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Analysis of Bubble Presence in Cryptocurrency Markets

DIPLOMA THESIS

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Declaration of Authorship

I hereby proclaim that I wrote my diploma thesis "Analysis of Bubble Presence in Cryptocurrency Markets" on my own under the leadership of my supervisor using only the listed resources and literature.

Prague, January 10, 2019 ....

Yulia Rebrova

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Abstract

This master thesis focuses on the analysis of the cryptocurrency market in 2016-2019 period and aims to confirm the presence of bubbles in this market. First, there are performed SADF and GSADF tests recommended as being able to detect the presence of financial bubbles as well as to indicate the starting and the end date using a date-stamping procedure. Based on the outcomes of the tests performed over the twelve major cryptocurrencies, according to their market capitalization, it can be concluded that there were bubbles present which burst around the break between 2017 and 2018 and there are bubbles started in 2019 for a few cryptocurrencies. Second, there is applied a framework called Log-Periodic Power Law model which is suggested as being able to capture the end of the bubble ex-post and ex-ante together with the price development. Consequently, the Log-Periodic Power Law model was able to capture the time of crash for different cryptocurrencies with high accuracy at the end of 2017 and beginning 2018. It can be stated that the cryptocurrency market had a price exuberance resulted in the bubble burst. For the bubbles started in 2019, the prediction with expanding rolling window was able to mimic the price evolution better compared to the single one prediction period. The critical times of crash was again determined quite precisely but the prediction horizon was short. Overall, the framework captures the speed of the price acceleration and the log-periodic oscillation which differ significantly from one cryptocurrency to the other. This means that the price evolution of cryptocurrencies has different patterns during the bubble period and the price behavior cannot be simply generalized. However, based on the results of calibration of the Log-Periodic Power Law model it is a promising framework the price evolution in the cryptocurrency market which is historically prone to high volatility.

Keywords

Cryptocurrency market, bubble, SADF test, GSADF test, LPPL model, critical point, log-periodic oscillation

JEL Classification

C22, C53, G01, G17

Abstrakt

Diplomová práce se zabývá analýzou kryptoměnových trhů mezi lety 2016 a 2019 a jejím cílem je potvrzení výskytu bublin na těchto trzích. Nejprve jsou provedeny SADF a GSADF testy, které jsou doporučovány k detekci výskytu finančních bublin a k indikaci počátečního a konečného data prostřednictvím definované metody date-stamping. Na základě zmíněných testů na dvanácti hlavních kryptoměnách (podle tržní kapitalizace) se dá shrnout, že se bubliny vyskytly a praskly na přelomu roku 2017 a 2018; zároveň u některých kryptoměn se bubliny začaly objevovat v roce 2019. Následně je aplikován Log-Periodic Power Law model, který by měl být schopen zachytit konec bubliny ex-post a ex-ante společně s vývojem ceny. Ve výsledku byl „Log-Periodic Power Law“ model schopen s velkou přesností zachytit čas prasknutí bublin různých kryptoměn ke konci roku 2017 a začátku roku 2018. Dá se konstatovat, že trh kryptoměn prošel cenovou exuberancí, která vyústila v prasknutí bubliny.

Pro bubliny, které začaly v roce 2019, predikce s expanding rolling window byly schopny napodobit vývoj ceny lépe než predikce jen pouze pro jedno období. Kritické časy prasknutí bublin byly opět určeny zcela přesně, avšak horizont predikce byl krátký. Celkově tento koncept zachycuje rychlost cenové akcelerace a log-periodickou oscilace, které se liší pro jednotlivé kryptoměny. Znamená to, že cenový vývoj kryptoměn má různé vzorce v průběhu období výskytu bublin a cenové chování se tudíž nedá jednoduše zobecňovat. Nicméně výsledky kalibrace log-periodic power law model ukazují slibný způsob odhadu cenového vývoje na trzích kryptoměn, které jsou historicky známe vysokou volatilitou.

Klíčová slova

Trh krytoměn, bublina, SADF test, GSADF test, LPPL model, kritické body, log-periodická oscilace

JEL klasifikace

C22, C53, G01, G17

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Acronyms

ADA	Cardano
ADF	Augmented Dickey Fuller Test
AIC	e Information Criterion
BCH	Bitcoin Cash
BNB	Binance Coin
BSADF	Backward Standard Augmented Dickey Fuller Test
BSV	Bitcoin SV
BTC	Bitcoin
C2C	Cryptocurrency to cryptocurrency
ETH	Ethereum
FOMO	Fear of missing out
FX	Foreign Exchange
GDP	Gross Domestic Product
GSADF	Generalized Standard Augmented Dickey Fuller Test
HP	Hodrick Prescott
ICO	Initial Coin Offering
IPO	Initial Public Offering
JLS	Johansen-Ledoit-Sornette
KPSS	Kwiatkowski-Phillips-Schmidt-Shin Test
LPPL	Log-Periodic Power Law
LTC	Litecoin
OTC	Over-the-counter
PSY	Phillips, Shi and Yu
PWY	Phillips, Wu and Yu

RMSE	Root Mean Squared Error
SADF	Standard Augmented Dickey Fuller Test
SD	Standard deviation
TRX	Tron
USD	United States Dollar
USDT	Tether
XLM	Stellar
XRP	Ripple

Introduction

Cryptocurrency market has been attracting plenty of attention lately not only from professional investors and investment funds, but also from individuals who are hardly familiar with investments. For the past few years many cryptocurrencies have been providing higher average rate of return than conventional financial instruments in securities market. Furthermore, mass media has been actively covering news about cryptocurrency market and blockchain technologies. Aforementioned factors have generated enormous amount of publicity. Yet, trading in such a market have implied higher risks and losses in comparison with traditional markets. Huge price falls in Bitcoin or any other cryptocurrency is nothing new. However, the number of participants and transactions in the market together with the total market capitalization have been growing at alarming rates. That is why crypto economy cannot be overlooked anymore and needs further investigation. Some experts and economists mentioned that some cryptocurrencies resemble signs of financial bubbles, dating back to 2010. Nowadays, the market is often compared with The Tulipmania or with The Dot-Com Bubble. Some experts claim that it might be the largest one in the history and might cause a dramatic downturn in the world's economy. The others suggest that high volatility and instability are common attributes of development of new markets.

The importance and relevance of the chosen topic can be explained by a relative novelty of revolutionary crypto economy, by rapid development of blockchain technologies and by the lack of research on the cryptocurrency bubbles. Ongoing changes in regulations and market participants behavior contribute to significant shifts in the market. Many governments discuss how blockchain technologies can be fully integrated into a future day-to-day life.

The subject of the thesis is the cryptocurrency market as a whole. Furthermore, some cryptocurrencies, chosen based on their market capitalization, will be analyzed separately and then compared.

The aims of the thesis are to evaluate whether the current situation in the cryptocurrency market resembles a financial bubble and to test the market data using various available methods for bubble presence.

To accomplish the task, the thesis defines a set of **objectives**:

- Definition of a “financial bubble”, “cryptocurrency” and specifics of blockchain technologies;
- Determination of factors which can cause bubble bursts;
- Analysis of historical and present price dynamics in the crypto economy;
- Overview of available behavioral, econometric and mathematical methods for financial bubbles modeling;
- Collection and gathering of data needed to provide conclusions;
- Practical development and application methods for testing and modeling;
- Comparison of the methods and achieved results.

The aims and objectives define the structure of this thesis. In order to provide successfully reliable conclusion, there will be used analysis, synthesis, comparison, historical analogues, summarization, behavioral economics, econometric and mathematical methods.

Hypothesis

Certain methodologies can precisely date-stamp the beginning and the end of the bubble. Other cryptocurrencies, called “altcoins’, experience analogous behavior to Bitcoin during observed bubbles. The bubbles occurred in 2017 have similar behavior to those bubbles in making in 2019 which have not been terminated. In addition, price evolution of highly volatile cryptocurrencies can be captured by existing econometric models.

1 Financial Bubbles

1.1 Definition of Financial Bubbles

Financial markets have a critical role in everyone's life, even in the life of a person not engaged in trading activities. That is because the financial markets drive global economic trends, optimism or pessimism in numerous markets; hence, they significantly affect people's wellbeing and prosperity. However, financial markets, as all markets, sometimes deviate from fundamental theoretical concepts and some individuals behave irrationally while making decisions. A financial bubble, which is also called an economic bubble or an asset bubble, can be a consequence of a particular divergence. A bubble burst or price crashes can occur in one market, can also spill over to other markets and cause a downturn in the world economy. There are well-known examples like the Dot-Com Bubble, the tech bubble, which burst in 2000 and the global financial crisis 2008 which started in the US subprime mortgage market. "*An asset bubble occurs when the price of a financial asset or commodity rises well above either historical norms or its intrinsic value, or both*" (Picardo, 2018). The cause could be that market participants overestimate a potential of a new technology or a particular market segment. Alternatively the term can be described as "*a situation in which asset prices appear to be based on implausible or inconsistent views about the future.*" (Krugman, 2013). Yet, there is another definition of financial bubbles which incorporates a psychological side of human behavior and that is why denied by some experts. A financial bubble is "*a situation in which news of price increases spurs investor enthusiasm, which spreads by psychological contagion from person to person, in the process amplifying stories that might justify the price increases, and bringing in a larger and larger class of investors who, despite doubts about the real value of an investment, are drawn to it partly by envy of others' successes and partly through a gamblers' excitement*" (Shiller, 2017). In the context of cryptocurrency market, all definitions should be taken into account to reflect on them in the practical part of the analysis. Clearly, to define the term "bubble" is hard, because various definitions are controversial. For the Log Periodical Power Law model, which will be used in the thesis, proposed by Johansen et al. (2000) a bubble is "a faster-than-exponential" growth, coming from a positive feedback. In less traditional way the bubble is "*a faster-than-exponential increase in asset prices, that reflects positive feedback loop of higher return anticipations competing with negative feedback spirals of crash expectations*" (Filimonov & Sornette 2013).

The Dutch Tulip Mania is referred to the first asset bubble in the history, dated back to 17^{th} century. It is regularly hard to observe or determine intrinsic values in real-time markets because of a high volatility and rapid markets growth. It is common that a presence of bubbles is identified retrospectively, once an unexpected price crash happened. It is so called bubble burst which occurs when a price skyrocketing is followed by a strong decline and a new market price is the way below the maximum price of the asset. The actual characteristic problem going hand in hand with bubbles is that the equilibrium price is not formed under normal market conditions, based on economics theory of supply and demand.

Many economists believe that main root of bubbles is a dramatic deviation of prices of financial instruments and commodities from their intrinsic values caused by high liquidity and irrationality of market participants. Yet, there is an evidence that bubbles exist which are not speculative. For instance, research suggests that bubbles may arise without bounded rationality, uncertainty or speculation. On the contrary, market agents are aware that high return rates are not endless, knowing that a market crash will be followed. However, taking into account high risks, they take an advantage of the situation and gaining huge profits. Non-speculative bubbles are possible because market participants use probabilistic models like Markov Switching Model to predict switching between state of low price volatility to high volatility with a certain probability over time. Less conventional theories illustrate that such developments in market can be sociologically driven.

1.2 Characteristics and Factors of Financial Bubbles

1.2.1 Social and psychological factors

- **Greater Fool Theory** which claims that the event of market crash is caused by permanently optimistic market participants, the fools, how sell already overvalued assets at higher prices to other participants, the greater fools. Such a speculative spiral continues unless there is no greater fool to buy mispriced assets. The situation leads to a market downfall and a price drop. The theory, however, have not been confirmed by empirical research.
- **Extrapolation** is a method used for prices prediction based on an assumption that existing trends will last continuously in the future. Investors extrapolate high returns into the future overlooking a possibility of a change in the trend direction as well as related risks. At some point of time, such a misleading extrapolation leads to higher expectations, overbidding of assets and bubble creation.
- **Herd behavior** of investors chasing last market trends. These market participants rely on decision-making of larger groups of individuals and mimic their behavior.

They copy investment strategy not trying to justify its correctness because of the fear missing out high returns of assets. People do not conduct any deeper analysis or assessment of risks. The collapse of markets is caused by spurious herding and the price of assets climbing much higher than its realistic intrinsic value.

- **Moral hazard** of large entities taking advantage of their dominant position. If there is a cartel in the market which pours into significant capital reserves into a particular asset, its actions can be taken by smaller firms as a signal to follow the trend. The bubble is inflated since the growing demand accelerates the asset price. Once the price reaches its peak the cartel immediately sells the asset which leads to a devastation and bankruptcy of smaller companies unable to stand the price crash.

1.2.2 Key characteristics of asset bubbles

- Significant deviations of market ratios from historically known ones or typical within a particular industry. During housing bubble, prices for real estate were untypically high compared to income. The unusual high price to earnings ratio for stocks means that investors are paying more for a dollar of earnings;
- Abnormal usage of debt leverage for buying assets;
- Providing more loans for riskier borrowers with lower credit scores or uncollateralized loans and mortgages;
- Lower ability of borrowers to repay loans compromised due to the expected price increase in the future;
- Using poor reasons or being too optimistic explaining the growth in asset prices like "housing prices can only go up";
- Intensive marketing of the asset as well as active generation of publicity by media;
- Current account imbalances, resulting greater savings than investments, which lead to growing volatility of capital flows.
- Low interest rates boosting lending and lending up.

The theory of financial instability presented by Minsky (1992) can explain the development of a market turbulence and behavior of market participants throughout the bubble creation.

1.2.3 Five steps of asset bubbles:

1. **Displacement** happens when investors and traders are admired by new patterns in markets like an innovative technology, a know-how or interest rates which are all-time low historically. The most representative showcase of displacement is the decrease in the federal funds rates from 6.5% in May, 2000, to 1% in June, 2003. Within the three-year period, the interest rate on 30-year fixed-rate mortgages

dropped by 2.5 p.p. to a rate of 5.21% which was the historical minimum. Such a shift led to the beginning of the housing bubble.

2. **Boom** is a phase when prices increase slowly after displacement starts and then reach momentum when more and more investors participate in the market, shifting the market to the boom stage. At this stage, media get involved and provide active coverage for the asset which gained such a high popularity. Fear of missing out on the opportunity of a lifetime to invest in the asset forces more speculations, which leads to the alarming growth of the number of participants.
3. **Euphoria** is the stage when caution about the asset arises since prices grow extremely high. For instance, at the peak of the Dot-Com bubble in 2000, the cumulated value of all technology stocks traded on the Nasdaq was larger than the GDP of the majority of the countries in the world. At this phase, new measurements and methods are used to validate the fierce increase in prices.
4. **Profit Taking** stage is when some institutional and large investors note some signs of the bubble presence and start to sell out their positions and gaining profits. It is difficult to predict a moment of a bubble burst as well as extremely risky, so many investors having short positions can suffer losses for a long period. It is worth to notice that a minor event can cause a bubble cease to exists. For instance, sometimes warning signs are ignored by markets as it happened in August 2007, when BNP Paribas stopped withdrawals from funds with significant exposure to the U.S. subprime mortgages since it was unable to evaluate their holdings. Yet, this negative event was overlooked, because in a few months equity markets peaked again.
5. **Panic** is a final phase when asset prices crash as quickly as they rose. All participants, investors and speculators, whose values of holdings are falling rapidly are trying to sell those assets at any price because they met margin call already. Since supply excesses demand, prices fall dramatically. The most memorable global panic in financial markets was in October 2008, a few weeks later when Lehman Brothers announced bankruptcy and within a month world equity markets lost over 20% of the total market capitalization.

2 Cryptocurrency Market

2.1 Definition of Cryptocurrency

A cryptocurrency is a digital asset which uses cryptography to make financial transactions secure, to have a control over the creation of new units and to unable verification of asset transfers. Cryptocurrencies are also called as alternative currencies since they have various legal statuses depending on laws and banking systems in different countries. In some countries they are allowed for use and trade, in others they are restricted or even banned. Bitcoin is the first decentralized cryptocurrency and the most prominent one, having the highest market capitalization, as of 23rd December 2019. Since the creation of Bitcoin, there were more than 4000 altcoins, alternative versions of bitcoin (Vigna, 2017), which appeared in the crypto market.

The most important property of cryptocurrencies is decentralized control. Cryptocurrencies are based on a blockchain which provides the validity of each crypto's coins. Cryptographic technologies and sophisticated encryption algorithms are implemented to secure payments of online transactions and data exchanges. Blockchains' design is safe and secure which makes almost impossible a double spending, one digital token can be used more than once, only 51% attack can hack the system. Cryptocurrency is created by the whole cryptocurrency environment collectively, where governments or central banks cannot intervene, and is not backed by any asset like gold, yet it is believed it has a value itself. Generally speaking, all users are responsible for stability and development within a network. Miners use extensive capacities and sophisticated machine learning algorithms to validate and timestamp transactions for a fee, to add the information to distributed ledgers. Distributed ledgers contain blocks where all information is stored. Once any information is recorded in the block, it cannot be simply altered without the change of following blocks that has to be approved be the majority of network. Hence, the system reliably functions itself and cryptocurrencies do not need a trusted third party to function properly or control them.

2.2 Cryptocurrencies in The Context of Monetary Theory

While cryptocurrencies is an innovative and purely digital concept with no backing by government, it is important to see how such currencies can fit into monetary theory for fiat money. If some of the monetary theory concepts are hold, it might make cryptocurrencies a form of e-cash which can replace fiat money used today. If the theory cannot be applied to the cryptocurrencies, they will be used for speculation mainly as it happens today. Cryptocurrencies are not localized nor have a specific geography for use and not limited to any particular virtual economy, so its' circulation is not limited. Due to this feature and decentralization within network, money supply control nor monetary policy can be applied as to traditional currencies.

Traditional money has three most well-known function. First, it should be generally accepted as a medium of exchange. Secondly, it should be a unit of account so individuals can compare the cost of goods and services. Thirdly, it should be a store of value that is stable throughout the time. For instance, the Central Bank of Canada concluded in the research that cryptocurrencies do not fully have this functions in place. Mining of cryptocurrencies in a way is a money emission for fiat currencies, yet it requires some hardware and comes with high electricity consumption. When price of Bitcoin grows significantly, miners become more active and invest more in hardware, to increase a computational power needed for the Bitcoin production. Since the Bitcoin network adjusts the difficulty of the cryptographic task to solve to mine coins, when more miners come to scene with more computational power, the difficulty increases. Thus, this leads the mining to become less lucrative and the price of the Bitcoin goes down again. Such situation is not common for traditional money.

To have a better perspective on digital money leveraging blockchain technologies, Quantity Theory of Money can be used to unfold some issues related to cryptocurrencies and their likelihood of replacing fiat money. Famous Fisher's equation is:

$$MV = PY. \tag{2.1}$$

Taking Bitcoin as the most famous cryptocurrency with the highest potential, M is the money supply which is equal to predefined 21 millions of Bitcoins. Unfortunately, while quantity of Bitcoin is fixed, the available amount decreases due to the loss of private keys. Hence, the available money supply can reach zero at some point in the future. Money creation rule for Bitcoin is fixed, the number of Bitcoin can be easily predicted for any future point of time, unless the whole Bitcoin project is altered. Same is for the inflation of M, because it is open source data about how many Bitcoins are created per a block produced. Turning to V, the velocity of money, it is very difficult to calculate for Bitcoin or other cryptocurrency, due to the lack of reliable source. Every single transaction is recorded without a possibility of double spending, since the network is pseudo-anonymous, nobody knows if the transaction was done to purchase something or it was a transfer between two accounts of a single person, or between exchanges for speculation purposes. So the transactions with no real impact cannot be cleaned out like FX volumes which are not considered in GDPs of counties. Looking into P, the price of the goods and services, with its decrease the purchasing power grows. Purchasing power for Bitcoin have been varying dramatically over the past few years. Lastly, Y is the total economic output which is goods and services produced for further exchange. It is crucial that very limited number of goods and services are bought or sold in Bitcoin, everything is paid in fiat money, so Y is highly dependent on fiat world for Bitcoin.

In conclusion, for the fiat money the Equation 2.1 a growth in money supply leads to a growth of general price level P that decreases a purchasing power. Whereas, for Bitcoin or generally for cryptocurrencies the outcomes can be different compared to fiat currency.

2.3 Advantages and Disadvantages of Cryptocurrencies

It is admitted that blockchain technology is revolutionary. The technology can become widespread because it can be implemented not only in banking, but also in almost any industry due to its functionality. Disintermediation is beneficial because systems do not require trusted third-parties, enabling users to control all their transactions and activities. By nature blockchain technology is tamperproof, decentralization makes it secure and resistant to attacks, no one can block or suspend someone's account like bank accounts.

On the other hand, there are plenty of disadvantages which might affect future development of the cryptocurrency markets together with prices. First of all, uncertainty tight to regulation and legal status. Since the development of the technology is rapid, financial institutions and governments do not address this issue fast enough, so the future is unclear. That is a major deterrent factor for cryptocurrency global adoption. Cryptocurrencies are criticized because of a popularity in darknet markets, money laundering and justice obstruction. Amongst other cryptocurrency weaknesses is lower scalability compared to traditional banking. In addition, cryptocurrency market is highly volatile, low liquid, there were several manipulations taken place recently. It remains to be seen how shortcomings will be addressed in the future.

2.4 Cryptocurrency Exchanges

Some cryptocurrencies are traded publicly and some privately. There are two main kinds of exchanges for cryptocurrencies. There is a so called fiat exchange which enables the transfer of US Dollars, Euros, and other most common government-backed currencies into traded cryptocurrencies. Such platforms charge a fee for their services of around 1% for each transaction. Exchanges play a key role for high market liquidity and enable a comparison between digital money with traditional ones. The second type is cryptocurrency to cryptocurrency exchange, C2C, which enables a trade of cryptocurrencies with each other. Alongside with public exchanges, many cryptocurrencies are traded privately, a large OTC cryptocurrency market exists which is hard to gauge.

2.5 Current Market Situation

2.5.1 Price dynamics

Price dynamics for the major cryptocurrencies between the end 2015 and July 2019 is represented in Figure 2. 1. Many cryptocurrency prices were fluctuating dramatically and volatility was present during the whole period. In November 2017, Bitcoin price reached its all-time high, 20000 USD. Yet, it is seen that some cryptos have shown similar trends meaning that there was a correlation between those currencies.

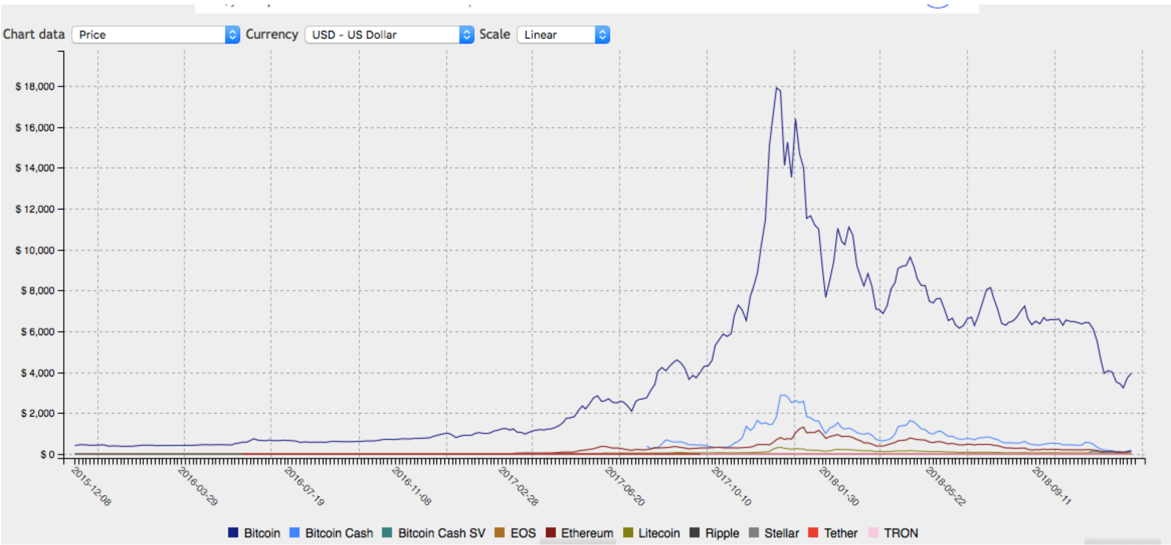


Figure 2. 1 Price chart of the 10 major cryptocurrencies 12.2015 - 06.2019
(Source: https://www.cryptocurrencychart.com/top/25)

2.5.2 Price correlation

The respective correlation of cryptocurrencies is shown in the correlation coefficient matrixes below. In 2016, pairwise correlation for already existing cryptocurrencies was not exceeding a value of 0.63, which is exhibited in Table 2. 1 between Bitcoin and Litecoin. The rest of digital coins was not that strongly correlated. The majority of cryptocurrencies with the largest market capitalization in 2018 and 2019 even did not exist in 2016. In 2017, relationships became much stronger compared in 2016 values. All cryptocurrencies were positively correlated.

