9:

$$y_t = y^* - \gamma(r^* + \pi_t + \alpha_\pi(\pi_t - \pi^*) + \alpha_y(y_t - y^*) - E_t[\pi_{t+1}] - r^*)$$

and finally the expected inflation term was replaced using an equation for the adaptive expectations $E_t[\pi_{t+1}] = \pi_t$:

$$y_t = y^* - \gamma(r^* + \pi_t + \alpha_\pi(\pi_t - \pi^*) + \alpha_y(y_t - y^*) - \pi_t - r^*). \tag{3.10}$$

After a slight algebraic rearrangement[2] 3.10 can be transformed into the desired form:

$$y_t = y^* - [\gamma\alpha_\pi/(1 + \gamma\alpha_y)](\pi_t - \pi^*). \tag{3.11}$$

If α_π were lower than zero, than the curve of aggregate demand 3.11 would be upward sloping. The increased inflation would cause increased demand, which would trigger an inflationary spiral. More on Taylor rule properties can be found for example in Woodford [42].

[1] The equation of the aggregate demand is taken from Mankiw [21]

[2] Details in appendix A

Transforming 3.9 by putting the inflation terms π_t together, it can be rewritten to the following form:

$$i_t = r^* + \pi^* + \beta_\pi(\pi_t - \pi^*) + \beta_y(y_t - y^*), \tag{3.12}$$

where $\beta_\pi = 1+\alpha_\pi$, so in this case the Taylor principle is met when β_π is higher than 1.

For the econometric purposes equation 3.12 can be further amended to:

$$i_t = \beta + \beta_\pi \pi_t + \beta_y(y_t - y^*) \tag{3.13}$$

where $\beta = r^* + \pi^*(1 - \beta_\pi)$.

As far as the econometric procedure, the ordinary least squares is possible to use since the equation 3.13 is linear or two stage least squares when the regressors are expected to be endogenous.

Taylor [37] resorted in 1999 to ordinary least squares to estimate parameters of 3.13 to examine U.S monetary history.

The results suggest considerable tendency of the response parameters β_π, β_y to grow over time among periods of the International Gold Standard Era, Bretton Woods Era and post-Bretton Woods Era. Value of β_π grew over time from negligible 0.019 in the period 1879–91 to high 1.533 in the Greenspan period 1987–97.

3.3 Interest rate smoothing

The inflation and the output gap, which are the leading variables entering the Taylor rule, might show a considerable volatility over time. The volatility would require according to the Taylor rule a frequent and violent adjustment of the short term interest rate. However, there are numerous reasons why the central banks should react to the changes more gradually, which can be achieved in context of the Taylor rule by adding a smoothing parameter.

Orphanides [25], for example, emphasises the importance of gradual adjustment of the nominal interest rate due to the possible measuring error of the macroeconomic variables. Before the data revision is done, it is difficult to distinguish between a measurement error and an economic shock. Prudent policy makers should realize this and avoid overreaction, which may become source of instability.

Kydland and Precott [19] showed that when an authority like central bank operates without a certain commitment it is not further able to control expectations of the private sector. The individuals form rational expectations and then discretionary monetary policy seemingly optimal now may not be optimal in the next period — the central bank faces the so called time inconsistency[3] problem. Woodford [41] showed that in the context of a simple model of optimizing private-sector behavior sticking to a simple inertial instrumental rule is the optimal behaviour.

Furthermore, Goodfriend [12] argues that volatile nominal short term interest rates might cause instabilities at the financial markets as the markets overreact to the change in the reference interest rate, which can cause a heavy reallocation of the assets in portfolios and abrupt changes in cost of the credit and ultimately hurt the real economy. Also a quick and deep reversal of trend of change in reference interest rate might cause a loss of the central bank credibility.

Another supportive reason for adding the smoothing parameter to the Taylor rule might be the fact that monetary policy makers change the key nominal interest rates by small steps, usually by 0.25 or 0.50 percent point per meeting and the time among consecutive changes in the interest rates is normally a question of weeks or months.

The interest rate smoothing is defined as:

$$i_t = (1-\rho)i_t^* + \rho i_{t-1} + v_t, \tag{3.14}$$

where ρ is the smoothing parameter, whose value is between zero and one, and i_t^* is the recommended interest rate given by the policy rule.

In most empirical papers the smoothing parameter is very high around 0.8 and highly statistically significant.

3.4 Forward-Looking Taylor Rule

Due to the lags in the transmission mechanism, it takes time before the changes in the monetary policy conditions affect the real economy. That is why the forward-looking policy rule, dealing with forward-looking variables, might characterize the nature of the monetary policy better.

In this context the basic Taylor rule is backward-looking, as it allows the central bank to change the reference interest rate based only on the lagged inflation and output gap. On the other hand the forward-looking modifications

[3] The time inconsistency is often used as a supportive argument in a broader discussion whether the monetary policy should be rather rule following or discretionary, a stylized summary of arguments why policy rule might be more desirable can be found for example in Taylor [38].

allow the central bank to include a wide range of information that is expected to participate in forming expectations about future.

Clarida, Gali and Gerlter [6] propose following simple linear forward–looking rule:

$$i_t^* = i^* + \beta_\pi(E[\pi_{t,k}|\Omega_t] - \pi^*) + \beta_y E[x_{t,k}|\Omega_t] \tag{3.15}$$

where $\pi_{t,k}$ is the annual inflation between periods t and t+k, $x_{t,k}$ denotes an average output gap between periods t and t+k. E is the rational expectations operator; the expectations are based on the information set Ω_t available at the time t, the time when the interest rate is set. Finally i^* is target nominal interest rate when both inflation and output gap deviations from the targets are zero or in other words equilibrium nominal interest rate.

According to Clarida, Gali and Gertler [6, 7], the monetary rule like 3.15 has a considerable empirical and theoretical appeal since its approximate or sometimes even exact forms are optimal rule for the central banks that have quadratic loss function over inflation and output.

To get a relation for the real interest rate, 3.15 can be rewritten to obtain:

$$r_t^* = r^* + (\beta_\pi - 1)(E[\pi_{t,k}|\Omega_t] - \pi^*) + \beta_y E[x_{t,k}|\Omega_t] \tag{3.16}$$

with r^* to be the equilibrium real interest rate. The crucial point is, that similarly as in the back-ward looking Taylor rule 3.12 the coefficient β_π needs to be higher than one, otherwise a self-fulling burst of inflation may occur. Same logic applies to the response parameter β_y that needs to be higher than zero.

Defining $\beta = i^* - \beta_\pi \pi^*$ and consistently with it transforming 3.15 yields:

$$i_t^* = \beta + \beta_\pi E[\pi_{t,k}|\Omega_t] + \beta_y E[x_{t,k}|\Omega_t] \tag{3.17}$$

which is more convenient for the econometric estimation. Further under the consideration of the interest rate inertia 3.17 is modified by substituting it to 3.14:

$$i_t = (1-\rho)(\beta + \beta_\pi E[\pi_{t,k}|\Omega_t] + \beta_y E[x_{t,k}|\Omega_t]) + \rho i_{t-1} + v_t. \tag{3.18}$$

In the forward-looking version of the Taylor rule with rational expectations it is always needed to take into account endogenous regressors because the current value of reference interest rate is affected by the future variables, which are at the same time affected by the previous values of the reference interest rate.

In the literature, this endogeneity problem in equation 3.18 is often solved by using Generalized Method of Moments as it is used in the case of Clarida, Gali and Gertler [6, 7] and it will be also used in this thesis as the main econometric method.

3.5 Additional modifications

Within the original specification either backward-looking or forward-looking, only variables of the inflation and the output gap enter the Taylor rule. So it offers plenty of room for modifications - to add other regressors that are expected to have an impact on the reference interest rate as exchange rates or money growth.

