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Title of the Master's Thesis:

Game Theory in Managerial Decision Making: Applications to the Microfinance Industry

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Declaration of Authenticity

I hereby declare that the Master's Thesis presented herein is my own work, or fully and specifically acknowledged wherever adapted from other sources. This work has not been published or submitted elsewhere for the requirement of a degree programme.

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Prague, May 13, 2020

Signature: Kristine Gyulbudaghyan

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Game Theory in Managerial Decision Making: Applications to the Microfinance Industry

Abstract:

The microfinance industry demonstrated fast growth in recent years. On the one hand, small loans are available for a wider audience, especially for those who cannot get access to bank loans. On the other hand, microfinance companies face higher risks of default when they provide unsecured loans to people with low-income levels. This raises a question of optimal design for credit contracts together with the valid criteria for decision making.

Game theory proved to be a useful theoretical concept helping managers to make optimal decisions within strategic interactions with other agents.

In this thesis, we aim to design an optimal credit agreement for microfinance credit organizations to maximize its expected profit when the future decision of a borrower to repay or not is unknown. We propose incentives for borrowers to exert enough effort for being able to repay debt in the future, which may help to reduce the moral hazard problem for the microfinance credit organization.

Key words:

Game theory, microfinance, moral hazard

Table of Contents

1. Introduction

1.1. Research Motivation

The notion of "strategy," originally a military term, appeared in the 1960s in the managerial literature and the world of business (Hammoudi and Daidj, 2018, p 16). In 1965, "SWOT" (*Strengths, Weaknesses, Opportunities, Threats*) was defined as follows: "It is the founding model of strategic management which highlights strategic analysis under two angles: external with the market and internal with the firm (Learned, Christensen, Andrews, 1965).

The word "strategy" has inspired a variety of authors and, consequently, led to several definitions. According to the authors of the book *Game theory approach to managerial strategies and value creation* (Hammoudi and Daidj, 2018, p 6) definitions of strategy can be classified based on certain criteria/logic specific to strategic management:

- Firm-environment relation (external diagnosis)
- Resources-competencies (internal diagnosis of the firm)
- Resource allocation

Furthermore, the game theory focuses on the relations between a firm and its environment by providing an opportunity to use its competitive recourses in play. Hammoudi and Daidj (2018, p 25) provide an excellent induction of how companies can use game theory to explain mechanics related to strategic behavior from top-down. For example, a firm must decide to train staff or not, training will increase productivity but will also require a financial investment. Thus, there will always be arbitration. Further on, downstream, game theory concerns all the decisions evolving around pricing, production, publicity level, relations with supplier, market-entry, localization of activities, and many more. Furthermore, companies tend to face complex situations; thus, the decision-making process becomes particularly tricky. A company can overcome these difficulties by looking closely at the interdependencies on the market, which will help it quickly realize that specific strategies should never be used as they generate lower gains compared with others. Practice like this will help the firm to simplify the decision-making process by merely reducing the number of options. The idea behind this practice applies to the notion of the domination of one strategy over another (Hammoudi and Daidj, 2018, p 29).

Moreover, Asikomurwa and Mohaisen (2015), in their paper about *Game theory and business intelligence in strategic business decisions*, emphasize that game theory helps to find the right strategies which consecutively lead to the right strategic decisions for the business. Furthermore, making business decisions is a balance between data and opinion, where opinion is the best possible professional judgment. Authors of the paper strongly believe that game theory gives a possibility to look at the past, using current data and predict the future for a business. CEOs and managers using business intelligence tools can benefit from the insights of game theory by designing a game that is right for their organizations. Further on, no specific game theory strategy is perfect because various situations within the organization have to be handled in unique ways. The bottom line is that game theory can be used very efficiently as a tool for decision making, whether in political, economic, psychological, or business settings (Asikomurwa and Mohaisen, 2015).

