University of Economics, Prague

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Convergence of Output and Wages in Europe

Diploma Thesis

Author: Bc. Lukáš Jordán
Supervisor: Ing. Pavel Potužák, Ph.D
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I hereby declare on my word of honour that I have written the diploma thesis independently with using the listed literature.

Bc. Lukáš Jordán

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Abstract

The diploma thesis aims to analyse the convergence within Europe. The analysis of convergence is done in the sense of β and σ as well, not only for real output per capita but also for wages. The neoclassical model of growth (the RCK model) predicts the unconditional convergence of -0.020 and -0.068 for α amounts to 0.70 and 0.35, respectively. Besides, it can be shown that the growth model implies σ-convergence. The graphical and empirical part verifies the presence of the convergence processes and also that the output is more converged than wages. The final speed equals to -0.017 and -0.040 for unconditional and conditional convergence, respectively. The speed of wage convergence is not precisely determined. The only finding of wages is that they converge faster than output. In addition, the σ-convergence is confirmed for both variables.

Key Words: β-convergence, σ-convergence, neoclassical growth model, output, wages

JEL classification: O41, O47, O52

Abstrakt

Diplomová práce si klade za cíl analyzovat konvergenci zemí v rámci Evropy. Analýza konvergence se provádí ve smyslu β i σ nejen pro proměnnou reálný produkt na obyvatele, ale i pro proměnnou reálné mzdy. Neoklasický model růstu (RCK model) předpovídá nepodmíněnou rychlost konvergence -0,020, případně -0,068 pro α rovno 0,70, případně 0,35. Kromě toho lze ukázat, že implikací růstového modelu je i σ konvergence. Grafická i empirická část potvrzuje přítomnost konvergenčních procesů a také, že produkt je konvergován více než mzdy. Výsledná rychlost se rovná -0,017, respektive -0,040 pro nepodmíněnou respektive podmíněnou konvergenci produktu. Rychlost konvergence mezd se nepodařilo jednoznačně určit, jediným závěrem tak může být, že mzdy konvergují rychleji než produkt. Pro obě veličiny se také potvrdila σ konvergence.

Klíčová slova: β konvergence, σ konvergence, neoklasický model růstu, produkt, mzdy

JEL klasifikace: O41, O47, O52

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'A key economic issue is whether poor countries or regions tend to grow faster than rich ones...'

Barro and Sala-i-Martin (1992)

Introduction

The theory of convergence is the prominent topic in the discussion of the long-run growth (Barro and Sala-i-Martin, 1992). The convergence of output was investigated several times since the 1990s. However, the wage convergence debate has been arising in the present, for example, in Naz, Ahmad, and Naveed (2017). Although the output might be decently converged, this does not imply that wages converge as well. The household well-being usually depends on wages rather than output. The further noteworthy reason for researching the convergence of wages is that, in 1993, the European Union established the European Single Market which would tend to lead to a strong convergence in prices including a price of labour.

The first part of the thesis is compounded of two sections, the literature review and the Ramsey-Cass-Koopmans neoclassical growth model. First, the literature review discusses the historical evolution and the current debate of the theories of convergence and, as the dynamic panel data is used, appropriate estimators thereof. Second, the neoclassical theory of growth is derived, wherein the β-convergence is demonstrated. Besides, it is shown that the growth model also implies σ-convergence. Moreover, this section handles with the error caused by the log-linearisation by using the Taylor approximation.

The further part consists of three sections, the introduction of the dataset, the graphical analysis, and the empirical analysis. First, as stated before, the dataset is formed by dynamic panel data for European countries. It contains, among others, real GDP per capita in PPP as a proxy variable for output per capita, and real wages in PPP, which are used for both the graphical and the empirical analysis. Second, the graphical analysis provides three figures for the subjected variables. The first figures portray the relationship between the growth and the level of the indicators. The remaining pictures show the σ-convergence in two ways, the evolution of dispersions in time, and maps of Europe depicting distributions of both output and wages. Third, the empirical analysis shows estimations of the speed of β-convergence and σ-convergence. The speed of convergence is estimated by estimators of the difference GMM, the system GMM, the pooled-OLS, the random-effect, the fixed-effect, and the first-difference, whereas estimations of σ-convergence are done by the OLS.

The thesis aims to evaluate the convergence of both real output per capita and real wages in Europe. The convergence is investigated in two senses, β-convergence and σ-convergence.

1 Literature review

The first pieces of papers concerning the analysis of convergence of output originate in the 1980s, but the beginning works were far from the current studies. First, the most crucial differences lie in the lack of formality in the convergence debate. The convergence regression equations were not derived from the theoretical model of growth. Second, economists did not distinguish between conditional and unconditional convergence and focus merely on unconditional convergence, which may have led to some misunderstandings (Islam, 2003). For example, a highly cited article written by Baumol (1986) estimated the speed of convergence for 16 highly developed countries whereof the very long data was available. The author confirmed the convergence hypothesis and extended the findings even beyond the edge of the free-market countries. The author concluded that there are, at least, three convergence groups, including centrally-planned. DeLong (1988) criticised the analysis of Baumol (1986) as it suffered from the sample selection bias. The sample of 16 countries was a priori converged. The choice needs to be made ex-ante rather than ex-post, meaning that countries must have appeared to have the potential to converge at the initial point in order to be involved in the analysis of convergence. Nonetheless, as Islam (2003) said, Baumol's (1986) paper had represented a significant piece of work in further debate.

Later on, Barro and Sala-i-Martin (1992) and Mankiw, Romer, and Weil (1992) formally derived the convergence processes in the neoclassical growth model and the Solow growth model, respectively. These findings opened a space for new theories in the convergence debate, such as β-convergence versus σ-convergence or conditional versus unconditional convergence, which shall be discussed formally in section 3. Both Barro and Sala-i-Martin (1992) and Mankiw, Romer, and Weil (1992) mainly focused on β-convergence.

Barro and Sala-i-Martin (1992) research the convergence across the United States and other 98 countries. The dataset comprises over 20 years. First, the analysis provides strong evidence for the convergence within the United States. U.S. states have been converging with a speed of around 2%. As no additional variables for the steady-state control are included, U.S. states seem to be sufficiently homogeneous to converge unconditionally. Second, convergence across 98 other countries only occurs if the regression model incorporates additional variables denoting a steady state. The speed of convergence is similar to that of U.S. states, which is 2%. Although the neoclassical Solow growth theory supports the conclusion that economies converge, authors find a discrepancy. The theory requires the value of the capital share of output, α, to be around 0.8,[1] but α hardly reaches that value. Thus, gained results do not match some theoretical findings.

Mankiw, Romer, and Weil (1992) suggest the solution of the discrepancy between the speed of convergence predicted by the Solow model and the actual speed of convergence obtained from empirical analysis. Authors introduce the augmented Solow model. The model incorporates both physical and human capital. A capital share of output can be doubled by including human capital in the production function. Mankiw, Romer, and Weil (1992) propound the idea that each production factor contributes one third to the whole production. Since both capital shares of output are equal to around 0.3, the overall capital share approaches 0.6, which predicts the speed of convergence of 2%. Their empirical analysis covers almost the whole world (apart from centrally planned economies) over the period 1960-1985. The evidence shows that adding the human capital in the regressions provides more significant results that tend to be closer to the predicted value.

Previously mentioned papers are done by the cross-sectional analysis, for example, Barro and Sala-i-Martin (1995) (chapter 11) use the panel-data approach to estimate the speed of convergence for the European, Japanese, and U.S. regions. The findings do not vary from the cross-sectional analysis, and the results confirm the convergence across investigated regions with the speed of 2-3%. Nevertheless, Caselli, Esquivel, and Lefort (1996) criticise both approaches. Authors claim that both techniques suffer from endogeneity bias. They suggest using the generalised method of moments proposed by Arellano and Bond (1991). The estimated speed of convergence rapidly grows to 10%. However, Bond, Hoeffler, and Temple (2001) come with their piece of evidence. The Arellano and Bond's (1991) approach appears to be biased in growth model regressions. Time series of output are likely to exhibit a high persistence and under these conditions, the Arellano and Bond's (1991) method fails. Therefore, Bond, Hoeffler, and Temple (2001) recommend using either the level GMM invented by Arellano and Bover (1995) or the system GMM developed by Blundell and Bond (1998). This recommendation is not a general law so that

[1] By keeping reasonable values of all other variables, such as population growth, growth of technologies, the depreciation rate, and the saving rate.

they propound a rule how to decide whether to use Arellano and Bond (1991), Arellano and Bover (1995), or Blundell and Bond (1998), which is incorporated in section 6.

Yamarik (2006) uses the general method of moments (GMM) proposed by Arellano and Bond (1991) to estimate the speed of convergence within the U.S. states. Despite the fact that the Arellano and Bond's (1991) approach is used, it satisfies (although the author is not likely to be aware of that fact) the rule of Bond, Hoeffler, and Temple (2001). The method vanishes an omitted-variable bias and an endogeneity bias. The paper consists of three models, the Solow model, the open-economy version of the Solow model, and the augmented Solow model. The former suggests (using standard values of parameters) the speed of convergence of 2%, and the latter predicts 4%. The open-economy Solow model lies in the middle. The Yamarik's (2006) empirical analysis concludes that the speed of convergence equals to 4%, which contradicts the findings of Barro and Sala-i-Martin (1992), Mankiw, Romer, and Weil (1992), and also Caselli, Esquivel, and Lefort (1996), and confirms the outcome of the basic Solow model.

Higgins, Levy, and Young (2006) study the convergence on the U.S. county-level instead of country level. This approach increases the number of observations to 3,058, whereas similar papers exploit merely around 100-150 observations. The second advantage of the county-level approach allows investigating convergence within regional groups of countries with keeping a decent number of observations for each group. Third, 41 different variables can be used to study their influence on the balanced growth path. The broad sample provides a sustainable degree of freedom for such an extensive number of independent variables. In order to avoid inconsistency linked with the OLS method, Higgins, Levy, and Young (2006) use a three-stage least squares method (3SLS) instead of the GMM methods. This approach estimates the speed of conditional convergence between 6% and 8%, whereas the OLS estimator finds the value of 2%, which is a standard outcome thereof. It appears that both methods provides significantly different estimations.

The contribution of Young, Higgins, and Levy (2008) is one of the σ-convergence studies which is not paid attention as that of β-convergence. This paper confirms that β-convergence does not imply σ-convergence. Authors find the statistical evidence for β-convergence, however, σ-convergence is rejected for the majority of U.S. regions. Remarkably, it appears that the richest countries form a σ-convergence club. The difference between β and σ convergence is discussed in section 2.

Naz, Ahmad, and Naveed (2017) focus on the convergence of wages in the European Union. They attempt to answer three questions. First, do wages converge within the EU? Second, do EU regions tend to converge unconditionally or do they have their particular steady state? Third, do borders play a role in the convergence of wages? They use dataset covering NUTS2 level over the 1996-2016 period, which counts 203 observations. The analysis was based on a methodology of unit root tests. Their findings are as follows. First, wages within the EU tend to converge in terms of average wages. Second, the EU regions converge to their own steady state rather than to a mutual steady state. This validates solely the conditional convergence. Third, borders appear to matter. Regions on borders do not seem to converge. Although the European Union is attempting to create a common labour market, social barriers persist and prevent workers from moving, which keeps unequal prices of labour in the EU.

1.1 Conclusion of the literature review

The prominent attention is paid to the β-convergence. The most significant pieces of work are represented by Barro and Sala-i-Martin (1992), and Mankiw, Romer, and Weil (1992) in which the theoretical derivation of the β-convergence is shown. Despite that, the estimated speed of convergence appears to be biased as the authors exploit the OLS method that proposes the value of approximately 2%. Further studies, e.g. Yamarik (2006), improve the estimation by the generalised methods of moments propounding a faster convergence of 4%. The focus on wages emerges in recent studies, for example, Naz, Ahmad, and Naveed (2017) find barriers for wage convergence in spite of the effort to form the common labour market by the European Union.

2 Definitions of convergence

There are many ways to define convergence of countries. This section covers four definitions, namely β-convergence, σ-convergence, conditional convergence, and unconditional convergence.

2.1 β-convergence

Sala-i-Martin (1996) defines the β-convergence as follows: *"There is (absolute) β-convergence if poor economies tend to grow faster than rich ones"* (Sala-i-Martin, 1996:p.1020). The definition has the following mathematical expression.

$$\log\left(\frac{y_{i,t+T}}{y_{i,t}}\right)\frac{1}{T} = \alpha - \beta\log(y_{i,t}) + \epsilon_{i,t} \tag{1}$$

Parameter β denotes the relationship between a level of output and a growth rate of output. The convergence occurs only if $\beta > 0$, which means that the relationship between a level and a growth of output needs to be negative. However, this condition does not ensure the existence of convergence, as can be seen on Fig 1.

2.2 σ-convergence

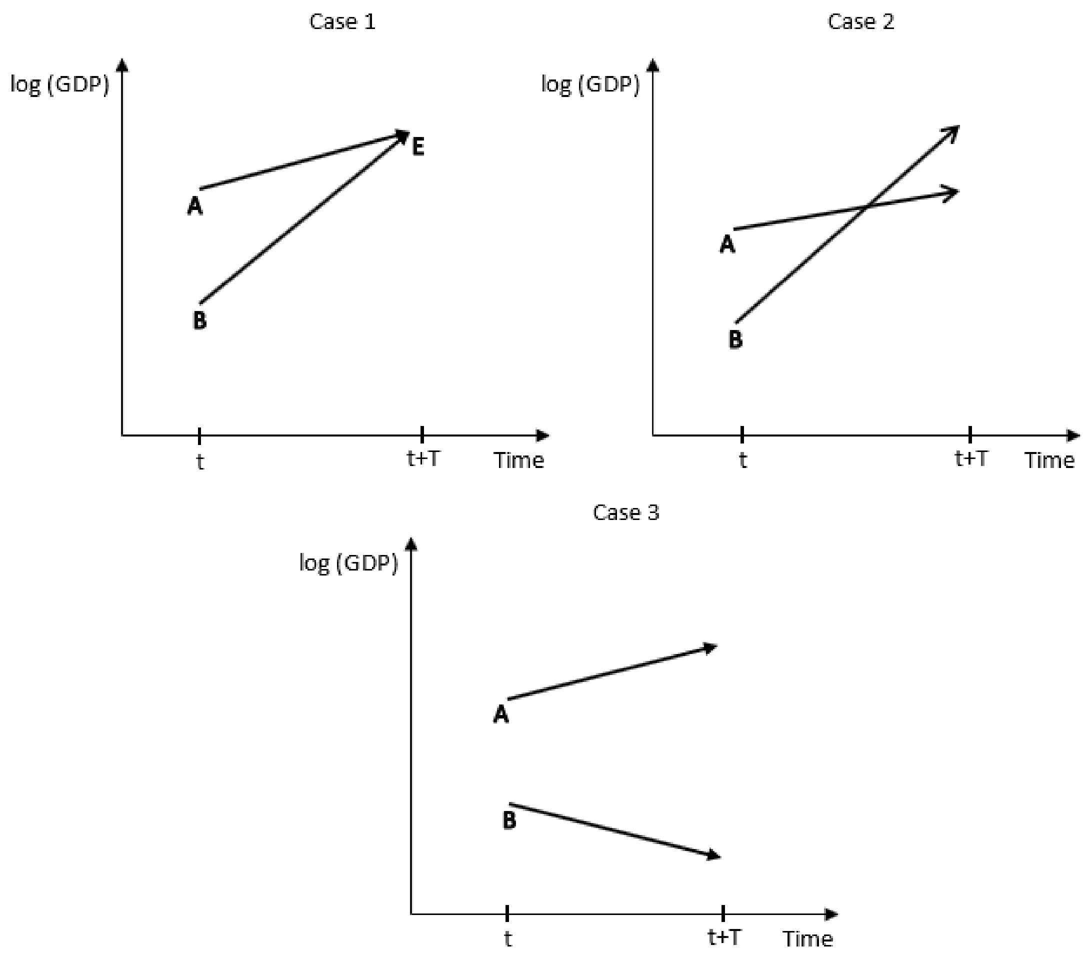
The definition of the σ-convergence says that: *"A group of economies are converging in the sense of σ if the dispersion of their real per capita GDP levels tends to decrease over time"* (Sala-i-Martin, 1996:p.1020), which leads to equation (2).

$$\sigma_{t+T} < \sigma_t, \tag{2}$$

σ_t represents a standard deviation of the logarithm of real output per capita at time t. As was mentioned before, if a group of economies converge in the sense of β-convergence, it does not mean convergence in the sense of σ-convergence. Sala-i-Martin (1996) adds that β-convergence is a necessary but not sufficient condition of the σ-convergence. As σ-convergence coincides with β-convergence, σ-convergence is a sufficient proof of the convergence of countries.