Concerns about precision of potential output estimation highlighted by Orphanides [28] lead Beckworth and Hendrickson [2] to formulate the policy rule adjusting reference interest rate based on the nominal GDP instead of the output gap and the inflation gap. Orphanides [27] replaced the output gap by the unemployment gap, however, in this specification the estimation of the NAIRU is also connected with an inevitable imprecision. Laubach [20] utilized a Phillips curve-type regression to estimate the NAIRU and concluded that the uncertainty around the NAIRU estimates is very high and in line with previous research.

4. Generalized Method of Moments

This section is predominantly based on Hamilton [14], Cameron and Trivedy [4] and Heij et al [16].

4.1 Basic intuition behind GMM

Suppose a linear regression model that is defined as $y = x^T\beta + \epsilon$, where x^T is transposed K x 1 vector of regressors and β is K x 1 vector of unknown parameters that is to be estimated using the method of moments. A supposition that the error term ϵ has a zero mean conditional on regressors, formally written: $E[\epsilon|x] = 0$, leads to moment conditions $E[x(y - x^T\beta)] = 0$. Method of moments estimator is then a solution to the sample mean conditions with N observations, that is defined as follows:

$$\frac{1}{N}\sum_{t=1}^{N} x_t(y_t - x_t^T\beta) = 0.$$

This can be solved easily as there is K moment conditions and K unknown parameters. However, on additional supposition, for instance that the error term ϵ is conditionally symmetric, thus $E[\epsilon^3|x] = 0$, the estimation of β is based on 2K conditions:

$$\begin{bmatrix} E[x(y - x^T\beta)] \\ E[x(y - x^T\beta)^3] \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

The Method of moments would then try to find solution to sample conditions $N^{-1}\sum_{t=1}^{N} x_t(y_t - x_t^T\beta) = 0$ and $N^{-1}\sum_{t=1}^{N} x_t(y_t - x_t^T\beta)^3 = 0$.

With 2K moment conditions and only K unknown parameters all the conditions can not be fulfilled and thus there is not any analytical solution. So the Generalized Method of Moments tries not to solve all the moment conditions at once, but tries to get them as close to zero as possible minimizing quadratic loss form:

$$Q(\beta) = \begin{bmatrix} E[x(y - x^T\beta)] \\ E[x(y - x^T\beta)^3] \end{bmatrix}^T W_N \begin{bmatrix} E[x(y - x^T\beta)] \\ E[x(y - x^T\beta)^3] \end{bmatrix}$$

where the W_N is 2K x 2K positive, symmetric weighting matrix determining weight of each moment condition. In this respect the OLS estimator is a special case of the generalized method of moments as well as the method of instrumental variables where also no analytical solution does exist if there are more instruments than regressors.

4.2 Formal, general notation

In a general notation a set of r population moment conditions for q parameters is defined as:

$$G(\theta_0) = E[\mathrm{h}(\mathrm{w}_t, \theta_0)] = 0$$

where h(•) is an r x 1 vector function[1], w_t is a term containing explained and explanatory variables and potential instrumental variables z_t and θ_0 is a vector of q true values of the parameters.

The corresponding sample moments would then be:

$$G_N(\theta) = \frac{1}{N}\sum_{t=1}^{N} \mathrm{h}(\mathrm{w}_t, \theta) \tag{4.1}$$

and on assumption of over-identification: r > q, GMM estimator then minimises quadratic form:

$$\mathrm{GMM}(\hat{\theta}) = \underset{\theta}{\operatorname{argmin}}\, Q_N(\theta),$$

where

$$Q_N(\theta) = G_N(\theta)^T W_N G_N(\theta). \tag{4.2}$$

The GMM estimator is dependant on the choice of the weighting matrix so its optimal choice is crucial for the minimization of the quadratic form.

To derive the expression for GMM estimator it is needed to differentiate $Q_N(\theta)$ with respect to θ to derive first order conditions. First, however, it is convenient to get approximation of $G_N(\theta)$ by taking first order Taylor expansion around the true value θ_0:

$$G_N(\theta) \approx G_N(\theta_0) + D_N(\theta - \theta_0), \tag{4.3}$$

where D_N is r x q matrix of first derivatives $\partial G_N \theta / \partial \theta^T$. Substitution of 4.3 into 4.2 yields:

$$Q_N(\theta) \approx (G_N(\theta_0) + D_N(\theta - \theta_0))^T W_N (G_N(\theta_0) + D_N(\theta - \theta_0)). \tag{4.4}$$

After multiplication the first derivative of 4.4 with respect to θ is given by:

$$\frac{\partial Q_N(\theta)}{\partial \theta} = G_N(\theta_0)^T W_N D_N + D_N^T W_N G_N(\theta_0) + 2 D_N^T W_N D_N (\theta - \theta_0)$$

[1] In case of ordinary least squares h(•) would be $x(y - x^T\beta)$

Since the first two term are vectors[2], the first order condition simplifies to: $D_N^T W_N G_N(\theta_0) + D_N^T W_N D_N(\hat{\theta} - \theta_0) = 0$ which expressed for $\hat{\theta}$ leads to:

$$\hat{\theta} = \theta_0 - (D_N^T W_N D_N)^{-1} D_N^T W_N G_N(\theta_0), \tag{4.5}$$

which states that the estimator equals the true value minus an estimation error.

4.3 GMM properties and optimal weighting matrix

If the law of large number is considered, then the sample moment converges in probability to the population moment as N approaches infinity:

$$G_N(\theta) = \frac{1}{N}\sum_{t=1}^{N} \mathrm{h}(\mathrm{w}_t,\theta) \rightarrow G(\theta) = E[\mathrm{h}(\mathrm{w}_t,\theta)]. \tag{4.6}$$

On supposition that the moment conditions are correctly specified the law of large numbers is sufficient for GMM estimator to be consistent for any symmetric, positive definitive weighting matrix - details can be found in Hansen [15]. From the law of the large numbers it follows that the sample moments evaluated at the true parameters $G_N(\theta_0)$ converges in probability to the population value $G(\theta_0) = 0$ so the error term in 4.5 is zero and thus $\hat{\theta} = \theta_0$. If the central limit theorem holds for $\mathrm{h}(\mathrm{w}_t,\theta)$, then

$$\sqrt{N} G_N(\theta) = \frac{1}{\sqrt{N}}\sum_{t=1}^{N} \mathrm{h}(\mathrm{w}_t,\theta) \rightarrow N(0,S),$$

where S is the asymptotic variance of $\mathrm{h}(\mathrm{w}_t,\theta_0)$:

$$S = \lim_{N\rightarrow\infty} NE[G_N(\theta)G_N^T(\theta)]. \tag{4.7}$$

If the data obey both the law of large numbers and central limit theorem, then the asymptotic distribution of GMM estimator is for any symmetric positive definite weight matrix W defined as:

$$\sqrt{N}(\hat{\theta} - \theta_0) \rightarrow N(0,V),$$

where the asymptotic variance $V = (D^T W D)^{-1} D^T W S W D (D^T W D)^{-1}$ where D is the probability limit of D_N for N $\rightarrow \infty$. It applies to efficient GMM estimator, that it chooses W_N such, that it minimises the asymptotic variance V. The optimal weighting matrix turns out to be S^{-1}, so the moments with a high variance have smaller weight and vice versa. The best moments are those with small S and large D. Large D, derivative of the moments, means that the moments bear important information, in other words the moment conditions

[2] $dim(D_N)$ = r x q, dim(W) = r x r, $dim(G_N(\theta_0))$ = r x 1 and therefore $dim(G_N(\theta_0)^T W_N D_N)$ = 1 x q and $dim(D_N^T W_N G_N(\theta_0))$ = q x 1

are very violated when the vector of the parameters θ is not on its true values θ_0.