Over the past years, business leaders came to realize that economic growth and success can also be achieved through social values. Corporate sustainability became one of the essential components of the company's competitive advantage. Many global leaders such as Toyota, Philips, Shell, and many more incorporated the concept of corporate sustainability in their everyday operations (Jaehun and Prabhu, 2013, p 50).

The same way can be considered an effective microfinance program with a corporate sustainability approach for alleviating poverty. Microfinance institutions (MFIs) were designed to provide more flexible solutions to consumers with unstable or seasonal income, which is particularly prevailing among microentrepreneurs in developing countries. Despite the flexibility that microfinance institutions provide, they can experience difficulty in maintaining the sustainability of their practices. This difficulty could arise due to many factors, for instance, information asymmetries. In many economic journals and articles, the phenomena of asymmetries are studied deeply, for example, the article written by Barboni (2017) shows how microfinance institutions can overcome asymmetries through offering rigid and flexible repayment schedule. Lab-in-the field games conducted with Indian entrepreneurs confirm the model predictions by showing that risk-averse individuals choose a rigid contract, as it is cheaper than the flexible contract, and high-risk consumers are more likely to take up a flexible repayment schedule. The paper concludes that it is possible to design a screening instrument that will allow lenders to know in advance who the flexible contract can be offered to (Barboni, 2017). One of the most common definitions of information asymmetries refers to one side possessing more information than the other side, so if MFIs could find a way to predict the type of their consumers, then they can significantly increase their profits.

1.2. Literature Review

As of December 2010, over 3652 MFIs were reaching more than 200 million people, most of whom were among the poorest when they took their first loan (Maes and Reed, 2012). The presented information is momentous, given the obstacles that poor people had to overcome in order to get financing. The success of microcredit initiatives is based on the methodology and flexibility provided by the institutions. Widely used methodologies include group lending, dynamic incentives, lending with collateral, and regular-repayment schedules. Group lending represents a group of people who are jointly responsible for individual loans. The idea of dynamic incentives helps the borrowers to get access to future loans as an enticement to repay the current one (Shapiro, 2015).

In the beginning, one of the most popular methodologies was group lending; however, more recently, there was a shift towards other aspects of microfinance. Fischer and Ghatak (2010) cite various factors that affected the shift from group lending, such as decreased reliance by major MFIs and increasing costs connected with group lending. Another exciting idea is represented in the American Economic Journal stating that "although many microfinance institutions rely on the borrowers to monitor their contracts jointly, on the contrary, those group-based mechanisms tend to be vulnerable to free-riding and collusion" (Giné, Jakiela, Karlan, and Morduch, 2010, p 61).

Providing financing to low-income communities can be an extremely tricky business. MFIs typically lack full information about their consumers and find it impossible to provide loan contracts; on the other hand, the consumers can lack sufficient credit histories with commercial banks. Here we see the idea of moral hazard and adverse selection, which can be an obstacle for MFIs to lend profitably. Despite these difficulties, during the last three decades, microfinance advocates found workable mechanics which will allow providing small, uncollateralized loans to needy consumers. Repayment rates on those loans exceeded 95 percent, and by 2007, the year after Muhammed Yunus and Grameen Bank won the Nobel Peace Prize, MFIs were serving about 150 million people around the world (Yunus and Muhammed, 1999).

Furthermore, Sandar Win (2018) cites in her paper the importance of the adequate assessment of credit risk, which is not only critical to long-run banking institutions but also the economy as a whole. Credit risk can be defined as a situation where the borrower will fail to meet obligations with agreed terms and conditions. Furthermore, the failure to manage the credit risk will result not only in accounting loss but also, in transactional loss and forgone opportunity costs. Hence, customer relationship management (CRM) became one of the top priorities of banking institutions.

If we think about the lending practices from borrowers' perspectives, we can assume that borrowers, themselves, are decision-makers whose choices interact with MFIs. This interaction can have an essential effect on actions leading to the conflict or utilizing cooperation. This idea also sheds light on the study of strategic decision-making named game theory. As presented by Dixit and Nalebuff (2010), in their book named *The Art of strategy*, game theory can be found in the simplest of situations. For example, the struggle to save more money is a game of one's current self against the short-run future self, which is inclined to overspend. The current self commits to behaving better and, at the same time, this commitment should be stable, meaning the future self should be denied from the possibility to deviate.