Figure 1 shows possible scenarios of economic growth from a convergence perspective. Case 1 reveals the situation when countries converge. Both countries are growing toward the steady state, E. Since country B starts with a lower level of output, B is growing faster until it reaches the steady state. Since both countries are growing toward the same

Figure 1: Cases of the convergence process



Source: Sala-i-Martin (1996), modified

point, dispersions of their outputs are decreasing, and both the β-convergence and the σ-convergence appear. This situation may be considered as an implication of the neoclassical growth theory. Case 2 is almost identical to case 1, country B is growing faster than country A but they do not converge to the same point. In this situation, only the β-convergence might occur. The σ-convergence cannot appear, because the dispersion of output is not solely decreasing over time. Finally, in case 3, country A is merely growing while the level of output of country B starts at the lower level and is falling (not necessarily, it may slightly grow). In other words, countries converge neither in the sense of β-convergence nor in the sense of σ-convergence.

2.3 Unconditional convergence vs conditional convergence

Terms conditional and unconditional convergence are mainly related to β-convergence. Islam (2003) defines unconditional convergence, the so-called absolute convergence, as a situation when all economies converge to the same steady state. In other words, the sign of the β-coefficient is negative even if the regression equation does not include other variables describing the steady state of each country. On the other hand, conditional convergence, the so-called relative convergence, admits that economies might have their own steady states. In order to find a piece of evidence of the β-convergence, one needs to incorporate additional variables to obtain a negative sign of β coefficient. Thus, conditional β-convergence is a weaker concept than the absolute version. In reality, economies have different characteristics such as the population growth, the growth of technology, and the capital share of output. Therefore, economies do not converge to the same steady state, if they are not homogeneous enough. Only countries with the same parameters tend to converge towards one steady state.

Equation (1) represents the unconditional convergence, whereas equation (3) illustrates conditional convergence, where $X_{j,i,t}$ denotes the vector of j variables that describes the steady state (Sala-i-Martin, 1996).

$$\log\left(\frac{y_{i,t+T}}{y_{i,t}}\right)\frac{1}{T} = \alpha - \beta\log(y_{i,t}) + \Psi_j X_{j,i,t} + \epsilon_{i,t+T} \tag{3}$$

3 Neoclassical model of economic growth

The neoclassical model of economic growth was developed by Ramsey (1928) and extended by Koopmans (1963), and Cass (1965). The model is also known as the Ramsey-Cass-Koopmans (RCK) model. The RCK model is the contemporary long-run and very long-run model of economic growth. Convergence processes of output, capital, and wages are the direct implications of this model. The convergence of all three variables can be mathematically expressed. A noteworthy fact is that the corresponding speeds of convergence of capital, output, and wages have the same value. This section shows a concise introduction to the RCK model, and mathematical derivation of speeds of convergence of both output and wages. Detailed mathematical steps are incorporated in Appendix A.

The RCK model assumes that an economy consists of two sectors, firms and consumers, no government, and no foreign economies. Firms produce a single homogeneous output, $Y(t)$, created by three production factors, the level of technology $A(t)$, labour $L(t)$, and capital $K(t)$. By producing output, firms want to maximise their profits. The output is generated by an invariable production function, $F[K(t),A(t)L(t)]$. The production function needs to be twice differentiable with constant returns to scale, have a positive marginal product, and a diminishing marginal rate of substitution between factors. As a closed economy and no government are assumed, the output is either consumed or invested. The level of technology steadily grows over time by g per cent per period. The growth of the capital stock at time t consists of the difference between investment, $I(t)$, and depreciation, $\delta K(t)$, where δ denotes a depreciation rate. The process is known as the law of motion of capital. And finally, the labour force increases by n per cent per period (Barro and Sala-i-Martin, 1992; Barro and Sala-i-Martin, 1995).

The second sector, consumers, want to maximise the present value of their lifetime utility. The lifetime utility function of a representative agent is a function of her consumption, $U[C(t)]$. The utility function needs to be non-decreasing, twice differentiable with a positive marginal utility and a negative second derivative. The limit of the marginal utility goes to infinity, meaning that each person wishes to avoid a meagre consumption at any time. The number of consumers corresponds to the size of the labour force (Barro and Sala-i-Martin, 1992; Barro and Sala-i-Martin, 1995).

For the purpose of this thesis, the constant-relative-risk-aversion (CRRA) utility function is employed,

$$u(t) = \frac{C(t)^{1-\theta} - 1}{1-\theta}, \tag{4}$$

where θ is a coefficient of a risk aversion. The bigger θ, the greater risk aversion. Additionally, the Cobb-Douglas labour-augmented production function is used,

$$Y(t) = K(t)^{\alpha}[A(t)L(t)]^{1-\alpha}, \tag{5}$$

as Barro and Sala-i-Martin (1995) propose, where α denotes a capital share of output. Moreover, Barro and Sala-i-Martin (1995) show that the labour-augmented type of the Cobb-Douglas production function is the only possible version so as to obtain a solution in which economies are steadily growing in a steady state.

3.1 First-order conditions

Cass (1965) suggests that the model may be solved as a benevolent social planner problem who wishes to maximise social welfare measured by the utility function. Equations from (6) to (9) are implications of assumptions of the model and have been already discussed. The parameter ρ denotes the subjective discount rate of which the representative agent discounts the future.

$$\max_{C} U[C(t)] = \int_0^\infty \frac{1}{(1+\rho)^t} \frac{C(t)^{1-\theta} - 1}{1-\theta} L(t)\, dt, \tag{6}$$

subject to

$$\dot{K}(t) = I(t) - \delta K(t) \tag{7}$$

$$Y(t) = I(t) + C(t) \tag{8}$$

$$Y(t) = K(t)^\alpha [A(t)L(t)]^{1-\alpha} \tag{9}$$

$$K(0) \geq 0 \tag{10}$$

$$\lim_{t\to\infty} [K(t) e^{-t \int_0^t r(\tau) d\tau}] \geq 0 \tag{11}$$

Equation (10) excludes a solution in which the capital would be negative. Last but not least, equation (11), the transversality condition, means that the present value of capital must be zero in infinity. If one were willing to hold capital at the end of days, it would not lead to an optimal solution, because one can increase utility by consuming possessed assets. This claim also excludes the opposite situation. No one can end in the debt in the present value in infinity because, as being said, no one would be willing to hold assets. In other words, there are no remaining savings and debts at the end of the days from the present value prospective. That leads to the optimal growth path (Barro and Sala-i-Martin, 1995).

As mentioned before, the RCK model might be solved as a benevolent social planner problem. That can be done by using the present-value Hamiltonian approach. The lower-case letter denotes the same variable in the per-effective-worker term, which needs to be used to obtain a solution that does not depend on the size of the population.

$$\mathscr{H} = B\,e^{-\Omega t}\,\frac{c(t)^{1-\theta} - D}{1-\theta} + \lambda\big[k(t)^{\alpha} - c(t) - (n+g+\delta)k(t)\big], \tag{12}$$

where B denotes $A(0)^{1-\theta}L(0)$, Ω signifies $\rho - (1-\theta)g - n$, and D stands for $\frac{1}{(A(0)e^{g})^{1-\theta}}$. First-order conditions are as follows: (Barro and Sala-i-Martin, 1992; Barro and Sala-i-Martin, 1995)

$$[c]: \qquad B\,e^{-\Omega t}c(t)^{-\theta} - \lambda = 0 \tag{13}$$

$$[k]: \qquad \lambda\big[\alpha k(t)^{\alpha-1} - (n+g+\delta)\big] = -\,\dot{\lambda} \tag{14}$$

$$[\lambda]: \qquad k(t)^{\alpha} - c(t) - (n+g+\delta)k(t) = \dot{k}. \tag{15}$$

The combination of the first-order conditions creates a system of non-linear differential equations. Equation (16) signifies the corresponding Euler equation (EE). The EE expresses the optimal consumption path with respect to the level of capital. Equation (17) represents the budget constraint (BC). The BC shows the allocation of output between consumption and gross investment. The gross investment consists of net investment, depreciation, and equipping new labour force and technology with capital.

$$\frac{\dot{c}(t)}{c(t)} = \frac{\alpha k(t)^{\alpha-1} - (\rho+\delta+\theta g)}{\theta} \tag{16}$$

$$\frac{\dot{k}(t)}{k(t)} = k(t)^{\alpha-1} - c(t)k(t)^{-1} - (n+g+\delta) \tag{17}$$

3.2 β-convergence of output

As the solution of the RCK model is the system of nonlinear differential equations, the first step needs to be log-linearisation. This step ensures that the final solution has an analytical form. Both equations ought to be transformed into a matrix, where hat letters denote a percentage deviation of the variable from the steady state (Barro and Sala-i-Martin, 1992; Barro and Sala-i-Martin, 1995).

$$\begin{pmatrix} \dot{\hat{k}} \\ \dot{\hat{c}} \end{pmatrix} = \begin{pmatrix} \rho - n - (1-\theta)g & (n+g+\delta) - \frac{\rho+\delta+\theta g}{\alpha} \\ \frac{(\alpha-1)(\rho+\delta+\theta g)}{\theta} & 0 \end{pmatrix} \begin{pmatrix} \hat{k} \\ \hat{c} \end{pmatrix} \tag{18}$$

The matrix equation (18) represents the system of linear differential equations with constant coefficients. Such differential equations always have an analytical solution.

$$\begin{pmatrix} \log[k(t)] \\ \log[c(t)] \end{pmatrix} = \begin{pmatrix} \log[\bar{k}] \\ \log[\bar{c}] \end{pmatrix} + \begin{pmatrix} e^{\beta_1 t} & e^{\beta_2 t} \\ e^{\beta_1 t}\frac{A_{21}}{\beta_1} & e^{\beta_2 t}\frac{A_{21}}{\beta_2} \end{pmatrix} \begin{pmatrix} C1 \\ C2 \end{pmatrix}, \tag{19}$$

Equation (19) is the solution of the system of linear differential equations, where C_1 and C_2 denote integrative constants, and β_1 and β_2 represent eigenvalues of the matrix of constant coefficients. Letters with bar signify values of the variables in the steady state. Barro and Sala-i-Martin (1995) say that due to the transversality condition, C_1 cannot be bigger than zero. People would be decreasing consumption and increasing the level of capital. That would lead to the destruction of the economy. Also, C_1 cannot be lower than zero. This case would destruct the economy as well. People would be increasing consumption by decreasing the amount of capital and the economy would collapse. It implies that C_1 must be zero. That finding falls apart the matrix equation into two single equations. Then, the value of C_2 can be calculated by evaluating $t = 0$. Equation (20) and equation (21) show the convergence processes of capital and consumption.

$$\log k(t) = \log \bar{k} + e^{\beta_2 t}(\log k(0) - \log \bar{k}) \tag{20}$$

$$\log c(t) = \log \bar{c} + e^{\beta_2 t}(\log c(0) - \log \bar{c}) \tag{21}$$

The second step deals with βs. From equation (22), it arises that the first β must be positive and the second one negative. Let us indicate β_2 as the negative one. The economic theory defines $-\beta_2$ as the speed of convergence. The speed of convergence expresses how fast economies tend to approach the steady state (Barro and Sala-i-Martin, 1992; Barro and Sala-i-Martin, 1995).

$$\beta_{1,2} = \frac{[\rho - n - (1-\theta)g] \pm \sqrt{[\rho - n - (1-\theta)g]^2 - \frac{4(\alpha-1)(\rho+\delta+\theta g)}{\theta}\left[(n+g+\delta) - \frac{\rho+\delta+\theta g}{\alpha}\right]}}{2} \tag{22}$$

The final step proves that the convergence of output is the same as the convergence of capital. Since $\log y(t) = \alpha \log k(t)$, the convergence equations of output and capital must be equal. Inserting $\log y(t)$ into equation (20) creates equation (23), the convergence equation of output. (Barro and Sala-i-Martin, 1992; Barro and Sala-i-Martin, 1995).

$$\log y(t) = \log \bar{y} + e^{\beta_2 t}(\log y(0) - \log \bar{y}) \tag{23}$$

3.3 β-convergence of wages

In order to obtain the convergence-of-wages equation, the formula for wages needs to be found. In the competitive economy, wages are determined by the marginal product of labour.

$$W = \frac{\partial Y}{\partial L} = (1-\alpha)K^{\alpha}A^{(1-\alpha)}L^{-\alpha} \tag{24}$$

Then, to be comparable with the rest of the model, wages need to be transformed into the per-effective-worker form.

$$w = \frac{W}{A} = (1-\alpha)k^{\alpha} \tag{25}$$

Equation (26) expresses the log-linearisation of equation (25).

$$\hat{w}(t) = \alpha\,\hat{k}(t). \tag{26}$$

Equation (27) displays the derivative of equation (26) with respect to time.

$$\dot{\hat{w}}(t) = \alpha\,\dot{\hat{k}}(t), \tag{27}$$

Inserting equations (26) and (27) into the matrix equation (18) substitutes capital for wages in the system.

$$\begin{pmatrix} \frac{1}{\alpha} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \dot{\hat{w}} \\ \dot{\hat{c}} \end{pmatrix} = \begin{pmatrix} \rho - n - (1-\theta)g & \alpha[(n+g+\delta) - \frac{\rho+\delta+\theta g}{\alpha}] \\ \frac{1}{\alpha}[\frac{(\alpha-1)(\rho+\delta+\theta g)}{\theta}] & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\alpha} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{w} \\ \hat{c} \end{pmatrix} \tag{28}$$

Equation (28) can be multiplied (from the left side) by the inverse matrix of $\begin{pmatrix} \frac{1}{\alpha} & 0 \\ 0 & 1 \end{pmatrix}$ and rearranged.

$$\begin{pmatrix} \dot{\hat{w}} \\ \dot{\hat{c}} \end{pmatrix} = \begin{pmatrix} \rho - n - (1-\theta)g & \alpha[(n+g+\delta) - \frac{\rho+\delta+\theta g}{\alpha}] \\ \frac{1}{\alpha}[\frac{(\alpha-1)(\rho+\delta+\theta g)}{\theta}] & 0 \end{pmatrix} \begin{pmatrix} \hat{w} \\ \hat{c} \end{pmatrix} \tag{29}$$

Equation (29) figures the analogous system of differential equations of wages and consumption as the system of capital and consumption. As the speed of convergence amounts to an eigenvalue, and eigenvalues are calculated by a determinant, and since the determinant of system (27) is identical to the determinant of system (16), the solution of the system of differential equations of wages and consumption must coincide with the solution for capital and consumption. It implies that the speed of convergence of capital is the same as the speed of convergence of wages. The equation of wage convergence appears to be equal to that of capital.

$$\log w(t) = \log \bar{w} + e^{\beta_2 t}(\log w(0) - \log \bar{w}) \tag{30}$$

3.4 Calibration

The crucial part of the model construction is the calibration of parameters. The RCK model exploits six parameters, n, g, δ, ρ, θ, and α that need to be evaluated at particular values. This subsection proposes values of those based on the previous analyses and pieces of work related to Europe. All parameters are recapitulated in table 1.