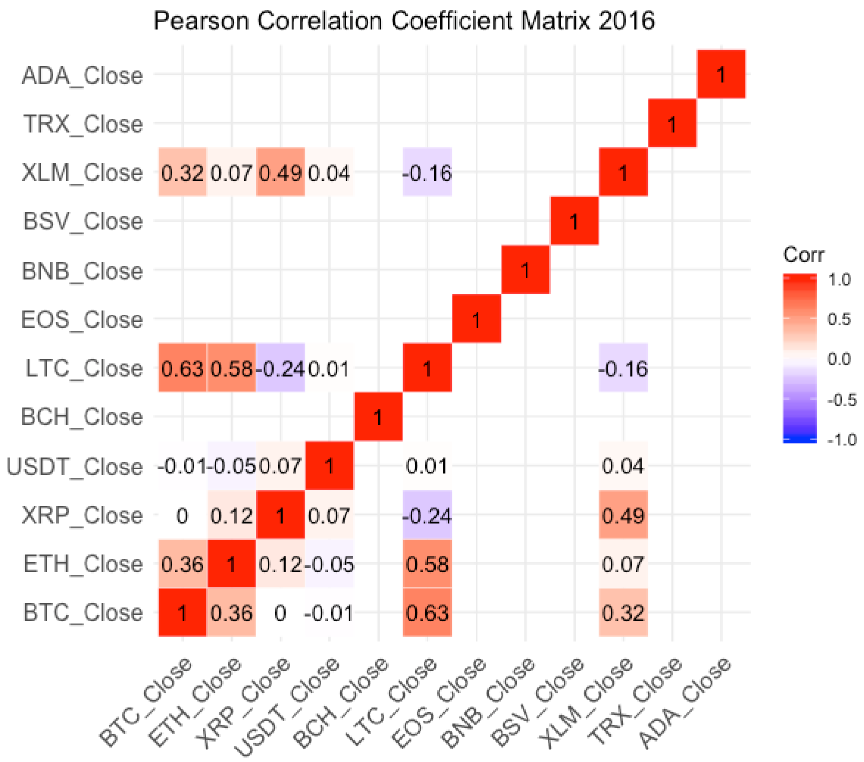


Table 2. 1 Pairwise correlation heatmap in 2016
(data: coinmarketcap.com, individual work)

Table 2. 2 indicates that correlation for many pairs overcame a value of 0.9 and the lowest correlation coefficient was 0.28 between Ripple and Tether. After a peak of the market in the break between 2017 and 2018, the strong correlation persisted in the market, yet the overall picture had changed.

Table 2. 3 shows that Tether was negatively correlated with the majority of the coins, the relationship between some of the pairs became stronger, some pairs became slightly less correlated. Overall, the number of Initial Coin Offerings (ICOs) was growing on year-on-year basis. It can be concluded, that in general cryptocurrencies were getting more popular and there were higher capital inflows on the cryptocurrency exchanges. Those factors, could have led to creation of bubbles in the cryptocurrency market.

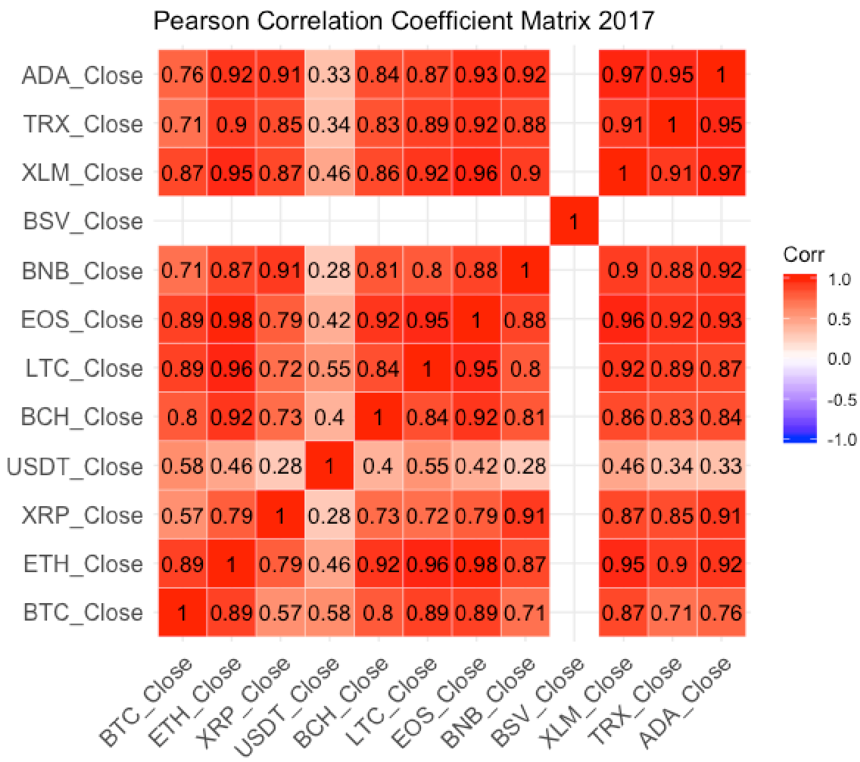


Table 2. 2 Pairwise correlation heatmap in 2017
(data: coinmarketcap.com, individual work)

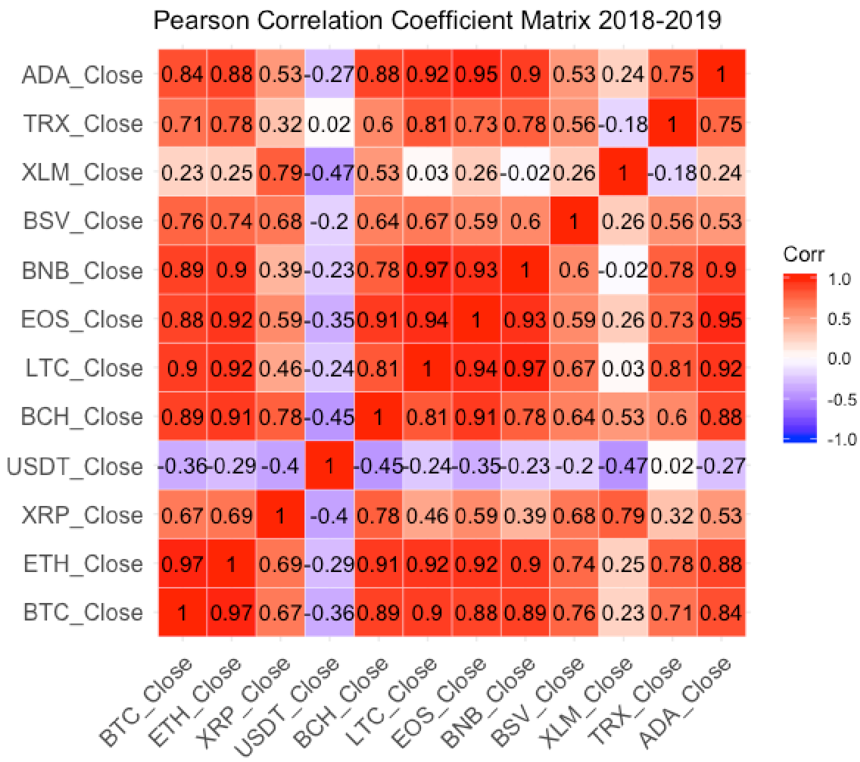


Table 2. 3 Pairwise correlation heatmap in 2018-2019
(data: coinmarketcap.com, individual work)

2.5.3 Market capitalization

Figure 2. 2 below shows cryptocurrency index for the last three years, between 2015 and 2018. It is obvious that starting from May 2017 total market capitalization began to climb up with abnormal rates. Such a trend lasted till the peak was reached at the beginning of 2018. Then there was an extremely high volatility in the market, which might represent a panic, yet it will be analyzed in the practical part of the thesis.

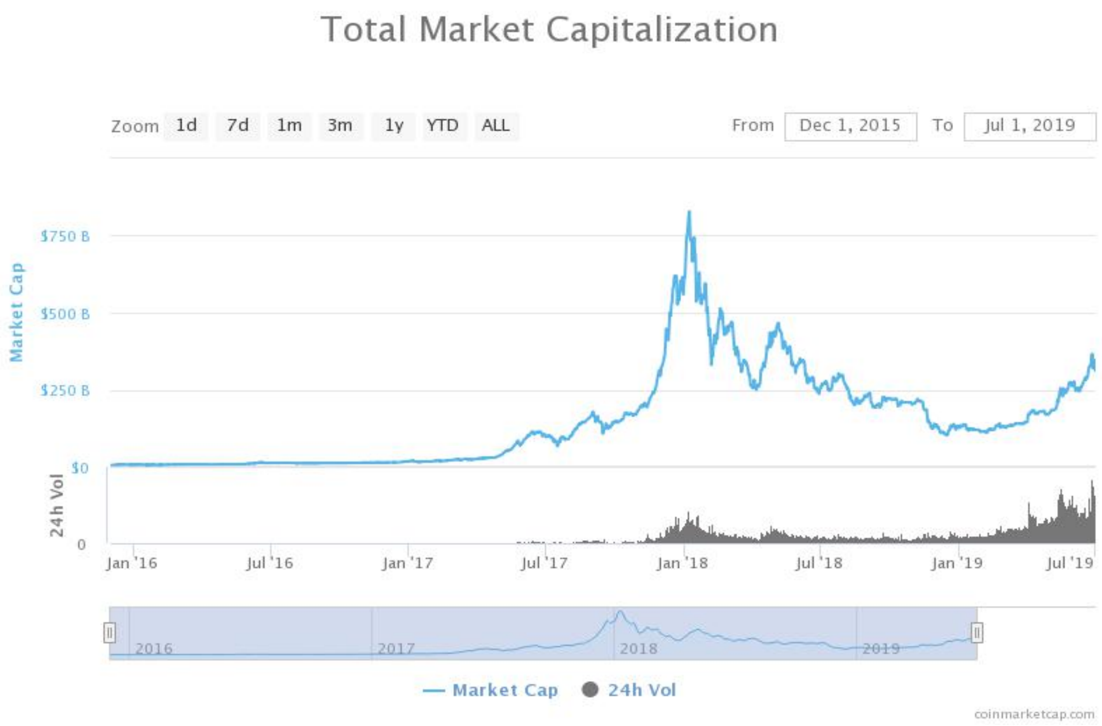


Figure 2. 2 Total market capitalization in cryptocurrency market 12.2015-06.2019
(Source: https://coinmarketcap.com/charts/)

While bitcoin is still a dominant cryptocurrency, it can be noticed from Figure 2. 3 that there were some shifts in the market power, especially in June 2017, when the gap between Bitcoin and Ethereum was close to disappear. Bitcoin lost its power overtime with decreasing relative market capitalization and provided room for other cryptocurrencies to develop. Afterwards, the dominance starter to be stronger, yet never has reached its maximum. The market started to be more turbulent from 2017.

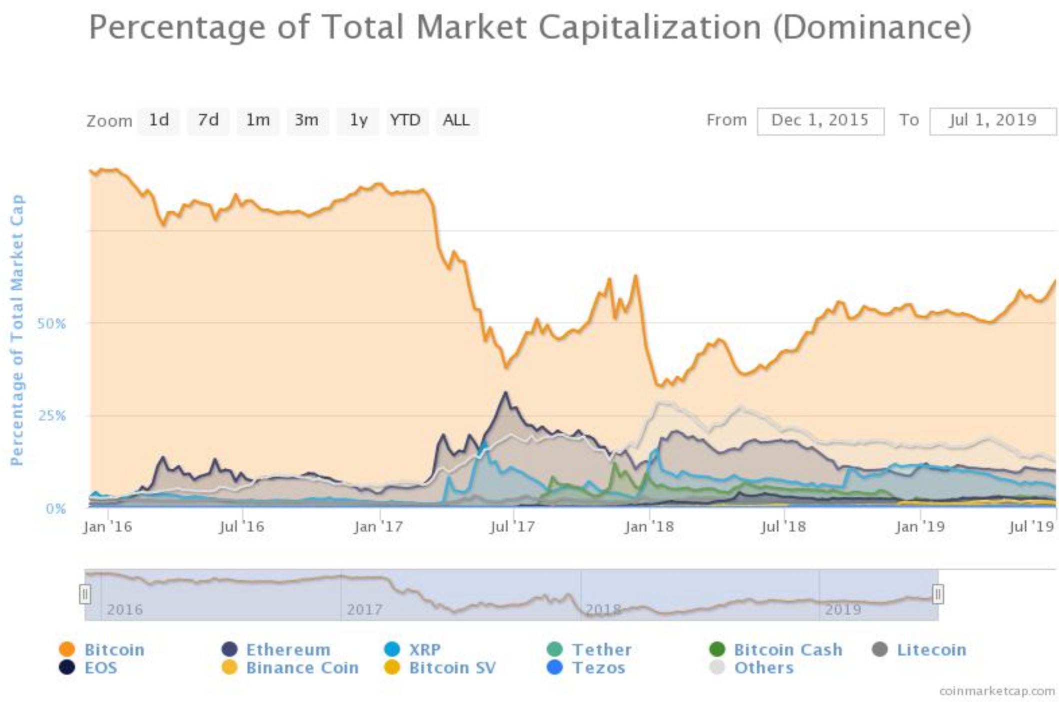


Figure 2. 3 Market capitalization share in cryptocurrency market 12.2015-06.2019 (Source: coinmarketcap.com)

2.6 Cryptocurrency Market vs Dot-Com Bubble

Cryptocurrencies and ICOs are often compared with the historical Dot-Com bubble which occurred in 1990s, because it was a revolutionary innovation, which emerged and grew rapidly, possessed technological uniqueness and changed the business. After the price crash and further analysis of the causes of the bubble burst, it was acknowledged that in the Dot-Com market there were many companies with not well-developed, ineffective and week business models.

Today amongst top 2000 largest public companies in the world at least 50 have been involved in blockchain technologies and have made progress by the end 2018. However, according to Spiceworks report, 56% of large enterprises are planning to implement emerging blockchain-enabled solutions.

In comparison, on the peak of the Dot-Com bubble, 126 firms added to their name ".com", while 57 had already removed it between 2000 and 2001. Such a trick had been used by over a 100 companies, changing their names to something that evokes blockchain or crypto to take advantage of the current hype.

2.6.1 KodakCoin example

For instance, as presented in Figure 2. 4 a legendary company Kodak announces a KodakCoin at the beginning 2018, after the introduction share prices tripled, soaring from around 3.13 USD to 10.7 USD. That was a successful example of gaining momentum and brand recovering. Yet, the after catching the fire, the current share price decreased 2.7 USD, as of 24th December 2018.

Figure 2. 4 Eastman Kodak Company Stock Price
(Source: finance.yahoo.com)

2.6.2 Riot Blockchain Inc. example

Another example is Riot Blockchain Inc., previously known as Bioptix, which was not involved into cryptocurrency business until October 2017 and worked in a veterinary sector. It seemed that company diversified its portfolio of activities, yet blockchain had never been plan to become a primary or a core business for the company. Looking at Figure 2. 5, from 8 USD, prior to a name change, the share price climbed over 40 USD, questioning the reason behind it.

Companies and corporations containing any blockchain related “buzzwords” in their names with greater ease attract investors and encourage funding. However, many companies have not developed business models or competitive advantages, but rather focused only on the growth of token prices after. Businesses believe that blockchain technology will work itself generating profits.

Figure 2. 5 Riot Blockchain, Inc. Stock Price
(Source: finance.yahoo.com)

2.7 Fraudulent Activity in The Cryptocurrency Market

Cryptocurrencies and blockchain have become number one topic on tech conferences. Furthermore, there was developed a large community of crypto fans, which led to high demand for topic related events like meetups and talks. Nowadays all people discuss crypto, including children. Marketing budget for ICO campaigns sometimes reaches 80% of pre-ICO funding.

Another issue is a growing number of scams due to insufficient professionality and knowledge of market participants. Fraudulent ICOs, fake wallets, pyramid or Ponzi schemes, pump and dump groups as well as cloud mining are the most common ones. All this affects market prices, reliability and trust in the market.

Since ICO is a way of capital-raising similar to Initial Public Offering which is not subject to current regulations and such an offering does not fall under the definition of securities, there is a growing number of illegal activities like fraud.

Pump and dump mechanisms are another phenomenon of crypto market which is yet unavoidable. Well-organized individuals, typically using private groups on messengers like Telegram, choose one coin as a target to drive price up. They start selling and buying between each other small volumes of cryptocurrencies, sometimes using bots.

Besides, the actors actively engage into spreading news about growing price on social media to trig a fear-of-missing-out, FOMO, of unprofessional investors. Once the target price is reached, the actors initiate a dump sale and crash the price. This harms unprofessional investors and incur massive losses even for large investors. Massachusetts Institute of Technology in their review claim that pump and dump schemes are estimated 7 million USD in daily volume. Cloud mining schemes which provide individuals with some server space for mining as well as processing power and electricity and promise outstanding returns. Some mining enthusiasts lack knowledge, that each time the difficulty of mining rises and gives lower returns.

2.8 Regulation Issues

The People's Bank of China prohibited ICOs 4th September 2017, cryptos like Bitcoin and Ethereum crashes steeply in response to such actions which is displayed in Figure 2. 6.

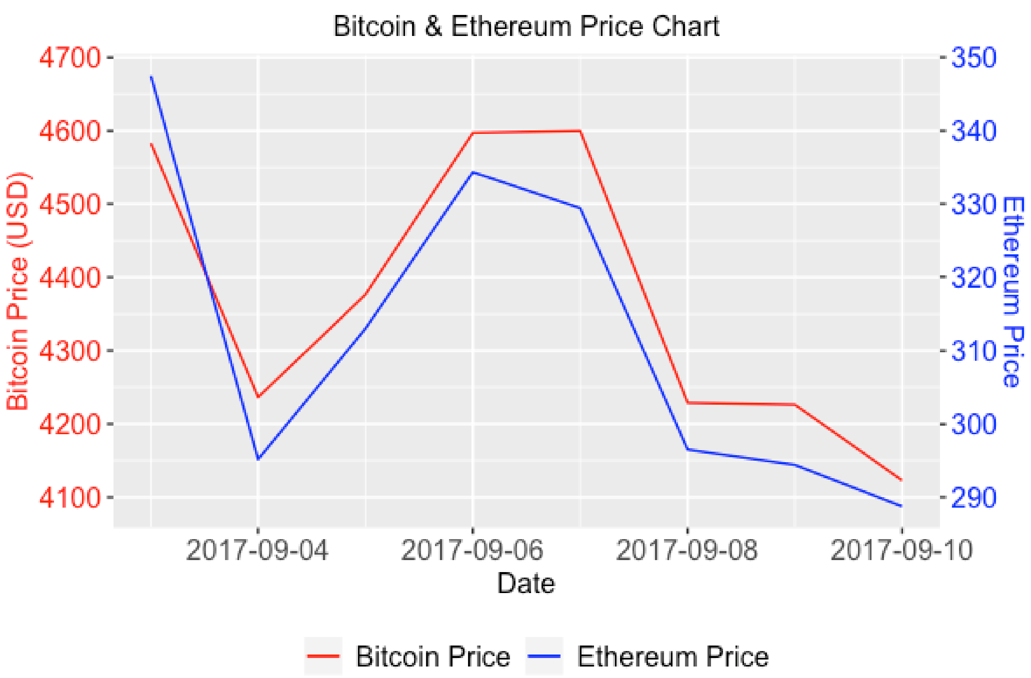


Figure 2. 6 Price reaction to the announcement of People's Bank of China (data: coinmarketcap.com , individual work)

In early 2018, Google together with Facebook, Twitter, LinkedIn and others banned all advertisement of ICOs on their platforms. Unprecedent and unforeseen changes of regulations heavily affect cryptocurrency markets.

2.9 Herding Behavior

2.9.1 Influencers

Market is also affected by influencers, sometimes to a ridiculous extreme. They can drive prices up with a single tweet. John McAfee, successful tech entrepreneur, provides investment advice on Tweeter, Figure 2. 7. There is a correlation between his promotional posts and prices spikes. Digibyte reacted with a price jump of 60% within few minutes after advise given by McAfee, whereas the price burst more than tripled in half an hour.

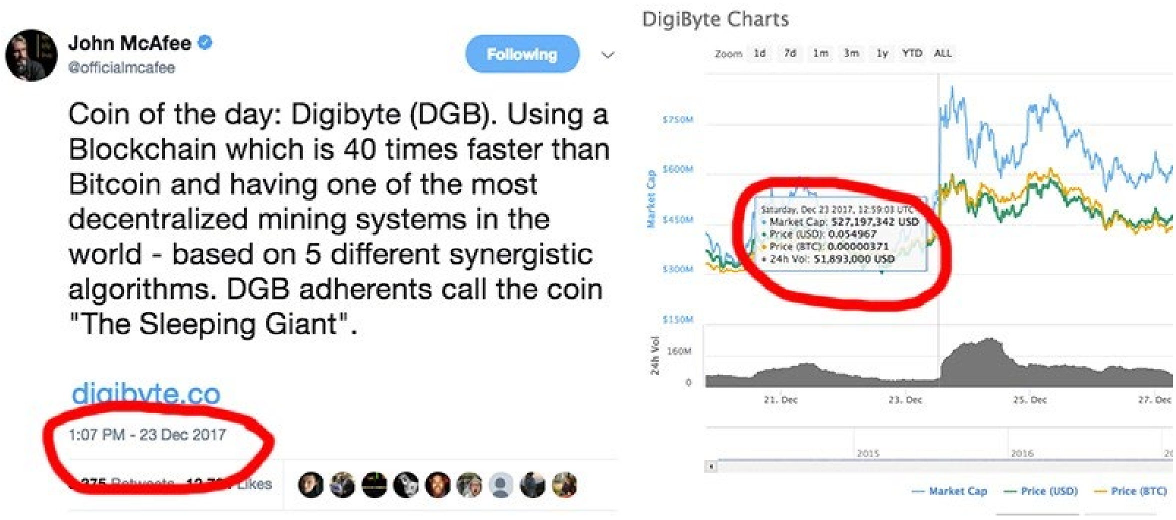


Figure 2. 7 Screenshots of market response to John McAfee Twitter post (Source: https://www.vice.com/en_us/article/9knnpz/john-mcafee-twitter-coin-of-the-day-cryptocurrency-markets)

2.9.2 Speculation over Vitalik Buterin death

Another example of manipulation of prices is a headline about a death of Vitalik Buterin, the creator of the second most valuable crypto coin on the market Ethereum, posted on 4Chan website 25th June 2017. The post was deleted soon, however, the story went viral and rumors started to spread quickly on other sources. The news was notorious and caused hoax on the market. Within hours Ethereum lost 4 billion USD of its total market value and a price crash over 10%. A meltdown was provoked by fake news from 4Chan user which was later disapproved by Vutalik Buterin himself and the price started to recover. Ethereum is decentralized and yet the reaction of market participants displayed little understanding of such a concept and showed how vulnerable cryptocurrencies are.