On the other hand, future incentives can change, and such actions that change the game to ensure a better outcome for the player are called strategic moves. Authors, Dixit and Nalebuff (2010), believe that commitments, threats, and promises that players possess will not improve the outcome of the game if they are not credible. A way to make the communication credible is to agree to pay the penalty if one fails to follow it through. Reputation for credibility can also be lost if one tries a strategic move in the game and then backs off. Future rivals can remember the player's past actions in dealing with others and can establish certain beliefs. Therefore, players have the incentive to establish a reputation that makes the future strategic moves credible (Dixit and Nalebuff, 2010).

1.3. Research Objective and Research Questions

Various authors have studied the implementation of a game-theoretic approach towards bank loan repayments. For instance, Paliński (2015) represented a model of loan repayment in the form of a signaling game in which the borrower sends a signal (proposed amount of the loan), and the bank receives the signal and takes action (accepts or rejects the proposal).

Similarly, this thesis examines the relations between the borrower and the lender by establishing concepts of game theory in a core. As a result, the game-theoretic model designs the optimal credit agreement such that it provides enough incentives for borrowers to repay debt fully and on time. The data that is used throughout this thesis has significant value as it creates a base for decision-making for the author. The information is taken from an Armenian microfinance credit organization that has been present in the country already for seven years. The author aims to design an optimal game-theoretic model which should guide the firm in their decision making and will answers questions like:

- How can the firm set an optimal credit agreement with the borrower?
- What factors affect (or do not affect) the loan repayment rate in the company's portfolio?
- How can the firm increase the loan repayment rate through supervision?

This thesis proposes an approach that will guide the lender through the decision-making process of the credit agreement parameters, such as loan amount and interest rate. The lender will face situations like; high-risk customers pretending to be low-risk ones to get better conditions, or moreover, once the loan is given, the borrower might decide not to pay, and this is known as moral hazard problem. In this thesis, the author aims to propose such agreements between the lender and borrower that will help to alleviate the moral hazard problem and will keep the borrower's incentives on the initial level.

1.4. Outline

This thesis consists of two main parts: theoretical and practical. The theoretical part begins with chapter 2, which provides the reader with a general introduction about game theory and its concepts. In the beginning, episodes start with the basic game-theoretic models and discuss ideas of classical authors that studied game theory, like Fudenberg and Tirole (1991), further on, the thesis progresses to more specific and advanced models. Primary topics and subtopics include discussion of fundamental theories such as cooperative and noncooperative games, normal and extensive form games, the concept of equilibrium and Nash equilibrium. The theoretical part advances with the discussion of sequential games and focuses on extensive form games with imperfect information. For almost every model, there is a game example that helps to illustrate the theory to the reader.

Furthermore, the theoretical part examines concepts, such as; Bayesian Nash and the perfect Bayesian equilibrium. Chapter 3 continues with theory by introducing the reader to the idea of asymmetric information. Topics discussed in asymmetric information chapter include moral hazard and adverse selection. Afterward, the chapter focuses on the concept of signaling through insurance game, which, later on, is used as a base to construct one of the game models for Firm X (Armenian microfinance institution). The theoretical part of this thesis serves as a good base for analysis and provides ideas for the construction of the optimal game models for the practical part.

Moving on with the last chapters, chapter 4 presents the practical part of the thesis, including a comprehensive overview of the banking system of Armenia, macroeconomic indicators, and microfinance institutions and their lending practices. Chapter 5 takes a closer glimpse into the operations of Firm X and its loan portfolio and continues by interpreting the data and extracting meaningful insights. Furthermore, with the help of statistical tools, the author tries to find dependencies between types of loans and factors influencing the loan to become overdue. After all the necessary analysis is conducted, and the theoretical base is established, the author designs several game models using the factors which led the loan to become overdue. Game models help to propose a structure of a game such that no borrower will have the incentive to deviate, or even if the borrower varies, the firm can have enough resources and information to control its clients. The thesis concludes with chapter 6 by underlying the most critical findings, stating work limitations, and giving some future recommendations to the firm.