The θ parameter is taken from a survey that is written by Havranek et al. (2015). Authors of the paper collect the elasticity of intertemporal substitution, $\frac{1}{\theta}$, from 169 studies. The survey covers 17 European countries, among others. First, the aggregate elasticity of substitution is computed as an average of the above mentioned countries, which amounts to 0.465. Subsequently, the coefficient of the risk aversion, θ, equals to an inverse number of the calculated aggregate elasticity of substitution. The corresponding coefficient of risk aversion is 2.151. The value has two interpretations. First, people in Europe are risk-averse. Second, they prefer consumption smoothing over time, which implies no 'perverse' shape of the utility function.

The parameter of n can be obtained from the real data. The thesis uses indicators of the total population in 38 analysed countries over the 1995-2018 period taken from the World Bank (2019f). The growth rate is calculated as a weighted average, where the weight is the fraction of population of the country on the total population in particular year. The final growth rate of the population in Europe equals -0.04% per year. This value has a severe consequence. The sum of g and δ must be greater than 0.04 in order that the RCK model would not fail.

Nadiri and Prucha (1996) estimate the depreciation-rate parameter, δ. The estimated value of the depreciation rate of the physical capital equals 0.059. Although authors investigate the U.S. economy, Mankiw, Romer, and Weil (1992) argue that the depreciation rate does not vary across countries, or at least in countries sharing the know-how about the physical capital. The value of 0.059 can per se eliminate the negative growth rate of the population so that the RCK model is stable.

A lot of authors claim that the rate of technological growth g lies between 0.01 and 0.02, e.g. Mankiw, Romer, and Weil (1992), Barro and Sala-i-Martin (1992), Lucas (1988).

Let us simply use the average of both values, which means $g = 0.015$.

Evans and Sezer (2005) research the discount rate for 19 European countries. They suggest that the average utility discount rate in Europe equals to 0.01.

Table 1: Values of parameters of the RCK model

Parameters	Values	Description
n	-0.040	Population growth
g	0.015	Technology growth
δ	0.059	Depreciation rate
ρ	0.010	Subjective discount rate
θ	2.151	Coefficient of a risk aversion
α	0.350 | 0.700	Capital share of output
β	-0.068 | -0.020	Speed of convergence

Source: Barro and Sala-i-Martin (1992); Evans and Sezer (2005); Havranek et al. (2015); Lucas (1988); Mankiw, Romer, and Weil (1992); Bank (2019f); own calculations

Finally, the parameter α is underlying with regards to the speed of convergence. The broader the capital is considered, the bigger the share of output, α, and the slower the convergence process. Mankiw, Phelps, and Romer (1995) say that although the growth accounting suggests the capital share of output equals approximately one-third of the total income and labour gains two thirds, it may not be implausible that the values differ. If capital, k, is thought to be a compound variable of physical and human capital, the share thereof increases. Since the growth accounting counts the share of human capital as a part of the labour earnings, this rearrangement reduces the labour share of output. If one thinks of capital being only physical, the capital share of output is 0.35. This share creates the relatively fast speed of convergence, -0.068. The speed of convergence predicts that the product and wages reduce a half of the the gap between the initial and the steady state values in 10.2 years. On the contrary, should broad capital be considered, the capital share of output increases to 0.70. It reduces the speed of convergence to -0.020, and the half-life time extends to 34.7 years.

3.5 Error due to the Taylor approximation

Since the Dynare software (the extension of Matlab) is exploited, and because Dynare can work only with discrete models, the discrete version of the Ramsey-Cass-Koopmans neoclassical model is used for the numerical methods, as described in Brida, Cayssials, and Pereyra (2014) and Angeletos (2013) and extended into the per effective worker form.

Reiss (2000) analyses the error caused by the Taylor approximation. He claims that a direct comparison between the non-linear speed of convergence and the linear one can be calculated only at single points. Because of that issue, he proposes to compare the non-linear half-life, T, and the linear half-life, $\tilde{T}$. The relationship between those might be expressed as follows: should the order of the Taylor approximation increase, the linear half-life will approach the non-linear half-life, $\tilde{T} \to T$.

Reiss (2000) continues the analysis with the claim: if the gap between output per effective worker in time t, $y(t)$, and the initial output per effective worker, $y(0)$, equals to the half of the gap between the initial output per effective worker, $y(0)$, and the output per effective worker in the steady state, $\bar{y}$,

$$[y(t) - y(0)] = \frac{1}{2}[\bar{y} - y(0)], \tag{31}$$

the time required to obtain half-life in linear model[2] equals to

$$\tilde{T} = \frac{\log \frac{1}{2}}{-\beta}. \tag{32}$$

The half-life time in the non-linear model can be expressed by equation (33).

$$T = y^{-1}\left[\frac{\bar{y} + y(0)}{2}\right] \tag{33}$$

Table 2 displays the relative and the absolute gap between the linear and the non-linear model for different values of α and relative distances from the steady state, λ_0. Re-

[2]It can be done by substituting $y(t)$ for equation (23).

iss (2000) creates a similar table. The relative gap is defined as $\mu = \frac{\tilde{T}-T}{T}$. Equation (33) implies that the error mainly depends on the initial value of output, here expressed in the relative distance from the steady state, λ_0, and the parameter α that is, as mentioned above, the biggest moderator of the convergence speed and thereby the half-life time. As values in table 2 are similar to those of Reiss (2000), the finding may be generalised into three statements. First, unsurprisingly, the error decreases as the distance from the steady state decreases. Second, the error also decreases as the share of the capital, α, approaches the value of approximately 0.5. Third, if the share is high ($\alpha > 0.5$), the log-linearisation tends to err less in the relative form but the absolute value of the error grows.

Table 2: Errors due to the Taylor approximation with respect to λ_0 and α

α	0.35		0.70	
λ_0	μ [%]	$\tilde{T}-T$ [years]	μ[%]	$\tilde{T}-T$ [years]
0.9	5.46	0.62	-1.52	-0.61
0.8	10.88	1.19	-4.11	-1.70
0.7	14.68	1.55	-5.94	-2.51
0.6	22.22	2.20	-9.03	-3.93
0.5	21.12	2.87	-12.27	-5.54
0.4	36.29	3.22	-16.49	-7.82

Notes:

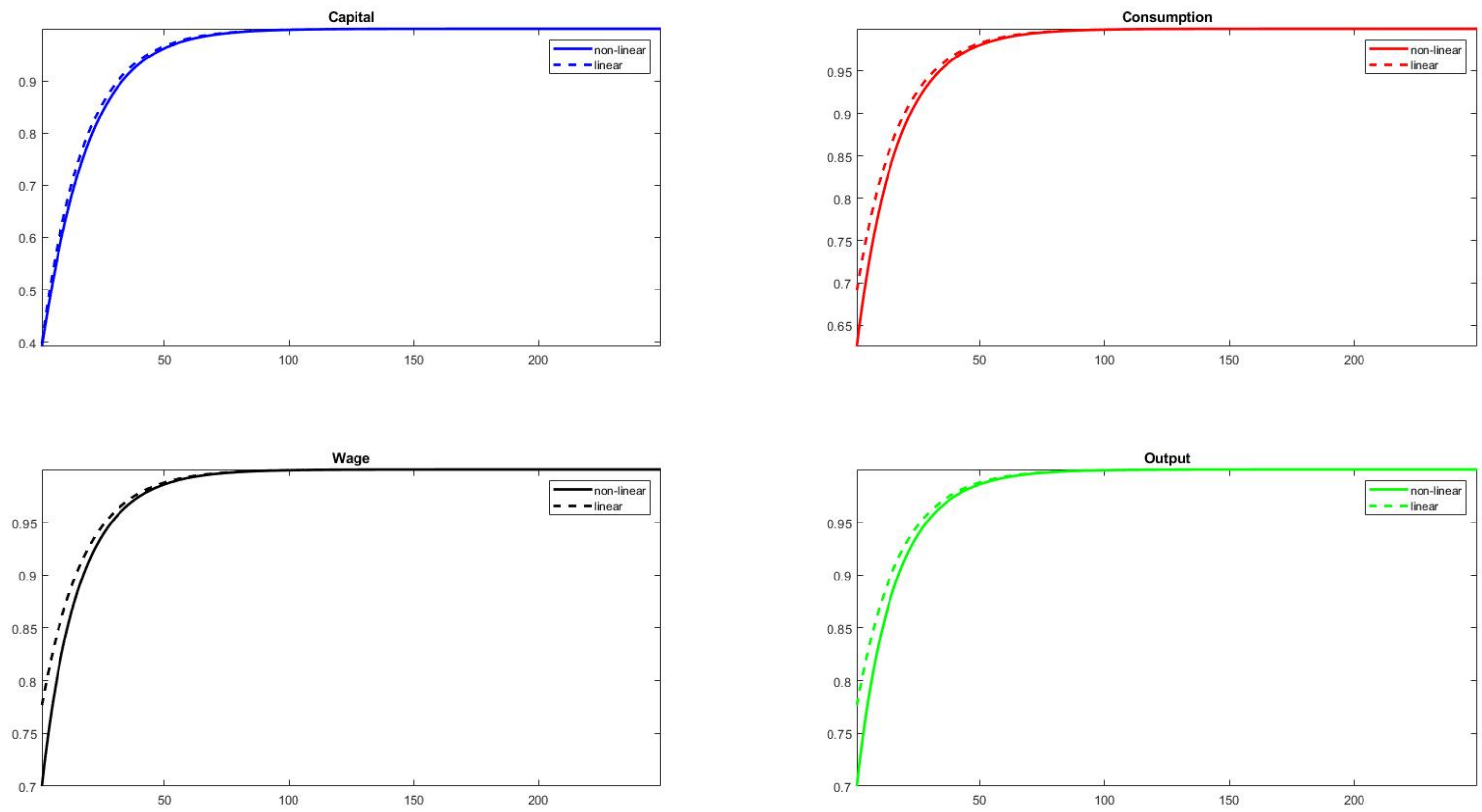
$\tilde{T}-T$	absolute number of years of which both methods differ
μ	relative number of years of which both methods differ
λ_0	percentage deviation of the initial value from the steady state

Source: own calculations

Figures 2 and 3 depict the saddle path of capital, consumption, output, and wage for both $\alpha = 0.35$ and $\alpha = 0.70$, respectively. The solid line denotes the non-linear model and the dashed line corresponds to the linear model. The initial points of output and wage are on 70 % of their steady-state value. As mentioned before, the saddle paths of the linear model with $\alpha = 0.70$ err less relatively but more in absolute values than of that with $\alpha = 0.35$. It lasts over 200 years to reach the steady state[3] in the latter, whereas in the former, economies achieve the steady state in 90 years, which means 2.2 times faster. The

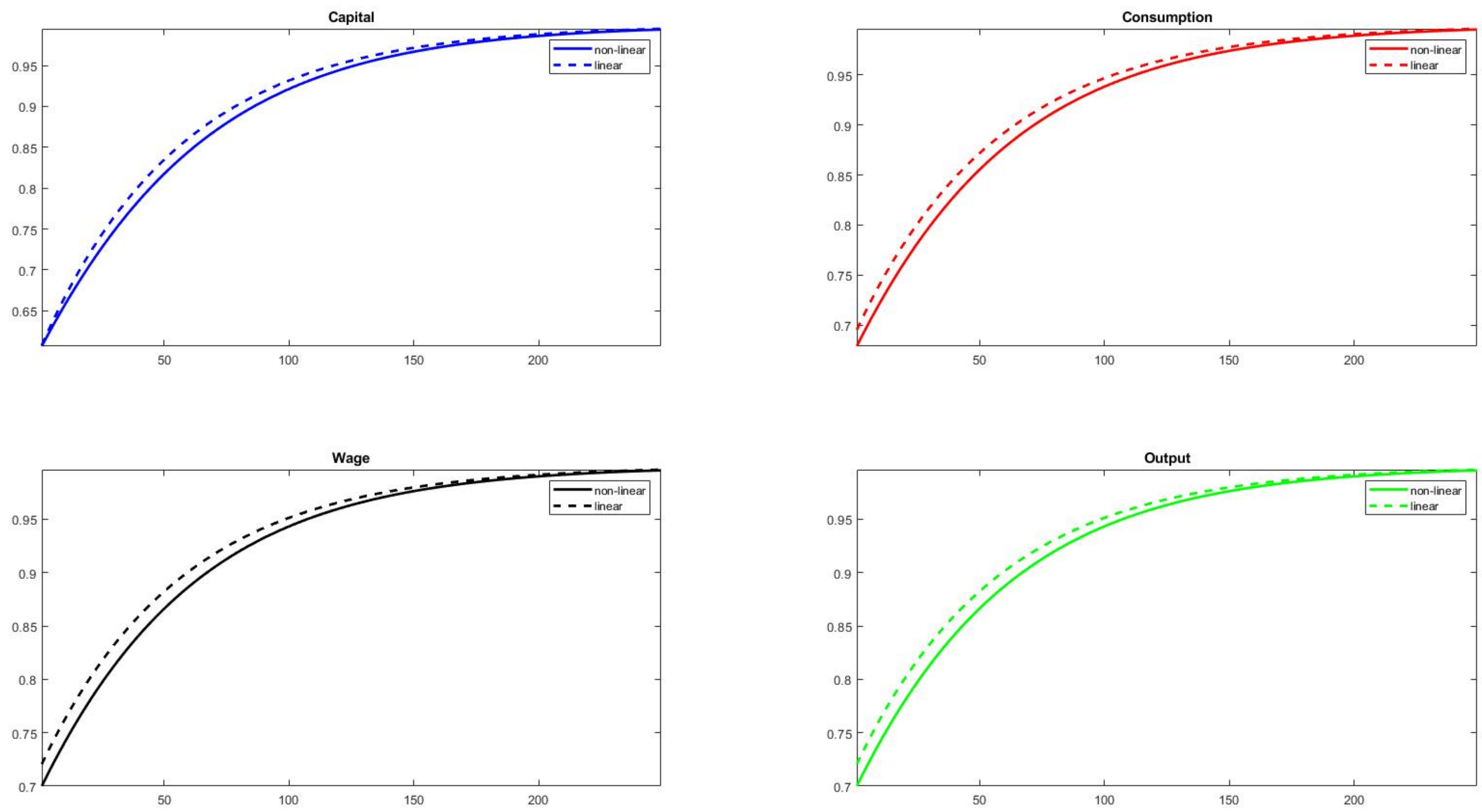
[3]Economies cannot reach the steady state in the finite horizon. They may only reach the epsilon-neighbourhood of the steady state.