2.9.3 Introduction of Libra by Facebook

The most recent example of potential price exuberance is an appeared information about Facebook intended their own cryptocurrency which was released in May 2019. The formal announcement about a Facebook coin Libra occurred 18th June 2019. Over May and June Bitcoin and Ethereum price more than doubled. Bitcoin price overcame 13000 USD and Ethereum price reached 337 USD respectively. Figure 2. 8 visualizes the situation comprehensively. Other coins reacted in the same fashion. In June the prices topped for 2019. The tech giant, Facebook, brought optimism to the cryptocurrency market what stimulated to price jumps and speculative investments. Afterwards, Libra faced a lot of criticism from lawmakers who raised regulatory concerns. In addition, the blockchain's distributed ledger is supposed to be used, however, Libra will not be decentralized, nor rely on cryptocurrency mining meaning that Libra is more of a traditional currency than a trendy cryptocurrency. Hence, it can be claimed that many people did not try to understand the underlying technology, but solely relied on behavior of the crowd when making buying decisions. Once, there were more information available, the prices in the cryptocurrency market started to decrease.

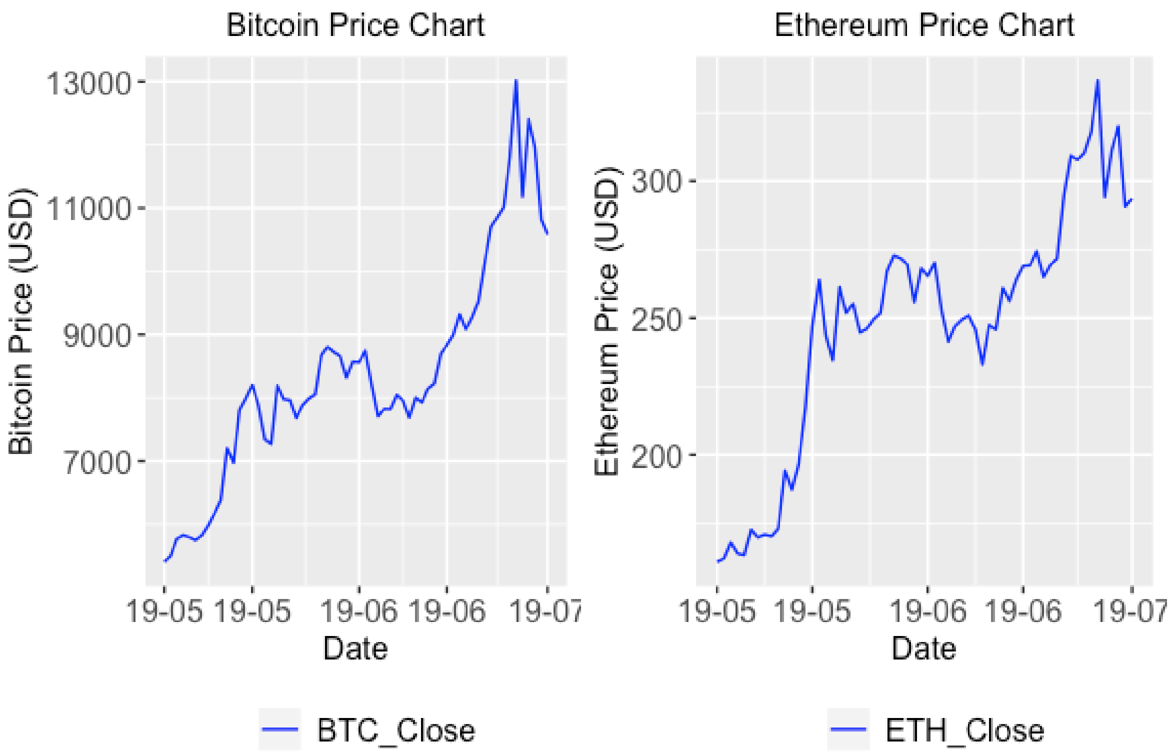


Figure 2. 8 Bitcoin & Ethereum price charts after an announcement of Libra (data: coinmarketcap.com, individual work)

The aforementioned examples remind similar behavior patterns to behavior during the Dot-Com bubble. Price manipulations come from different sources, yet some market participants actively pursuit market trends not engaging in analysis of fundamental values of cryptocurrencies and react to some news like a herd, potentially possessing herd behavior.

2.9.4 Google Trends

Google Trends is a website where information about the popularity of different search queries can be found. The data contain the relative search frequency of various queries over time. To demonstrate the market behavior, “Bitcoin currency”, “Ethereum” queries were chosen for further analysis and comparison to the price evolution of Bitcoin and Ethereum. In 2016, the interest in the mentioned queries was stable for both digital currency as shown in Figure 2. 9 and Figure 2. 10. In 2017, when the extreme price upswings started to occur, the search frequency data showed the same behavior for bitcoin which was driven by the responsiveness of the public to the market news. The Ethereum, the growth of Google trends was outpacing the price growth. Public interest was ahead of the price spikes. From the beginning of the year till June 2017, Ethereum price grew from 8.38 USD to 400 USD , whereas the interest from 2% to 74%.

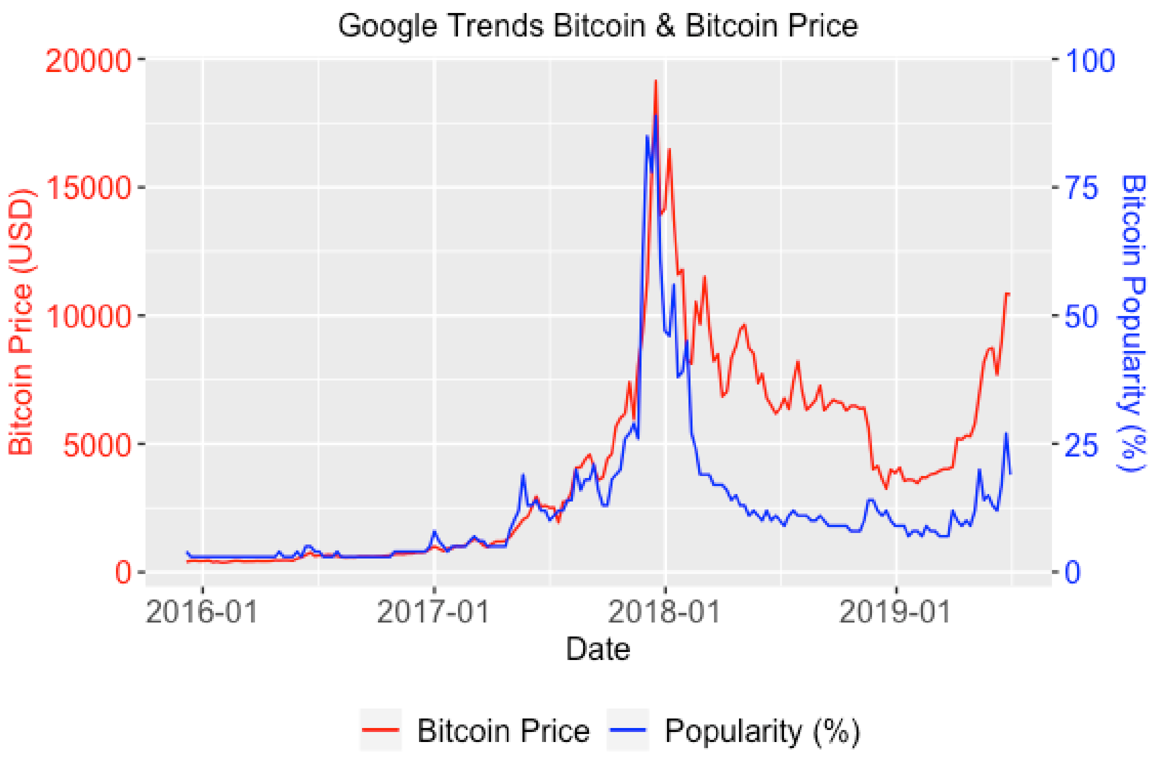


Figure 2. 9 Evolution of Google Trends for Bitcoin compared to its prices
(data: trends.google.com, individual work)

After the crash of prices in early 2018, the interest in the cryptocurrency market dropped dramatically and never was as high as when prices were around all-time-high over Google search engine users. For Bitcoin, Figure 2. 9 exhibit that although prices in 2018 and 2019 were higher than prices till the middle 2017 and high volatility was observed, the search queries number leveled off below 25%.

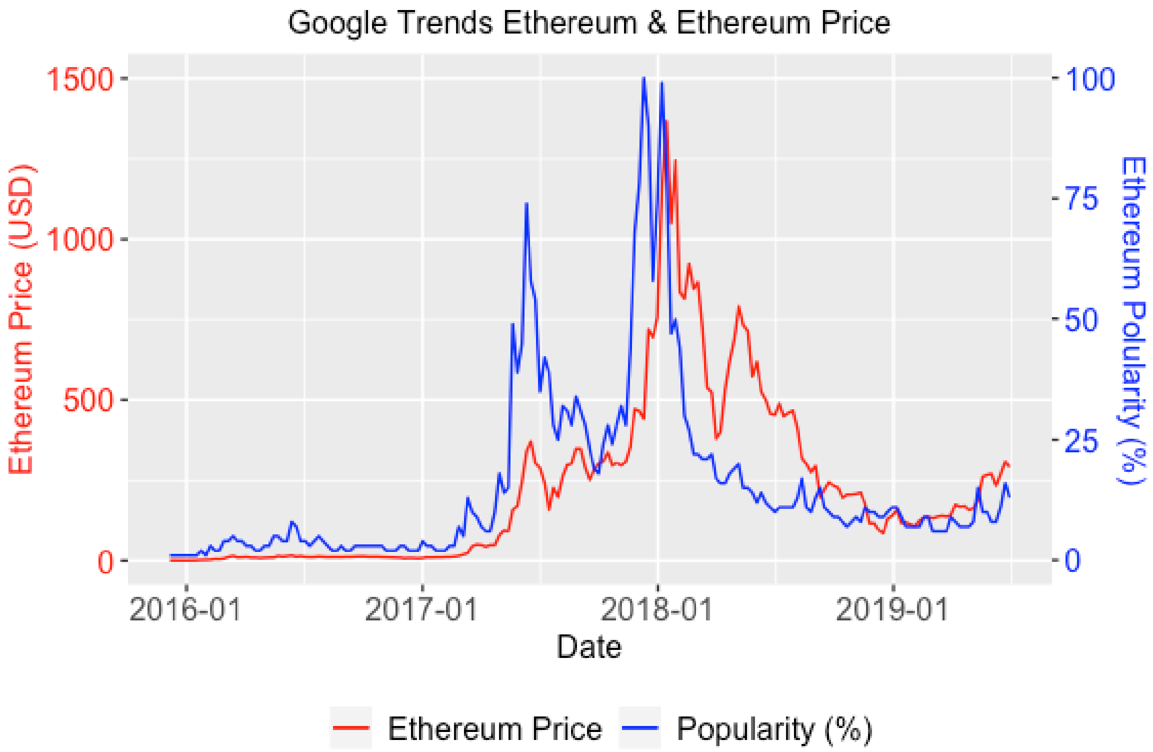


Figure 2. 10 Evolution of Google Trends for Ethereum compared to its prices (data: trends.google.com, individual work)

Looking at Figure 2. 10, the same pattern occurred with Ethereum, however, Google trends data copied the shape of price dynamics more. It is crucial to understand, whether market participants were involved a coordination mechanism and a collective decision making led to the price exuberance over a given period. It is clear that people had been searching for news and media which in turn have played an important role into setting expectations and growing hysteria over blockchain projects. Since, the majority of crypto-investors have weak prior knowledge and limited in processing information, they were aiming to generate easy profits overlooking their private information. Instead, a large number of traders were imitating the investment decisions of individuals who made it previously possessing a consensus mechanism. From the Figure 2. 8, the hypothesis is that herding behavior could trigger the creation of speculative bubbles and this should be investigated further in the practical part of the thesis.

3 Econometric Approach

3.1 Definition of a Rational Bubble

To derive a rational bubble mathematically, at first assume that each individual maximizes expected utility over a period of time. An optimization problem looks as:

$$Max\ E_t\{\sum_{i=0}^{t} \beta^i u(c_{t+i})\}$$

where c_{t+i} is consumption of a single item, β^i is a discount factor for future consumption, $u(c_{t+i})$ is an increasing utility function, E_t is the conditional expectation operator which depends on an information set which affects present and past values of the variables in the model. y_t is an endowment for each period. Individuals can not only consume goods, but also purchase shares x_{t+i} at the price of P_{t+i} per share with a dividend of D_{t+i}. The budget constraint for an individual at time t+i is

$$c_{t+i} = y_{t+i} + (P_{t+i} + D_{t+i})x_{t+i} - P_{t+i}\, x_{t+i+1}.$$

The first-order condition for the utility maximization problem:

$$E_t\{\beta u'(c_{t+i})[P_{t+i} + D_{t+i}]\} = E_t\{u'(c_{t+i-1})P_{t+i-1}\}.$$

The right-hand side of the equation is the marginal utility from selling a share at period $t + i - 1$ whereas the left-hand side represents the present value of $t + i$ expectation of the marginal utility from selling a share next period.

For the simplicity, in this simple asset pricing model is assumed that required rate of return is constant under risk neutrality. Then maximization problem could be also written

$$\beta E_t(P_{t+i} + D_{t+i}) = E_t(P_{t+i-1})$$

Under the no-arbitrage condition and risk neutrality assumptions, in the equilibrium, where interest rate, r, is constant over a set period of time

$$E_t(P_{t+i-1}) = \frac{1}{1+r} E_t(P_{t+i} + D_{t+i})$$

The price of an asset at $t + i - 1$ equals the expected discounted payoff at the next period, $t + i$.

The next equation (3.1) is used for empirical test for bubble presence. The current market price of the asset, P_t, then can be decomposed into two parts. The first represents a cumulative sum of expected future products of dividends in infinite period of time which is also called a market-fundamentals solution. The second part represents non-fundamental bubble component B_t:

$$P_t = \sum_{i=1}^{\infty} \left(\frac{1}{1+r}\right)^i E_t(D_{t+i}) + B_t \tag{3.1}$$

$$E_t(B_{t+1}) = (1+r)B_t.$$

Financial time series has its specifics which have to be bear in mind. Such aspects like non-stationarity of time series, information asymmetry, heteroscedasticity as well as changes in dividend generation process and discount factor can affect robustness of empirical analysis. If there is no asymmetry, individuals are risk-neutral, discount factor is constant then

$$P_t = \sum_{i=1}^{\infty} \left(\frac{1}{1+r}\right)^i E_t(D_{t+i}) + \lim_{i\to\infty} \left(\frac{1}{1+r}\right)^i P_{t+1}. \tag{3.2}$$

If the second part of the equation (3.2), bubble component, is not zero, then an individual due to discount factor can sell the asset at the expense of lower utility. After a log-linear transformation of the expression of bubble component can be denoted as

$$b_t = \lim_{i\to\infty} \rho^i E_t(p_{t+i} - d_{t+i})$$

where $p_{t+i} = \log(P_{t+i})$, $d_{t+i} = \log(D_{t+i})$, $\rho = 1/[1 + e^{\overline{p-r}}]$, $\overline{p-r}$ is the average logarithm of price-to-dividend ratio. In this model, if p_t and r_t are at most $I(1)$ processes, $b_t \neq 0$, then only the presence of a bubble can explain explosiveness of $p_t - d_t$. Hence, some test of bubble presence test for an explosive behavior in $p_t - d_t$ (Campbell and Shiller, 1988).

When actual price is higher than fundamental price, there is a positive bubble part, then it means that investors assume that overpaying for the asset will pay off because it is expected that bubble will inflate. Market participants are ready to pay premium in addition to fundamental price since they expect the premium to be greater in the next period.

3.2 Hodrick-Prescott Filter

In order to use econometric approaches and analyze the market of cryptocurrencies, the should be defined some quantifiable criterions which signalize the bubble is present in the particular market. Financial bubble is tightly related to a price volatility which is abnormally high within a specific period of time. The difference between deviations which adhere to the creation of bubble and some temporarily deviations depends on the type of a particular financial asset. Yet, International Monetary Fund states that the bubble, during the boom cycle, is any large changes of price which have to be in the top 25-quantile of all recorded peak-peak price increases in the data sample.

Also, price bubble is defined as " *a period in which aggregate real asset prices are more than 10 percent above their recursively estimated Hodrick Prescott trend*." (Detken & Smets, 2004)

Hodrick Prescott (HP) Filter is a tool for smoothing time series which is commonly used in real business cycle theory for obtaining an estimate of a series affected by long-term trend component by removing a cyclical component. This method was used for the first time for the analysis of U.S. business cycle after war. HP filter is a two-sided linear filter which calculates the smoothed series s of y while minimizing the variance of y around s with a penalty which constrains the second difference of s. HP filter is an optimization problem which minimizes the following:

$$\min_{s}(\sum_{t=1}^{T}(y_t - s_t)^2 + \lambda \sum_{t=2}^{T-1}[(s_{t+1} - s_t) - (s_t - s_{t-1})]^2)$$

where $y_t = s_t + c_t + \epsilon_t$ decomposed into a trend component, s_t, a cyclical component c_t and an error term ϵ_t. The first part of the problem penalizes the cyclical component, whereas the second part penalizes discrepancies in the growth rate of the trend component.

Raw data has different frequencies, so for different types of data different lambda are recommended:

$$\lambda = \begin{cases} 100 \; for\ annual\ data \\ 1{,}600 \; for\ quarterly\ data \\ 14{,}400 \; for\ monthly\ data \\ 43{,}200 \; for\ daily\ data \end{cases}$$

3.3 PWY (Phillips, Wu and Yu, 2011) and PSY Tests (Phillips, Shi and Yu, 2015)

PWY and PSY both use recursive and rolling ADF-test to identify bubbles and date-stamp them with some variations in specification. The null hypothesis states that there is a unit-root. The alternative hypothesis states that the process is mildly explosive. Authors assume that the process is a random walk with a drift:

$$y_t = dT^{-\eta} + y_{t-1} + \varepsilon_t$$

where d is a constant, n is a coefficient that determines the drift magnitude, while T is the sample size which approaches infinity, ε_t is an error term.

The equation bellow (3.3) is the one which is tested for the bubble presence, alternatively saying for an explosive behavior:

$$y_t = \mu + \delta_T y_{t-1} + \sum_i^p \phi_i \Delta y_{t-i} + \varepsilon_t \quad (3.3)$$

where $\delta_T = 1 + cT^{-\theta}$ c $c > 0$ и $0 < \lambda < 1$, y_t is exogenous variable (e.g. the price of cryptocurrency), μ is an intercept, p is a number of lags ε_t is an error term.

The hypothesis for the test can be written as follows:

$$H_0 \colon \delta = 1,$$
$$H_1 \colon \delta > 1.$$

PSY test defines Δy as following regression model:

$$\Delta y_t = \alpha_{\lambda_1 \lambda_2} \lambda_1 \lambda_2 + \rho_{\lambda_1 \lambda_2} y_{t-1} + \sum_{i=1}^{p-1} \gamma_{\lambda_1 \lambda_2}^i \Delta y_{t-i} + \varepsilon_t.$$

And t-statistics is calculated as:

$$ADF_{\lambda_1 \lambda_2} = \frac{\hat{\rho}_{\lambda_1 \lambda_2}}{S.E.(\hat{\rho}_{\lambda_1 \lambda_2})}.$$

PWY test, more known as SADF test, was developed using a variation of a right-tail ADF test in 2011 by Phillips, Wu and Yu. It is a relatively new bubble detection strategy which is able to determine the beginning date and the termination date. The SADF test recursively calculates the ADF statistics. Starting point is fixed and a window is expanding each time, at the beginning of the test the window size set manually to an arbitral value, $\lambda_w = \lambda_0$. The first observation in a dataset is a starting point of the test, $\lambda_1 = 0$. The last observation is the end point, λ_2. The regression is estimated recursively and each time an ADF statistics is calculated and assigned as $ADF_{\lambda 2}$, until the window expanded to the size of λ_2, so $\lambda_w = \lambda_2$ and the whole sample is used. The SADF statistics is the supremum value of $ADF_{\lambda 2}$ sequence:

$$SADF(\lambda_0) = \sup_{\lambda_2 \epsilon [\lambda_0, 1]} \{ADF_{\lambda_2}\}.$$

The SADF test initiates a repeated ADF test on a forward sample sequence which expands each iteration. The procedure is presented in Figure 3. 1 bellow:

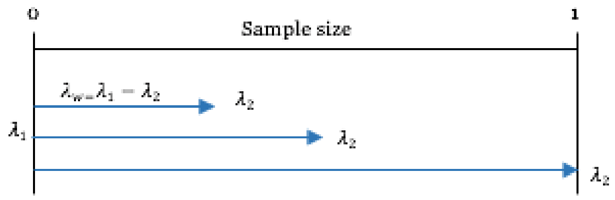


Figure 3. 1 SADF Procedure

For a date-stamping, PWY suggests to compare each element of the series of $ADF_{\lambda 2}$ estimates to the critical values of a right-tail ADF to detect a starting point of the bubble at time T_{r2}. The estimated date when the bubble bursts is denoted as $T_{\hat{\lambda}_e}$ and it is the first chronological point in the sample when $ADF_{\lambda 2}$ exceeds the critical value. Whereas $T_{\hat{\lambda}_r}$ is the end point of the bubble, which occurs after $T_{\hat{\lambda}_e}$ and when $ADF_{\lambda 2}$ falls below the critical value:

$$\hat{\lambda}_e = \inf_{\lambda_2 \epsilon [\lambda_0, 1]} \{\lambda_2 : ADF_{\lambda_2} > cv^{\beta T}{}_2\}$$

$$\hat{\lambda}_f = \inf_{\lambda_2 \epsilon [\hat{\lambda}_e, 1]} \{\lambda_2 : ADF_{\lambda_2} < cv^{\beta T}{}_2\}$$

where $cv_{\lambda_2}^{\beta T} = 100(1 - \beta_t)$ of a sup ADF statistics based on $[T_{r2}]$. Time series can include no bubble, a single bubble, or multiple bubbles. If during the date-stamping procedure SADF test shows more periods which indicate a bubble, the PSY test has to be considered for a more precise results.

PSY test is one of the most recent tests for bubbles detection proposed by Phillips, Shi and Yu in 2013. However, the PWY and PSY tests use the same testing variable, the key difference is the rolling window setting. The PSY test is known as generalized SAFD (GSADF) and windows are set more flexibly as well as the starting point. In the PSY test the rolling window widths is changed by a forward recursive progression. Thus, the procedure differs from PSY and looks as follows:

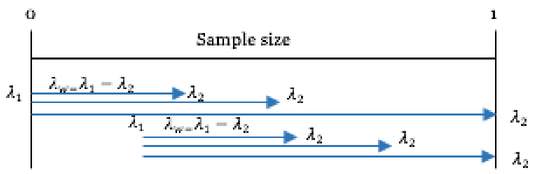


Figure 3. 2 GSADF Procedure

GSADF test is used for testing the null hypothesis about non-existence of the bubble. Formally, the statistics is defined as:

$$GSADF(\lambda_0) = \sup_{\substack{\lambda_2 \epsilon [\lambda_0, 1] \\ \lambda_1 \epsilon [0, \lambda_2 - \lambda_0]}} \{ADF_{\lambda_1 \lambda_2}\}.$$

The null hypothesis is rejected when GSADF exceeds a critical value. Then GSADF applies a date-stamping strategy that estimates the start and end dates of a bubble or bubbles. The aforementioned modifications expand the range of subsample data which makes GSADF test to the test superior to SADF test, because it is more accurate in detection of several bubbles in the dataset.