2. Game Theory

Game theory is a science of strategy. It was initially developed during the 1920s and had a fast growth during World War II as there was a need to develop new ways of thinking in the area of military strategy (Nicholson and Snyder, 2010, p 176). Any situation which involves strategic decision making can be presented as a game. It attempts to identify logically and mathematically the actions which need to be taken by players for gaining the best outcomes in a wide array of games. All the games share the characteristic feature of interdependence, meaning the outcome for each player depends on the strategies of the rest of the participants. Strategic interactions include constraints on the players' actions and interests but do not indicate which actions the players do take. A solution is a systematic description of the outcomes that might arise in a family of games (Osborne and Rubinstein, 2008). The underlying assumptions in game theory stress that decision-makers perform reasonable decisions taken into account other players' strategies.

The models of games have been extensively applied in a wide range of classes of real-life situations. For instance, the idea of Nash equilibrium has been used to study political and oligopolistic competition.

2.1. Basic Concepts of Game Theory

Nicholson and Snyder (2010, p 176) state that all the games include four essential components, such as players, strategies, payoffs, and information. A player can be either a single individual or a group of individuals who act as decision-makers. As a decision-maker, the player is aware of his/her alternatives, has clear preferences, and has some forms of expectations about any uncertainties. Strategies include the players' choices throughout the game and represent a contingent plan of action based on what other players are doing. Strategies are based on actions, and those actions can range from very simple to many complex ones. The returns that the players receive by the completion of the game is called payoffs. These payoffs can be both; explicit and implicit types. Explicit payoffs represent the utility players obtain from explicit monetary payments (Nicholson and Snyder, 2010).

Moreover, the same authors explain that implicit payoffs represent the inner satisfaction that the players receive from playing the game. Players also have some knowledge before making their movies, called their information. For example, in simultaneous-move games, players do not have any information about other players' actions, and they move simultaneously. On the contrary, in sequential-move games, the first player does not know others' actions while making a move, but other players, in this scenario, have the opportunity to observe the first player's action. In some games, namely in games with incomplete information, players have the opportunity to observe and learn things that others might not be aware of. For instance, in card games, the player knows only his/her own cards and has the opportunity to learn throughout the game and, that knowledge will influence the whole course of the game. John Harsanyi developed the theory of incomplete information in his "Games with incomplete information played by 'Bayesian' players," 1967. In games with complete information, players know the payoffs of individual actions of other players, said otherwise, players are fully aware of the rules of the game and also know the utility functions of other players. Game with perfect information represents a model where all players know the structure of the game and are entirely informed of all the events that have previously taken place. An excellent example of a game with perfect information can be chess, where each player can inspect other player's moves and pieces on the board. Finally, in games with imperfect information player who makes a move does not have complete knowledge about the full history of the game (Nicholson and Snyder, 2010, p 177).

Once players are defined, there can be present two model types: non-cooperative and cooperative. Non-cooperative games are those in which the sets of possible actions of individuals are driven by self-interest; therefore, binding agreements are not possible. Cooperative games, on the other hand, deal with situations where agreements are binding and represent the best joint course of action.

Nicholson and Snyder (2010) defined that there exist two ways of describing the games: strategic (normal) form games and extensive form games. Normal form games are represented in a table or matrix. The number of rows indicates one player's strategies, and the number of columns represents the other player's strategies. Extensive form games are usually represented in the form of a game tree, and actions proceed from top to bottom. The tree form contains all the information about the game. It establishes who moves when, where the moves lead to, and what moves are available to each player. Lastly, the payoffs for each player are represented at each end.