Figure 2: The saddle path of capital, consumption, output, and wage, for $\alpha = 0.35$ and $\lambda_0 = 0.70$



Source: own calculations

Figure 3: The saddle path of capital, consumption, output, and wage, for $\alpha = 0.70$ and $\lambda_0 = 0.70$



Source: own calculations

non-linear approximation is done by the Newton's method.

3.6 From the theory to the regression

First of all, the convergence equations of output and wages are expressed in the unit of per effective worker. However, the only observable data provides with the per capita indicators. The equations need to be transformed into the per-capita term denoted by the superscript, x^c. The thesis follows the instructions suggested by Barro and Sala-i-Martin (1995). Detailed mathematical steps are shown in Appendix B.

Recall equation (23), infinitesimally rearranged. Because the model solely exploits the negative eigenvalue, β_2, the subscript of the β is no longer needed, and the minus sign is used instead.

$$\log y_t = e^{-\beta t} \log y_0 + (1 - e^{-\beta t}) \log \bar{y}$$

Equation (23) implies that the average growth of output per effective worker during the period T equals:

$$\frac{1}{T} \log \frac{y_t}{y_0} = \frac{1 - e^{-\beta T}}{T} \log \frac{\bar{y}}{y_0}. \tag{34}$$

Obtaining equation (35) consists of three steps, setting T at one, transforming the per-effective-worker unit into the per-capita term, and finally, adding random disturbance, u_t.

$$\log \frac{y_t^c}{y_{t-1}^c} = g + (1 - e^{-\beta}) \log \bar{y} - (1 - e^{-\beta}) \log y_{t-1}^c + (1 - e^{-\beta}) g(t-1) + u_t \tag{35}$$

Barro and Sala-i-Martin (1992) propose to incorporate a dummy variable that captures external shocks, e.g. the global recession in 2007 that might have a significant impact on output. The s_t denotes the year when a shock occurs, and ϕ signifies the effect thereof in the particular year.

$$u_t = \phi s_t + \epsilon_t \tag{36}$$

Combining equations (35) and (36) creates the regression equation of model 1.

Regression MODEL 1:

Model 1 describes the unconditional convergence. Stable variables, g and $(1 - e^{-\beta}) \log \bar{y}$, are hidden in the intercept, a_1.

$$\log \frac{y_t^c}{y_{t-1}^c} = a_1 - (1 - e^{-\beta}) \log y_{t-1}^c + (1 - e^{-\beta}) g(t-1) + \phi s_t + \epsilon_t \tag{37}$$

The regression equation for wages is identical.

Regression MODEL 2:

Model 2 estimates the conditional convergence. Countries are divided into two groups. The first group contains countries of the former Eastern Bloc, and the second includes the rest as the graphical analysis has suggested (see section 5). Parameters κ_1 and κ_2 measure the impact of dummy variables representing the groups. Analogously, Barro and Sala-i-Martin (1995) uses dummy variables to control for different steady states in the U.S.

$$\log \frac{y_t^c}{y_{t-1}^c} = a_1 - (1 - e^{-\beta}) \log y_{t-1}^c + (1 - e^{-\beta}) g(t-1) + \kappa_1 \text{west} + \kappa_2 \text{east} + \phi s_t + \epsilon_t \tag{38}$$

The regression equation for wages is also identical.

Regression MODEL 3:

Model 3 is based on model 1 but the output in the steady state is rewritten in the parameters form, which brings new variables, $\log \frac{I_t}{Y_t}$, and $\log(n_t + g + \delta)$. In addition, the model controls for the effect of government spending and net export, $\log \frac{G_t}{Y_t}$, and $\log \frac{X_t}{Y_t}$, where κ_3 and κ_4, respectively, measure the effect thereof. The two latter measure the effect of the fiscal policy and the international openness as Cavenaile and Dubois (2011) do.

$$\begin{gathered} \log \frac{y_t^c}{y_{t-1}^c} = g + (1 - e^{-\beta}) \frac{\alpha}{1-\alpha} \log \frac{I_t}{Y_t} - (1 - e^{-\beta}) \frac{\alpha}{1-\alpha} \log(n_t + g + \delta) \\ -(1 - e^{-\beta}) \log y_{t-1}^c + (1 - e^{-\beta}) g(t-1) + \kappa_3 \log \frac{G_t}{Y_t} + \kappa_4 \log \frac{X_t}{Y_t} + \phi s_t + \epsilon_t \end{gathered} \tag{39}$$

Model 3 for wages is slightly different, it uses $\log y_t^c$ instead of $\log \frac{I_t}{Y_t}$, and $\log(n_t + g + \delta)$. This modification also changes the value of the intercept.

$$\begin{aligned} \log \frac{w_t^c}{w_{t-1}^c} = a_3 + (1 - e^{-\beta}) \log y_t^c - (1 - e^{-\beta}) \log w_{t-1}^c + (1 - e^{-\beta}) g(t-1) \\ + \kappa_3 \log \frac{G_t}{Y_t} + \kappa_4 \log \frac{X_t}{Y_t} + \phi s_t + \epsilon_t \end{aligned} \tag{40}$$

Note that some variables have the identical parameter (or only the sign differs), which might be a decent specification test of the regression results.

3.7 Theory of the σ-convergence

This subsection follows the analysis of Barro and Sala-i-Martin (1995) assuming that the data generation process comes from equation (35),

$$\log \frac{y_{i,t}^c}{y_{i,t-1}^c} = a_t - (1 - e^{-\beta}) \log y_{i,t-1}^c + u_{i,t},$$

where a_t equals to $g + (1 - e^{-\beta}) \log \bar{y} + (1 - e^{-\beta}) g(t-1)$.

By subtracting $\log y_{t-1}^c$ from both sides and supposing the unconditional convergence it can be obtained:

$$\log y_{i,t}^c = a - e^{-\beta} \log y_{i,t-1}^c + u_{i,t}. \tag{41}$$

Now, the dispersion can be gained by subtracting the average of equation (41) from the same equation, summing with respect to i, dividing both sides by N, which is the number of individuals, and squaring.

$$\sum_{i=1}^{N} \left(\log \frac{y_{i,t}^c}{\bar{y^c}_t} \right)^2 = \sum_{i=1}^{N} e^{-2\beta} \left(\log \frac{y_{i,t-1}^c}{\bar{y^c}_{t-1}} \right)^2 + \sum_{i=1}^{N} 2 \operatorname{cov}(\log y_{i,t-1}^c, u_{i,t}) + \sum_{i=1}^{N} u_{i,t}^2. \tag{42}$$

The $\mathrm{cov}(\log y^c_{i,t-1}, u_{i,t})$ is assumed to be 0, meaning that equation (42) may be rewritten into the final form.

$$\sigma_{y^c_t} = e^{-2\beta}\sigma_{y^c_{t-1}} + \sigma_{u_t} \tag{43}$$

Equation (43) is also used for both output and wages in the regression analysis in section 6.

4 Dataset

The thesis exploits wide panel data for EU countries and Albania, Belarus, Bosnia and Herzegovina, Iceland, Macedonia, Montenegro, Norway, Serbia, Switzerland, and Ukraine throughout years of 1995-2017. The analysis uses two dependent variables, four control variables, and two regional dummy variables. The first dependent variable is the real GDP per capita at constant prices of 2011 in the purchasing power parity (PPP) measured by the international dollar[4] between 1997 and 2017. The second dependent variable is the real average annual wage at constant prices of 2018 in PPP measured by the US dollar in the 1996-2018 period.

As mentioned before, the regression analysis employs four control variables: the net export of goods and services, the government final consumption expenditure, the gross capital formation, and the growth rate of population. Apart from the latter, all those indicators are measured as a fraction of the GDP during 1996-2018 period. The growth rate of population is measured in a percentage change. Furthermore, the analysed countries are separated into two groups according to whether the country was a part of the Eastern Bloc. These groups form two dummy variables.

Data was taken from databases of the World Bank and the OECD. The final number of years used in each regression depends on the shortest time series so that the panel data is balanced. Not every regression model has the full sample of countries. Some variables do not have the appropriate number of observations available in public databases.

[4] Quasi currency developed by the World Bank to measure indicators in PPP

4.1 Descriptive statistics

Table 3: Descriptive statistics of exploited variables

	Obs	Mean	Median	SD	Min	Max
GDP per capita	798	29,646	27,890	16,837	4,064	97,864
Wage	598	36,930	38,097	17,293	8,969	66,504
Net Export	598	2.69	2.04	8.41	-20.67	34.05
Government	598	19.97	19.68	3.17	10.91	27.94
Investments	598	23.16	22.69	4.41	10.22	41.54
Growth rate of population	572	0.006	0.006	0.01	-0.054	0.066

Source: OECD (2019); Bank (2019a); Bank (2019b); Bank (2019c); Bank (2019d); Bank (2019e); Bank (2019f), own calculations

Table 3 shows descriptive statistics of the analysed data. The variable HDP contains 798 observations of all 38 countries from 1997 to 2017. The average GDP per capita equals 29,646 international dollars. The lowest value of output was measured in Albania in 1997. On the opposite, the highest value was recorded in Luxembourg in 2007. It is noteworthy that the real GDP per capita in PPP of Luxembourg, Cyprus, Spain, Finland, Greece, Italy, Norway, and Ukraine in 2017 have not reached the level of real GDP per capita in PPP in 2007 yet.

The average wage variable numbers only 598 observations. It incorporates 26 countries over the 1996-2018 period. The average wage among countries amounts to 36,930 dollars. The lowest annual average wage was recorded in Lithuania in 1996 and equalled to 8,969 dollars. Contrary, people in Iceland earned 66,504 in 2018, which is the highest value of the sample.

The average net export has a value of 2.69% of GDP. The net export varies from -20.67% of GDP to 34.05% of GDP. The government spend 19.97% of GDP on average. The minimum value and the maximum value of government spending are 10.91% of GDP, and 27.94% of GDP, respectively. Investments form 23.16% of GDP on average and range between 10.22% of GDP and 41.54% of GDP. Finally, the variable of the population growth rate, the maximum and the minimum value equal to -0.054% and 0.066%, respectively with the average growth of 0.006% per year.

5 Graphical analysis

The graphical analysis provides three figures illustrating β and σ convergence for both output per capita and wages. This analysis shows a relationship between the growth and the level of GDP per capita and wages, an evolution of dispersion of GDP per capita and wages in time, and a map of Europe figuring the distribution of GDP per capita and wages.

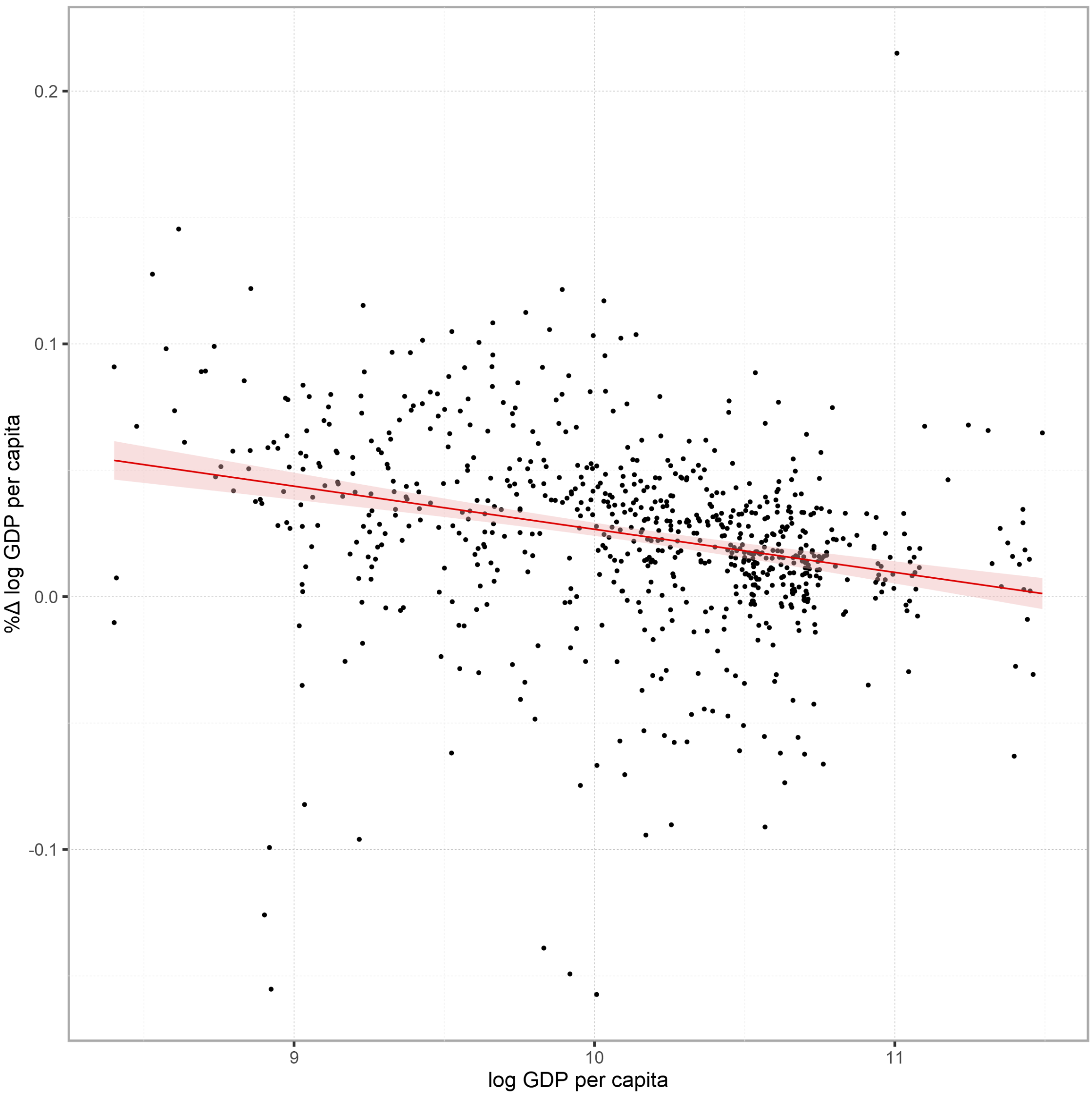
5.1 β convergence

Figures 4 and 5 illustrate the relationship between the growth of GDP per capita and wages, respectively, in PPP in a logarithm captured by the y-axis, and the level of GDP per capita and wages, respectively, in PPP in a logarithm captured by the x-axis over the whole period and across countries from the sample. Should the relationship appears to be negative, countries unconditionally converge in the sense of β.