Phillips et al. (2011) suggests a backward SADF (BSADF) test for a date-stamping procedure for higher accuracy of bubble identification. The BSADF test is similar to the GSADF test, yet it has an opposite test direction. Formally, BSADF test takes a fixed end point λ_2 instead of λ_0 starting point for GSADF.

Formally, the bubble period estimates are defined as

$$\hat{\lambda}_e = \inf_{\lambda_2 \epsilon [\lambda_0, 1]} \{\lambda_2 \colon \mathrm{BS}ADF_{\lambda_2}(\lambda_0) > \ cv^{\beta T \lambda_2}{}_2\}$$

$$\hat{\lambda}_f = \inf_{\lambda_2 \epsilon [\hat{\lambda}_e, 1]} \{\lambda_2 \colon BSADF_{\lambda_2}(\lambda_0) < \ cv^{\beta T \lambda_2}{}_2\}$$

where BSADF (r_0) for $\mathrm{r}_2 \epsilon [\mathrm{r}_0, 1]$ is a backward sup ADF statistics. If $\mathrm{BS}ADF_{\lambda_2}(\lambda_0)$ is larger than the corresponding critical value at time $T\lambda_2$, this particular point denoted as $\widehat{T\lambda}_e$ is the date of bubble origination. If it is smaller than the critical value, that is the date of bubble termination denoted as $\widehat{T\lambda}_f$.

$$G\mathrm{SADF}\ (\lambda_0) = \sup_{\lambda_2 \epsilon [\lambda_0, 1]} \{BSADF_{\lambda_2}(\lambda_0)\}$$

and $cv_{\lambda_2}^{\beta T} = 100(1 - \beta_t)$ critical value of sup ADF statistics for $[T\lambda_2]$ observations.

Phillips et al. (2011) recommend that the bubble duration should be longer than $L_T = \log T$ months. This condition helps to eliminate short-term volatility and price corrections of financial instruments as well as it takes data frequency into account (Phillips et al 2013).

3.4 Log-Periodic Power Law Model

3.3.1 Introduction

Log-Periodic Power Law Model (LPPL) is a model which has gotten a lot of attention in financial markets because of its many successful predictions. This approach was introduced by Johansen et al. (2000) and Sornette (2003). The key assumption of the Johansen-Ledoit-

Sornette (JLS) model is that there are only two types of agents in the market. The first type is traders who have rational expectations and identical preferences. They can be represented by a single agent. The second type is irrational agents possessing herding behavior, so called “noise” traders. In the model, traders create networks where their trading behavior depends on the decisions of other traders in networks as well as on external influencers. The traders can be only in two states: buy or sell. Those interactions can make agents create groups which possess self-similar behavior which might lead the market to a bubble at some point of time. Particular situation might appear when the bubble burst is close to occur and market participants become self-organized, and self-similar behavior becomes prevailing over disorder.

Besides, another crucial assumption in the model is that a bubble can be self-sustained process, because of the positive feedbacks which are created by increasing risk and the agents’ interactions. According to Johansen and Sornette (2001) a price crash is not a certain in the model, but described by a probability. The hazard rate $h(t)$ — the probability per unit of time that crash will take place in the next instant, given that it has not yet occurred. Market agents continue investing, because the risk of price crash to occur is compensated by the return generated by the bubble. In this scenario, the probability that the bubble vanishes smoothly is low. The hazard rate $h(t)$ measures the likelihood that a large number of market participants will take the same sell position simultaneously imitating the behavior of other traders in the network. In this case, coordination in the behavior will lead to a position which the market will not be able to satisfy without the significant price drop due to imbalance of sell and buy orders. The LPPL model provides an ability to detect bubbles and forecast their most likely end.

3.4.1 Macroscopic modeling

According to Stanley (1971) and Goldenfeld (1992), a simple way to describe a process imitation amongst market participants is by assuming that the hazard rate $h(t)$ can be represented by the following equation:

$$\frac{dh}{dt} = Ch^{\delta} \tag{3.4}$$

where a constant C is greater than zero, and $\delta > 1$ is the average number of interactions amongst traders minus one. From the equation (3.4), h^{δ} increases when interactions among traders increases and decreases when interactions decreases. If the equation (3.4) is integrated, the dependence of the hazard rate on power law is the following:

$$h(t) = \left(\frac{h_0}{t_c - t}\right)^{\alpha}, \alpha = \frac{1}{\delta - 1}$$

where t_c is the critical time when a crash is the most probable. The condition $0 < \alpha < 1$ is crucial for the critical time t_c to happen in finite time as well as for the price not to increase indefinitely when $t \rightarrow t_c$. In addition, h^α represents an amplitude of the power law acceleration.

3.4.2 Price dynamics

An equation (3.5) for price dynamics is derived from the hazard rate (3.24) which depicts certain features of the model and is introduced in Subsection 3.4.1 earlier. The rational agent given by Johansen et al. (2000) is risk neutral and has rational expectations. Hence, following the efficient market hypothesis, the asset price *p(t)* follows a martingale process, $E_t[p(t')] = p(t)$, $\forall t' > t$, where $E_t[]$ is the conditional expectation given all information available up to time *t*. For the market equilibrium, the given equation is a necessary condition for no arbitrage. The price dynamics thus can be expressed by:

$$dp = \mu(t)p(t)dt + \sigma(t)p(t)dW - kp(t)dj$$

where $\mu(t)$ denotes a drift at time t, dj is a jump, such that $j = 0$ before a crash and $j = 1$ after the crash took place. In addition, $k \in (0, 1)$ is a fixed percentage by which the asset price falls during a crash. The strength of the jumps is dependent of the hazard rate $h(t)$. Hence:

$$E[dp] = \mu(t)p(t)dt - kp(t)h(t)dt. \tag{3.5}$$

If the no-arbitrage condition holds with rational expectations, then $E[dp] = 0$, so that $\mu(t)p(t)dt - kp(t)h(t)dt = 0$. Thus, $\mu(t) = kh(t)$. Implementing the substitution to the equation 3.5, the differential equation of the price dynamics before the price crash can be expressed as $d(\ln p(t)) = kh(t)$. The solution looks as follows:

$$\ln\left[\frac{p(t)}{p(t_0)}\right] = k\int_{t_0}^{t} h(t')dt'. \tag{3.6}$$

Simply is the higher the chance of the crash is, the faster the asset price must grow in order to compensate the increased risk of crash for traders in the market.

Suggested by JLS (2000), the hazard rate *h(t)* has to be corrected by log-periodic oscillations which are accelerating because their frequency explodes when it gets to the critical point. The hazard rate behaves according to the following equation:

$$h(t) \approx B_0(t_c - t)^{-\alpha} + B_1(t_c - t)^{-\alpha}\cos[\omega\ln(t_c - t) + \varphi']$$

where $\omega/2$ is the log-frequency and φ' defines the oscillation. Using equation (3.6) , the price dynamic before critical time is:

$$\ln[p(t)] \approx \ln[p(c)] - \frac{\kappa}{\beta}\{B_0(t_c - t)^{\beta} + B_1(t_c - t)^{\beta}\cos[\omega\ln(t_c - t) + \phi]\}.$$

According to Fantazzini and Geraskin (2011), the main LPPL equation can be rewritten in order to be more suitable for fitting a financial time series in to following way:

$$\ln[p(t)] \approx A + B(t_c - t)^{\beta}\{1 + C\cos[\omega\ln(t_c - t) + \phi]\} \tag{3.7}$$

where $A > 0$ is the price value at the critical time $p(t_c)$, $B < 0$ represents the power law, the increase of the price over a unit of time reaching the critical point t_c, $C \neq 0$ is the proportional magnitude of the oscillations around the exponential growth, β should lie between zero and one to guarantee a finite price at the critical point t_c and defines the power law acceleration of prices, ω determines the frequency of oscillation during bubble, $\phi \in (0, 2\pi)$ and is a phase parameter. Equation 3.7 is the fundamental equation for the Log Periodic Power Law which explains the time-based growth of asset prices before the critical time. It is pointed out that A, B, C, and ϕ, are just units of distribution of β and ω parameters.

3.4.3 LPPL fitting procedure

Estimating LPPL parameters is not a trivial task but this can be done by minimizing the sum of squared residuals, implementing the Least-Squares Method:

$$S(t_c, \beta, \omega, \phi, A, B, C) = \sum_{t=t_1}^{t_n}[y_t - f(t)]^2 =$$
$$= \sum_{t=t_1}^{t_n}[\ln[p(t)] - A - B(t_c - t)^{\beta} - C(t_c - t)^{\beta}\cos[\omega\ln(t_c - t) + \phi]]^2.$$

Minimization of a nonlinear cost function with many variables is difficult because of a presence of multiple local minima. Nevertheless, the optimization problem can considerably less complex when noticing that three linear A, B, C can be slaved to the four nonlinear parameters t_c, β,ω, and ϕ. It can be proved that

$$\min_{t_c,\beta,\omega,\phi,A,B,C} S(t_c, \beta, \omega, \phi, A, B, C) \equiv \min_{t_c,\beta,\omega,\phi} S_1(t_c, \beta, \omega, \phi),$$

where

$$S_1(t_c, \beta, \omega, \phi) \equiv \min_{A,B,C} S(t_c, \beta, \omega, \phi, A, B, C).$$

3.4.4 New LPPL fitting procedure

While the previous model gives us a good representation of price dynamics, it faces an burdensome fitting procedure and has several local minima of the function due to an relationship between log-frequency ω and phase ϕ. The new calibration method was proposed by Filimonov and Sornette (2013), which transforms the original LPPL formulation from a function of 4 nonlinear and 3 linear parameters into the model with 3 nonlinear parameters and 4 linear ones. Such a transformation makes fitting procedure less complex as well as enhances its stability. The transformed cost function, makes the LPPL model more appropriate for empirical data, because it promises smoother properties which eases a process of finding a global minima. The enhancement of JLS helps to get rid of the interdependence between the phase constant ϕ and log-frequency ω and the newly formulated LPPL expression is

$$\ln[p(t)] = A + B(t_c - t)^{\beta} + C_1(t_c - t)^{\beta}\cos(\omega\ln(t_c - t)) + C_2(t_c - t)^{\beta}\sin(\omega\ln(t_c - t)) \quad (3.8)$$

where $C_1 = C\cos\phi$, $C_2 = C\sin\phi$. As written in the last equation (3.8) , the formula for $p(t)$ in LPPL model consequently contains only 3 nonlinear — t_c, ω, β. To get the parameter estimates, the same Least-Squares method is applied and modified LPPL model is consequently rewritten as

$$F(t_c, \beta, \omega, A, B, C_1, C_2) = \sum_{t=t_1}^{t_n}[\ln[p(t)] - A - B(t_c - t)^{\beta} - C_1(t_c - t)^{\beta}\cos(\omega\ln(t_c - t)) - C_2(t_c - t)^{\beta}\sin(\omega\ln(t_c - t))]^2 .$$

Slaving the 4 linear parameters A, B, C_1, C_2 to the 3 nonlinear t_c, β, ω, the nonlinear optimizations problem

$$\{\widehat{t_c}, \hat{\beta}, \widehat{\omega}\} = \arg\min_{t_c,\beta,\omega} F_1(t_c, \beta, \omega)\,,$$

where

$$F_1(t_c, \beta, \omega) \equiv \min_{A,B,C_1,C_2} F(t_c, \beta, \omega, A, B, C_1, C_2).$$

3.4.5 Choosing a starting date for fitting an LPPL model

According to the authors of the LPPL model, the first point must be the one with the lowest price value when the bubble started. Since the definition does not clearly specify how to distinguish starting point as well as there are many local minima in time series, the results of the date-stamping procedure of GSADF tests will be used to choose the right time period before the bubble burst to estimate the parameters.

It is worth mentioning that LPPL model does not do a projection of price dynamics after the price crash and thus estimated parameters are no longer valid after estimated critical point $\widehat{t_c}$ which can differ from actual one due to some uncertainty. It can be denoted as $\widehat{t_c} = t_c^{actual} + \varepsilon$, where $\widehat{t_c}$ is a predicted output of the LPPL framework and ε is an error term.

The price behavior after the bubble burst is out of scope of this thesis.

3.4.6 Parameters' range recommendations

The final part of LPPL model specification before the calibration and selection of the model can be done is to enforce some additional constraints of individual estimated parameters of the model. In addition to aforementioned reasonable constraints on $A > 0$, $B < 0$, and $C \neq 0$ which are based on theoretical assumptions of economic models, there are further restrictions suggested based on empirical evidence of various historical market crashes occurred in the past. To ensure that the hazard rate $h(t)$ is positive over time, Bothmer and Meister (2003) imposed the condition that $|C| < 1$. For the modified version there is a slight difference — $|C_1| < 1$ same as $|C_2| < 1$. Obviously, $t_c > 0$ must hold and t_c must be larger than the last point of the given dataset.

Additionally, Lin et al. (2009) advised on further restrictions, by stating $6 \leq \omega \leq 15$ so that the log-periodic oscillations are neither too fast nor too slow. Later, this constraint was narrowed by Sornette (2003a) who found that $\omega = 6.36 \pm 1.56$. Besides, another limitation was introduced after more research by Bree and Joseph (2010) which suggests $\beta = 0.33 \pm 0.18$. Those restrictions will be debated and reviewed during the fitting process handled over available data.

3.4.7 Model selection

The drawback of the LPPL model is that it can be fitted on any time series even without the presence of log-periodic oscillations in data. Hence, R^2 is not reliable enough to judge the quality of the model. For each of 12 cryptocurrencies an LPPL model will be developed. For each time series, the model will estimate seven parameters and the model with the lowest value of root mean squared error (RMSE) between the actual and fitted values will be considered the best among others. RMSE formula is the following:

$$RMSE = \sqrt{\frac{\sum_{t=1}^{T}(y_t - f(t))^2}{T}}$$

Another problem which can arise from the developed model is non-stationary residuals. For this reason, the Kwiatkowski-Phillips-Schmidt-Shin test (KPSS test) which was introduced in 1992 will be used. The null hypothesis states that time series is stationary and alterative that it has a unit-root. The KPSS test statistics is defined as

$$KPSS = T^{-2} \sum_{t=1}^{T} S_t^2 / s^2(l)$$

where $S_t = \sum_{i=1}^{t} e_i$, $t = 1, 2, \ldots, T$, $e_i = y_t - \bar{y}$ and $s^2(l)$ is a consistent estimator of the long-run variance. The test will be applied to residuals in order to check for a spurious regression, a regression in which independent series are found seemingly related, yet only might have temporal relationship. If the null hypothesis of KPSS test is accepted of 5% significance level then the residuals of the model are stationary and the regression is not spurious.

Another criterion to be used to compare estimated model is Akaike information criterion (AIC). AIC calculates the quality of each particular model relative to the other developed models.

$$AIC = N \ln \left(\frac{\sum_{t=t_1}^{t_n} (y_t - f(t))^2}{N} \right) + 2k$$

where k is the number of fitted parameters and N is the number of observations. The lower is the AIC, the more preferable the model is among other ones.

4 Empirical Analysis

This section will contain five parts. The first one shortly describes a dataset used for the empirical analysis. Next two focuses on SADF and GSADF tests which verify bubble presence in each time series and then provide date-stamping procedure which helps to indicate the beginning and the termination date. Those time intervals will be used for the next part of this section as a input that eases the search of the best LPPL model fit. The forth part is dedicated the best LPPL model fits for bubbles in 2017 for each cryptocurrency. The last part concentrates on bubble prediction.

4.1 Dataset Description

For an empirical analysis, there were chosen top 12 cryptocurrencies by market capitalization as of September 2019. Namely: Bitcoin, Ethereum, Ripple, Tether, Bitcoin Cash, Litecoin, EOS, Binance Coin, Bitcoin SV, Stellar, Tron, Cardano. The data for the respective cryptocurrencies were web scraped from www.coinmarketcap.com from December 2015 or later from their official ICO for Cordano, Bitcoin Cash, EOS, Binance Coin, Bitcoin SV, or Stellar. The last observation taken is dated 1st July 2019. The closing cryptocurrency prices where used in the practical part of the thesis. Descriptive statistics for the whole dataset is presented in Table 4. 1. It can be observed that the given data can be characterized by high volatility. For instance, bitcoin relative standard deviation was over 50%, whereas Binance Coin relative standard deviation exceeded 90%. Time series covers slightly over two and a half year, and except Tron which Max/Min ratio is low (1.18), the Max/Min ratio of the other cryptocurrencies varies from 5.58 for Bitcoin SV to 1762.68 for Ethereum. It is clear that prices for digital coins had been changing significantly over the given time interval.

Cryptocurrency	Ticker	Median	Min	Max	Mean	SD	Skewness	Kurtosis
Bitcoin	BTC	3620.8	359.19	19497.4	4174.4	2149.9	1.4	4.17
Ethereum	ETH	142.40	0.7922	1396.4	224.55	58.996	1.02	2.99
Ripple	XRP	0.2333	0.0041	3.38	0.2929	0.0562	1.06	3.099
Tether	USDT	1.0000	0.9136	1.08	1.0005	0.0101	0.54	3.91
Bitcoin Cash	BCH	514.52	77.37	3923.1	713.36	125.58	0.63	2.01
Litecoin	LTC	41.12	3	358.34	54.62	33.393	0.81	2.53
EOS	EOS	5.2800	0.493	21.54	5.6627	1.6599	0.46	2.11
Binance Coin	BNB	10.00	0.0999	38.82	11.056	10.37	0.65	2.065
Bitcoin SV	BSV	75.01	42.75	238.34	95.51	51.132	1.64	4.306
Stellar	XLM	0.03302	0.0014	0.8962	0.1042	0.0361	2.31	9.263
Tron	TRX	0.0257	0.0014	0.2206	0.0318	0.0059	-0.07	2.988
Cardano	ADA	0.0870	0.0185	1.11	0.1563	0.0208	0.31	2.988

Table 4. 1 Data descriptive statistics for the major 12 cryptocurrencies

4.2 Application of PWY (SADF) test

In order to implement LPPL model later, SADF or GASDF tests are used to test each individual time series for bubble presence and define the start and end dates of bubbles. The crucial information for the future LPPL calibration is a beginning of the bubble which is otherwise hard to state as noted in the Subsection 3.4.5 For the SADF and GASDF test, Monte Carlo simulation with 1000 repetitions were applied in order to calculate critical values for 90%, 95%, 99% significance level and date-stamping procedure. In any output table like Table 4. 2 the starting date and the end can be found, together with SADF test values and 95% critical values. If SADF test statistics is greater than the critical value then the null hypothesis of having a unit root is rejected. This means a presence of at least one bubble in the given time series. SADF test is suitable for an identification of a single periodically collapsing bubble. Whereas, the GSADF test should be used for an identification of multiple periodically collapsing bubbles, if more bubbles are identified from the date-stamping procedure in SADF test. “Too short bubbles” are omitted from the output tables because some discrepancy might occur when implementing randomness generator within software. In addition, the short-term price volatility can cause a similar effect.

4.2.1 Bitcoin and Ethereum results

The results from Table 4. 2 clearly shows for Bitcoin and Ethereum the bubble presence on 95% significance level because the null hypothesis is rejected. Both cryptocurrencies indicate bubbles but in case of the bubble in occurring in 2017 for Bitcoin the end of the bubble is dated 20th January 2018, whereas for Ethereum the end is dated 3rd February 2018. Figure 4. 1 and Figure 4. 2. helps to visualize the timeline of the bubbles.

Cryptocurrency	Origination and end date of a bubble	SADF-test statistics	95% Critical Value
Bitcoin	06.06.2016-20.06.2016, 23.12.2016-05.01.2017, 21.02.2017-16.03.2017, 26.04.2017-20.01.2018	7.694789	1.509340
Ethereum	22.02.2016-30.03.2016, 13.03.2017-12.07.2017, 05.08.2017-03.02.2018	11.1161	1.509340

Table 4. 2 SADF test results for Bitcoin and Ethereum

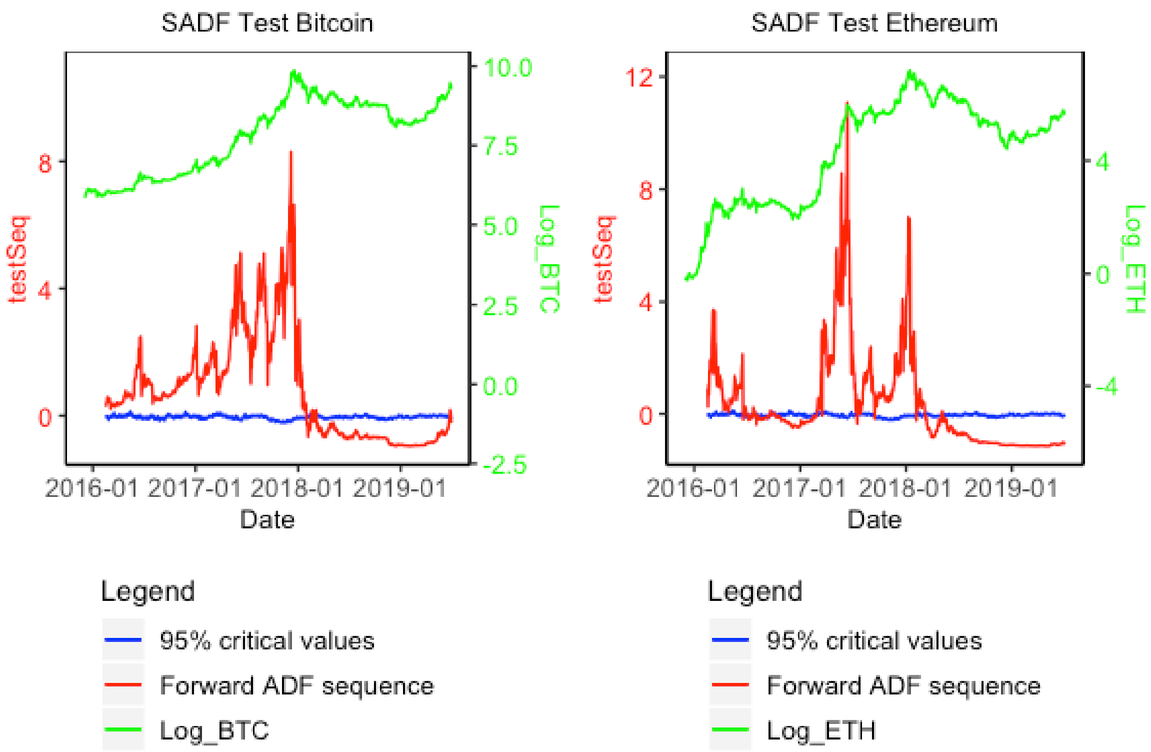


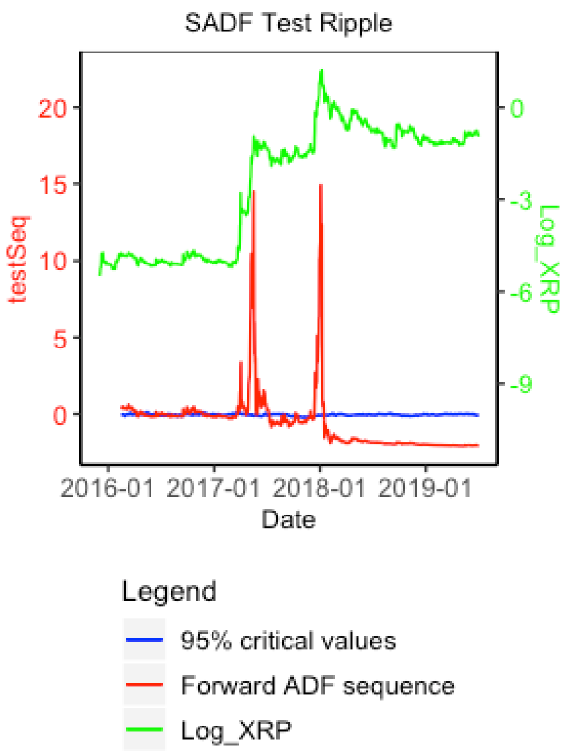
Figure 4. 1 SADF date-stamping procedure for Bitcoin and logarithm of its price
Figure 4. 2 SADF date-stamping procedure for Ethereum and logarithm of its price

4.2.2 Ripple and Tether results

The results from Table 4. 3 tells that there was at least one bubble present for Ripple, however, six periods in total are considered by SADF test as bubbles. The major two spikes in prices correspond to spikes in 95% ADF test sequence as shown in Figure 4. 3. SADF test for Tether has not confirmed the bubble presence but rather a high volatility of prices locked in the specific price range. Figure 4. 4 depicts the situation with Tether accurately.