2.2. Dominant Strategies

Osborne and Rubinstein (2008, p 3), assume that the structure of the game is a piece of common knowledge for the players, meaning that each player is aware that other players know the "rules of the game." In simultaneous strategic-form games, each player has his/her strategies and is separated from other players; meaning player has no chance to observe other actions. Each participant also knows the structure of the game and is aware that so does the opponent and vice-versa.

Two-player strategic form finite games are presented in the form of a matrix that represents the player's payoff (or utility) functions, as shown in *table 1*. The representation of a game in the matrix below is described as a normal form game that represents a two-player game involving two pure strategies each: U, D (up, down) for player 1, and L, R (left, right) for player 2.

	L	**R**
U	4,3	5,1
D	2,1	8,4

Table 1: Two-player strategic form finite game; source: Osborne and Rubinstein, 2008

The first player's actions are identified with the rows, and the second player is with columns. The two numbers in the boxes are formed by rows *r* and by columns *c*, which represents players' payoffs. If we have a look at *table 1*, then we can easily say that strategy (D, R) is strictly dominant as it is generating the highest payoffs for both players (8,4).

The point is that although (D, R) is better than (U, L), when player 2 decides not to choose the dominated strategy then (U, L) is better than (D, R) given that there is a one percent chance that player 2 plays L. So, this example provides an understanding that rationality in the sense of not playing strictly dominated strategy is common knowledge and is subject to sensitiveness to small uncertainties about behavioral assumptions that players make about each other (Osborne and Rubinstein, 2008, p 13).

Further on, Osborne and Rubinstein (2008), also provide an explanation on the difference between the analysis of games and the analysis of single-player decisions. In a single-player decision-making process, the only uncertainty involves the possibility of moves, and the decision-maker is assumed to have fixed beliefs. On the contrary, in a game, there are several decision-makers involved, and the expectations players have about their opponents are internal, resulting in a change of actions of all players once there is a change in the game. The idea of equilibrium does not necessarily mean that each player's best choice will lead to mutual best results. And indeed, one of examples is the Prisoners' Dilemma, where participants are drawn into adverse outcomes by following their best private interests.

2.3. Equilibrium and Nash Equilibrium

Authors of the book *Intermediate microeconomics and its application*, Nicholson and Snyder (2010), define market equilibrium at the point where supply equals the demand, and no participant has an incentive to change his/her behavior. The most comprehensive approach used for defining equilibrium in the games is named after John Nash, who developed the idea in the 1950s. In the concept of game theory, equilibrium is defined by the best-response strategy. Moreover, the latter defines the strategy that generates the highest payoff for the player, given what other players are doing.

Unfortunately, not all games have strict dominance, and this is when the idea of Nash Equilibrium comes to place. The idea of Nash has the advantage of existing in a wide variety of games and is widely used in the concept of strategic games. So, more straightforward, Nicholson and Snyder (2010) described Nash equilibrium as a set of strategies so that each player's strategy is the best response to the other players' strategies, and the decision-making of players depends on the knowledge of equilibrium. In economics, the very best interpretation of strategies can determine the choice of output and price level, which come across in Cournot and Bertrand games, respectively. The idea of Nash equilibrium was present in the two of the first games that have been studied, namely the Bertrand (1883) and Cournot (1838) models of oligopoly. Nicholson and Snyder (2010) further elaborate that in the Bertrand model, firms simultaneously choose prices and then make their output choice of how much to produce once prices are known. In the Cournot model, firms simultaneously choose the quantities that they will produce. In both games, firms choose their best response functions by taking into account the actions of their opponents. Moreover, it is common to refer to the equilibria of the models as "Bertrand equilibrium" and "Cournot equilibrium," but it is more helpful to think of them as Nash equilibrium of two different games (Nicholson and Snyder, 2010, p 414).

Further on, authors Osborne and Rubinstein (2008, p 35) defined that the idea of Nash equilibrium involves how the game will be played, meaning that all players take into consideration that if a particular Nash equilibrium will occur then, no player has an incentive to play differently. Thus, Nash equilibrium has a property that players can predict it and also predict that their opponents can predict it, and so on. *Table 2* shows how players wish to coordinate their behavior but have conflicting interests.