The idea behind is similar to the weighted least squares applied when the error variance is not constant — the observations with the lesser precision are weighted less and the one with the higher precision are weighted more. For $W = S^{-1}$ the asymptotic variance V simplifies to: $V = (D^T S^{-1} D)^{-1}$.

The variance matrix is not known and must be estimated, which is often done by a two-step or iterative procedure. For the sake of simplicity and clarity let the sample moment $G_N(\theta) = \frac{1}{N}\sum_{t=1}^{N} \mathrm{h}(\mathrm{w}_t,\theta) = \frac{1}{N}\sum_{t=1}^{N} \mathrm{h}_t$ and $\hat{\mathrm{h}}_t$ to be the observations on h_t for t = 1,...,N. At true values, the variance matrix can be defined as the sum of autocovariances matrices: $S = \sum_{j=-\infty}^{\infty} \Gamma_j = \Gamma_0 + \sum_{j=1}^{\infty}(\Gamma_j + \Gamma_j^T)$, where $\Gamma_{-j} = \Gamma_j^T$ and where $\Gamma_j = E\{[\mathrm{h}(\mathrm{w}_t,\theta_0)][\mathrm{h}(\mathrm{w}_{t-j},\theta_0)]^T\}$.

To show this more in detail, recall from the equation 4.7 that the variance matrix is at the same time defined as: $S = \lim_{N\to\infty} NE[G_N(\theta)G_N^T(\theta)]$, where N$E[G_N(\theta)G_N^T(\theta)]$ written in detail equals $\frac{N}{N^2}E[(\mathrm{h}_1,\mathrm{h}_2,\ldots\mathrm{h}_N)(\mathrm{h}_1,\mathrm{h}_2,\ldots\mathrm{h}_N)^T]$

$$= \frac{1}{N}\begin{bmatrix} & E(\mathrm{h}_1\mathrm{h}_1^T) & + & E(\mathrm{h}_1\mathrm{h}_2^T) & + & E(\mathrm{h}_1\mathrm{h}_3^T) & +\ldots & + & E(\mathrm{h}_1\mathrm{h}_N^T) \\ + & E(\mathrm{h}_2\mathrm{h}_1^T) & + & E(\mathrm{h}_2\mathrm{h}_2^T) & + & E(\mathrm{h}_2\mathrm{h}_3^T) & +\ldots & + & E(\mathrm{h}_2\mathrm{h}_N^T) \\ + & E(\mathrm{h}_3\mathrm{h}_1^T) & + & E(\mathrm{h}_3\mathrm{h}_2^T) & + & E(\mathrm{h}_3\mathrm{h}_3^T) & +\ldots & + & E(\mathrm{h}_3\mathrm{h}_N^T) \\ & \vdots & & \vdots & & \ddots & & & \vdots \\ + & E(\mathrm{h}_N\mathrm{h}_1^T) & + & E(\mathrm{h}_N\mathrm{h}_2^T) & + & E(\mathrm{h}_N\mathrm{h}_3^T) & +\ldots & + & E(\mathrm{h}_N\mathrm{h}_N^T) \end{bmatrix}$$

$$= \frac{1}{N}\begin{bmatrix} & \Gamma_0 & + & \Gamma_{-1} & + & \Gamma_{-2} & +\ldots & + & \Gamma_{-(N-1)} \\ + & \Gamma_1 & + & \Gamma_0 & + & \Gamma_{-1} & +\ldots & + & \Gamma_{-(N-2)} \\ + & \Gamma_2 & + & \Gamma_1 & + & \Gamma_0 & +\ldots & + & \Gamma_{-(N-3)} \\ & \vdots & & \vdots & & \ddots & & & \vdots \\ + & \Gamma_{(N-1)} & + & \Gamma_{(N-2)} & + & \Gamma_{(N-3)} & +\ldots & + & \Gamma_0 \end{bmatrix}$$

$= \frac{1}{N}[N\Gamma_0 + (N-1)\Gamma_{-1} + (N-1)\Gamma_1 + (N-2)\Gamma_{-2} + (N-2)\Gamma_2 \cdots + \Gamma_{(N-1)} + \Gamma_{-(N-1)}]$.

The autocovariances are estimated as:

$$\hat{\Gamma}_j = \frac{1}{N}\sum_{t=j+1}^{N} \hat{\mathrm{h}}_t\hat{\mathrm{h}}_{t-j}^T \tag{4.8}$$

for j = 0,1,...,k, where k is the maximum length of the lag. The covariance S is then estimated by:

$$\hat{S} = \hat{\Gamma}_0 + \sum_{j=1}^{k} w_j(\hat{\Gamma}_j + \hat{\Gamma}_j^T) \tag{4.9}$$

where w_j are the weights put on the lags. When $w_j = 1$, then weights on each lag length is the same. However, often it is better, when the weights on more distant lags are smaller; to achieve this, popular method is to use the so called Bartlett weights: $w_j = 1 - j/(k+1)$.

Since neither the optimal weighting matrix nor the values of parameters are known, the estimation usually begins with setting an arbitrary weighting matrix such as an identity matrix. First estimation of θ is then performed:

$$\mathrm{GMM}(\hat{\theta}^{(1)}) = \underset{\theta}{\mathrm{argmin}}(G_N(\theta))^T G_N(\theta).$$

The estimated vector $\hat{\theta}^{(1)}$ is used to get $\hat{h}_t = h(w_t, \hat{\theta}^{(1)})$. $\hat{h}_t$ is then used in 4.8 to compute $\hat{\Gamma}_j$ for j = 0,1,...k and finally $\hat{S}$ is estimated by 4.9, the inversion of $\hat{S}$ is then set as a new weighting matrix W_N to perform once more:

$$\mathrm{GMM}(\hat{\theta}^{(2)}) = \underset{\theta}{\mathrm{argmin}}(G_N(\theta))^T W_N G_N(\theta).$$

The iterative method of computation repeats the estimation until the time, when in the sth step $\hat{\theta}^{(s)} \approx \hat{\theta}^{(s+1)}$.

4.4 Test of overidentifying restrictions

In a model with more moment conditions than needed for estimation of θ, it is possible to test the validity of these overidentifying restrictions. For that purpose Hansen suggested a test of closeness of $N^{-1}\sum_{t=1}^{N} \mathrm{h}(\mathrm{w}_t, \hat{\theta})$ to zero, which is the test of H_0: $E[h(w, \theta_0)] = 0$. Hansen [15] showed that the overidentifying restrictions test statistics (further noted as J-stat.) takes the form:

$$\left(\frac{1}{N}\sum_{t=1}^{N} \hat{\mathrm{h}}_t\right)^T \hat{S}^{-1} \frac{1}{N}\sum_{t=1}^{N} \hat{\mathrm{h}}_t$$

and under the null hypothesis it is asymptotically distributed as $\chi^2(r-q)$. If the test statistic is high then the population conditions does not equal zero and the GMM estimator is inconsistent.

4.5 Putting GMM and Taylor rule together

The fact, that the rational expectations assumption is imposed on the forecasts entering the Forward-Looking Taylor rule allows one to construct moment conditions and consequently to use in case of the overidentification the generalized method of moments.