Cryptocurrency	Origination and end date of a bubble	SADF-test statistics	95% Critical Value
Ripple	16.02.2016-02.05.2016, 14.06.2016-20.07.2016, 05.08.2016-29.08.2016, 12.09.2016-22.11.2016, 23.03.2017-09.07.2017, 25.11.2017-14.01.2018	14.92256	1.509340
Tether		0.4697883	1.509340

Table 4. 3 SADF test results for Ripple and Tether



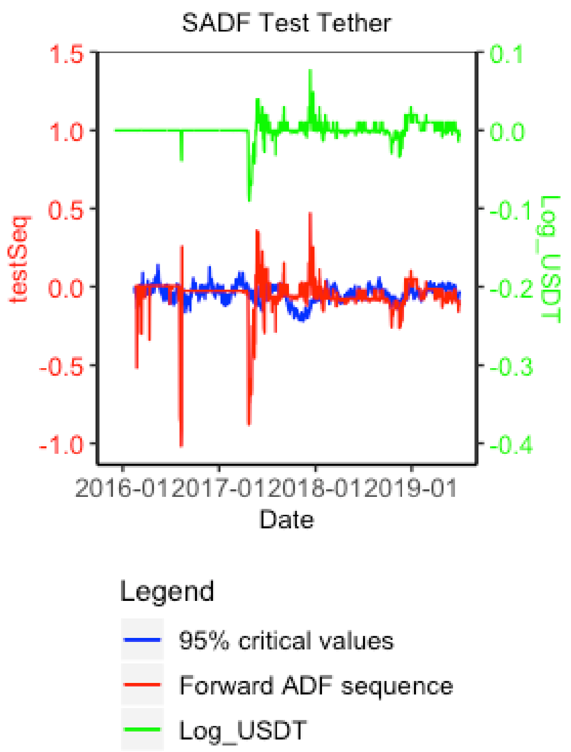


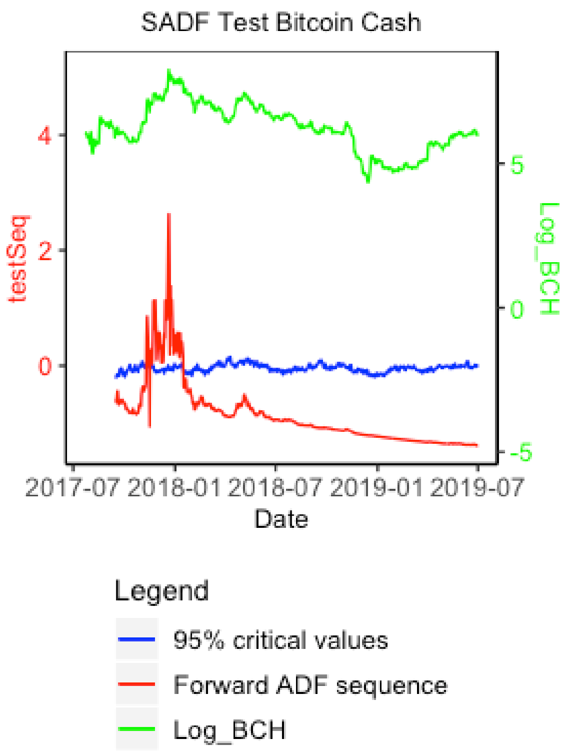
Figure 4. 3 SADF date-stamping procedure for Ripple and logarithm of its price
Figure 4. 4 SADF date-stamping procedure for Tether and logarithm of its price

4.2.3 Bitcoin Cash and Litecoin results

Bitcoin Cash almost right after its ICO had shown that prices are mildly explosive which indicates the bubble starting the first half of November 2017 as summarized in Table 4. 4. Litecoin price in the same time had three period of bubble dated back in 2017 which might be a sign that the bubble lasted the majority of the year. Further analysis will follow using GSADF test. From the date-stamping procedure shown in Figure 4. 5 and Figure 4. 6 the bubbles finished around the middle of January.

Cryptocurrency	Origination and end date of a bubble	SADF-test statistics	95% Critical Value
Bitcoin Cash	10.11.2017-15.01.2018	2.629208	1.492414
Litecoin	28.05.2016-20.06.2016, 03.04.2017-25.05.2017, 29.05.2017-13.09.2017, 09.11.2017-13.01.2018	12.06917	1.509340

Table 4. 4 SADF test results for Bitcoin Cash and Litecoin



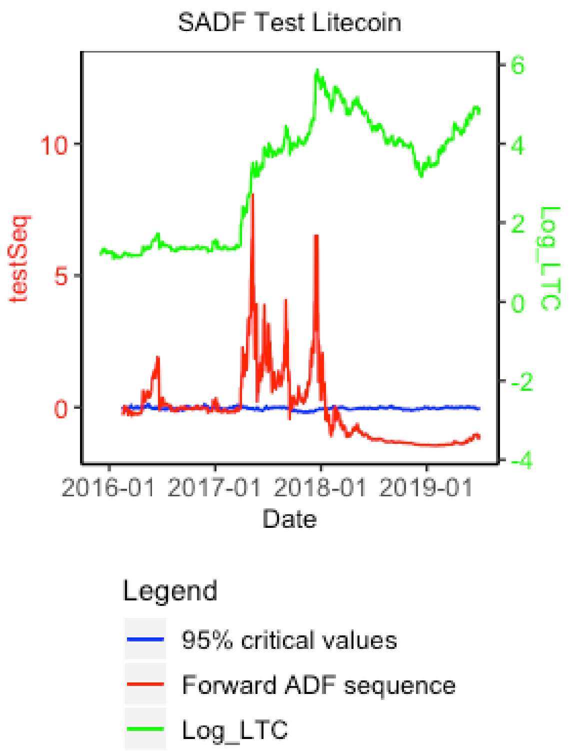


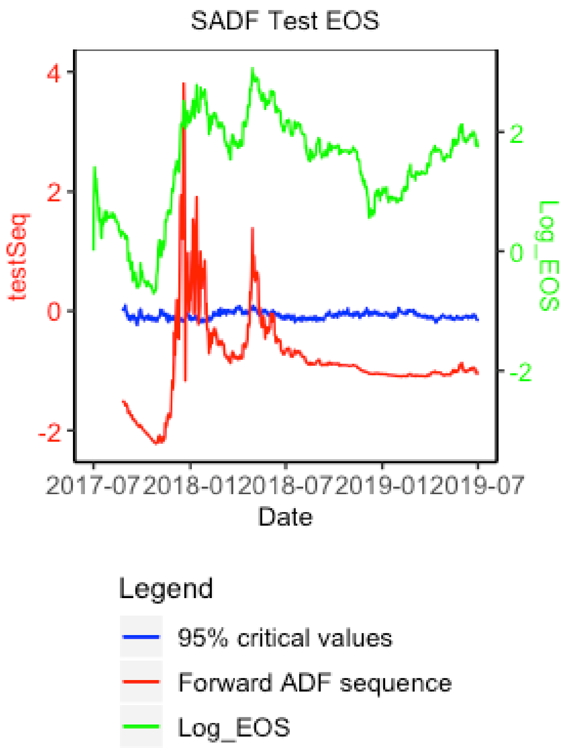
Figure 4. 5 SADF date-stamping procedure for Bitcoin Cash and logarithm of its price
Figure 4. 6 SADF date-stamping procedure for Litecoin and logarithm of its price

4.2.4 EOS and Binance Coin results

Similarly, the at least one bubble was present for EOS and Binance Coin as presented in Table 4. 5. After the bubble burst at the very end of January 2018, EOS prices grew again in April 2018 and this is detected by SADF test as a bubble as shown in Figure 4. 7. Whereas Binance Coin prices after the termination of the bubble in January 2018, started to remind the creation on the bubble in April 2019 as presented in Figure 4. 8.

Cryptocurrency	Origination and end date of a bubble	SADF-test statistics	95% Critical Value
EOS	05.12.2017-31.01.2018, 24.04.2018-13.05.2018	3.809606	1.415386
Binance Coin	02.10.2017-03.11.2017, 30.11.2017-20.01.2018, 01.04.2019- ongoing	3.563866	1.480517

Table 4. 5 SADF test results for EOS and Binance Coin



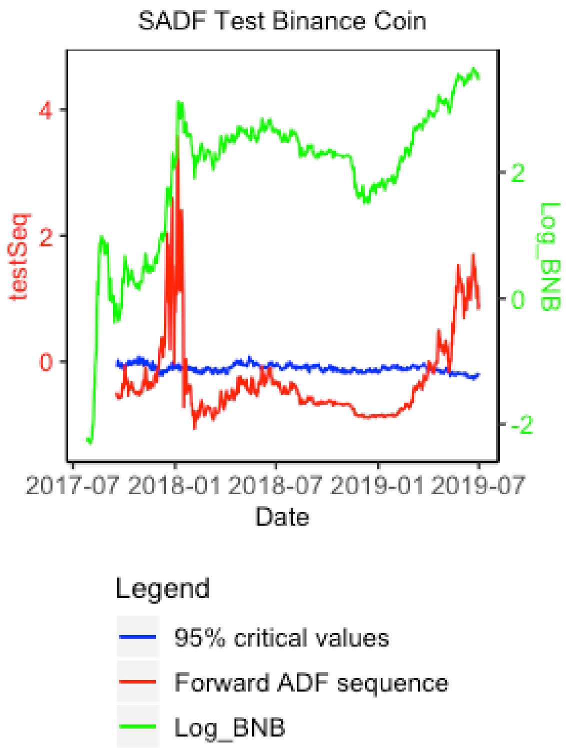


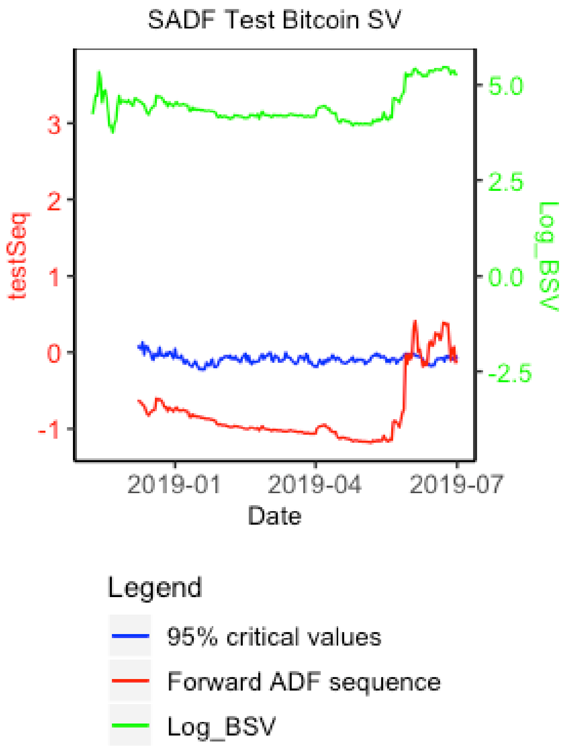
Figure 4. 7 SADF date-stamping procedure for EOS and logarithm of its price
Figure 4. 8 SADF date-stamping procedure for Binance Coin and logarithm of its price

4.2.5 Bitcoin SV and Stellar results

For Bitcoin SV the null hypothesis is not rejected because the SADF-test statistics is lower than 95% critical value in Table 4. 6. Stellar similarly to the other aforementioned cryptocurrencies had a bubble busted dated back to January 2018. In Figure 4. 9 and Figure 4. 10 the date-stamping procedure is visualized and periods of bubbles follow the high price growth.

Cryptocurrency	Origination and end date of a bubble	SADF-test statistics	95% Critical Value
Bitcoin SV	31.05.2019-29.06.2019	0.4139689	1.374077
Stellar	04.05.2017-24.05.2017, 28.11.2017-15.01.2018	12.48889	1.509340

Table 4. 6 SADF test results for Bitcoin SV and Stellar



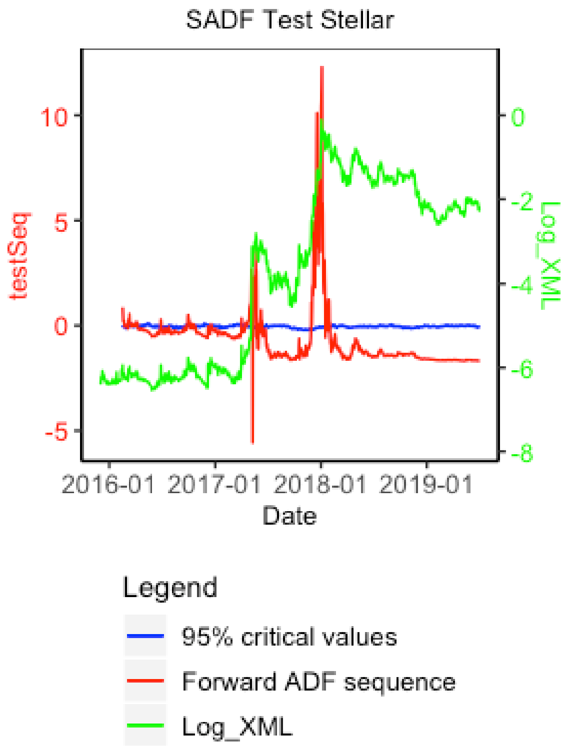


Figure 4. 9 SADF date-stamping procedure for Bitcoin SV and logarithm of its price
Figure 4. 10 SADF date-stamping procedure for Stellar and logarithm of its price

4.2.6 Tron and Cardano results

From Table 4. 7 is obvious that the null hypothesis is not rejected to Tron and that Cardano prices indicated the bubble at the break between 2017 and 2018. Figure 4. 11 and Figure 4. 12 captures the development of this cryptocurrencies over time.

Cryptocurrency	Origination and end date of a bubble	SADF-test statistics	95% Critical Value
Tron	07.12.2017-08.01.2018	0.4139689	1.374077
Cardano	24.11.2017-26.01.2018	5.213202	1.992269

Table 4. 7 SADF test results for Tron and Cardano

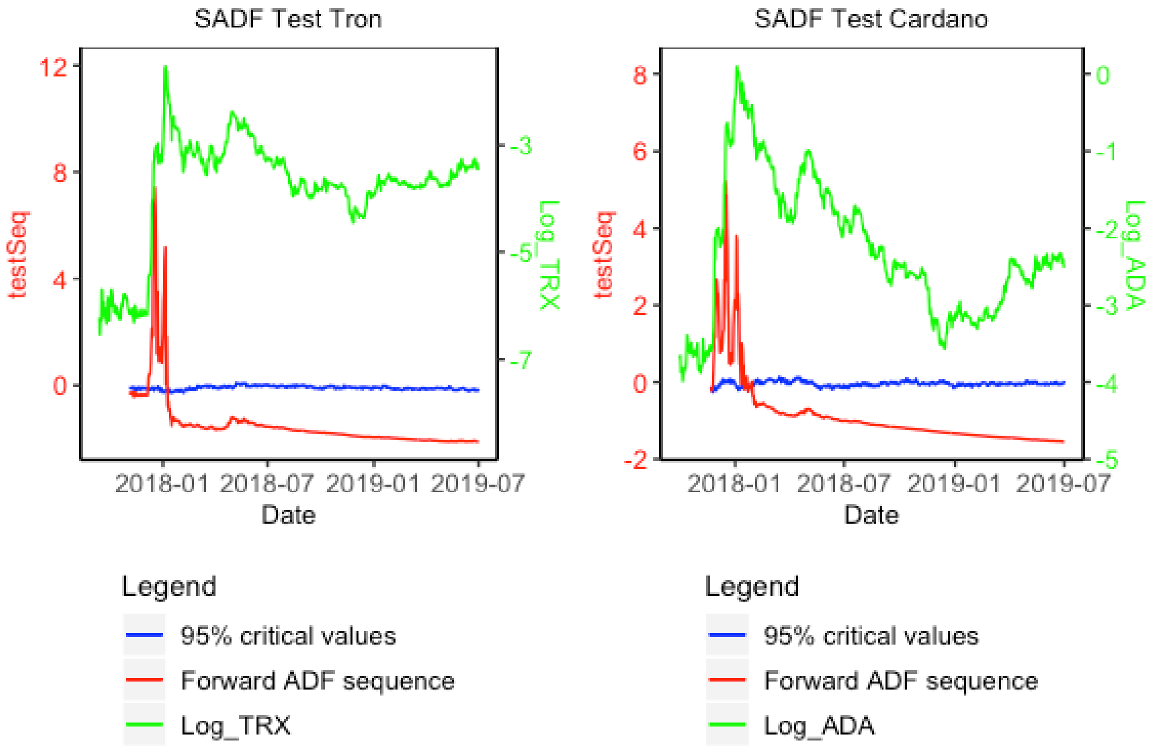


Figure 4. 11 SADF date-stamping procedure for Tron and logarithm of its price
Figure 4. 12 SADF date-stamping procedure for Cardano and logarithm of its price

Except Tether, Bitcoin SV, and Tron, all other cryptocurrencies rejected the null hypothesis which means in the given time series at least one bubble were present. For the currencies existed prior 2017, there were periods of a relatively short duration characterized as bubbles. For instance, in Bitcoin and Ethereum time series those "bubble-like" periods occurred in 2016 which did not last longer than even a month. Hence, this might indicate periods of high volatility. On the other hand SADF test for Ripple indicated a three-month-long period in 2016 resembling a bubble. All the cryptocurrencies that existed in 2017 had in common a prices explosive behavior which occurred on the break between 2017 and 2018. The duration of bubbles was different, for the Bitcoin it was almost nine-month-long period, for Ethereum — five months, for the other digital currencies they were shorter. However, the termination varied only slightly. All bubbles terminated by early February 2018. For Binance Coin and Bitcoin SV, SADF test also indicated a bubble presence in 2019.

From the graphical representation of the SADF results and log prices, it can be noted that bubble periods detected by the date-stamping procedure in the most of the cases correspond to an abnormal growth of the prices. The termination of bubbles often lied near a price crash or local price crash. As SADF test confirmed, a Tether price evolution did not feature the bubble and the graphic representation indicates that Tether time series did not have a trend. Prices were rather locked in the range between 0.9136 USD and 1.08 USD. This cryptocurrency will be excluded from the further analysis. Nevertheless, Bitcoin SV and Tron will be further analyzed by more reliable GSADF test since the graphic representation indicate similar patterns as other cryptocurrencies where bubbles were found.

4.3 Application of PSY (GSADF) test

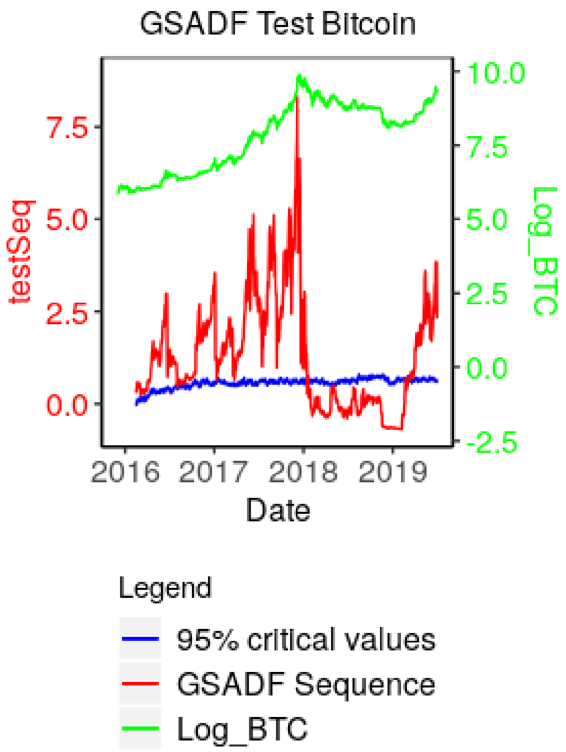
Since more periods resembling bubbles and explosive behavior of prices were detected in each of the time series, the GSADF as more robust approach to bubble detection is applied to clarify more precise periods of bubbles. Instead of SADF, the GSADF test results will be used as an input for LPPL model in order to help find a start date. The results of GSADF differs from the previously received in SADF test. Excluding Tether, for all cryptocurrencies the null hypothesis are rejected because the GSADF statistics are larger than respective 95% critical values. Hence, bubbles were present in each time series of cryptocurrency closing prices. Looking closer into results, the GSADF test detected the bubble over bitcoin prices lasted around two years from the beginning of 2016. It is the longest lasting bubble indicated by the test. In the same fashion, the bubbles were detected in the break between 2017 and 2018. It can be stated that in the cryptocurrency market the bubble which burst at the beginning of 2018 were present. This period of time will be taken for the development of LPPL models and its calibration. Bubbles which are or were present in 2019 will be investigated and used for the forecasting of the bubbles in the cryptocurrency market.

4.3.1 Bitcoin and Ethereum Results

Cryptocurrency	Origination and end date of a bubble	GSADF-test statistics	95% Critical Value
Bitcoin	16.02.2016-20.01.2018, 09.03.2019- ongoing	8.32848	2.390561
Ethereum	16.02.2016-23.03.2016, 27.02.2017-05.04.2017, 13.04.2017-20.04.2017, 26.04.2017-09.07.2017, 08.08.2017-07.09.2017, 23.11.2017-03.02.2018, 15.05.2019-22.05.2019, 22.06.2019-29.06.2019	13.15982	2.390561

Table 4. 8 GSADF test results for Bitcoin and Ethereum

Table 4. 8 confirms multiple bubbles for Bitcoin and Ethereum. Whereas Bitcoin showed two long period of bubbles over time Ethereum had more fragmented results. Nevertheless, GSADF test for both cryptocurrencies similarly depicted major price crashes as the end of the bubble. Figure 4. 13 and Figure 4. 14 compliment the information in the table above.