	A	**B**
A	2, 1	0, 0
B	0, 0	1, 2

Table 2: Game with Nash Equilibrium; source: Osborne and Rubinstein, 2008

The game has two Nash equilibria (A, A) and (B, B). There are two steady states: one in which both players choose (A, A) and another one where both players choose (B, B) (Osborne and Rubinstein, 2008, p 37).

2.4. Strategic Form Games

Nicholson and Snyder (2010, p 178) point out also a that a strategic form game involves three elements: the pure-strategy, set of players, and payoff functions, which provide utility for given strategies. Strategies can be of two forms; pure strategy and mixed strategy. The pure strategy indicates that all moves are determined in advance, and mixed strategy does not explicitly determine which move will be taken in given situation, but it assigns probability to a particular pure strategy for the game.

Every player's strategy is to maximize his/her payoff function, which involves making other players either worse or better off (Fudenberg and Tirole, 1991, p 4). Decisions are made independently and simultaneously, meaning that each player does not have the opportunity to observe what other players are doing. On the contrary, sequential-move extensive model games specify an order of events; each player has the chance to observe other actions, not only at the beginning of the game but also throughout the whole game course. In general, for simultaneous move games, it is more common to use the payoff matrix, and for sequential move games, the extensive form is widely used, known as a decision tree (Nicholson and Snyder, 2010, p 180).

Nicholson and Snyder (2010) strongly believed that another essential component in the games is information which matters in each type of a game. In cooperative games, players usually have complete information about other players, while in non-cooperative games, players might not be fully informed.

The number of players also varies through the games, starting with two-player, three-player, or n-players games being possible. In two-player zero-sum games, the sum of the utilities in a constant setting is equal to 0.

$$\sum_{i=1}^{n} \mathrm{U(i)} = 0$$

A classic example illustrating a zero-sum game with mixed strategies is called the Matching Pennies game again presented by the authors Nicholson and Snyder (2010, p 185). The game is straightforward; it consists of two layers; A and B, and each player secretly chooses whether to leave a penny with its head or tail up. Afterward, players simultaneously expose their choices. A wins B's penny if both coins match, either two tails or two heads, and B wins A's penny if the coins do not match. The game can be represented in both forms, a normal or extensive form, and the unique feature about the game is that two players' payoffs in each box add to zero, and that is why it is called a zero-sum game. A game in normal form is represented in *table 3*, where all possible outcomes corresponding to four boxes are equally likely to occur with probability ¼. By using the formula for expected payoffs, we can calculate both players' expected payoffs, which will equal at the end to zero.

		B	
		Heads	Tails
A	Heads	1, -1	-1, 1
	Tails	-1, 1	1, -1

Table 3: Matching pennies game in normal form; source: Nicholson and Snyder, 2010

So, A's expected payoffs will equal the probability-weighted sum of the payoffs in each outcome.

$$(¼) \times (1) + (¼) \times (-1) + (¼) \times (-1) + (¼) \times (1) = 0$$

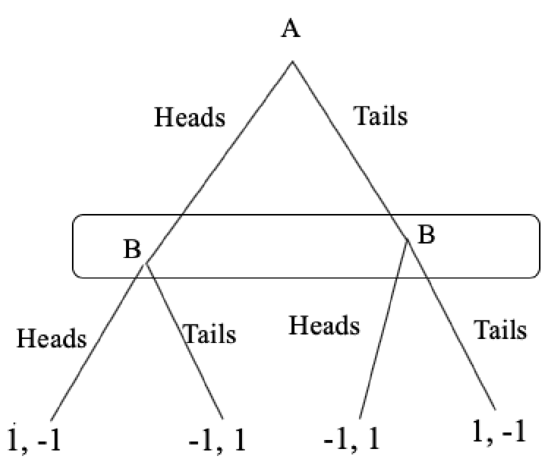


Figure 1: Matching pennies game in extensive form; source: Nicholson and Snyder, 2010

Furthermore, similarly can be calculated B's expected payoffs, which is equal to zero. The set of strategies by playing Heads or Tails with an equal chance (½ and ½), is the mixed-strategy Nash equilibrium of the game. The same idea of the game is represented in *figure 1* but extensive form.