The basic forward-looking Taylor rule from 3.18 is:

$$i_t = (1-\rho)(\beta + \beta_\pi E[\pi_{t,k}|\Omega_t] + \beta_y E[x_{t,k}|\Omega_t]) + \rho i_{t-1} + v_t. \tag{4.10}$$

The expected values $E[\pi_{t,k}|\Omega_t]$ and $E[x_{t,k}|\Omega_t]$ are not known. Fortunately enough, the fact, that a certain realized value $\pi_{t,k}$ can be rewritten into the forecast $E[\pi_{t,k}|\Omega_t]$ and a certain forecast error m_t which under the rational expectations fulfills:

$$E[\mathrm{m}_t|\Omega_t] = 0$$

which in words means that the agent with rational expectations makes no systematic errors, makes it possible to rewrite 4.10 by replacing the unknown forecast by the realized values to:

$$i_t = (1-\rho)(\beta + \beta_\pi \pi_{t,k} + \beta_y x_{t,k}) + \rho i_{t-1} + \mathrm{u}_t, \tag{4.11}$$

where the error term u_t contains the forecast errors of output gap and inflation and exogenous disturbance v_t:

$$\mathrm{u}_t = -(1-\rho)(\beta_\pi(\pi_{t,k} - E[\pi_{t,k}|\Omega_t]) + \beta_y(x_{t,k} - E[x_{t,k}|\Omega_t])) + v_t$$

Rational expectations implies: $E[\mathrm{u}_t|z_t] = 0$, where z_t is a vector of variables from the central bank information set Ω_t uncorrelated with u_t. Based on these variables the central bank forecasts inflation and output but the central bank does not adjust monetary conditions directly to them.

Equation 4.11 and $E[\mathrm{u}_t|z_t] = 0$ implies following moment conditions:

$$E[i_t - (1-\rho)(\beta + \beta_\pi \pi_{t,k} + \beta_y x_{t,k}) - \rho i_{t-1}|z_t] = 0$$

and thus

$$E[z_t \mathrm{u}_t] = E[z_t(i_t - (1-\rho)(\beta + \beta_\pi \pi_{t,k} + \beta_y x_{t,k}) - \rho i_{t-1})] = 0.$$

The corresponding sample moments take the form:

$$G_N(\theta) = \frac{1}{N}\sum_{t=1}^{N} z_t(i_t - (1-\rho)(\beta + \beta_\pi \pi_{t,k} + \beta_y x_{t,k}) - \rho i_{t-1}) = 0.$$

5. Data

As popularized by Orphanides [26], the data that enter the Taylor Rule, like GDP and inflation, have a tendency to be revised over time. The estimation can be therefore performed with the real time data, the data that the monetary policy makers had at disposal during the decision–making process, or alternatively with the revised data. The most precise up-to-date data will be used for the purpose of this thesis. All the time series (except of GDP) used in the estimation are on a monthly basis and all the data are at disposal at The ECB Statistical Data Warehouse up to the figures of Global Price Index of All Commodities which were obtained from the database of Federal Reserve Bank of Saint Louis (FRED).

All the time series up to the shadow interest rate spans from January 2000 to the end of 2018. The shahow interest rates has been at disposal since September 2004.

5.1 Output gap

The output gap is in this thesis calculated as a percentage deviation of the real GDP from the potential output. GDP, however, unlike the rest of used variables, is not reported on a monthly basis; figures are reported quarterly and annually. Performing the estimation with monthly periodicity many authors overcome this obstacle using a GDP proxy - Index of industrial production (IIP), but it may not seem to be very appropriate, as the share of the industrial production on the total GDP is in the advanced countries relatively low and continues diminishing. So in this thesis besides the Index of industrial production, the time series of quarterly GDP is transformed by cubic spline to monthly time series and used as well.

Each segment of a time series transformed by cubic spline is represented by a cubic polynomial and the adjacent points of the lower and higher frequency series have the same level, first and second derivative. [30]

Another problematic point is the sole estimation of the potential output. A very popular smoothing method is a Hodrick-Prescott filter decomposing time series y_t into a growth component g_t identified as the potential output and cyclical component c_t: $\mathrm{y}_t = \mathrm{g}_t + c_t$ by minimizing the cyclical deviation from the trend:

$$\min_{\{g_t\}_{t=-1}^{N}} \{\sum_{t=1}^{N} c_t^2 + \lambda \sum_{t=1}^{N} [(g_t - g_{t-1}) - (g_{t-1} - g_{t-2})]^2\}$$

where λ is a parameter describing how much the variability in the growth component is penalized. The larger λ the more smoothed the time series is. The conventional wisdom chooses values for λ at 1600 for quarterly data and 14400 for monthly data. [17]

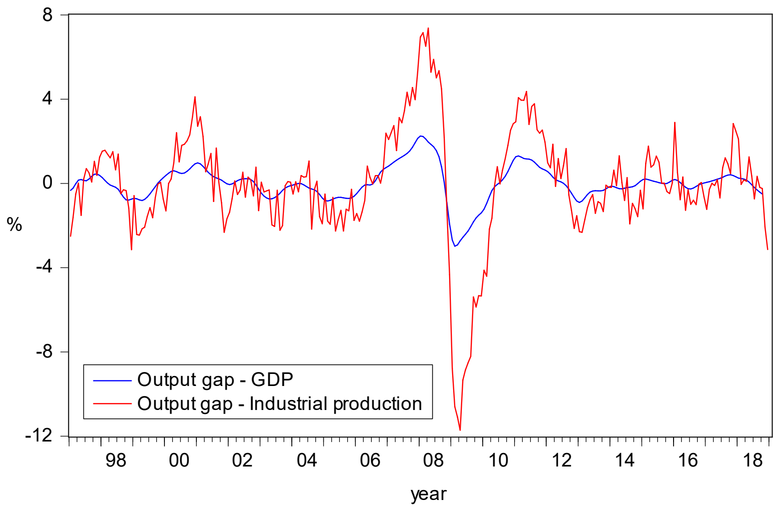

Figure 5.1: Estimated output gap

The figure 5.1 shows that the estimated output gaps depending on the used data differ significantly from one another in terms of volatility. For example, the economic contraction in 2008–2009 is nearly more than tripled if measured by Index of industrial production.

5.2 Inflation

To measure the inflation, a year-to-year change of Harmonised Index of Consumer Prices (HICP) is used. As discussed in 1.3 it might be also convenient to let the core inflation enter the Taylor Rule as well. A year-to-year change of the harmonised Index of Consumer Prices excluding energy and processed food is chosen as a proxy of the core inflation.

The graph 5.2 summarizes inflation in last two decades. After reaching the peak in 2008 the inflation slumped into a negative territory. The deflationary pressure, as mentioned in chapter 2.4, appeared once more between 2014 and 2015, at the time when the ECB launched its Quantitative Easing.

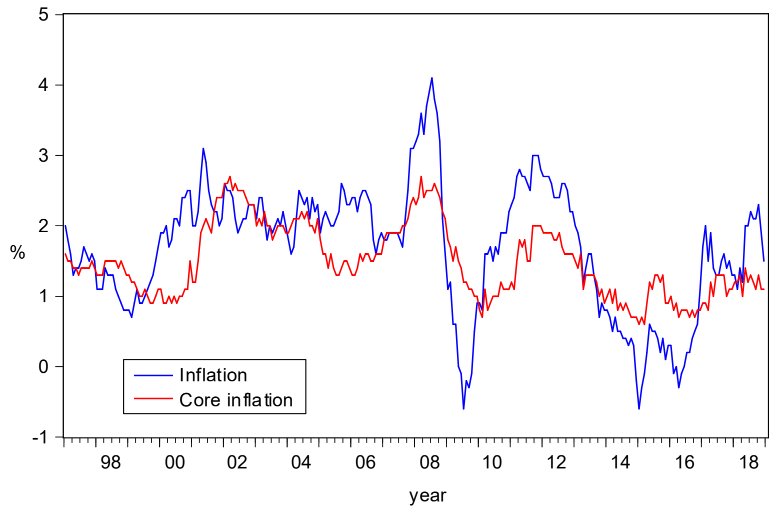

Figure 5.2: Inflation and core inflation

5.3 Interest rates

There is a wide variety of possible candidates for the policy interest rate. The first is the main refinancing rate on Main refinancing operations. This interest rate, however, shows no volatility over time, so the interest rate at the interbank market as such, Euro Interbank Offered Rate (Euribor), seems more convenient. Nominal Euribor of 3-month maturity is used.