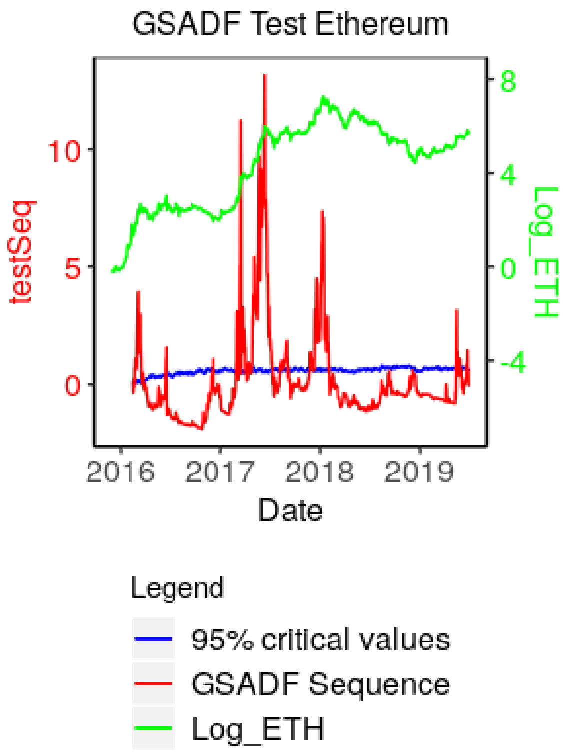


Figure 4. 13 GSADF date-stamping procedure for Bitcoin and logarithm of its price
Figure 4. 14 GSADF date-stamping procedure for Ethereum and logarithm of its price

4.3.2 Ripple results

The GSADF test results for Ripple in Table 4. 9 confirms the results of SADF test. The date-stamping procedure in Figure 4. 15 is also similar to the one of SADF test.

Cryptocurrency	Origination and end date of a bubble	GSADF-test statistics	95% Critical Value
Ripple	23.03.2017-25.06.2017, 13.12.2017-13.01.2018	17.3151	2.390561

Table 4. 9 GSADF test results for Ripple

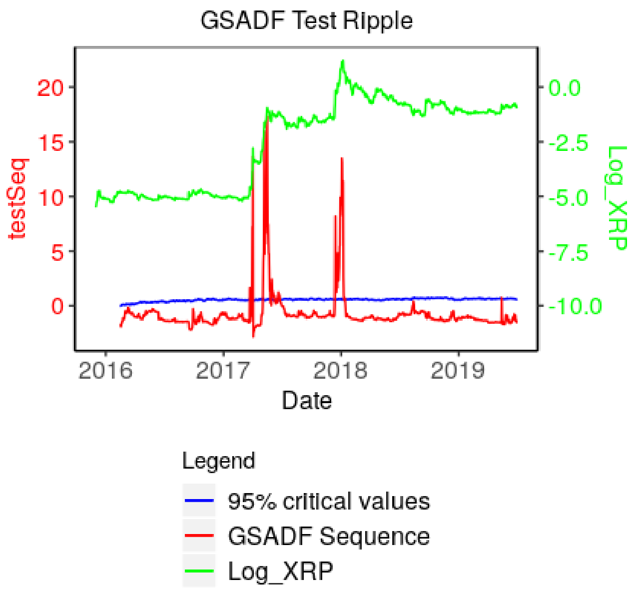


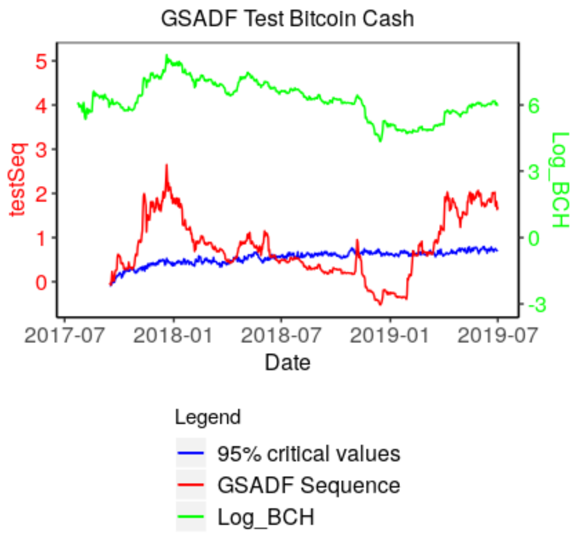
Figure 4. 15 GSADF date-stamping procedure for Ripple and logarithm of its price

4.3.3 Bitcoin Cash and Litecoin results

The results in Table 4. 10 confirms multiple bubbles for Bicoin Cash and Litecoin. Although, Bitcoin Cash bubble terminates at the end of June 2018 and covers a very long period of price drop, the major drop occurred at the beginning of 2018 as shown in Figure 4. 16. Litecoin price evolution indicated more periods of bubbles with shorter duration as can be seen in Figure 4. 17.

Cryptocurrency	Origination and end date of a bubble	GSADF-test statistics	95% Critical Value
Bitcoin Cash	15.09.2017-20.06.2018, 08.02.2019- ongoing	2.649607	2.317406
Litecoin	25.04.2016-27.04.2016, 27.05.2016-20.06.2016, 30.03.2017-13.09.2017, 16.11.2017-15.01.2018, 20.11.2018-16.12.2018, 07.03.2019-20.04.2019, 11.05.2019- ongoing	13.15547	1.521981

Table 4. 10 GSADF test results for Bitcoin Cash and Litecoin



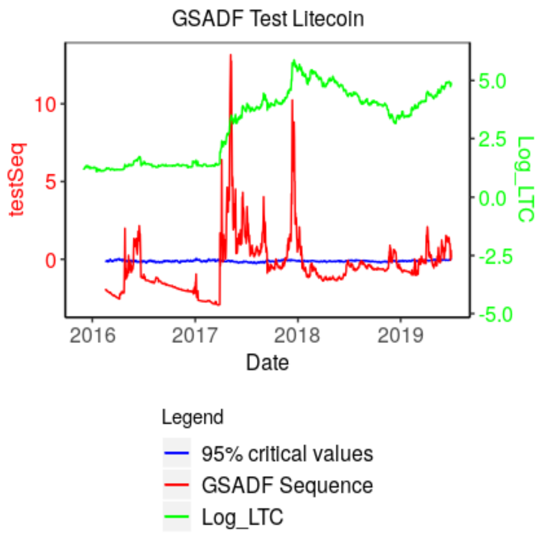


Figure 4. 16 GSADF date-stamping procedure for Bitcoin Cash and logarithm of its price
Figure 4. 17 GSADF date-stamping procedure for Litecoin and logarithm of its price

4.3.4 EOS and Binance Coin results

Table 4. 11 contains information regarding the results of GSADF tests for EOS and Binance Coin. The null hypotheses are rejected since the GSADF-test statistics is greater the 95% critical values. Multiple bubbles occurred for EOS in 2018 as well as two are dated back in 2019. For both coins, the bubble burst terminated in January 2018 was captured. Figure 4. 18 and Figure 4. 19 helps to understand the situation graphically.

Cryptocurrency	Origination and end date of a bubble	GSADF-test statistics	95% Critical Value
EOS	03.11.2017-29.01.2018, 23.04.2018-10.05.2018, 20.11.2018-27.12.2018, 02.04.2019-20.04.2019, 14.05.2019-02.06.2019	7.239904	1.457202
Binance Coin	04.12.2017-15.01.2018, 17.11.2018-17.12.2018, 09.02.2019- ongoing	7.00961	1.501696

Table 4. 11 GSADF test results for EOS and Binance Coin

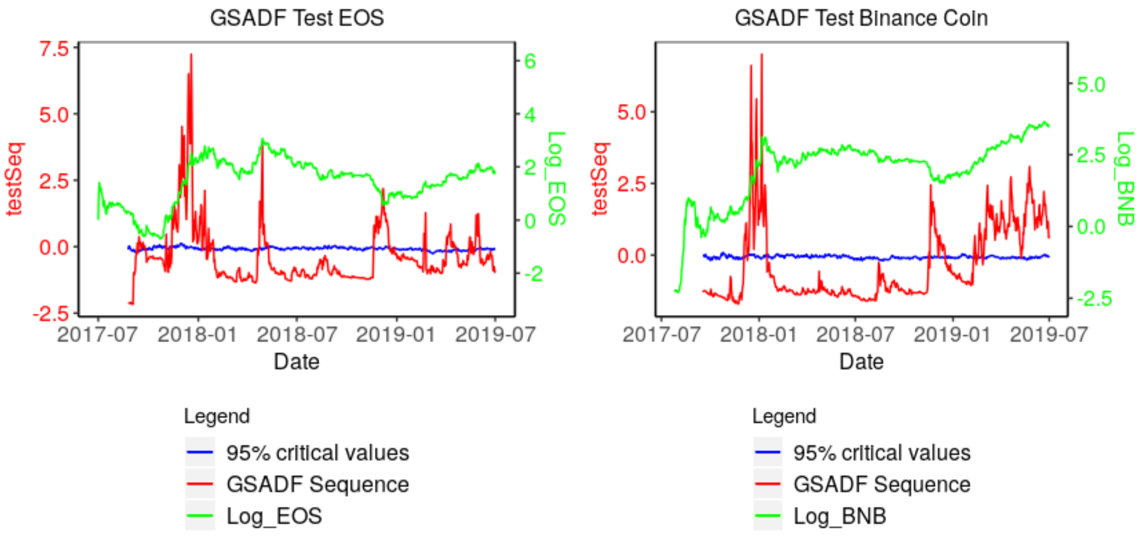


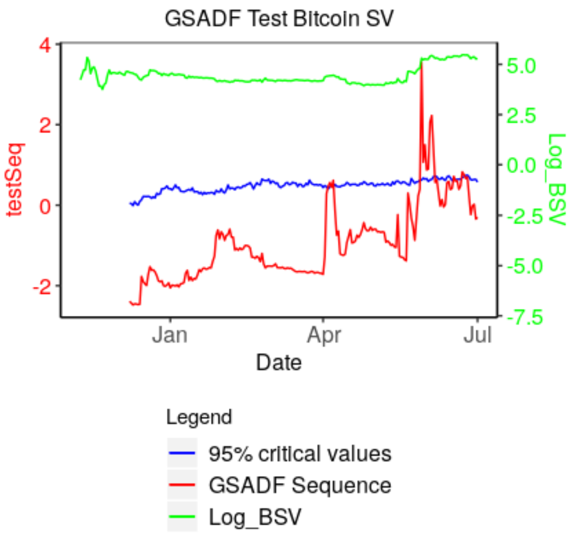
Figure 4. 18 GSADF date-stamping procedure for EOS and logarithm of its price
Figure 4. 19 GSADF date-stamping procedure for Binance Coin and logarithm of its price

4.3.5 Bitcoin SV and Stellar results

GSADF test as more reliable test compared to SADF helps to confirm the bubble presence for Bitcoin SV as seen in Table 4. 12. GSADF-test statistics is greater than 95% critical value, hence, the null hypothesis is rejected. The bubble captured by the Figure 4. 20. Multiple bubbles for Stellar are also observed but the results differ from the results of SADF test which can be from Figure 4. 21.

Cryptocurrency	Origination and end date of a bubble	GSADF-test statistics	95% Critical Value
Bitcoin SV	29.05.2019-05.06.2019	3.554054	2.096807
Stellar	01.05.2017-22.07.2017, 27.11.2017-11.06.2018, 05.12.2018-27.12.2018, 28.01.2019-18.02.2019	7.932484	1.597100

Table 4. 12 GSADF test results for Bitcoin SV and Stellar



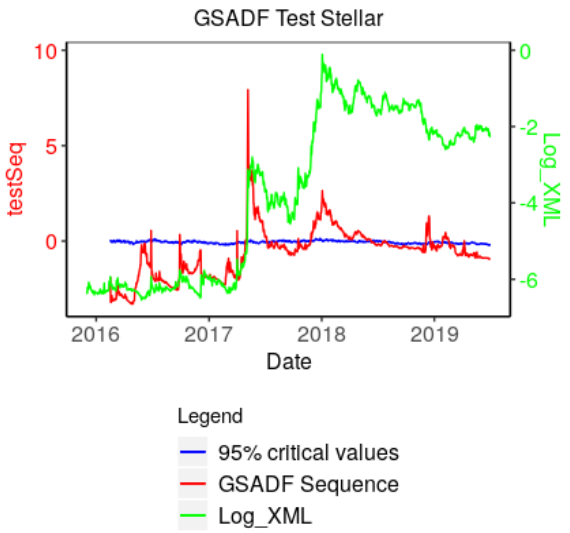


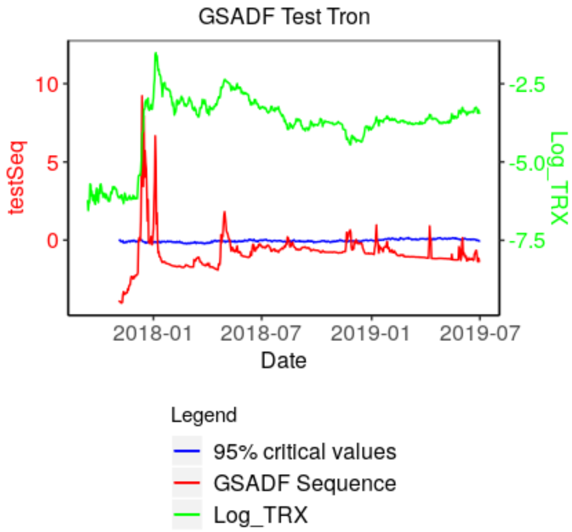
Figure 4. 20 GSADF date-stamping procedure for Bitcoin SV and logarithm of its price
Figure 4. 21 GSADF date-stamping procedure for Stellar and logarithm of its price

4.3.6 Tron and Cardano results

Turning to Tron and Cardano, Table 4. 13 shows three main periods of bubble presence for both cryptocurrencies. Based on the data, almost immediately after the ICOs at the end of 2017, cryptocurrencies had bubbles corresponding to a very high growth followed by the drop at the beginning of 2018. Figure 4. 22 and Figure 4. 23 show the date-stamping procedure capturing this situation quite accurately.

Cryptocurrency	Origination and end date of a bubble	GSADF-test statistics	95% Critical Value
Tron	08.12.2017-07.01.2018, 24.04.2018-09.05.2018, 20.11.2018-07.12.2018	9.222364	1.470921
Cardano	27.11.2017-14.01.2018, 20.11.-2018-17.12.2018, 22.03.2019-19.04.2019	6.164368	1.476998

Table 4. 13 GSADF test results for Tron and Cardano



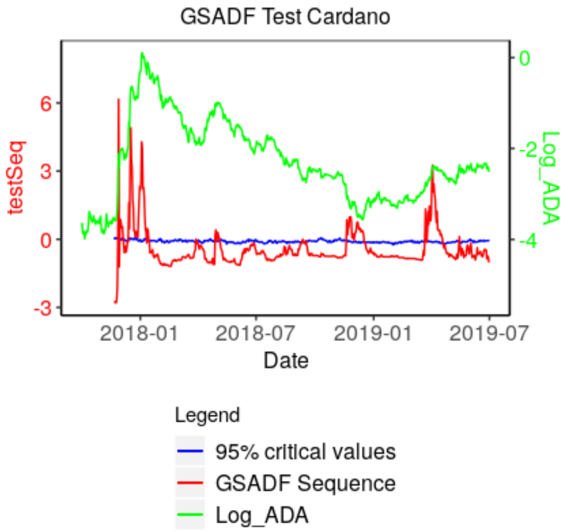


Figure 4. 22 GSADF date-stamping procedure for Tron and logarithm of its price
Figure 4. 23 GSADF date-stamping procedure for Cardano and logarithm of its price

The graphs make it simple to understand when bubbles occurred. When the red line representing a GSADF sequence is above the simulated 95% critical values it consequently means the period of the bubble presence. GSADF test detected more periods compared to SADF test which correspond bubbles with different duration. The hypothesis about Bitcoin SV and Tron stated in the Subsection 4.2 was confirmed, bubbles for both cryptocurrencies were detected. Hence, they will be further analyzed and LPPL model will be applied to them.

4.4 The Best LPPL Model Fit

In this part the modified version of the LPPL model is applied to the 2017 bubbles. As previously suggested the search of the starting date begins with using the periods of bubble presence found during the date-stamping procedure in GSADF test. Sequentially, the time range is altered to achieve lowest possible RMSE values simultaneously satisfying imposed constraints on the estimated parameters together with stationarity of residuals. For the purpose of the LPPL model closing log-prices are used for the model development.

4.4.1 Bitcoin and Ethereum

In Table 4. 14 and Table 4. 15 below results are presented for the best models found for Bitcoin and Ethereum. The final starting date in the best LPPL model fit for bitcoin in the end varied significantly from the suggested in GSADF test. On the other hand, for Ethereum the starting date was very close to the proposed one. Importantly, the obtained time t_c is predicted quite accurately. Whereas the estimated critical time t_c is 29 days later than actual critical time for Bitcoin and does not depict the real market situation very precise, the prediction of t_c for Ethereum is identical to actual time of crash. Value of KPSS p-values allow us not to reject the null hypothesis of stationary residuals for both cryptocurrencies.

Crypto ticker	Starting Date	End Date	Crash Date (estimated)	Crash actual	AIC	RMSE	KPSS statistics	p-value
BTC	15.08.2016	27.11.2017	14.01.2018	16.12.2017	-2064.89	0.10953	0.0784	>0.1
ETH	20.11.2017	04.01.2018	13.01.2018	13.01.2018	-209.273	0.07606	0.0515	>0.1

Table 4. 14 LPPL model statistics for Bitcoin and Ethereum

With regards to the estimated parameters of the modified LPPL model, the estimated frequency $\hat{\omega}$ lies in the suggested distribution given in the Subsection 3.4.6. However, the estimated parameter $\hat{\beta}$ which explains the speed of the price acceleration before the bubble burst for Bitcoin is not within advised range introduced by Bree and Joseph (2010). Nevertheless, the parameter satisfies initial constraints. The model for Ethereum satisfy all introduced constraints.

Crypto ticker	$\hat{\beta}$	$\hat{\omega}$	$\hat{t_c}$ (days)	$\hat{A}$	$\hat{B}$	$\widehat{C_1}$	$\widehat{C_2}$
BTC	0.59435	5.59972	517	9.9641	-0.09385	-0.003568	-0.001762
ETH	0.33602	5.28814	54	7.7094	-0.41915	-0.028824	-0.036087

Table 4. 15 LPPL model estimation results for Bitcoin and Ethereum

From the graphics displayed in Figure 4. 24 and Figure 4. 25, the price dynamics can be captured and the quality of fit. The trend is depicted for Bitcoin, however, the final disturbance of the price is not explained very well. For Ethereum, the developed model captures oscillation with high accuracy and the fitted line lies very closely to the actual one.

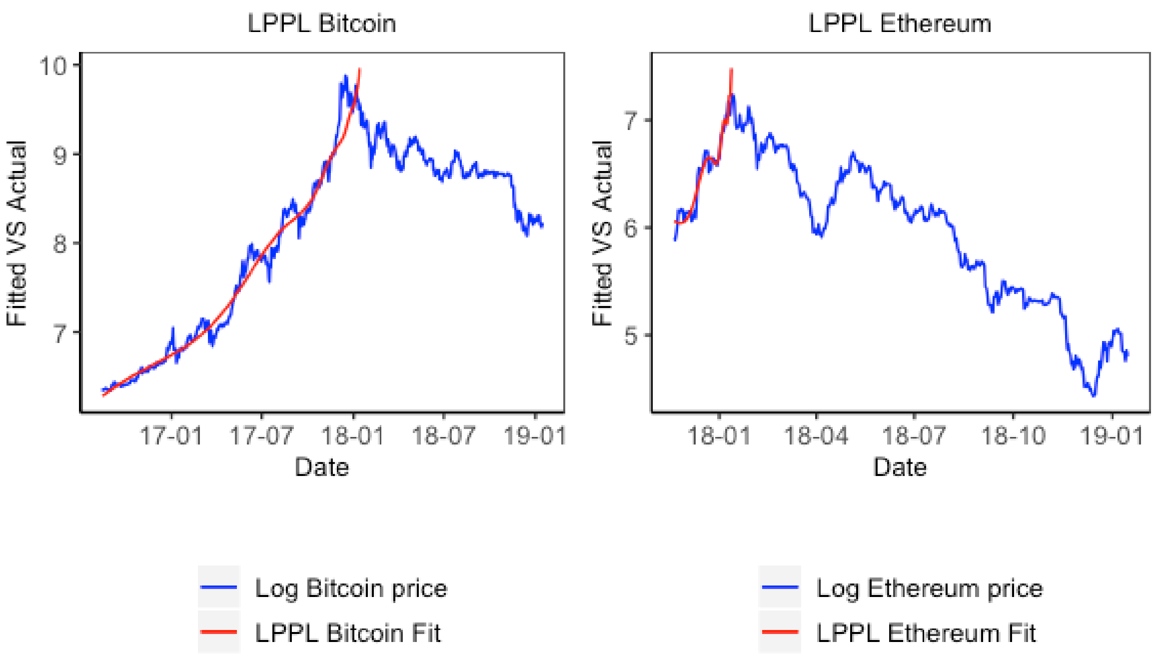


Figure 4. 24 Fitted LPPL model vs actual of Bitcoin log prices
Figure 4. 25 Fitted LPPL model vs actual of Ethereum log prices

4.4.2 Ripple and Bitcoin Cash

The estimation returned for Ripple and Bitcoin Cash is gives a close prediction of t_c which differs from an actual point of crash for -2 days for Ripple and for +14 days for Bitcoin Cash respectively as shown in Table 4. 16. During the model development process, it was challenging to reduce the RMSE values and find a good fit. Due to this fact the starting date was deliberately taken far from the GSADF suggestion. On the contrary, the starting date for Bitcoin Cash was precisely suggested by the date-stamping procedure. Again, residuals of the fitted model are stationary according to the KPSS p-values on 5% significance level.

Crypto ticker	Starting Date	End Date	Crash Date (estimated)	Crash actual	AIC	RMSE	KPSS statistics	p-value
XRP	22.08.2017	01.01.2018	09.01.2018	07.01.2018	-336.10	0.2655	0.3984	0.0779
BCH	15.09.2017	18.12.2017	03.01.2018	20.12.2017	-209.27	0.0761	0.0515	>0.1

Table 4. 16 LPPL model statistics for Ripple and Bitcoin Cash

The estimates of $\hat{\beta}$ presented in Table 4. 17 lies much lower than or much higher than recommended interval for Ripple and Bitcoin Cash respectively. For Ripple the power law growth $\hat{\beta}$ can be potentially explained by almost vertical growth or price jump during a short period of time. Other estimated parameters are in the given intervals. Values of $\hat{\omega}$ are very similar which can lead to similar log-periodic oscillations for both digital currencies.

Crypto ticker	$\hat{\beta}$	$\hat{\omega}$	$\hat{t_c}$ (days)	$\hat{A}$	$\hat{B}$	$\widehat{C_1}$	$\widehat{C_2}$
XRP	0.01127	6.34384	132	45.72	-45.0143	-0.029516	-0.109982
BCH	1.74132	6.19732	110.01	7.57	-0.00075	-0.000205	-0.000249

Table 4. 17 LPPL model estimation results for Ripple and Bitcoin Cash

Visually, in the Figure 4. 26 the turbulence of the prices for Ripple was not coped well by the model which led to some misspecification at the start, nonetheless, later the fit was very close to actual values. Overall, oscillations are depicted well for Bitcoin Cash, but not the final log-price value at the point of the collapse as given in Figure 4. 27.