In an extensive form game, we cannot make two players' moves to appear on the same level as the structure of the tree is not allowing that. So instead, we randomly choose one player to start, in *figure 1* it is player A, and another player, B to be a second-mover. An oval circle around B's decision points indicates that B does not have an opportunity to see which action A has taken and cannot observe which decision point has been reached when he/she makes his/her own decision.

In a two-player zero-sum game, whatever one player wins, leads the other one losing. In this case, we could consider that players are pure opponents, and even though these structure games were widely studied in game theory, most of the games are non-zero-sum games (Nicholson and Snyder, 2010).

2.5. Sequential Games

Sequential games differ from simultaneous games in the sense that the player who moves after another can learn information about the game up to that point and also observe the actions other players have chosen. Based on this information, the player can make more sophisticated strategies instead of choosing a single action. This strategy can become a contingent plan and be adjusted based on other players' actions (Nicholson and Snyder, 2010). In general, sequential games are of great interest because they help to better model the reality before making a decision. Real examples include; the producers' decision about how much to produce by, firstly, observing the demand in the market, or how duopolies observe each other's actions before introducing new goods to the market.

2.6. Extensive Games with Imperfect Information

Osborne and Rubinstein (2008, p 199) define that when some players do not identify the payoffs of others, the game is said to have imperfect information. Authors further explain that extensive games with imperfect information allow players to have only partial information about past actions while taking immediate action. The model is broad and allows to study scenarios not only related to players being imperfectly informed about one another but also situations in which a player is uncertain whether another player had acted or situations when the player took action and forgot about it. If we take parallels between extensive and strategic form games, we would assume that in a strategic game, while taking action, the player is not aware of others' actions. Furthermore, in Bayesian games, the players neither know other players' actions nor their private information. In extensive games with perfect information, the players are not informed about the future actions that their opponents would take (Osborne and Rubinstein, 2008, p 89).

On the contrary, in extensive games with imperfect information, the players not only have insufficient information about future actions but as well, as present and past actions of the other players. While choosing actions, players form some expectations about the unknowns. However, these expectations can differ in different models. Unlike, in strategic games, in the Bayesian model, the expectations are not derived strictly from the players' equilibrium behavior, as the players might face situations inconsistent with their behavior. Moreover, unlike extensive games with perfect information, the model studies not only players' future behavior and actions, but also past and present events (Osborne and Rubinstein, 2008, p 199). An example of an extensive game with imperfect information is shown in *figure 2*. First-mover is on top of the tree indicated by 1, and the lower player, player 2, is the second mover.

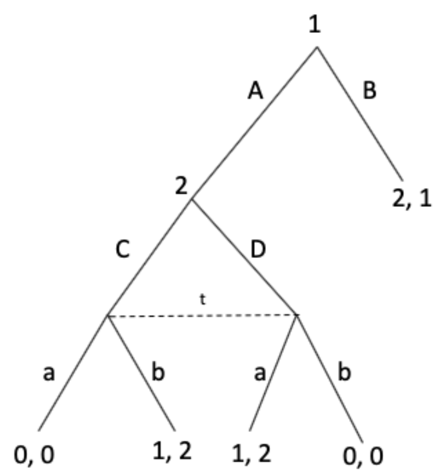


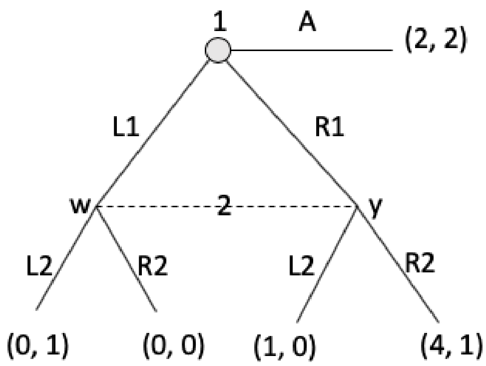
Figure 2: Extensive game with Imperfect Information; source: Osborne and Rubinstein, 2008