In June 2014 the main refinancing rate was lowered to 0.15 % and in March 2016 lowered to zero. The Euribor summarized in the figure 5.3 as a result of these changes in main refinancing rate fell in March 2016 to -0.3 % and has stayed there until these days. At the zero lower bound it is further pointless to take into account Euribor or MRO rate as a measurement of monetary policy, predominantly relying on unconventional measures.

Therefore the estimated shadow interest rates [1] by Wu and Xia [44] for eurozone are considered, too. The paper, in which the shadow interest rates are estimated, is from 2017. Wu fortunately regularly updates the estimations every month so the most up-to-date shadow interest rates are at disposal as well. For all that the estimation begins in September 2004.

The figure 5.3 suggests, that ECB unconventional monetary policy had an impact as if the main refinancing operations rate had been gradually falling to -6 %.

[1]The shadow interest rate can be downloaded from: `https://sites.google.com/view/jingcynthiawu/shadow-rates`.

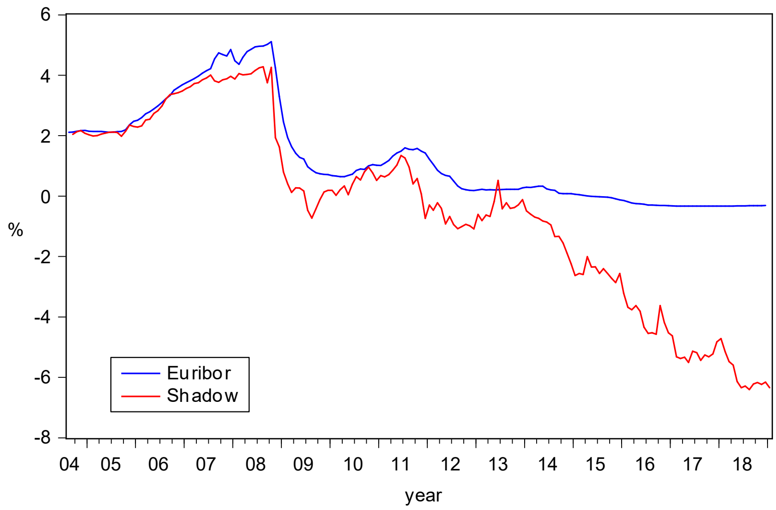

Figure 5.3: Euribor and shadow interest rate

5.4 Other variables

Among other variables, which might be useful when explaining the policy interest rates, nominal exchange rate between euro and dollar, money aggregate M3, long term interest rates, unemployment rates and world commodity price index are considered. For illustration also some WTI (West Texas Intermediate) oil prices will be mentioned and are taken from U.S. Energy Information Administration.

The choice of the long term interest rate is problematic. An intuitive choice would be a 10-year bond yield but the eurozone consists of 19 different countries. So instead of a bond of specific single country a long term interest rate for convergence purposes denominated in Euro with fixed composition for the Euro area might be useful proxy of long term interest rate. Yields of this proxy does not differ significantly from the yield of 10-year German bonds.

6. Estimation

6.1 Baseline specification

The baseline specification of the Taylor rule is estimated on the data from 2000, which is approximately the beginning of the Eurozone, to the end of 2014. After that the quantitative easing was launched. The estimation is performed on equation 3.18:

$$i_t = (1-\rho)(\beta + \beta_\pi E[\pi_{t,k}|\Omega_t] + \beta_y E[x_{t,k}|\Omega_t]) + \rho i_{t-1} + v_t \tag{6.1}$$

and further also the case without the interest rate inertia is considered for comparison. For the baseline model GDP as a measurement of output and HICP as a measurement of inflation is utilized.

The choice of horizons on the expected inflation and output gap is problematic, since there is not any consens about what the appropriate horizons should be. Therefore, in line with Clarida, Gali and Gertler [6] the horizon on the expected inflation is chosen to be one year (12 months) and zero on the output gap.

The last tricky part is the choice of instruments. On a quarterly basis Clarida, Gali and Gerlter [7] include in the information set four lags of inflation, output gap, the federal funds rate, the short-long spread, and commodity price inflation. Many authors after that closely followed the way how the instruments are chosen. Chadha et al. [31] consider for example four lags of output gap, inflation, the interest rate, log difference of a world commodity price index, real effective exchange rate and dividend-price ratio.

If the same method were applied to the estimation on a monthly basis, the information set would be very extensive - every variable would be included via twelve lags. Instead of 12 lags, Clarida, Gali and Gertler [6] consider first six and further 9th and 12th lags of output gap (x_t), inflation (π_t), world commodity inflation (o_t), interest rate (i_t) and exchange rate (q_t) - in total 40 instruments. All these variables might be useful in explaining expected inflation and at the same time there is not any big economic intuition why the central bank should change short term interest rate directly to them.

But in case of the ECB for example money growth included in the information set could be potentially problematic, because the ECB puts a big accent on the money growth, which is probably the legacy of German Bundesbank that used to operate under the money growth targeting regime. However, the money growth targeting regime has been rejected by the ECB.

Hensen and AAstrup [1] pointed out an econometric downfall of large instrumental set - the null hypothesis of the overidentifying restrictions test is practically impossible to reject. Their choice of instruments consists of first and second lag of inflation; first, second and third lag of interest rate and second, third, fourth and fifth lag of output gap.

For the baseline model of this thesis, the instrumental set from Clarida, Gali and Gertler [6] is slightly modified. The most important variables - inflation, output gap and interest rate are intact in a way supposed by Clarida, Gali and Gertler [6] but in order to reduce a little bit the number of the instruments, nominal exchange rate EUR/USD and world commodity inflation are added only with three lags. Above these, also three additional lags of long term interest rate are included in the baseline instrumental set – a wide variety of literature has been written on the importance of the long term interest rate (l_t) for the inflation expectations, see for example Fisher [9] and Goodfriend [11].

The baseline instrumental set looks as follows:

$$z_t = (x_{t-1} \dots x_{t-6}, x_{t-9}, x_{t-12}; \pi_{t-1} \dots \pi_{t-6}, \pi_{t-9}, \pi_{t-12}; i_{t-1} \dots i_{t-6}, i_{t-9}, i_{t-12}; o_{t-1} \dots o_{t-3}; q_{t-1} \dots q_{t-3}; l_{t-1} \dots l_{t-3})$$

where the world commodity inflation is obtained as a log difference of the world commodity price index. In total the baseline model contains 34 instruments including a constant.

As far as the econometric procedure, Newey-West (HAC) weighting matrix (with Bartlett weights as a method to weight autocovariances) robust to heteroskedasticity and autocorrelation, which according to Gerberding, Seitz and Worms [10] may result from the overlapping structure of the inflation forecasts errors, is used.

So the weighting matrix is set according to the mathematical relations from section 4.3 as follows:

$$\widehat{W}_N = \hat{S}^{-1}$$

where

$$\hat{S} = \hat{\Gamma}_0 + \sum_{j=1}^{k} w_j(\hat{\Gamma}_j + \hat{\Gamma}_j^T)$$

$$\hat{\Gamma}_j = \frac{1}{N} \sum_{t=j+1}^{N} \hat{\mathrm{h}}_t \hat{\mathrm{h}}_{t-j}^T$$

and w_j is in a form of Bartlett Kernel: $w_j = 1 - j/(k+1)$.

The table 6.1 summarises the estimated results for the nominal euribor as explained variable where also estimations with different data measurements are reported. The inflation and output gap response parameters have all the “right“ sign and are statistically significant in the baseline model. For example in the baseline case with the smoothing term the coefficient $\beta_\pi = 1.26$ means, that the central bank reacts to a one percent point increase in expected inflation by increasing the real interest rate by 0.26 percent point. The output gap response parameter then suggests, that the central bank reacts to a one point increase of the output gap by increasing nominal interest rate by 2.36 percent points. The degree of interest rate smoothing is very high and also highly statistically significant, in case of index of industrial production as high as 0.97. The values of ρ are in line with findings of Clarida Gali and Gerlter [6], where ρ moved between 0.87 and 0.97 but rather closer to the higher bound.