It seems that the GDP per capita converges as the OLS-regression (red) line is decreasing. The light red area represents a confidence interval. The confidence interval is narrow, which brings robustness to the OLS regression. The speed of convergence is slow and approximately 0.4% per year.

The behaviour of wages is nearly identical to the GDP per capita. They converge in the sense of β as well since the OLS-regression line is downward sloping. In addition, as the confidence interval clings to the regression line, the findings are robust.

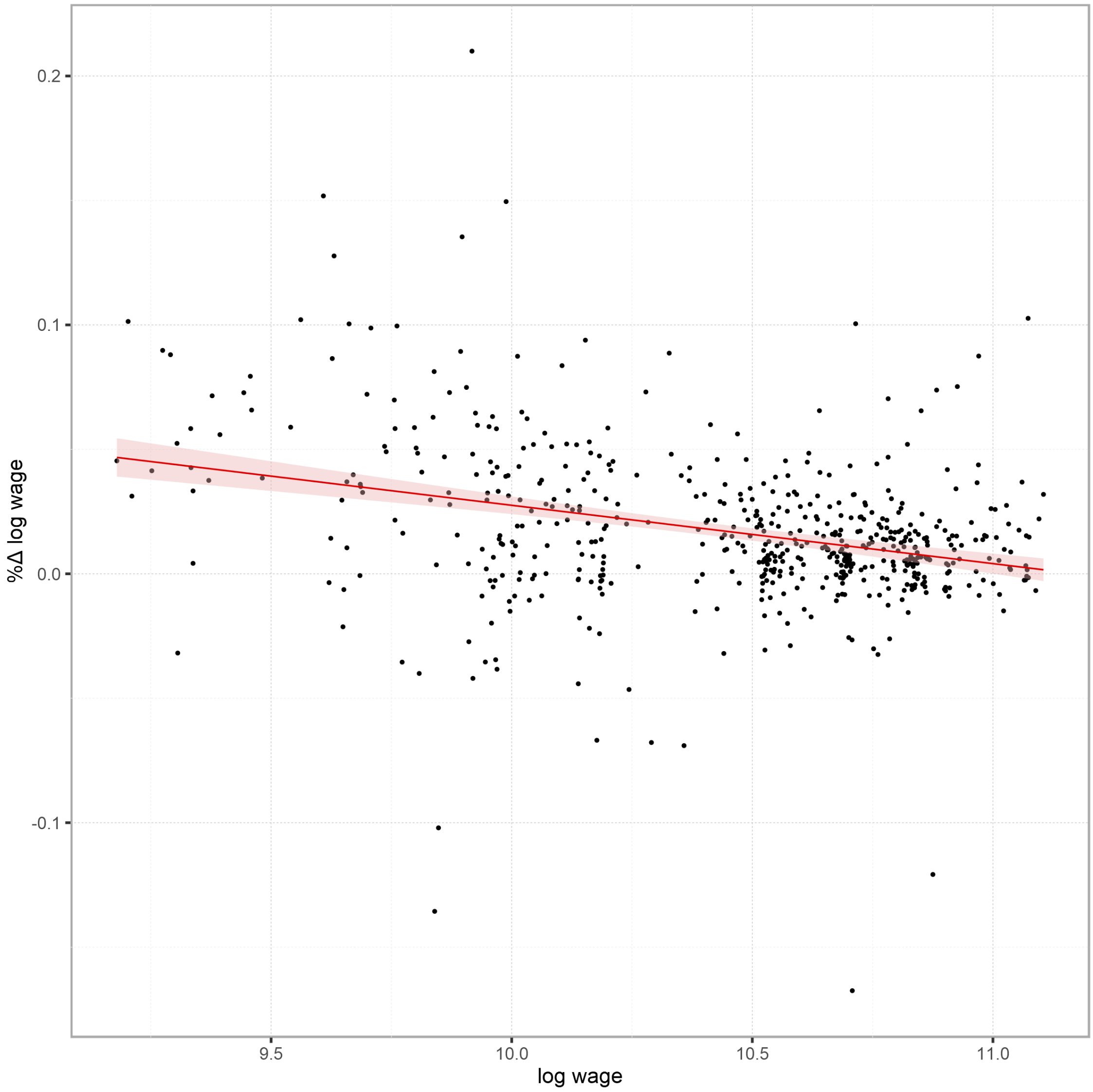
Figure 4: Relationship between the growth of GDP per capita and the level of GDP per capita



Source: Bank (2019b), own calculation

The figure contains the full sample (38 countries) over the 1997-2017 period.

Figure 5: Relationship between the growth of wages and the level of wages



Source: OECD (2019), own calculation

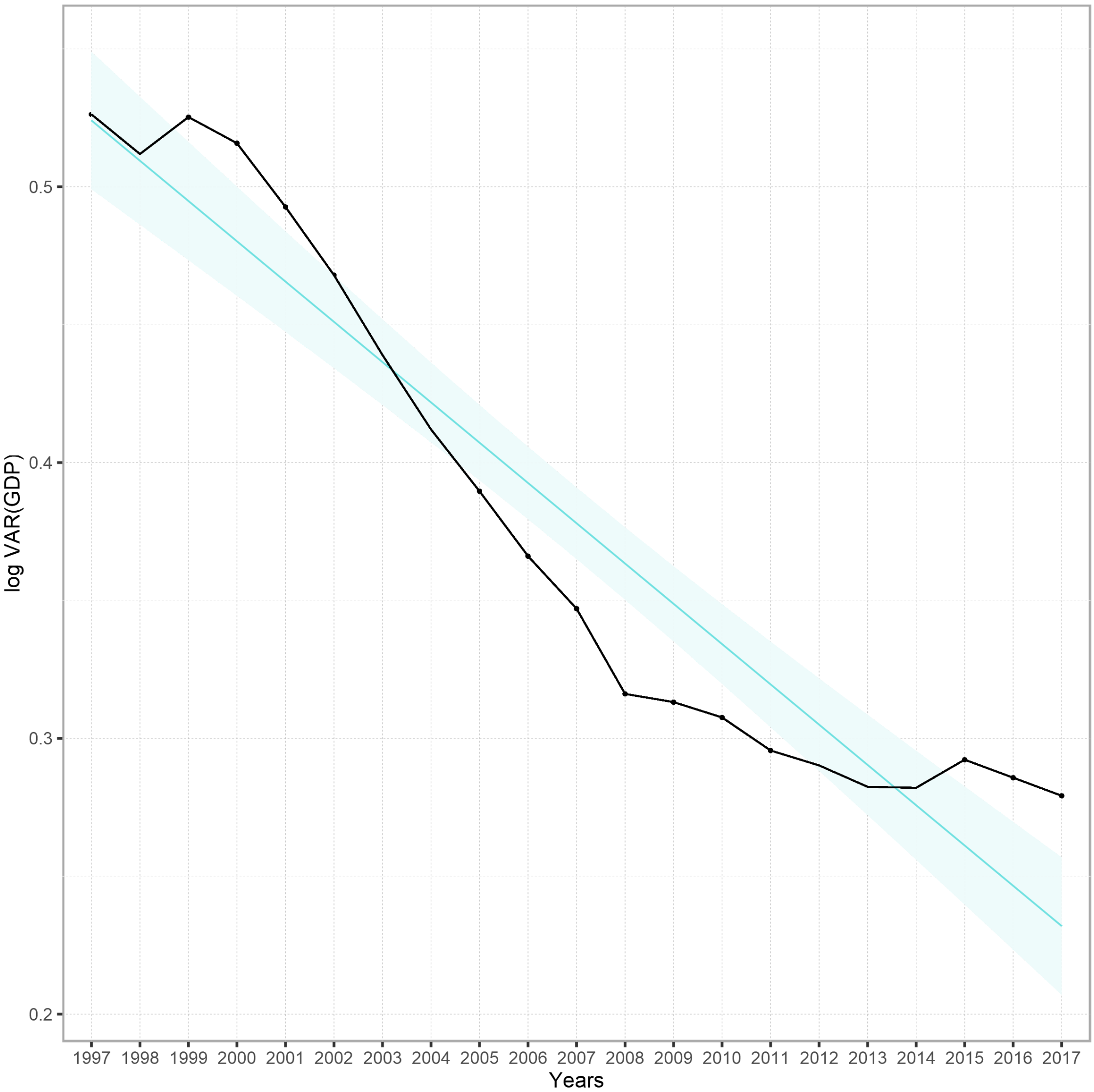
The figure contains 26 countries over the 1996-2018 period.

5.2 σ convergence

Figures 6 and 7 portray the evolution of the variance of GDP per capita and wages in time. The x-axis represents individual years, and the y-axis shows the logarithm of the variance of GDP per capita and wages in PPP, respectively, calculated across countries from the sample.

The evolution curve of the variance of GDP per capita decreases in time. The blue line displays the ordinary-least-square regression line. The downward-sloping shape proves the σ-convergence of the GDP per capita in Europe.

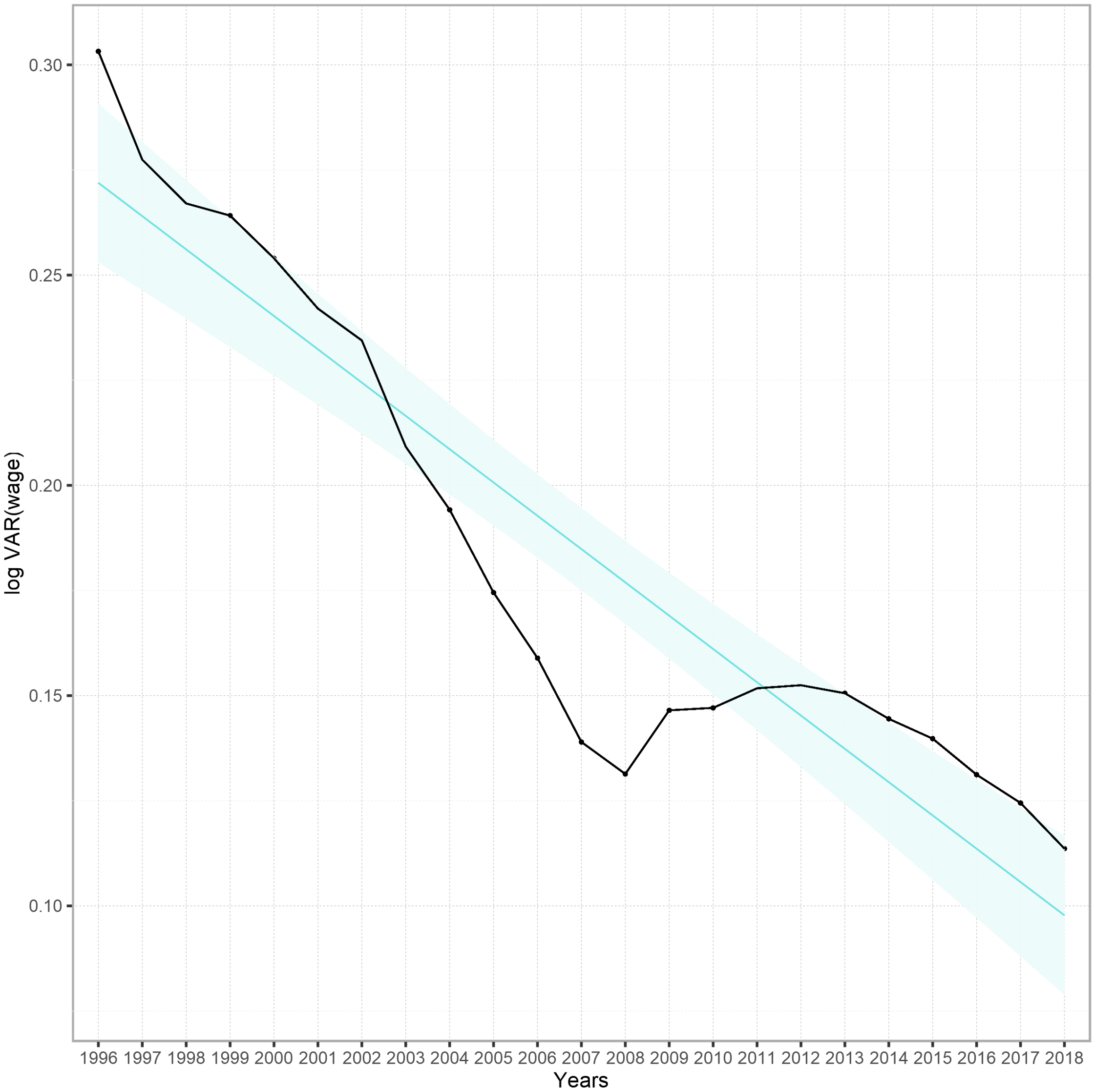
Figure 6: Illustration of sigma convergence of GDP per capita



Source: Bank (2019b), own calculation
The figure exploits the full sample of countries throughout the 1997-2017 period.

The evolution of wages appears to be similar to the GDP per capita. Nevertheless, one can observe an increase between 2008 to 2013, clearly occasioned by a temporary external shock of the global recession, which is an irrelevant issue in this analysis. The fitted values represented by the blue line are downward sloping, which leads to a conclusion that wages in Europe tend to converge in the sense of σ.

Figure 7: Illustration of sigma convergence of wages



Source: OECD (2019), own calculation
The figure uses 26 countries throughout the 1996-2018 period.