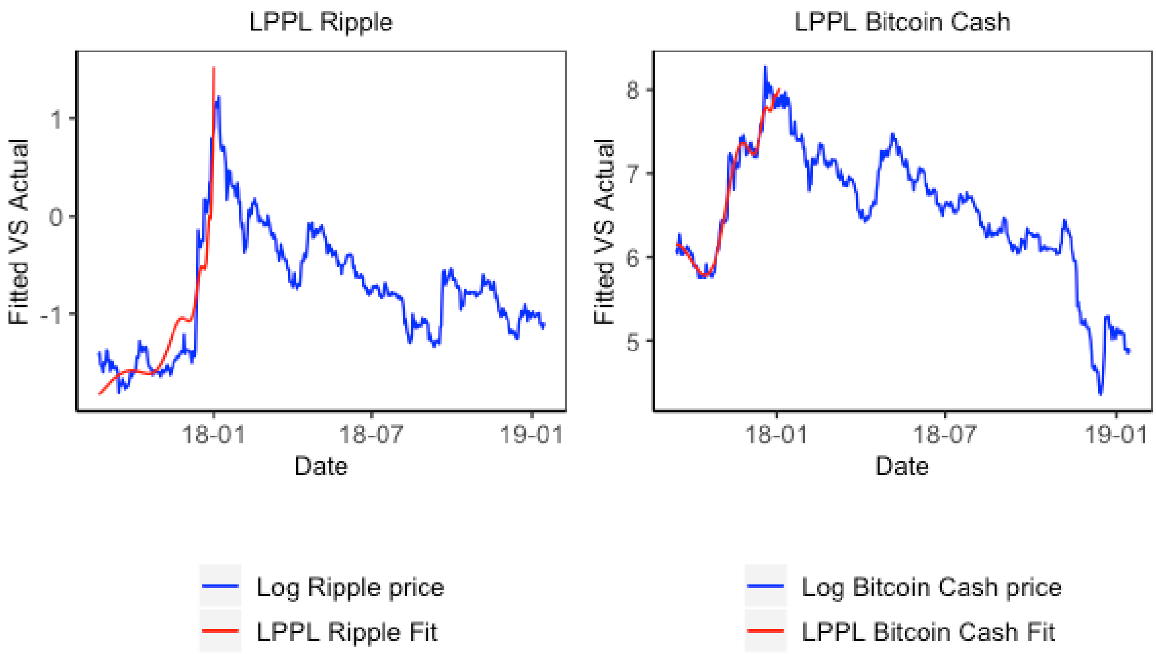


Figure 4. 26 Fitted LPPL model vs actual of Ripple log prices
Figure 4. 27 Fitted LPPL model vs actual of Bitcoin Cash log prices

4.4.3 Litecoin and EOS

Again, starting dates for Litecoin and EOS was similar to the GSADF output. Bubbles started at the first half of November 2017 as noted in Table 4. 18. The day of the crash for Litecoin predicted with -4 days from the real day of the crash. For EOS it is +5 days to the real day of the price collapse. The lowest RMSE are achieved and residuals are stationary.

Crypto ticker	Starting Date	End Date	Crash Date (estimated)	Crash actual	AIC	RMSE	KPSS statistics	p-value
LTC	01.11.2017	13.12.2017	14.12.2018	18.12.2018	-200.89	0.0822	0.0748	>0.1
EOS	13.11.2017	13.01.2018	18.01.2018	13.01.2018	-231.64	0.1379	0.033	>0.1

Table 4. 18 LPPL model statistics for Litecoin and EOS

As for estimated parameters $\hat{\beta}$, they are reportedly below and under the narrowed interval as shown in Table 4. 19, yet between zero and one as initially required. The remaining parameters are in the desired intervals. The outcomes of the model for $\hat{\omega}$ for EOS is at the lower bound of the interval.

Crypto ticker	$\hat{\beta}$	$\hat{\omega}$	$\hat{t_c}$ (days)	$\hat{A}$	$\hat{B}$	$\widehat{C_1}$	$\widehat{C_2}$
LTC	0.008076	5.152688	43.66	59.39637	-53.7333	-0.001648	0.037212
EOS	0.780385	4.914372	76.42	3.421256	-0.08744	0.0071847	-0.015128

Table 4. 19 LPPL model estimation results for Litecoin and EOS

It appears from the graphs, that LPPL model captures nicely the actual price development regardless of unsatisfied constraints for β, see Figure 4. 28. On the contrary, the fit for EOS is not able to cope up with high volatility and oscillations before the crash, see Figure 4. 29. This led to not a good fit of the price at the time of the crash.

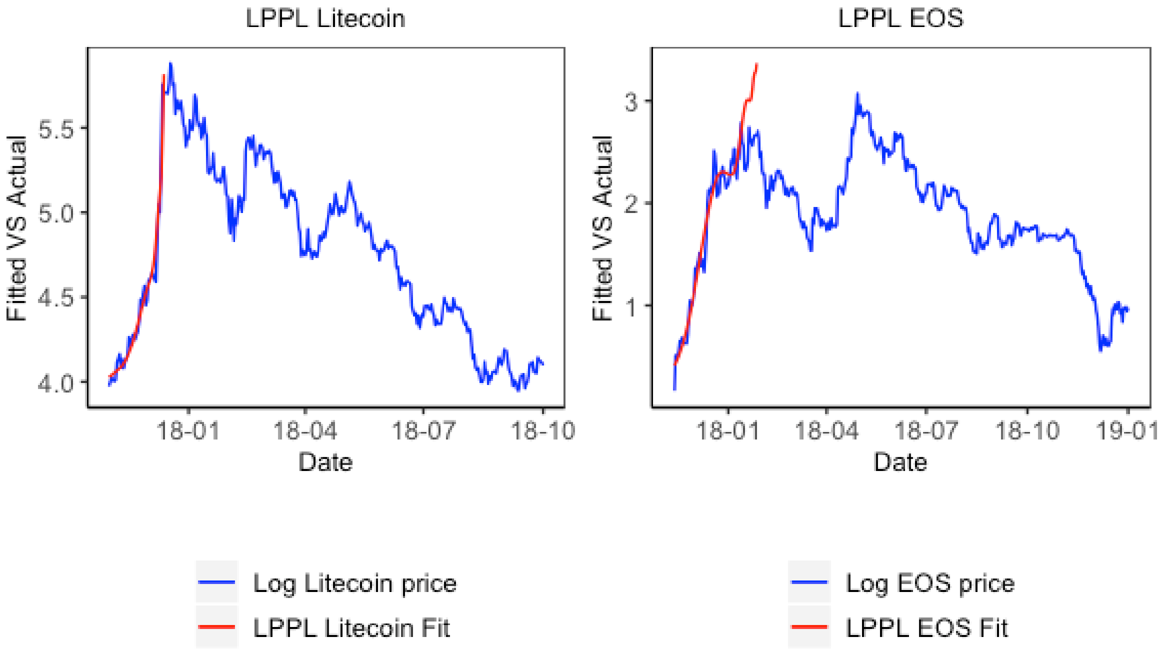


Figure 4. 28 Fitted LPPL model vs actual of Litecoin log prices
Figure 4. 29 Fitted LPPL model vs actual of EOS log prices

4.4.4 Binance Coin and Stellar

Similarly as in previous LPPL model fits, the dates of the bubble crash were determined with high accuracy. Binance Coin crash date estimate is +7 days to actual date of the burst and Stellar is +12 days respectively, see Table 4. 20. In order to achieve better performance of the model the starting dates were altered and does not correspond precisely to the GSADF outputs. Besides, residuals are stationary and overall the model can be considered as a robust one.

Crypto ticker	Starting Date	End Date	Crash Date (estimated)	Crash actual	AIC	RMSE	KPSS statistics	p-value
BNB	10.11.2017	10.01.2018	13.01.2018	06.01.2018	-230.54	0.1392	0.029	>0.1
XLM	30.09.2017	01.01.2019	15.01.2018	03.01.2018	-343.1	0.1497	0.031	>0.1

Table 4. 20 LPPL model statistics for Binance Coin and Stellar

Apart from estimation of an acceleration parameter $\hat{\beta}$, all parameters satisfy stated constraints in Table 4. 21. Estimates of $\hat{\beta}$ are less than one but are higher than the latest introduced constraints. High values of the speed of the price growth parameter might reflect in not explosive enough behavior of the prices.

Crypto ticker	$\widehat{\beta}$	$\widehat{\omega}$	$\widehat{t_c}$ (days)	$\widehat{A}$	$\widehat{B}$	$\widehat{C_1}$	$\widehat{C_2}$
BNB	0.43952	4.846355	64.78	4.095306	-0.62092	-0.013826	0.032708
XLM	0.80733	5.929883	107.74	-0.02178	-0.11105	-0.000644	0.01552

Table 4. 21 LPPL model estimation results for Binance Coin and Stellar

Clearly, the developed models do fit the price growth but do not capture oscillations with occasional price falls and quick jumps as shown in Figure 4. 30 and Figure 4. 31. The final log price is predicted well with a slight delay.

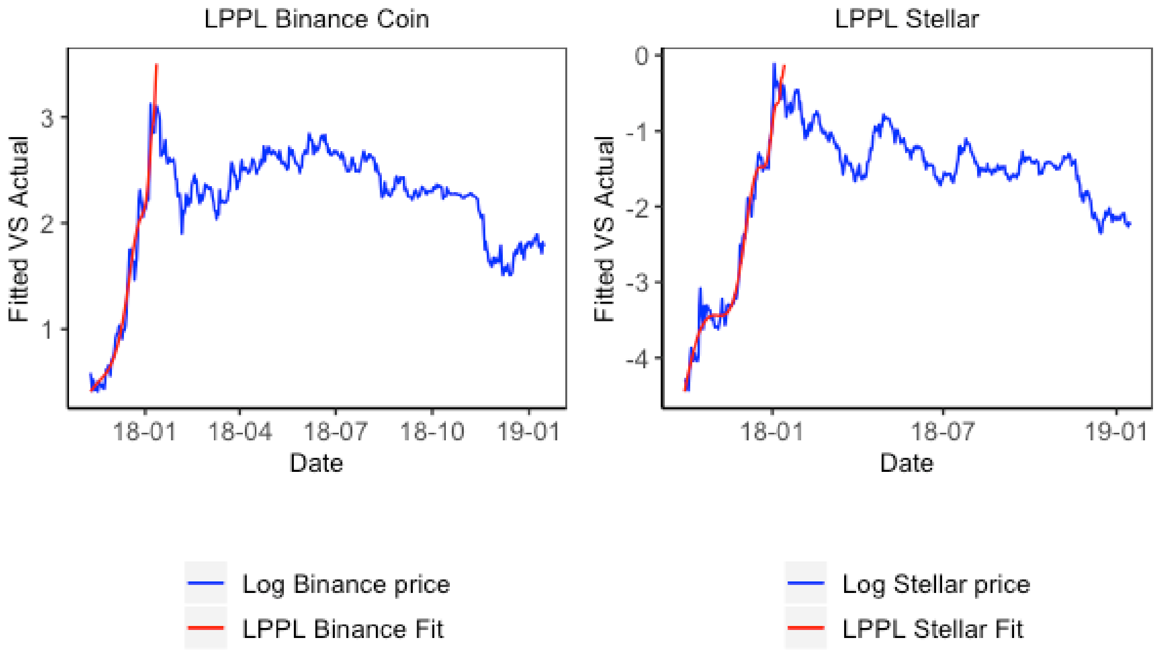


Figure 4. 30 Fitted LPPL model vs actual of Binance Coin log prices
Figure 4. 31 Fitted LPPL model vs actual of Stellar log prices

4.4.5 Tron and Cardano

Using starting dates near to the GSADF test results gave the fitted models with low RMSE numbers compared to numerous intervals tested for a good fit. The crash date for Tron was +12 days and for Cardano it was only +7 days. KPSS test on residuals indicated stationarity by accepting the null hypothesis, see Table 4. 22.

Crypto ticker	Starting Date	End Date	Crash Date (estimated)	Crash actual	AIC	RMSE	KPSS statistics	p-value
TRX	01.12.2017	08.01.2018	17.01.2018	05.01.2018	-126.6	0.1648	0.0351	>0.1
ADA	30.11.2017	02.01.2018	11.01.2018	04.01.2018	-121.8	0.1357	0.0487	>0.1

Table 4. 22 LPPL model statistics for Tron and Cardano

Again, the problem of the model is $\hat{\beta}$ for two cryptocurrencies which do not lie in the most modern interval suggested in the theoretical part. For Cardano $\hat{\omega}$ is out of desired range suggested by Johansen (2003). Yet, it is still in the initial interval between 6 and 13. The remaining parameters satisfy the imposed constraints, see Table 4. 23 for confirmation.

Crypto ticker	$\hat{\beta}$	$\hat{\omega}$	$\hat{t_c}$ (days)	$\hat{A}$	$\hat{B}$	$\widehat{C_1}$	$\widehat{C_2}$
TRX	0.63557	7.23528	47.898	0.30178	-0.48939	0.047555	-0.0754
ADA	0.9667	8.07114	42.64	0.17751	-0.06327	0.018178	-0.0036

Table 4. 23 LPPL model estimation results for Tron and Cardano

Consequently, the models captures accurately log-periodic oscillations as well as the fitted model cope up with actual price acceleration well. For Tron, however, the final value is not depicted close to the real global price peak, see Figure 4. 33.

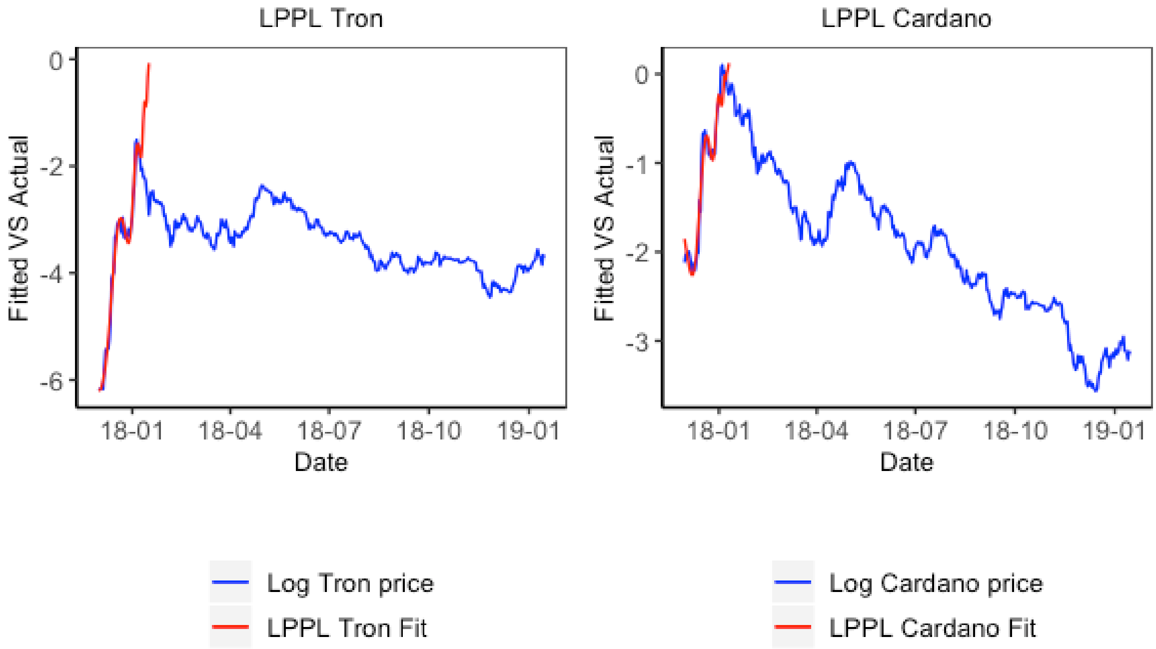


Figure 4. 32 Fitted LPPL model vs actual of Tron log prices
Figure 4. 33 Fitted LPPL model vs actual of Cardano log prices

4.4.6 Summary of the LPPL model fit

Taking for analysis 12 major cryptocurrencies in the market with the largest market capitalization, the bubble presence for 11 of them was confirmed. The modified LPPL fitting procedure was performed for the ones where bubbles were detected in 2017. During that year, major price jumps took place. From the achieved results, it can be concluded that LPPL model is quite powerful tool and GSADF inputs were helpful to achieve strong performance and lower RMSE. The performed analysis was done on datasets with a different length used as inputs. Hence, it is quite hard to compare the models between each other. Predictions varied in accuracy from -5 days to +14 days, for bitcoin although is +29 days. Developed models were able to grasp the price dynamics, however, it was problematic to satisfy the constraints for the model parameters. Especially $\hat{\beta}$ which are out of the stricter intervals for eight cryptocurrencies out of ten. It might be a specific of a cryptocurrency market which is a relatively new market on the financial arena. Therefore, bubbles were present as such but price acceleration were different in speed as well as in the frequency of log-oscillations. The minimum frequency was indicated for EOS with $\widehat{\omega} = 4.914372$. On the contrary, the maximum frequency for Cardano with $\widehat{\omega} = 8.07114$.

4.5 Predictions of Future Crashes

In this section, the LPPL model will be used to predict future crashes since GSADF test in the date-stamping procedure detected bubbles for many cryptocurrencies that started in 2019 and based on the results have not yet finished. For forecasting of the end of the bubbles, there are made more predictions with the rolling forward prediction window with the fixed starting day for each particular cryptocurrency. The window of fourteen days was chosen to capture price development more precisely. Biweekly predictions were done for each time series of log-prices, where each two weeks estimated parameters were changed. Those predictions were bind together and plotted against the prediction covering the whole period from the starting day till the end. Full results are presented in the subsections below.

4.5.1 Bitcoin

Based on the output of GSADF test from Table 4. 8, the bubble has started in the first half of March. This date is taken as a starting point of the forecasting and the time windows used for prediction are presented in Table 4. 24.

Overall, eight predictions were made. Figure 4. 34 nicely depicts results of predictions. It is obvious that consecutive predictions with expanding window capture more of log-periodic oscillations, whereas the prediction made for the whole period smoothly captures overall development without large swings. RMSE values tend to grow with larger sample, however, all values are significantly lower 0.1 which is much better results compared to the LPPL model of Bitcoin crash in 2017, Table 4. 14. Non-linear parameters varies dramatically among predictions as shown in Table 4. 25. Which can be explained by short-term changes of the price behavior and high volatility which persists throughout the whole period of the analysis. As a supper to such conclusions, log-periodic oscillations depicted by $\hat{\omega}$ varies from 2.9258 to 8.458. In addition, some of the parameters do not fulfill recommended constraints mentioned in the Subsection 3.4.6.

Num.	Starting Date	End Date	Crash Date (estimated)	RMSE	KPSS statistics	p-value
1.	09.03.2019	23.03.2019	24.03.2019	0.0046	0.0931	>0.1
2.	09.03.2019	06.04.2019	06.04.2019	0.0185	0.05491	>0.1
3.	09.03.2019	20.04.2019	23.04.2019	0.0259	0.05995	>0.1
4.	09.03.2019	04.05.2019	17.05.2019	0.0338	0.04432	>0.1
5.	09.03.2019	18.05.2019	31.05.2019	0.0394	0.0798	>0.1
6.	09.03.2019	01.06.2019	10.06.2019	0.049	0.135	>0.1
7.	09.03.2019	15.06.2019	25.06.2019	0.062	0.0897	>0.1
8.	09.03.2019	29.06.2019	02.07.2019	0.0608	0.077	>0.1

Table 4. 24 LPPL Model Prediction statistics 2019 for Bitcoin

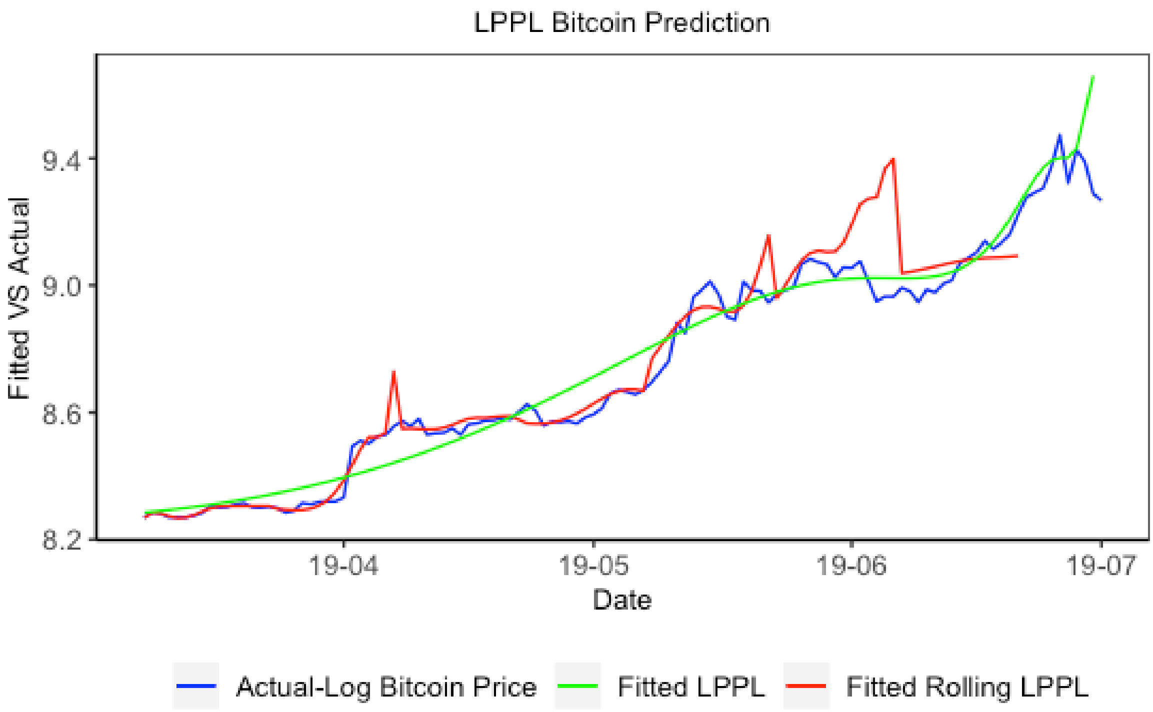


Figure 4. 34 LPPL Model Prediction 2019 for Bitcoin

Num.	$\widehat{\beta}$	$\widehat{\omega}$	$\widehat{t_c}$ (days)	$\widehat{A}$	$\widehat{B}$	$\widehat{C_1}$	$\widehat{C_2}$
1.	0.63557	6.41	16.45	8.31	-0.0000006	-0.00000005	-0.0000005
2.	0.01324	2.9258	28.77	15.76	-7.213	-0.04299	0.01767
3.	1.7766	5.6612	45.19	8.589	-0.00064	-0.00017	0.00018
4.	1.0117	5.9797	69.95	8.717	-0.00558	0.00058	0.00156
5.	0.035	6.308	83.08	18.11	-8.396	0.026	-0.0572
6.	0.3439	8.458	93.13	9.838	-0.328	-0.0128	0.0101
7.	1.672	4.798	108.9	9.096	-0.0004	-0.00008	-0.00003
8.	0.379	3.35	115.1	9.77	-0.224	0.0241	-0.0095

Table 4. 25 LPPL model estimation results for Bitcoin in 2019

4.5.2 Ethereum

Turning to Ethereum, GSADF test again indicated periods which remind bubbles. Nevertheless, the observed bubble started in the middle of May 2019 is much shorter duration compared to bitcoin. Focusing on prediction, the same methodology was applied and results are presented in Table 4. 26 and Table 4. 27 . RMSE values show low numbers and imply a good forecast. However from the graphical representation is clear that oscillations were not captured precisely by consecutive forecast, Figure 4. 35. Especially at the end of the prediction period the red line goes in the opposite directions with real numbers. On the opposite, the forecast for the whole time range mimics the shape of actual log-prices as well as the price development towards the end. Oscillations captured by $\widehat{\omega}$ do not vary significantly across developed models, although forecasts lie quite apart from each other.