The Taylor rule with interest rate inertia has a strong theoretical and also empirical appeal. Many papers uses only this specification. Interestingly, without the interest rate inertia the response parameters fall substantially. The coefficient $\beta_\pi = 0.70$ would mean that the central bank does not react to the increase in the expected inflation sufficiently to increase the real interest rate and thus the Taylor principle does not hold in this specification.

The J-stat. column suggests that the null of the test of overidentifying restrictions is not rejected in any of the specifications.

Table 6.1: Baseline specification and different data

Used data	ρ	β	β_π	β_y	J-stat.
GDP gap, HICP	0.96***	-0.43	1.26***	2.39***	17.84
(baseline)	(0.00)	(0.61)	(0.27)	(0.44)	(0.96)
GDP gap, HICP		0.96***	0.70***	0.67***	23.82
(baseline)		(0.35)	(0.15)	(0.14)	(0.81)
GDP gap, HICP core	0.95***	-2.57***	2.75***	1.39***	21.23
	(0.01)	(0.89)	(0.52)	(0.48)	(0.88)
GDP gap, HICP core		-2.08***	2.62***	0.09	16.72
		(0.36)	(0.19)	(0.14)	(0.98)
IIP gap, HICP	0.97***	-1.08	1.58***	0.89***	21.47
	(0.00)	(0.75)	(0.34)	(0.19)	(0.87)
IIP gap, HICP		0.21	1.08***	0.18***	22.26
		(0.43)	(0.20)	(0.04)	(0.87)

Note: HAC standard errors in the parentheses; *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$; at J-stat. column p value of the null hypothesis of the overidentifying restrictions test in the parentheses; baseline instrumental set contains first 6, 9th, 12th lagged values of output gap, inflation, short term interest rate; first three lags of log difference of world commodity price index; first three lags of nominal EUR/USD; first three lags of long term interest rate and a constant; when different data for output gap and inflation are utilized, instrumental set is modified accordingly

Some notable results arise from comparison of the response parameters of the baseline with the response parameters in the specifications with either core inflation or index of industrial production. The parameter β_y is much smaller and in the model without interest rate inertia and with core inflation even statistically insignificant. More precisely β_y in the baseline is 2.39 and in model with IIP it is 0.89, which is more consistent with the estimated results of Clarida, Gali and Gertler [6] who utilized IIP to measure the output and where the values and variation of the response parameter β_y were within dozens of basis points rather than single percent points. On the contrary the coefficient β_π tends to grow when alternative data measurement is considered.

The figure 6.1 and 6.2 depicts actual interest rates and the interest rates resulting from the estimated baseline Taylor rule. It is evident that in a model with the interest rate inertia the resulting interest rates follow very closely the actual path of the interest rates. Only a slight deviation occurs on the peak of the recent crisis and between 2010-2012 period. On the other hand the resulting interest rates from the Taylor rule without the interest rate inertia shows considerable deviations from the actual path. More interestingly, the resulting interest rates fall in 2008/2009 into a negative territory even though the official main refinancing rate as well as the Euribor stayed well above zero. Further in 2012-2014 period the resulting interest rate is already well above one percent point, even though the quantitative easing was launched at the beginning of 2015. It seems that ECB was at that time rather discretionary than following a simple policy rule without interest rate inertia.

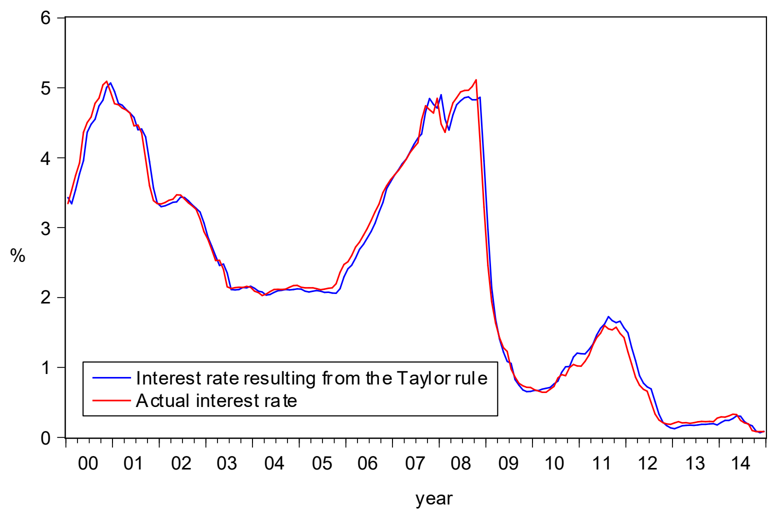

Figure 6.1: Actual and resulting interest rate of the baseline with ρ

The same outcome is supported also by the fogure 6.3, which shows the interest rates resulting from the Taylor rule estimated with usage of shadow interest rates on a sample from September 2004 to January 2017. The estimated response parameters are: $\beta_\pi = 1.18$, $\beta_y = 0.59$ and $\beta = -1.48$. All are statistically significant on conventional significance levels. The shadow interest rate path resulting from the Taylor rule deviates from the actual shadow

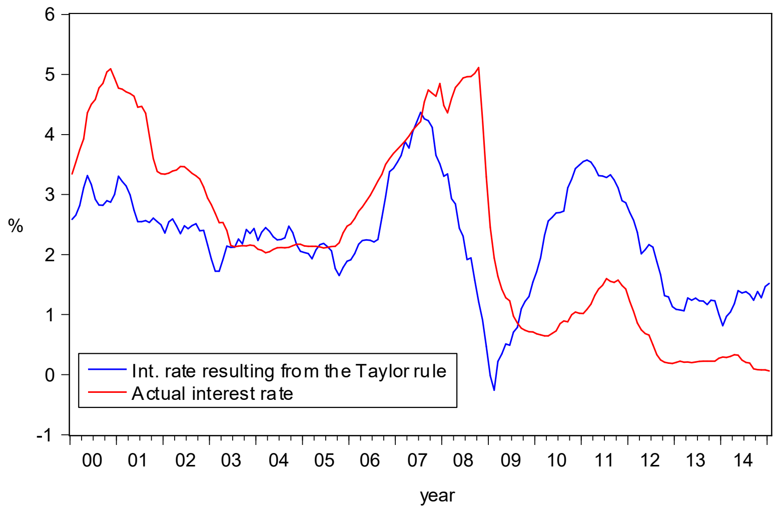

Figure 6.2: Actual and resulting interest rates of the baseline without ρ

interest rates since 2014. It seems that QE was indeed a discretionary reaction.

Between 2014 and 2015 the world was hit by a positive supply shock as the oil prices had collapsed. In July 2014 WTI oil prices were hovering around 103 USD per barrel and one year later in July 2015 WTI oil prices were already at 50 USD per barrel. It is reasonable to expect that this oil price collapse substantially weakened inflation dynamics and at the same time prevented the output gap from a steep fall into a negative territory. And because in the estimated policy rules here the output gap is with high and highly significant response parameter, the resulting path of the interest rate does not fall substantially.

If the attention is turned back to 2008 era, it can be seen that the resulting shadow interest rate is below -2 %. It might also suggest that the ECB was not very forward-looking during the crisis 2007-2009 since the resulting and actual shadow interest rate follow a similar path but with approximately a one-year long lag.