Figure 8 pictures a distribution of the GDP per capita across Europe in the years of 1997, 2004, 2011, and 2017. The darker the colour, the higher the GDP per capita. The colour is assigned according to the fraction of GDP per capita of particular countries to the one having the highest GDP per capita. To put it in another way, the fraction of the country that has the highest GDP per capita equals to one. Since Luxembourg is that country in all of the years, it is always the darkest one. It appears that the states are (according to the

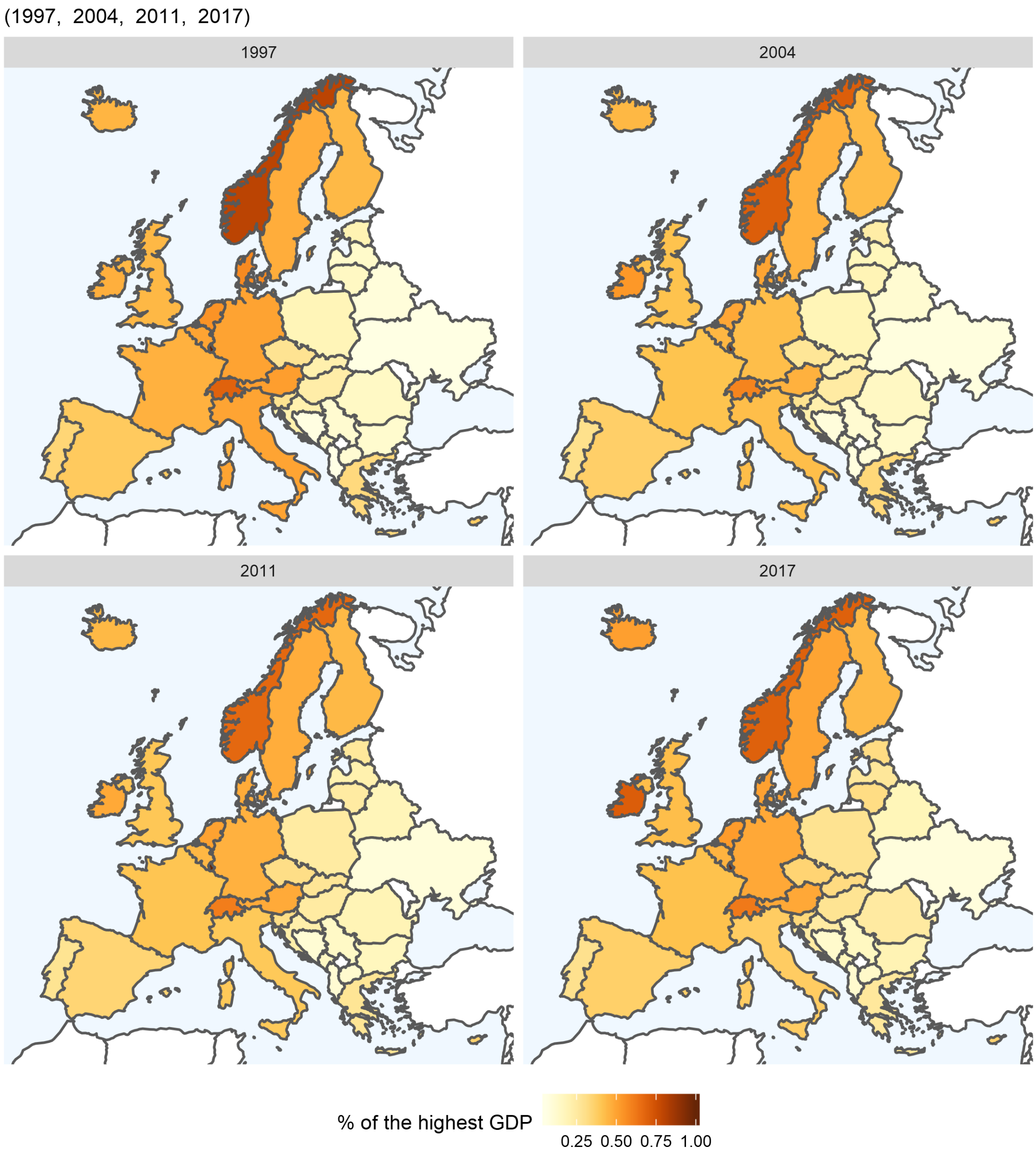
GDP per capita) divided into two groups, western and eastern, with the most significant gap in the year of 1997. Since 1997, eastern states have tended to darken, and besides that, some western countries have faded, in particular, Norway and Switzerland. Therefore, it might be concluded that the GDP per capita has been converging in the sense of σ.

Figure 9 portrays distribution of wages in Europe in the years 1996, 2000, 2008, and 2018. In the case of wages, the gap between western and eastern countries is even more noticeable creating a hotspot represented by Germany, Switzerland, Island, and states of Benelux, and a coldspot formed by Baltic republics. Although wages have been converging since 1997, they have not reached the level of convergence of the GDP per capita.

5.3 Findings of the graphical analysis

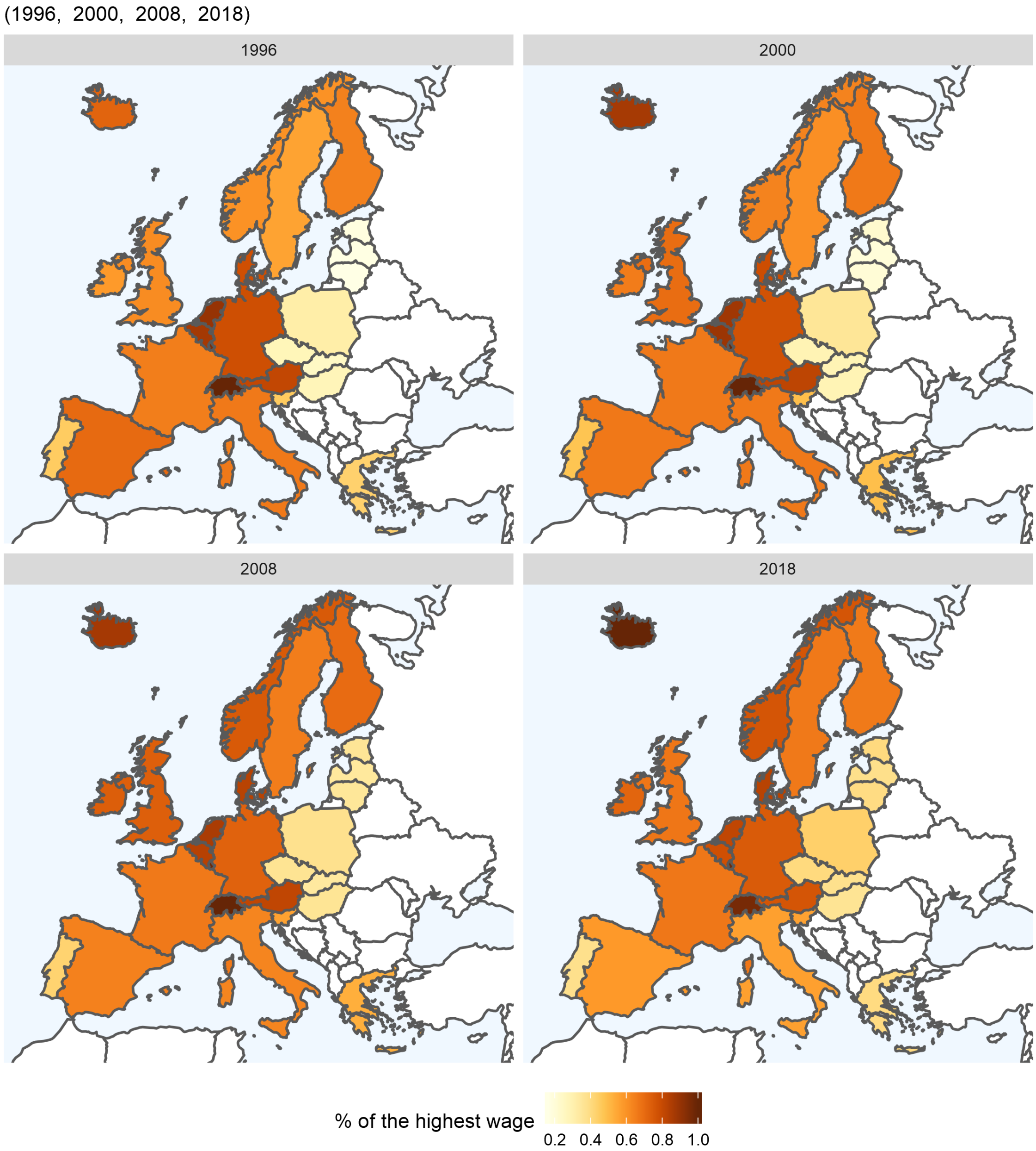
Based on the graphical analysis, both β and σ-convergence seem to take place. Despite the ongoing convergence, Europe is persistently divided into the richer west and the poorer east. The former iron curtain forms the imaginary border between the richer and the poorer world. Furthermore, the GDP per capita appears to be more converged than wages, which means that according to the neoclassical theory, wages in eastern countries are expected to rise faster than GDP. Moreover, some authors, for example Barro and Sala-i-Martin (1995), consider western countries to occur close to the steady state, meaning that their growth of GDP per capita and wages should cling to the growth of technology, g.

Figure 8: Map of GDP per capita in Europe



Source: Bank (2019b), own calculation

Figure 9: Map of wages in Europe



Source: (OECD, 2019), own calculation

6 Empirical analysis

6.1 β-convergence

The thesis uses the dynamic panel data approach to investigate the speed of convergence of output and wages. The crucial part of the analysis is to determine an appropriate estimator. Bond, Hoeffler, and Temple (2001) propound the rule whether to use the difference GMM (Arellano and Bond, 1991) or one of either the level GMM (Arellano and Bover, 1995) or the system GMM (Blundell and Bond, 1998). Bond, Hoeffler, and Temple (2001) follow Nickell (1981), who claims that the fixed effect estimator is downward biased, and Hsiao (1986) saying that polled-OLS estimator is upward biased. Bond, Hoeffler, and Temple (2001) propose a rule how to decide which of GMM estimators ought to be used. Should the value estimated by the D-GMM lie between values estimated by the pooled OLS and the fixed effect, the D-GMM estimator can be applied. Otherwise, the D-GMM estimator is biased, and either the L-GMM or the S-GMM are recommended. Abonazel (2016) argues that all three GMM estimators might suffer from bias. Therefore, he (and many others) propose corrected GMM estimators that are either consistent or at least more efficient. However, the implementation of those is not supported (according to my knowledge) in the R language. Nevertheless, since the bias of 'uncorrected' GMMs is almost negligible, the thesis does not take it into account.

The analysis uses six estimators (mainly for comparison purposes), the differences GMM, the system GMM, the pooled OLS, the random effect, the fixed effect, and the first difference to evaluate the speed of convergence and β-convergence itself. Models for both GDP per capita and wages are presented, as mentioned in section 3.6. All models are tested for autocorrelation and heteroscedasticity. As neither autocorrelation nor heteroscedasticity are rejected, the HAC standard errors need to be used. Additionally, models estimated by GMMs are tested of both by Sargan test to check the appropriateness of instruments and autocorrelation of the second order. Moreover, the panel data in the logarithmic terms do not indicate non-stationary behaviour according to the test proposed by Im, Pesaran, and Yongcheol (2003). However, some countries have the autocorrelation coefficients that approach one, therefore, the system GMM estimator may be more appropriate than the differential GMM.

Parameter β is obtained from the parameter of $\log y_{i,t-1}$ according to the regression equations. The standard errors are calculated by the delta method.[5]

6.1.1 Convergence of output

Table 4 displays the estimations of **model 1**, which represents the unconditional β-convergence for all six regression models. At first sight, the system GMM, the pooled OLS, and the random effects give almost identical results, taken into account that the second-order autocorrelation test of the system GMM does not reject the null hypothesis only at the level of 5%. The R^2 of the pooled OLS and the random effect are 37.4% and 36.2%, respectively. In addition, the Hausman test does not reject the null hypothesis, so models do not contain unobserved heterogeneity among countries. The pooled OLS and the random effect model are therefore more efficient than the fixed effect. The differential GMM model appears to be downward biased since 'Bond's rule' does not hold. The first-differences estimator seems significantly biased despite the fact that its R^2 is the highest. Overall, the unbiased speed of unconditional convergence is estimated by the S-GMM at 1.7%, which means that the half-life equals 40.77 years. Other estimators present biased results, the D-GMM estimates the β coefficient at -0.115, the pooled OLS at -0.017, random effect at -0.018, the fixed effect at -0.038, and the first difference at -0.378. All β parameters are significant at the level of 1%.

Table 5 shows the coefficients of **model 2**. There are merely the system GMM, the pooled OLS, and the random effect because the rest cannot handle the dummy variable, and the results would be the same as in model 1. The estimated models considerably vary. The speed of convergence of the system GMM is not significant. The coefficient of pooled-OLS β is not significant either. The only significant speed of convergence is seen in the random-effect model, -0.010, however, the value is upward biased and eminently small. The R^2s do not change in comparison with model 1. It appears that European countries are not divided into two convergence groups. As model 1 does not decline the unconditional concept of convergence, European countries might be homogeneous enough to

[5] For more information about the delta method, see, e.g., Oehlert (1992)

Table 4: MODEL 1 - Unconditional convergence speed of GDP per capita

	Dependent variable:					
	$\Delta \log y^c_{i,t,t-1}$					
	GMM		*OLS*			
	D	S	Pooled	RE	FE	FD
	(1)	(2)	(3)	(4)	(5)	(6)
Intercept		0.210***	0.209***	0.220***		0.010***
		(0.032)	(0.030)	(0.031)		(0.002)
$\log y^c_{i,t-1}$	−0.121***	−0.017***	−0.017***	−0.018***	−0.038***	−0.459***
	(0.020)	(0.003)	(0.003)	(0.003)	(0.010)	(0.051)
Lagged Trend	0.002***	−0.001**	−0.001***	−0.001***	−0.0002	
	(0.001)	(0.0003)	(0.0002)	(0.0002)	(0.0003)	
Shock_{2007}	0.031***	0.021***	0.021***	0.022***	0.023***	0.024***
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.003)
Shock_{2008}	0.003	−0.010	−0.009	−0.009	−0.006	0.016**
	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)
Shock_{2009}	−0.067***	−0.080***	−0.079***	−0.079***	−0.077***	−0.046***
	(0.007)	(0.008)	(0.008)	(0.008)	(0.008)	(0.008)
Shock_{2010}	−0.007	−0.010*	−0.010**	−0.010**	−0.009*	−0.0003
	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)
Shock_{2011}	−0.003	−0.006	−0.006	−0.006	−0.005	0.011***
	(0.004)	(0.004)	(0.004)	(0.004)	(0.004)	(0.002)
β	−0.115***	−0.017***	−0.017***	−0.018***	−0.038***	−0.378***
	(0.018)	(0.003)	(0.003)	(0.003)	(0.010)	(0.035)
Observations	722	760	760	760	760	722
R^2			0.374	0.362	0.359	0.510
Adjusted R^2			0.368	0.357	0.320	0.506
F Statistic			64.124***	427.536***	57.204***	124.172***
AR(2)	-1.646	-1.714*				
Sargan	37.076	35.293				

Note: *p<0.1; **p<0.05; ***p<0.01

Hausman Test: 4.712

The regressions use the full sample of countries over the period of 1997-2017.

grow towards one steady state.

Table 6 exhibits the regressions of **model 3**. First, the difference GMM estimator does not follow the Bond, Hoeffler, and Temple's (2001) rule, therefore, it is likely to be biased. Second, the pooled OLS and the random effect estimate the β of -0.015 and -0.021, respectively. Contrary, the fixed effect and the first difference predict a rapid speed of -0.112 and -0.415, respectively. The Hausman test concludes that fixed-effect estimators are more appropriate. However, this speed suggests that economies would reduce half of the gap between the initial point and the steady state in 5.82 years (FE) and 1.35 years (FD), which appears implausible. Third, according to Bond, Hoeffler, and Temple (2001), the system GMM seems to be the most accurate, thus, β equals -0.04. The R^2 vary from 48.1% to 67%, being greater than in model 1.