Num.	Starting Date	End Date	Crash Date (estimated)	RMSE	KPSS statistics	p-value
1.	15.05.2019	29.05.2019	31.05.2019	0.0277	0.081	>0.1
2.	15.05.2019	12.06.2019	19.06.2019	0.031	0.0439	>0.1
3.	15.05.2019	26.06.2019	06.07.2019	0.035	0.048	>0.1

Table 4. 26 LPPL Model Prediction statistics 2019 for Ethereum

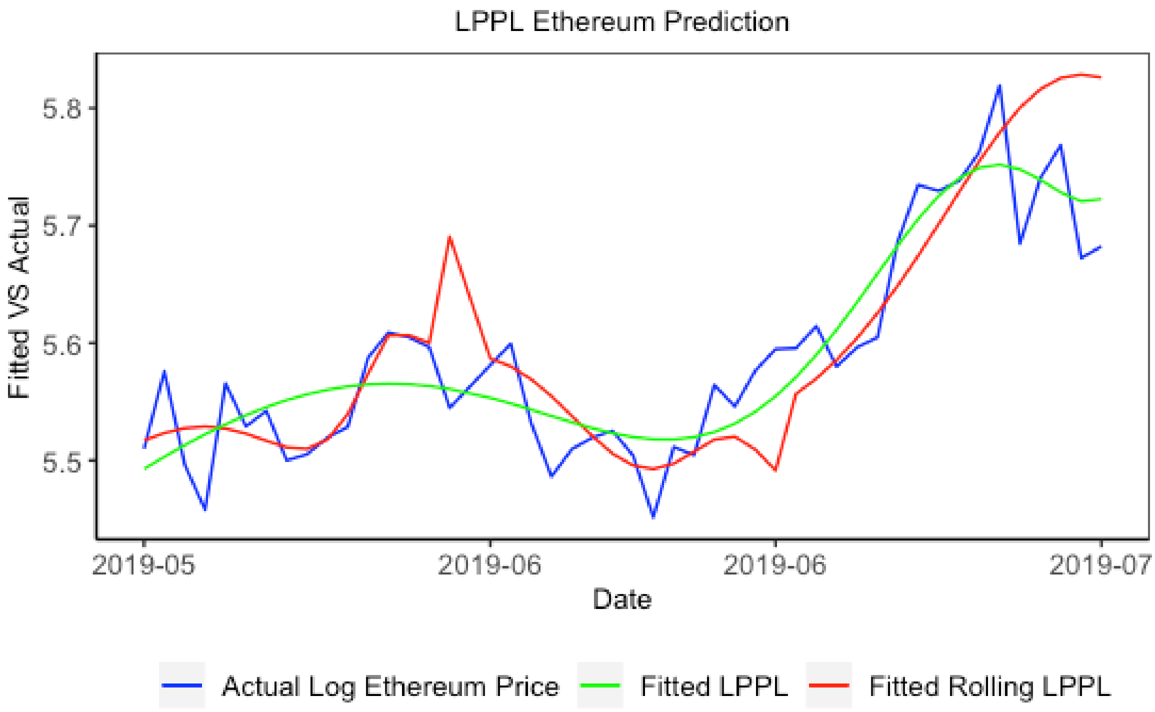


Figure 4. 35 LPPL Model Prediction 2019 for Ethereum

Num.	$\hat{\beta}$	$\hat{\omega}$	$\hat{t}_c$ (days)	$\hat{A}$	$\hat{B}$	$\widehat{C_1}$	$\widehat{C_2}$
1.	0.0116	4.5416	16.7	10.809	-5.15	0.0234	-0.0128
2.	0.921	5.706	35.08	5.48	0.0043	-0.0022	-0.002
3.	0.996	3.394	52.19	5.879	-0.012	0.004	0.00212

Table 4. 27 LPPL model estimation results for Ethereum in 2019

4.5.3 Bitcoin Cash

Predictions for Bitcoin Cash are supported by evidence in Table 4. 28 and Table 4. 29. As a starting point 8th February 2019 was chosen in accordance with GASDF test information from Table 4. 10. Estimated values of $\hat{\beta}$ and $\hat{\omega}$ are not consistent across predictions, nevertheless, rolling prediction copies actual price evolution which can be seen from Figure 4. 36. Prices jumps, occasional spikes and drops together with log-periodic oscillations correspond to original time series. The prediction done on the whole period has the highest RMSE value. In additions, visualization shows that LPPL model captures overall trend of the price increase but ignores high volatility over time. Some of the estimated parameters do not lie into suggested intervals, however expanding prediction window forecast shows high accuracy.

Num.	Starting Date	End Date	Crash Date (estimated)	RMSE	KPSS statistics	p-value
1.	08.02.2019	22.02.2019	24.02.2019	0.019	0.083	>0.1
2.	08.02.2019	08.03.2019	09.03.2019	0.031	0.08	>0.1
3.	08.02.2019	22.03.2019	25.03.2019	0.044	0.042	>0.1
4.	8.02.2019	05.04.2019	07.04.2019	0.069	0.084	>0.1
5.	8.02.2019	19.04.2019	29.04.2019	0.084	0.037	>0.1
6.	8.02.2019	03.05.2019	15.05.2019	0.089	0.054	>0.1
7.	8.02.2019	17.05.2019	25.05.2019	0.092	0.053	>0.1
8.	8.02.2019	31.05.2019	14.06.2019	0.0899	0.057	>0.1
9.	8.02.2019	14.06.2019	23.06.2019	0.091	0.064	>0.1
10.	8.02.2019	28.06.2019	25.07.2019	0.127	0.087	>0.1

Table 4. 28 LPPL Model Prediction statistics 2019 for Bitcoin Cash

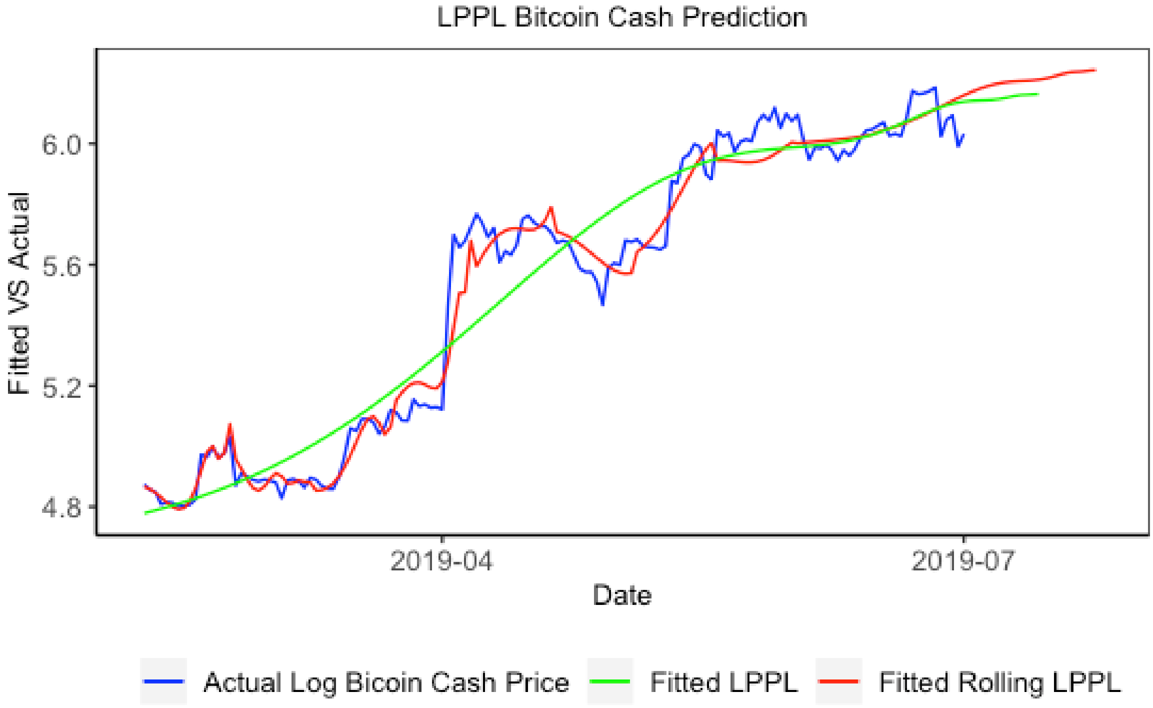


Figure 4. 36 LPPL Model Prediction 2019 for Bitcoin Cash

Num.	$\widehat{\beta}$	$\widehat{\omega}$	$\widehat{t_c}$ (days)	$\widehat{A}$	$\widehat{B}$	$\widehat{C_1}$	$\widehat{C_2}$
1.	0.199	4.246	16.63	5.39	-0.338	0.044	-0.0135
2.	1.294	6.711	29	4.88	0.00086	0.0013	-0.00203
3.	0.047	3.915	45.4	7.33	-2.118	0.044	0.0488
4.	0.007	4.5	58.05	35.97	-30.3	0.02157	-0.0659
5.	0.454	4.57	80.3	6.6495	-0.28	0.016	0.026
6.	0.114	3.92	96.05	8.99	-2.381	0.1168	0.1199
7.	0.558	4.687	106.8	6.225	-0.093	-0.002	0.0258
8.	0.821	7.347	126.26287	6.386	-0.0304	0.00169	-0.0052
9.	1.7	9.84	135.2	6.075	-0.00039	-0.00005	-0.000068
10.	1.389	4.597	167.06	6.243	-0.0011	0.00022	0.0001

Table 4. 29 LPPL model estimation results for Bitcoin Cash in 2019

4.5.4 Litecoin

Similarly to the precious examples, Litecoin prediction was done using the same approach. Expanding estimation window prediction accurately unfold price changes throughout the time being, yet the prediction lies far apart significantly at the end of the given period which is not what we are looking for. From Figure 4. 37 it can be noted that the prediction model deviated from actual price development during expanding of the prediction window. On the contrary, prediction made on the whole subset from 8th February till the end of June 2019 depicts the price going downhill and spike again in the end of June without depicting short-term volatility. Table 4. 30 and Table 4. 31 support graphics with real numbers and captures low values of RMSE and well as diverse combinations of estimated parameters which help to detect upward and downward trends combined with specific acceleration speed and log-periodic oscillations.

Num.	Starting Date	End Date	Crash Date (estimated)	RMSE	KPSS statistics	p-value
1.	11.05.2019	25.05.2019	27.05.2019	0.0387	0.094	>0.1
2.	11.05.2019	08.06.2019	10.06.2019	0.0459	0.0434	>0.1
3.	11.05.2019	22.06.2019	30.06.2019	0.047	0.051	>0.1
4.	11.05.2019	01.07.2019	05.07.2019	0.042	0.053	>0.1

Table 4. 30 LPPL Model Prediction statistics 2019 for Litecoin

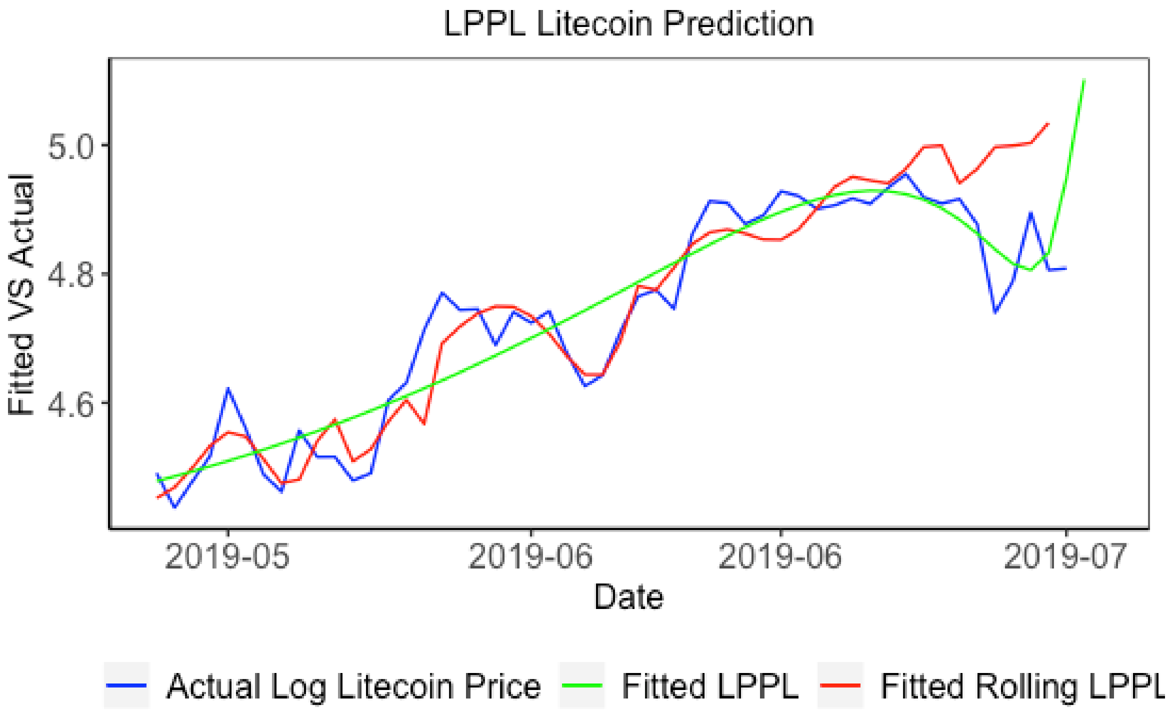


Figure 4. 37 LPPL Model Prediction 2019 for Litecoin

Num.	$\hat{\beta}$	$\hat{\omega}$	$\hat{t_c}$ (days)	$\hat{A}$	$\hat{B}$	$\widehat{C_1}$	$\widehat{C_2}$
1.	0.00085	9.28	16.6	39.43	-34.85	-0.0397	-0.0262
2.	0.2913	3.971	30.57	4.89	-0.1127	-0.0434	0.0052
3.	0.8978	12.182	50.96	5.08997	-0.0185	-0.0008	-0.0023
4.	0.2318	2.059	55.02	5.213	-0.229	0.0598	-0.0445

Table 4. 31 LPPL model estimation results for Litecoin in 2019

4.5.5 Binance Coin

Last but not least is the Binance Coin similarly with Bitcoin and Bitcoin Cash started 2019 with a bubble creation. Based on the outputs from Table 4. 32 and Table 4. 33, the Figure 4. 38 was created to support the predictions visually. Low RMSE across models tells us about accuracy of the models. Prediction with expanding time window provide speed acceleration and log-periodic parameters of $\hat{\beta}$ and $\hat{\omega}$ correctly takes evolution of actual log-prices. Besides that, the prediction with the whole period used as an input end up with similar prediction results without capturing the price exuberance.

Num.	Starting Date	End Date	Crash Date (estimated)	RMSE	KPSS statistics	p-value
1.	09.02.2019	23.02.2019	24.02.2019	0.0185	0.0792	>0.1
2.	09.02.2019	09.03.2019	13.03.2019	0.0486	0.0516	>0.1
3.	09.02.2019	23.03.2019	31.03.2019	0.054	0.0439	>0.1
4.	09.02.2019	06.04.2019	18.04.2019	0.0522	0.034	>0.1
5.	09.02.2019	20.04.2019	25.04.2019	0.0586	0.043	>0.1
6.	09.02.2019	04.05.2019	22.05.2019	0.061	0.039	>0.1
7.	09.02.2019	18.05.2019	28.05.2019	0.0733	0.041	>0.1
8.	09.02.2019	01.06.2019	05.06.2019	0.0728	0.0479	>0.1
9.	09.02.2019	15.06.2019	03.07.2019	0.0782	0.0475	>0.1
10.	09.02.2019	29.06.2019	09.07.2019	0.077	0.049	>0.1

Table 4. 32 LPPL Model Prediction statistics 2019 for Binance Coin

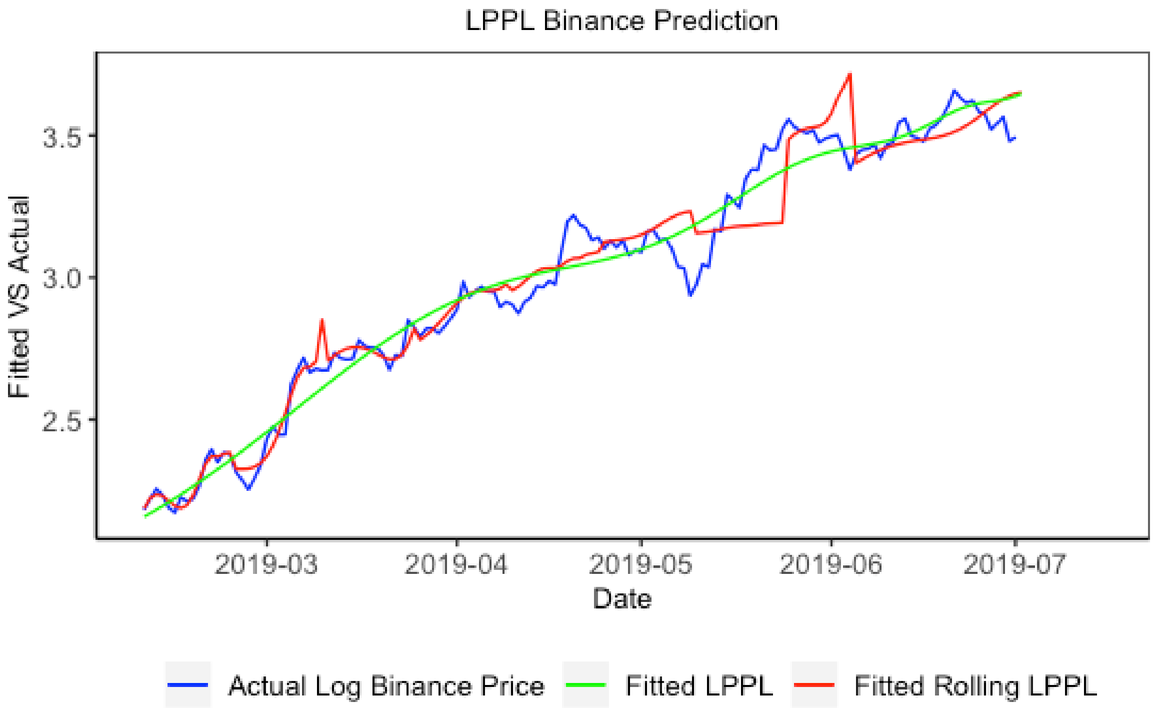


Figure 4. 38 LPPL Model Prediction 2019 for Binance Coin

Num.	$\hat{\beta}$	$\hat{\omega}$	$\hat{t_c}$ (days)	$\hat{A}$	$\hat{B}$	$\widehat{C_1}$	$\widehat{C_2}$
1.	1.738	4.231	15	2.384	- 0.00427	- 0.00046	- 0.00237
2.	0.0288	4.744	32.31	11.927	8.854	-0.0389	0.0421
3.	0.427	3.781	50.96	3.182	-0.161	-0.00648	-0.02979
4.	1.1003	7.048	68.42	3.073	-0.0082	0.00132	-0.00016
5.	1.208	8.443	75.15	3.09	- 0.0048	0.00075	0.00012
6.	1.268	8.239	102.1	3.2995	-0.003	-0.00027	0.00017
7.	1.799	1.0341	108.4	3.1942	-0.00022	0.000022	-0.000004
8.	0.639	3.4738	116.2	3.739	- 0.0674	0.00126	0.01053
9.	0.996	6.5276	144.06	3.759	-0.0103	-0.00051	-0.00092
10.	1.194	7.097	150.5	3.695	-0.0036	0.00016	0.000298

Table 4. 33 LPPL model estimation results for Binance Coin in 2019

4.5.6 Conclusion on predictions of future crashes

There were five cryptocurrencies analyzed in accordance with GSADF test which indicated ongoing bubbles in the given time series. Some other coins were not selected for this part of the thesis even the date-stamping procedure indicated potential bubbles due there a very short length. For each cryptocurrency the predictions of the modified LPPL model differed significantly. Models used for the predictions showed low RMSE values which was desirable, though this indicator might be potentially misleading, because predictions made on whole subsets of data without expanding prediction window were not capable of to capture periods of short-term volatility nor were able to deal with log-periodic oscillation. Those models provided very smoothed outputs compared to actual log-prices. Looking closer to predictions based on expanding prediction window, it can be noted that gradual expansion helped significantly to measure price ups and downs. That is why the coefficients $\hat{\beta}$ and $\hat{\omega}$ appeared to be very inconsistent amongst all cryptocurrencies taken for the analysis in this section. Taking into account that while finishing this analysis, the real data regarding the price development became available, the predictions of this section can be compared to the actual price evolution. The predictions suggest that all bubbles terminate by the end of July where prices reach maximum values. However, based on the current information it can be concluded that actually the highest prices throughout 2019 were observed in June. Both variations of LPPL were misled by input data and did not capture the end of the bubble very accurately for the cryptocurrencies chosen in this section. Hence, comparing two approaches used, it is clear that cryptocurrencies do not have specific patterns of behavior as such that is why predictions with expanding window gives better view on the price dynamic compared to the other approach used in this section. The second approach definitely captures a long-term development of the prices of the various coins. It is hard to achieve precision in predictions at the cryptocurrency market due to strong and quick shifts of the cryptocurrency prices. Since the LPPL model calculates $\widehat{t_c}$ itself, it is difficult to choose a prediction horizon as well as a starting point of the bubble.

Conclusion

The purpose of the thesis was to analyze the cryptocurrency market with the focus on twelve major cryptocurrencies by the market capitalization and apply available methods to detect bubbles at the market. The goal was accomplished by utilizing different available approaches and empirical results shed the light on the price evolution in the cryptocurrency market.

Based on some general analysis of market participants without some sophisticated models, it can be concluded that prices are highly affected by some social-behavior factors and the presence of noisy traders who are not well educated on the blockchain technology itself and who do not take into account intrinsic value of the cryptoasset. The market is prone to reaction to the news from influencers, herding behavior and is attractive for those who are interested in fraudulent activities. Vast majority of cryptocurrencies are used for speculations and not used instead of fiat money. It is hard to apply Monetary Theory concepts to the cryptocurrencies.

Nevertheless, during the empirical analysis, SADF and GSADF tests were used to detect whether there were or there are bubbles present in the cryptocurrency market. Using more robust results of GSADF tests, which are able to capture more than one period of bubble, there were detected bubbles across majority of cryptocurrencies taken for the analysis. Specifically, there were bubble bursts captured in the end of 2017 or in the beginning 2018, which can be a sign of the bubble presence in the whole market. Similarly five other cryptocurrencies indicated bubbles of a longer duration in 2019 which corresponded to the price exuberance. Going further the modified version of LPPL model was applied to the log-prices for predicting crashes based on the outputs from the GSADF results.

Overall, LPPL framework captured well the speed of the price acceleration and the log-periodic oscillation. The hard part of choosing the starting day of the bubble for LPPL model was eased by the implementation of the date-stamping procedure of GSADF tests. However, the LPPL model calibration was very sensitive to the chosen starting point which often led to the rejection on the model because of not fulfilling the constraints imposed on particular parameters from an economic theory. On the other hand, LPPL model estimated quite accurately critical point $\widehat{t_c}$ for bubbles 2017.

For bubbles detected in 2019, in order to avoid calibration based on one prediction the expanded rolling window were used. The comparison of two approached showed that rolling estimation much better mimics price evolution in terms of capturing oscillations and price acceleration. The general prediction based on a single window is not as precise as a previous method because it smoothed price fluctuations. The drawback of the model is that it estimates price evolution till the critical point t_c and afterwards the estimated parameters are invalid. In order to have longer prediction horizon, there should occur a large number of calibration to get better perspective on future price evolution.

In conclusion, some extensions and more advanced LPPL models might increase the quality of predictions. Since the cryptocurrency market does not have a long history and differs in characteristics compared to other financial instruments, there investigation on constraints imposed on the non-linear parameters in the LPPL models.

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