And finally the estimations allow to quantify the inflation target π^* that is by the ECB only vaguely defined as below but close to 2 %. From the equation 3.17:

$$i_t^* = \beta + \beta_\pi E[\pi_{t,k}|\Omega_t] + \beta_y E[x_{t,k}|\Omega_t]$$

it holds that: $\beta = i^* - \beta_\pi \pi^*$ and the equilibrium nominal interest rate equals the equilibrium real interest rate plus the inflation target: $i^* = r^* + \pi^*$. Substituting the second term into the first one yields:

$$\pi^* = \frac{\beta - r^*}{1 - \beta_\pi}. \tag{6.2}$$

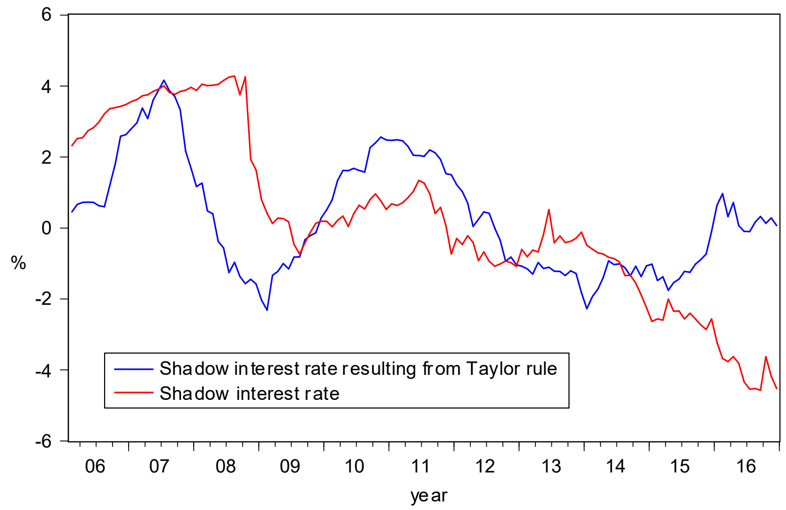

Figure 6.3: Actual and resulting shadow interest rate of the baseline without ρ

Alas, the equilibrium real interest rate is not known and like all the variables "with the stars" it can be only estimated. Following Clarida, Gali and Gertler [6] a sample mean of real interest rate is used as a rough proxy of the equilibrium real rate. The real rate is computed as the nominal Euribor minus the inflation.

The sample real rate is 0.36 for HICP and 0.67 for core HICP. Using them along with the estimated parameters β and β_π of the baseline model and plugging then into 6.2 yields inflation target 1.38 % for the model with ρ and 2 % in baseline model without ρ, which is pretty consistent with the verbal definition. Baseline model with core inflation yields inflation target 1.85 % with ρ and 1.69 % without ρ.

6.2 Robustness check

There are many possibilities how the potential instrumental set and horizons could look like, thus before the estimated results can be taken seriously, it is necessary to test how robust the baseline model is to the changes of the horizons and the instruments. The table 6.2 reports the estimations of various horizons. The estimations suggest that the the response parameters ρ, β and β_π are quite robust to these changes, but coefficient β_y shows great sensitivity to the changes of horizons on the output gap but it is quite robust along with other parameters to the changes in the expected inflation.

Now different instrumental sets are tested. The table 6.3 summarizes the re-

Table 6.2: Different horizons

Used data	ρ	β	β_π	β_y	J-stat.
π_{t+12}, x_t	0.96***	-0.43	1.26***	2.39***	17.84
(baseline)	(0.00)	(0.61)	(0.27)	(0.44)	(0.96)
π_{t+12}, x_t		0.96***	0.70***	0.67***	23.82
(baseline)		(0.35)	(0.15)	(0.14)	(0.81)
π_{t+12}, x_{t+1}	0.96***	-0.49	1.22***	2.79***	17.55
	(0.00)	(0.66)	(0.29)	(0.58)	(0.96)
π_{t+12}, x_{t+1}		0.93**	0.70***	0.55***	23.64
		(0.30)	(0.13)	(0.15)	(0.82)
π_{t+12}, x_{t+2}	0.97***	-0.71	1.24***	3.39***	18.00
	(0.00)	(0.76)	(0.33)	(0.88)	(0.95)
π_{t+12}, x_{t+2}		0.87**	0.73***	0.44***	23.61
		(0.36)	(0.15)	(0.13)	(0.82)
π_{t+12}, x_{t+3}	0.97***	-0.79	1.17***	4.30***	19.10
	(0.00)	(0.91)	(0.41)	(1.42)	(0.93)
π_{t+12}, x_{t+3}		0.78**	0.76***	0.32**	23.73
		(0.35)	(0.14)	(0.13)	(0.82)
π_{t+18}, x_t	0.96***	-0.13	1.17***	2.64***	20.23
	(0.00)	(0.38)	(0.18)	(0.34)	(0.91)
π_{t+18}, x_t		0.91***	0.76***	0.87***	25.12
		(0.30)	(0.13)	(0.15)	(0.76)
π_{t+24}, x_t	0.95***	-0.32	1.32***	2.66***	22.52
	(0.00)	(0.45)	(0.23)	(0.31)	(0.83)
π_{t+24}, x_t		0.49**	1.04***	1.09***	24.65
		(0.25)	(0.14)	(0.12)	(0.78)

Note: HAC standard errors in the parentheses; *** p < 0.01, ** p < 0.05, * p < 0.1; at J-stat. column p value of the null hypothesis of the overidentifying restrictions test in the parentheses; baseline instrumental set contains first 6, 9th, 12th lagged values of output gap, inflation, short term interest rate; first three lags of log difference of world commodity price index; first three lags of nominal EUR/USD; first three lags of long term interest rate and a constant sults with details about the instrumental sets in the note under the table. When only 10 or 19 instruments are involved in the information set, the estimated parameters up to ρ are very volatile but also quite interestingly the p value at the overidentifying restrictions test is, compared to all previously performed estimations, very low yet the null hypothesis can not be rejected at conventional significance levels. Also sole parameters β_π and β_y in the model with only 10 instruments are statistically significant only at either 0.1 % or 0.05 % significance level.

In the estimations with 28 instruments and more, however, the response parameters seem to be very robust to the changes of the instruments. When for example to the model with 28 instruments other 21 instruments are added, β_π falls only by 0.19 basis points and β_y rises by 0.28 basis points. Finally the Taylor rule with interest rate inertia seems to be more robust, because β_π, β_y have a tendency to fall as the number of instruments rises in the model without ρ, on the other hand in the Taylor rule with ρ the coefficients seem to be stabilized.

Table 6.3: Different instrumental sets

Num. of instruments	ρ	β	β_π	β_y	J-stat.
10	0.96***	-2.86	2.61**	1.62*	9.62
	(0.02)	(2.15)	(1.07)	(0.90)	(0.14)
10		-1.18	0.48*	1.82***	9.05
		(0.81)	(0.38)	(0.27)	(0.24)
19	0.96***	-2.61	2.42***	1.82**	14.71
	(0.01)	(1.75)	(0.85)	(0.72)	(0.47)
19		-0.61	1.56***	0.52**	12.44
		(0.45)	(0.23)	(0.21)	(0.71)
28	0.96***	-0.80	1.44***	2.22***	15.62
	(0.00)	(0.65)	(0.29)	(0.46)	(0.90)
28		0.76*	0.80***	0.63***	21.94
		(0.45)	(0.23)	(0.21)	(0.63)
34 (baseline)	0.96***	-0.43	1.26***	2.39***	17.84
	(0.00)	(0.61)	(0.27)	(0.44)	(0.96)
34 (baseline)		0.96***	0.70***	0.67***	23.82
		(0.35)	(0.15)	(0.14)	(0.81)
37	0.96***	-0.36	1.22***	2.51***	19.28
	(0.00)	(0.59)	(0.27)	(0.44)	(0.97)
37		1.09**	0.66***	0.68***	25.52
		(0.44)	(0.18)	(0.14)	(0.85)
49	0.96***	-0.45	1.25***	2.50***	22.96
	(0.00)	(0.48)	(0.21)	(0.38)	(0.97)
49		1.28***	0.55***	0.68***	27.70
		(0.44)	(0.17)	(0.13)	(0.98)

Note: HAC standard errors in the parentheses; *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$; at J-stat. column p value of the null hypothesis of the overidentifying restrictions test in the parentheses; 10 instruments - first three lags of output gap, inflation and interest rate and a constant; 19 instruments - first six lags of output gap, inflation and interest rate and a constant; 28 instruments - first six lags of output gap, inflation and interest rate, and first three lags of log difference of world commodity price index, first three lags of nominal exchange rate EUR/USD, and three lags of long term interest rate and a constant; 37 instruments - three lags of unemployment rate added to the baseline instrumental set; 49 instruments - baseline model instrumental set but with six lags of log difference of world commodity price index, six lags of nominal exchange rate EUR/USD and six lags of long term interest rate and unemployment

6.3 Added regressors

This section allows the central bank to respond to other variables besides the output gap and expected inflation by adding additional regressors to the baseline model with smoothing term, which was chosen because it had appeared to be more robust.