The parameters of the control variables behave as follows: First, the parameter of the investments variable is positive and significant for all estimators, which is consistent with the theory. The values vary from 0.056 to 0.190. Second, the parameter of the break-even investments is insignificant. Although, the theory predicts a negative sign and the same value as the investments have. Third, the parameter of the net export is not significant only in the pooled-OLS model. Finally, the parameter of the government spending variable appears to be significant only in half of the models, in the D-GMM, the fixed effect, and the first difference. The two latter do not have a prior expectation of the sign of the parameters.

6.1.2 Concluding remarks of β-convergence of output

According to the empirical analysis, not only the conditional convergence but also the unconditional convergence appear to occur. This analysis relies solely on the S-GMM estimator. The speed of unconditional convergence equals to -0.017, meaning that it reaches half-life in 40.77 years. The conditional convergence exhibits, not surprisingly, more than twice faster with the speed of -0.04, which means the half-life of 17.33 years. It might indicate that countries are grouped, either geographically or institutionally. Countries within the group tend to converge faster than outside the group. For example, based on the graphical analysis, Slovakia converges to the Czech Republic more quickly than to

Table 5: **MODEL 2 - Conditional convergence speed of GDP per capita**

	Dependent variable:					
	$\Delta \log y^c_{i,t,t-1}$					
	GMM		*OLS*			
	D	S	Pooled	RE	FE	FD
	(1)	(2)	(3)	(4)	(5)	(6)
Intercept	–	0.283	0.108**	0.138***	–	–
		(0.207)	(0.048)	(0.050)		
$\log y^c_{i,t-1}$	–	−0.025	−0.008	−0.010**	–	–
		(0.020)	(0.005)	(0.005)		
Lagged Trend	–	−0.001	−0.001***	−0.001***	–	–
		(0.0004)	(0.0002)	(0.0002)		
East	–	−0.002	0.014***	0.012**	–	–
		(0.020)	(0.005)	(0.005)		
Shock_{2007}	–	0.021***	0.021***	0.021***	–	–
		(0.004)	(0.003)	(0.003)		
Shock_{2008}	–	−0.008	−0.010	−0.009	–	–
		(0.007)	(0.006)	(0.006)		
Shock_{2009}	–	−0.077***	−0.080***	−0.080***	–	–
		(0.010)	(0.008)	(0.008)		
Shock_{2010}	–	−0.011*	−0.010**	−0.010**	–	–
		(0.006)	(0.005)	(0.005)		
Shock_{2011}	–	−0.006	−0.006	−0.006	–	–
		(0.005)	(0.004)	(0.004)		
β	–	−0.024	−0.007	−0.010**	–	–
		(0.020)	(0.005)	(0.005)		
Observations		760	760	760		
R^2			0.369	0.359		
Adjusted R^2			0.380	0.362		
F Statistic			59.211***	438.747***		
AR(2)		-1.542				
Sargan		35.526				

Note: *p<0.1; **p<0.05; ***p<0.01

The regressions use the full sample of countries over the period of 1997-2017.

Table 6: MODEL 3 - Conditional convergence speed of the GDP per capita

	Dependent variable:					
	$\Delta \log y^c_{i,t,t-1}$					
	GMM		*OLS*			
	D	S	Pooled	RE	FE	FD
	(1)	(2)	(3)	(4)	(5)	(6)
Intercept		0.282^{**}	0.054	0.092		0.011^{***}
		(0.129)	(0.060)	(0.069)		(0.002)
$\log y^c_{i,t-1}$	-0.187^{***}	-0.040^{***}	-0.015^{***}	-0.021^{***}	-0.119^{***}	-0.515^{***}
	(0.037)	(0.013)	(0.004)	(0.005)	(0.019)	(0.051)
Trend_{t-1}	0.003^{***}	-0.0003	-0.001^{**}	-0.0004	0.002^{***}	
	(0.001)	(0.0003)	(0.0002)	(0.0003)	(0.0004)	
$\log \frac{I_{i,t}}{Y_{i,t}}$	0.190^{***}	0.058^{***}	0.056^{***}	0.069^{***}	0.100^{***}	0.171^{***}
	(0.022)	(0.015)	(0.008)	(0.010)	(0.019)	(0.016)
$\log \frac{X_{i,t}}{Y_{i,t}}$	0.113^{***}	0.060^{*}	0.010	0.024^{**}	0.044^{**}	0.125^{***}
	(0.034)	(0.031)	(0.010)	(0.010)	(0.021)	(0.036)
$\log \frac{G_{i,t}}{Y_{i,t}}$	-0.102^{***}	-0.008	-0.015	-0.021	-0.101^{***}	-0.129^{***}
	(0.039)	(0.016)	(0.012)	(0.016)	(0.039)	(0.049)
$\log(n_t + g + \delta)$	-0.001	0.002	-0.002	-0.002	-0.001	-0.001
	(0.001)	(0.005)	(0.005)	(0.007)	(0.005)	(0.001)
β	-0.172^{***}	-0.038^{***}	-0.015^{***}	-0.021^{**}	-0.112^{***}	-0.415^{***}
	(0.031)	(0.013)	(0.004)	(0.004)	(0.017)	(0.033)
Observations	665	700	700	700	700	665
R^2			0.481	0.477	0.547	0.670
Adjusted R^2			0.473	0.469	0.516	0.664
F Statistic			57.951^{***}	627.312^{***}	71.872^{***}	132.506^{***}
AR(2)	-0.636	-1.631				
Sargan Test	32.608	32.447				

Note: *p<0.1; **p<0.05; ***p<0.01

Hausman Test: 103.41^{***}

The regressions use the full sample of countries over the period of 1997-2017.

Germany. Model 2 examines this hypothesis, but it was not proved. Yet, it does not reject the premise itself. Maybe there is a more complicated division that the graphical analysis cannot capture. Moreover, the graphical analysis indicates that the output is decently converged.

6.1.3 Convergence of wages

Table 7 describes **model 1** of wages, which explores the unconditional convergence. Remarkably, the difference GMM reject the convergence of wages with the value of 0.056, and the system GMM does not decline it only at the level of 10%. It can be said that the speed of convergence of wages is faster than output since each estimator, apart from the difference GMM, provides with a lower number. The R^2s are much lower than in the case of output, and it varies only from 13.9% to 28.8%. It is likely to relate to the fact that wages are not converged that much as output. The Hausman test proposes using the random-effect estimator rather than the fixed effect. However, the system GMM estimator provides with even faster speed, -0.082, than the fixed effect, -0.053. The pooled OLS and the first difference suggest the convergence speed of -0.025 and 0.507, respectively. To conclude, the convergence of wages is faster than output, but it cannot be decided which of models is unbiased if any.

Table 8 shows regressions of **model 2**. This model examines the conditional convergence of wages by using the dummy variable. The table displays only three regressions as the rest cannot handle the dummy variable. The system GMM estimator rejects the convergence hypothesis, and the pooled OLS and the random effect do not decline it only at the 10% and the 5% level with the speed of -0.016 and -0.023, respectively. The dummy variable 'East' is significant in neither of the models. It appears that the European countries cannot be divided into those two groups, although the graphical analysis suggests otherwise.

Table 9 portrays the coefficients of **model 3** describing conditional convergence based on the control variables. The difference GMM estimator does not reject the convergence process only at the 10% level with the speed of -0.296. The Hausman test proposes using the fixed effect estimators. In addition, the system GMM propounds the similar speed

Table 7: MODEL 1 - Unconditional convergence speed of wages

	Dependent variable:					
	$\Delta \log w^c_{i,t,t-1}$					
	GMM		*OLS*			
	D	S	Pooled	RE	FE	FD
	(1)	(2)	(3)	(4)	(5)	(6)
Intercept		0.879^{*}	0.287^{***}	0.308^{***}		0.008^{***}
		(0.508)	(0.044)	(0.042)		(0.001)
$\log w^c_{i,t-1}$	0.055	-0.082^{*}	-0.025^{***}	-0.027^{***}	-0.053^{***}	-0.507^{***}
	(0.049)	(0.049)	(0.004)	(0.004)	(0.011)	(0.031)
Trend_{t-1}	-0.001^{**}	0.0004	-0.0004^{*}	-0.0003	0.0001	
	(0.001)	(0.001)	(0.0002)	(0.0002)	(0.0003)	
Shock_{2007}	0.008	0.015	0.013^{*}	0.013^{*}	0.014^{**}	0.018^{***}
	(0.008)	(0.009)	(0.007)	(0.007)	(0.007)	(0.007)
Shock_{2008}	-0.017^{*}	-0.002	-0.012^{*}	-0.012^{*}	-0.010	0.008
	(0.009)	(0.007)	(0.007)	(0.007)	(0.007)	(0.006)
Shock_{2009}	-0.021	0.004	-0.017	-0.017	-0.015	0.005
	(0.016)	(0.014)	(0.013)	(0.013)	(0.013)	(0.011)
Shock_{2010}	-0.014^{**}	-0.005	-0.012^{**}	-0.012^{**}	-0.010^{*}	0.009
	(0.006)	(0.005)	(0.005)	(0.005)	(0.005)	(0.006)
Shock_{2011}	-0.021^{***}	-0.013^{*}	-0.021^{***}	-0.021^{***}	-0.020^{***}	0.001
	(0.006)	(0.007)	(0.005)	(0.005)	(0.005)	(0.004)
β	0.056	-0.079^{*}	-0.025^{***}	-0.027^{***}	-0.052^{***}	-0.410^{***}
	(0.052)	(0.045)	(0.004)	(0.004)	(0.011)	(0.020)
Observations	546	572	572	572	572	546
R^2			0.201	0.159	0.134	0.288
Adjusted R^2			0.191	0.148	0.082	0.280
F Statistic			20.306^{***}	106.495^{***}	11.884^{***}	36.286^{***}
AR(2)	-0.854	-0.747				
Sargan Test	24.628	12.039				

Note: *p<0.1; **p<0.05; ***p<0.01

Hausman Test: 5.131

The regressions use 26 countries of over the period of 1996-2018.

Table 8: **MODEL 2 - Conditional convergence speed of wages**

	Dependent variable:					
	$\Delta \log w^c_{i,t,t-1}$					
	GMM		*OLS*			
	D	S	Pooled	RE	FE	FD
	(1)	(2)	(3)	(4)	(5)	(6)
Intercept	–	-0.289	0.189^{**}	0.258^{***}	–	–
		(0.096)	(0.091)	(0.094)		
$\log w^c_{i,t-1}$	–	0.029	-0.016^{*}	-0.023^{**}	–	–
		(0.046)	(0.009)	(0.009)		
Lagged Trend	–	-0.001	-0.0005^{**}	-0.0004	–	–
		(0.001)	(0.0003)	(0.0003)		
East	–	0.045	0.010	0.005	–	–
		(0.036)	(0.008)	(0.008)		
Shock_{2007}	–	0.010	0.012	0.012	–	–
		(0.009)	(0.007)	(0.010)		
Shock_{2008}	–	-0.008	-0.012^{*}	-0.012	–	–
		(0.010)	(0.007)	(0.010)		
Shock_{2009}	–	0.001	-0.017	-0.017	–	–
		(0.015)	(0.013)	(0.019)		
Shock_{2010}	–	-0.007	-0.012^{**}	-0.012	–	–
		(0.007)	(0.005)	(0.007)		
Shock_{2011}	–	-0.015^{**}	-0.021^{***}	-0.021^{***}	–	–
		(0.008)	(0.005)	(0.007)		
β	–	0.029	-0.016^{*}	-0.022^{**}	–	–
		(0.047)	(0.009)	(0.009)		
Observations		572	572	572		
R^2			0.208	0.125		
Adjusted R^2			0.196	0.113		
F Statistic			18.455^{***}	80.626^{***}		
AR(2)		-0.855				
Sargan Test		13.558				

Note: *p<0.1; **p<0.05; ***p<0.01

The regressions use 26 countries of over the period of 1996-2018.

as the fixed effect, the former estimates -0.187 and the latter -0.184. Even though two estimators conclude this fast speed, this seems to be implausible. Economies would reach a half-life in 3.71 years and 3.77 years, respectively. The pooled OLS estimates the β at -0.060, the random effect at -0.083, and the first difference at (enormous) -0.703. The R^2s raise, contrary to model 1, to values between 25.6% and 49.4%. The conditional speed of convergence appears to be also faster than output's conditional speed. This supports the presumption that wages are less converged than output.

The parameters of the control variables are as follows: First, the GDP per capita is significant and positive in all models as the theory predicts. Additionally, the values ought to be identical (only the opposite sign) to the parameters of wages, which is (more or less) accomplished in the model of the system GMM and the random effect. Second, the impact of the government spending is significant only in the first difference model. Finally, the net export significantly affects merely the system GMM, the random effect, and the fixed effect model.

6.1.4 Concluding remarks of the β-convergence of wages

The empirical analysis provides with the pieces of evidence that suggest both the conditional convergence and the unconditional convergence. Wage convergence appears to be faster than that of output. Unfortunately, the speed of convergence is not unbiasedly estimated. Keeping the Bond, Hoeffler, and Temple's (2001) rule, the unconditional convergence ought to occur between -0.025 and -0.053, and the conditional convergence should be between -0.060 and -0.184. Analogously to output, the European countries are homogeneous enough to converge to their mutual steady state. However, countries also group in which they converge faster.

Table 9: MODEL 3 - Conditional convergence speed of wages

	Dependent variable:					
	$\Delta \log w^c_{i,t,t-1}$					
	GMM		*OLS*			
	D	S	Pooled	RE	FE	FD
	(1)	(2)	(3)	(4)	(5)	(6)
Intercept		0.145	0.145*	0.086		0.001
		(0.347)	(0.075)	(0.096)		(0.001)
$\log w^c_{i,t-1}$	−0.344	−0.187***	−0.060***	−0.083***	−0.184***	−0.703***
	(0.213)	(0.062)	(0.012)	(0.016)	(0.032)	(0.043)
Lagged Trend	−0.001	−0.0002	−0.0005**	−0.001**	−0.0002	
	(0.001)	(0.001)	(0.0002)	(0.0002)	(0.0004)	
$\log y^c_{i,t}$	0.365**	0.180***	0.048***	0.078***	0.159***	0.526***
	(0.171)	(0.067)	(0.015)	(0.022)	(0.039)	(0.075)
$\log \frac{G_{i,t}}{Y_{i,t}}$	0.005	−0.018	0.001	−0.0004	−0.013	0.194***
	(0.082)	(0.036)	(0.009)	(0.011)	(0.043)	(0.044)
$\log \frac{X_{i,t}}{Y_{i,t}}$	−0.062	−0.101***	−0.031	−0.057***	−0.082***	−0.067
	(0.050)	(0.038)	(0.022)	(0.021)	(0.021)	(0.061)
β	−0.296*	−0.171***	−0.058***	−0.080***	−0.168***	−0.532***
	(0.159)	(0.052)	(0.011)	(0.015)	(0.027)	(0.026)
Observations	546	572	572	572	572	546
R^2			0.256	0.241	0.294	0.494
Adjusted R^2			0.243	0.227	0.248	0.485
F Statistic			19.285***	177.728***	22.359***	58.029***
AR(2)	-0.114	-0.317				
Sargan Test	20.882	7.795				

Note: *p<0.1; **p<0.05; ***p<0.01

Hausman Test: 76.349***

The regressions use 26 countries of over the period of 1996-2018.

6.2 σ convergence

The regressions are set up based on equation (43). Since the panel data is transformed into the time series, the ordinary least squares (OLS) method is used. The prior stationary tests are not needed because the analysis follows a similar approach as stationary tests, e.g. the Dickey-Fuller test. If the regressions meet the assumption that the coefficient, $e^{-2\beta}$, equals to 0.967 ($\beta = -0.017$), it would reject the non-stationary concerns and confirms the unconditional convergence of output.

Table 10 shows σ-convergence of both GDP per capita and wages. First, the left column exhibits the regression of GDP per capita. The estimated autocorrelation coefficient equals 0.989. This value is greater than predicted and indicates a smaller speed of convergence. As this is not the most feasible approach to estimate the speed of convergence, this value plays a comparative role only. Since the autocorrelation occurs in the regression, the HAC standard error is used. Second, the right column demonstrates the results of the wage regression. The autocorrelation coefficient amounts to 0.977. It finds that the convergence speed of wages is greater than that of output. However, in this case, it is still lower than outlined by the analysis of β. The regression suffers neither from heteroscedasticity nor from autocorrelation. In general, the R^2s cannot be interpreted because of the lack of intercepts. To conclude, the regression analysis does not reject the σ-convergence hypothesis for both variables, thus, the European countries converge in the sense of β and σ as well.

Table 10: Regressions of the σ-convergence

	Dependent variable:	
	$\sigma_{y_t^c}$	σ_{w_t}
	OLS	
	(1)	(2)
$\sigma_{y_{t-1}^c}$	0.989^{***}	
	(0.032)	
$\sigma_{w_{t-1}}$		0.977^{***}
		(0.045)
Observations	21	23
R^2	0.979	0.955
Adjusted R^2	0.978	0.952
Residual Std. Error	0.058	0.042
F Statistic	933.451^{***}	462.173^{***}
Note:	*p<0.1; **p<0.05; ***p<0.01	

Conclusion

Although the neoclassical theory predicts the same level of convergence for output and wages (at least, by using the Cobb-Douglas production function), the reality might not be that clear. Wages convergence tends to lag behind output. The new discrepancy is arising: Is this issue a theory bug or policy markers fault? Indeed, the thesis can hardly answer this question. Because of that, the convergence debate appears to be topical and ought to be more examined.

The first parts discuss the literature review and the neoclassical growth model. The literature review presents the history and the actual convergence debate, and the appropriate estimators. The recent papers distinguish two types of convergence, β-convergence and σ-convergence. The β-convergence analysis investigates mainly the speed of convergence. The speed of convergence strongly depends on the estimator and the approach that are used. First, the cross-section analysis and the OLS estimators of the panel data conclude that the speed of convergence is approximately 2%. Second, the more sophisticated and less biased generalised methods of moments outline the speed between 4% and 6%.

The neoclassical growth model suggests the convergence speed of -0.068 and -0.020 if α equals 0.35 and 0.70 with the corresponding half-lives of 10.2 years and 34.7 years, respectively. Since the solution of the growth model is log-linearised, the analysis of the convergence contains the in-build error, which is shown graphically and numerically. The error tends to increase in two cases: first, the initial point recedes from the steady state, and second, α either increases from approximately 0.5 to 1 or decreases to zero, the latter has a more severe impact (in the percentage sense).

The next parts consist of the dataset description, the graphical analysis, and the empirical analysis. The graphical analysis displays three families of figures showing both real GDP per capita in PPP and real wages in PPP for β-convergence and σ-convergence. The first family of figures captures the relationship between the growth and the level of both variables. It concludes the negative relationship supporting the β-convergence. The second family pictures the evolution of the dispersion in time. It finds that the σ-convergence occurs in Europe. The third family depicts the map of European countries, exposing the distribution of GDP per capita and wages. Despite the evidence of the σ-convergence, there are permanent and significant differences between richer western countries and poorer eastern countries. That gap is more considerable for wages.

The empirical analysis examines three regressions of the β-convergence, the model of unconditional convergence, the model of conditional convergence determined by dummy variables, and the model of conditional convergence identified by control variables. Moreover, it also involves the regressions of the σ-convergence. Both convergence processes are verified for each of the variables. The speed of convergence of wages is not explicitly determined, however, it is faster than the speed of output. The output converges at the speed of 0.017 unconditionally and 0.04 conditionally according to the system GMM estimator. These numbers match the values predicted by the growth model if α amounts to 0.70. Besides, the σ-convergence hypothesis is also confirmed and coincides with the β-convergence analysis as the autoregression coefficients correspond to the capital share, α, of 0.70. These findings are in line with the literature. Thus, from the results, it can be presumed that wages shall grow faster than output, especially in the less developed areas of Europe.

The aims of the thesis are entirely fulfilled apart from the exact value of the β-convergence speed of wages as the empirical part concerning wages does not provide the eligible unbiased estimator. Nevertheless, the thesis succeeds to find the piece of evidence that wages converge faster than output per capita.

As a further extension, spatial regression might be used. This analysis would filter out the spatial correlation, and the results may be more precise and less biased. Moreover, the NUTS2 level can be used to increase the number of cross-section units a thereby the number of observations and degrees of freedom. It allows for separating countries by region and investigating each group individually or even within-county convergence. Furthermore, the question of why wages have not converged as much as of output ought to be answered.

A Appendix

The appendix A provides some details of the solution of the neoclassical growth model, which are not completely shown in section 3. Note that the function-of-time expression, (t) is omitted if no necessary.

A.1 From equation (6) to equation (12)

Equations $(\mathrm{A}.1)$, $(\mathrm{A}.2)$, $(\mathrm{A}.3)$, $(\mathrm{A}.4)$, $(\mathrm{A}.5)$, and $(\mathrm{A}.6)$ correspond to equations (6), (7), (8), (9), (10), and (11), respectively.

$$\max_{C} U(C) = \int_0^\infty \frac{1}{(1+\rho)^t} \frac{C^{\,1-\theta} - 1}{1-\theta} L\, dt, \tag{A.1}$$

subject to

$$\dot{K} = I - \delta K \tag{A.2}$$

$$Y = I + C \tag{A.3}$$

$$Y = K^\alpha (AL)^{1-\alpha} \tag{A.4}$$

$$K(0) \geq 0 \tag{A.5}$$

$$\lim_{t\to\infty} [K(t) e^{-t \int_0^\infty r(\tau) d\tau}] \geq 0 \tag{A.6}$$

All the equations must be rewritten into per effective worker form or rearranged.

Equation $(\mathrm{A}.1)$:

$$\begin{aligned} \log{(1+\rho)}^{-t} &\approx -t\rho \\ \frac{1}{(1+\rho)^t} &\approx e^{-\rho t} \end{aligned} \tag{A.7}$$

$$L(t) = L(0)e^{nt} \tag{A.8}$$

$$A(t) = A(0)e^{gt} \tag{A.9}$$

$$\max_{c} U(c) = B \int_0^\infty e^{-\Omega t} \frac{c^{1-\theta} - D}{1-\theta} dt, \tag{A.10}$$

where $B = A(0)L(0)^{1-\theta}$, $\frac{1}{D} = [A(0)\, e^{gt}]^{\,\theta-1}$, and $\Omega = \rho - (1-\theta)g - n$.

Equation $(\text{A}.2)$:

$$\dot{k} = k(t)^{\alpha} - c(t) - (n+g+\delta)k(t). \tag{A.11}$$

Equation $(\text{A}.3)$:

$$y(t) = i(t) + c(t) \tag{A.12}$$

Equation $(\text{A}.4)$:

$$y(t) = k(t)^{\alpha} \tag{A.13}$$

Equation $(\text{A}.5)$:

$$k(0) \geq 0 \tag{A.14}$$

Equation $(\text{A}.6)$ is redundant to treated here.

Combination of all equations can be expressed:

$$\mathscr{H} = B\, e^{-\Omega t}\, \frac{c(t)^{1-\theta} - D}{1-\theta} + \lambda\big[k(t)^{\alpha} - c(t) - (n+g+\delta)k(t)\big], \tag{A.15}$$

A.2 From equation (18) to equation (22)

Brzenina and Veselý (2012) show how to, in general, solve a system of differential equations with constant coefficients. This solution follows their steps.

Equation $(\text{A}.16)$ correspond to equation (18).

$$\begin{pmatrix} \dot{\hat{k}} \\ \dot{\hat{c}} \end{pmatrix} = \begin{pmatrix} \rho - n - (1-\theta)g & (n+g+\delta) - \frac{\rho+\delta+\theta g}{\alpha} \\ \frac{(\alpha-1)(\rho+\delta+\theta g)}{\theta} & 0 \end{pmatrix} \begin{pmatrix} \hat{k} \\ \hat{c} \end{pmatrix} \tag{A.16}$$

This equation is in the form of $x'(t) = Ax(t)$, which means that the searched solution needs to be a vector function:

$$x(t) = C_1 e^{\beta_1 t} v_1 + C_2 e^{\beta_2 t} v_2, \tag{A.17}$$

where C_1 and C_2 are integrative constants, β_1 and β_2 denote eigenvalues, and v_1 and v_2 signify eigenvectors. C_1 must equals 0, the explanation can be seen in section 3.

Eigenvalues:

The eigenvalues β_1 and β_2 equal to the roots of the characteristic polynomial of the matrix A.

$$\det(A - \beta \, \mathsf{E}\,) = 0, \tag{A.18}$$

The solution of the characteristic polynomial leads to a quadratic equation, meaning that there are two solutions.

$$\beta_{1,2} = \frac{\left[\rho - n - (1-\theta)g\right] \pm \sqrt{\left[\rho - n - (1-\theta)g\right]^2 - \frac{4(\alpha-1)(\rho+\delta+\theta g)}{\theta}\left[(n+g+\delta) - \frac{\rho+\delta+\theta g}{\alpha}\right]}}{2} \tag{A.19}$$

Eigenvectors:

The eigenvectors amount to the solution of the following system of the linear homogeneous equations.

$$(A - \beta_2 \, \mathsf{E}\,)v_2 = 0 \tag{A.20}$$

Since the determinant of $(A - \beta_2 E)$ is zero, there must be a non-trivial solution. Then, the first element may be set and the second one is calculated. The eigenvector v_1 would be analogical.

$$v_2 = \begin{pmatrix} 1 \\ \frac{A_{21}}{\beta_2} \end{pmatrix} \tag{A.21}$$

Final solution:

The combination of all pieces of knowledge provides:

$$\hat{k(t)} = e^{\beta_2 t} C2 \tag{A.22}$$

$$\hat{c(t)} = e^{\beta_2 t} \frac{A_{21}}{\beta_2} C2 \tag{A.23}$$

$\hat{k(t)}$ is rewritten to $\log k(t) - \log \bar{k}$, ($\hat{c(t)}$ is analogical).

$$\log k(t) = \log \bar{k} + e^{\beta_2 t} C2 \tag{A.24}$$

$$\log c(t) = \log \bar{c} + e^{\beta_2 t} \frac{A_{21}}{\beta_2} C2 \tag{A.25}$$

Now, the last part involves the solution of the 'Cauchy problem' for $t = 0$.

$$\log k(t) = \log \bar{k} + e^{\beta_2 t}(\log k(0) - \log \bar{k}) \tag{A.26}$$

$$\log c(t) = \log \bar{c} + e^{\beta_2 t}(\log c(0) - \log \bar{c}) \tag{A.27}$$

B Appendix

B.1 From equation (23) to regression equation of model 3

$$\log y_t = e^{-\beta t} \log y_0 + (1 - e^{-\beta t}) \log \bar{y}$$

First, $\log y_0$ needs to be subtracted from both sides of equation (23) and the whole equation must be divided by T.

$$\frac{1}{T} \log \frac{y_t}{y_0} = \frac{1 - e^{-\beta T}}{T} \log \frac{\bar{y}}{y_0} \tag{B.1}$$

The time change is set so that the difference is one period.

$$\log \frac{y_t}{y_{t-1}} = (1 - e^{-\beta}) \log \frac{\bar{y}}{y_{t-1}} \tag{B.2}$$

Both sides need to be transformed into the per capita form.

$$\log \frac{y_t^c A_{t-1}}{y_{t-1}^c A_t} = (1 - e^{-\beta}) \log \bar{y} + (1 - e^{-\beta}) \log \frac{A_{t-1}}{y_{t-1}^c} \tag{B.3}$$

On the LHS, technology variables might be cancelled out, only technology growth, g, remains. On the RHS, A_0 is normalised to 1 meaning that $g(t-1)$ is left.

$$\log \frac{y_t^c}{y_{t-1}^c} = g + (1 - e^{-\beta}) \log \bar{y} - (1 - e^{-\beta}) \log y_{t-1}^c + (1 - e^{-\beta}) g(t-1) + u_t \tag{B.4}$$

Equation (B.4) is the base for all models. Model 1 and model 2 can be obtained straightforwardly so further steps are omitted.

MODEL 3 - output:

Output in the steady state equals to:

$$\bar{y} = (\frac{\alpha}{\rho + \delta + \theta})^{\frac{\alpha}{1-\alpha}}. \tag{B.5}$$

Saving in the steady state amounts to:

$$\bar{s} = (n + g + \delta) \frac{\alpha}{\rho + \delta + \theta g} : \tag{B.6}$$

(Barro and Sala-i-Martin, 1992)

The neoclassical model implies that investments equal to savings, equation (B.7) may be obtained by combining equations (B.4), (B.5), and (B.6).

$$\log \frac{y_t^c}{y_{t-1}^c} = g + (1 - e^{-\beta}) \Big\{ \frac{\alpha}{1-\alpha} \Big[\log \frac{I_t}{Y_t} - \log(n_t + g + \delta) \Big] - \log y_{t-1}^c + g(t-1) \Big\} + u_t \tag{B.7}$$

MODEL 3 - wages:

Recall equation (B.4) in the version of wages.

$$\log \frac{w_t^c}{w_{t-1}^c} = g + (1 - e^{-\beta}) \log \bar{w} - (1 - e^{-\beta}) \log w_{t-1}^c + (1 - e^{-\beta}) g(t-1) + u_t \quad \text{(B.8)}$$

Wage is defined as the derivative of the production function with respect to labour.

$$w_t^c = (1 - \alpha) K_t^{\alpha} A_t^{1-\alpha} L_t^{-\alpha} = (1 - \alpha) y_t A_t = (1 - \alpha) y_t^c \quad \text{(B.9)}$$

Inserting equation (B.9) into equation (B.8) creates equation (B.10).

$$\begin{aligned} \log \frac{w_t^c}{w_{t-1}^c} = g + (1 - e^{-\beta}) \log(1 - \alpha) + (1 - e^{-\beta}) \log y_t^c - (1 - e^{-\beta}) \log w_{t-1}^c \\ +(1 - e^{-\beta}) g(t-1) + u_t \end{aligned} \quad \text{(B.10)}$$

All other adjustments are arbitrary or straightforward.

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