Not quite surprisingly, when deviation of money aggregate M3 growth from the reference value 4.5 % is added to the baseline model, it comes as highly statistically significant and also with the "right", positive sign - see table 6.4. So the ECB reacts to a one percent point increase of M3 growth above its reference value by increasing nominal interest rate by 0.16 percent point.

When nominal exchange rate EUR/USD is added as a regressor, the response coefficient is statistically insignificant.

The same result arises when a lagged inflation is added to the baseline. At the same time the response parameter β_π at the expected future value remains statistically significant and nearly unchanged along with the rest of the parameters. Therefore, it seems possible to conclude, that the ECB is indeed forward-looking rather than backward-looking. Clarida, Gali and Gertler [6] came to the same conclusion evaluating monetary policy of Bank of Japan, Bundesbank and Federal reserve in a period from 1974 to 1993. In all three cases the lagged inflation turned out to be statistically insignificant while the rest of the coefficients remained nearly unchanged, too.

Table 6.4: Added regressors

Added variable	ρ	β	β_π	β_y	β_{added}	J-stat.
none	0.96***	-0.43	1.26***	2.39***		17.84
baseline model	(0.00)	(0.61)	(0.27)	(0.44)		(0.96)
M3	0.95***	0.14	0.88***	1.88***	0.16***	17.88
	(0.00)	(0.50)	(0.24)	(0.28)	(0.05)	(0.97)
EUR/USD	0.96***	-0.27	1.10**	2.51***	4.54	16.82
	(0.00)	(0.61)	(0.29)	(0.52)	(3.53)	(0.95)
π_{t-12}	0.96***	-0.22	1.30***	2.42***	-0.15	18.24
	(0.00)	(0.86)	(0.32)	(0.50)	(0.43)	(0.93)

Note: HAC standard errors in the parentheses; *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$; at J-stat. column p value of the null hypothesis of the overidentifying restrictions test in the parentheses; baseline instrumental set contains first 6, 9th, 12th lagged values of output gap, inflation, short term interest rate, first three lags of log difference of world commodity price index, first three lags of nominal EUR/USD, first three lags of long term interest rate and a constant; in a model with M3 added as a regressor three lags of money growth are added to the instrumental set

Conclusion

The aim of this thesis was to estimate forward-looking Taylor rule for the European central bank from the time of establishment to the time of Expanded Asset Purchase Programme. The estimation was performed by the generalized method of moments. The baseline model with the interest rate inertia was formulated and estimated with various measurements of output and inflation. For the comparison also model without interest rate inertia was estimated. Robustness of the baseline model was then tested and finally additional regressors were added.

Both output and inflation response parameters from the estimation of the baseline model turned out to be statistically significant so it seems that the ECB responds to both output gap and the expected inflation. The central bank response to an increased expected inflation seems sufficient enough for the real interest rate to be increased as well so the Taylor principle seems to hold in the model with interest rate inertia assumption. This model also seems more robust than the model without the interest rate inertia where on the contrary, the response to the increase in the expected inflation seems to be insufficient for the Taylor principle to hold. The estimation of the smoothing term describing degree of the ECB interest rate inertia was very high and highly statistically significant. The results are also very sensitive to the choice of the data measurement. When the output gap is measured by the Index of industrial production (preferred by many authors), the estimated output gap response parameter is much smaller than in case of GDP measurement.

The actual interest rate path shows a considerable deviations from the interest rate path resulting from the Taylor rule without the smoothing term from the year 2010 onwards. When the shadow interest rate is used as the explained variable, the shadow interest rate path also shows significant deviation from the path resulting from the estimated Taylor rule especially during the era of recent crisis and then in 2014 onwards. The enhanced Credit Support was not launched until 2015 and it seems to be, according to the estimated Taylor rule, rather a discretionary reaction.

The estimation also allowed to quantify the inflation target officially only vaguely defined as „close but below 2 %“. The inflation target for the inflation measured by HICP seems to be 1.38 %, when the results from the model with interest rate inertia are used, and exactly 2 % when the results from model without interest rate inertia are utilized. The inflation targets for the core inflation are 1,85 % and 1.69 % respectively.

Not surprisingly, due to the Bundesbank, the ECB predecessor, sizeable emphasis on the money growth aggregate, the deviation of M3 aggregate growth from the reference value 4.5 % comes as statistically significant and with positive response parameter when added as a regressor to the baseline model. On the contrary the nominal EUR/USD exchange rate turns out to be statistically insignificant. The same outcome arises when a twelve-month lagged inflation is added to the baseline model, thus the forward-looking specification of the Taylor rule seems more preferable to the backward-looking one.

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Appendix

A. Detailed algebraic manipulation from sections 3.1 and 3.2

From section 3.1 the IS-PC-MR model with modified lag structure is:

$$\pi_t = \pi_{t-1} + \zeta(y_{t-1} - y^*) \tag{A.1}$$

$$(y_t - y^*) = -\zeta\lambda(\pi_{t+1} - \pi^*) \tag{A.2}$$

$$(y_t - y^*) = -\gamma(r_{t-1} - r^*) \tag{A.3}$$

Substituting A.1 into A.2 yields:

$$(y_t - y^*) = -\zeta\lambda(\pi_t + \zeta(y_t - y^*) - \pi^*)$$

$$\frac{1}{-\zeta\lambda}(y_t - y^*) = \pi_t + \zeta(y_t - y^*) - \pi^*$$

$$-\left(\zeta + \frac{1}{\zeta\lambda}\right)(y_t - y^*) = \pi_t - \pi^* \tag{A.4}$$

Now substituting A.3 into A.4:

$$-\left(\zeta + \frac{1}{\zeta\lambda}\right) - \gamma(r_{t-1} - r^*) = \pi_t - \pi^*. \tag{A.5}$$

Phillips curve A.1 can be now plugged back to A.5:

$$\left(\zeta + \frac{1}{\zeta\lambda}\right)\gamma(r_{t-1} - r^*) = \pi_{t-1} + \zeta(y_{t-1} - y^*) - \pi^*$$

$$r_{t-1} - r^* = \frac{1}{\gamma\left(\zeta + \frac{1}{\zeta\lambda}\right)}[(\pi_{t-1} - \pi*) + \zeta(y_{t-1} - y^*].$$

From equation 3.11 from section 3.2 the endogenized IS curve is:

$$y_t = y^* - \gamma\alpha_\pi(\pi_t - \pi^*) - \gamma\alpha_y(y_t - y^*)$$

And thus:

$$y_t - y^* + \gamma\alpha_\pi(\pi_t - \pi^*) + \gamma\alpha_y(y_t - y^*) = 0$$

$$(y_t - y^*)(1 + +\gamma\alpha_\pi) + \gamma\alpha_\pi(\pi_t - \pi^*) = 0$$

$$y_t = y^* - [\gamma\alpha_\pi/(1 + \gamma\alpha_y)](\pi_t - \pi^*).$$