In the game above, player 1 takes the first move, choosing between A and B. If player 1 chooses B, the game will end with payoff (2, 1). If the player chooses A, then it is player 2's move. Player 2 is informed that player 1 chose A, and will choose C or D. Afterward, it is player 1's turn to move, and by doing so, player 1 does not know whether player 2 chose C or D. This idea is indicated by the dotted line *t*, which connects the ends of histories after which player 1 moves for a second time, choosing an action from the set (a, b). The numbers under the histories indicate the players' payoffs, the first number being player 1's payoff, and second number player 2's (Osborne and Rubinstein, 2008).

2.7. Sequential Equilibrium

Sequential equilibrium has been criticized by Kohlberg and Mertens (1986) in order to allow strategically neutral changes in the game tree to affect the equilibrium. Further on, authors Fudenberg and Tirole (1991, p 96) compared two game models to define the idea behind the theory.

Looking at *figure 3a* and *figure 3b*, one can state that they are the same, only except for one irrelevant move named NA ("not across") in *figure 3b*.



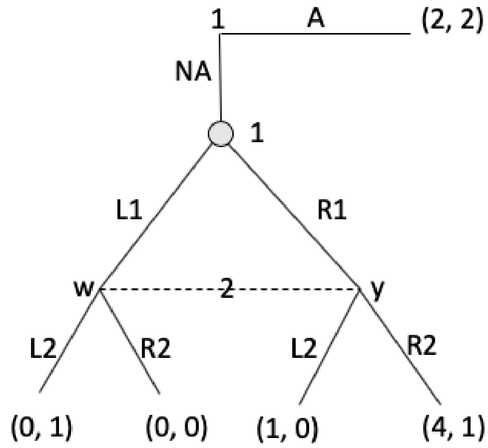


Figure 3a: Sequential game *3b: Simultaneous move subgame*

Source: Fudenberg and Tirole, 1991

A is a sequential equilibrium outcome in *figure 3a* but is not in *figure 3b*. In a simultaneous move subgame following NA, the only Nash equilibrium is found at (R1, R2), as R1 strictly dominates L1 for player 1. Thence, the only sequential equilibrium payoff is the one leading to the highest utility (4,1). The above figures also help to understand that the removal of a strictly dominated strategy affects the sequential equilibrium payoffs. If move L1 is deleted from *figure 3a*, the unique sequential equilibrium payoff will be (4, 1). We cannot also eliminate the idea that players might make mistakes at each information set. For example, in *figure 3b*, if player 1 makes a mistake of not playing A, he/she is still able to estimate that R1 will be a better outcome than L1. In *figure 3a*, player 1 can play either action (L1 or R1) by mistake, when intending to play A.

While talking about extensive games, we are taking into consideration only a player's strategy profile. When comparing it to sequential equilibrium, we take into account both; strategy profile and belief system, which specifies certain beliefs of the players in each information set. It is logical to consider the belief system as a component forming equilibrium, as it provides a model about players' expectations and their actions, given that those expectations are consistent with rationality.

In particular, actions that are not consistent with the strategy after the history of actions can be referred to as beliefs about what will happen in unexpected events. In the games with imperfect information, beliefs about unexpected events should include beliefs not only about the future but also about the past. So, in defining the sequential equilibrium, it should specify not only the players' strategies, but also their beliefs about the history of actions occurred.

Fudenberg and Tirole (1991) further state that players' beliefs can also be subject to constraints, including consistency with strategies, structural consistency, and common beliefs.

Consistency with Strategies

Players should have consistency in their strategies and beliefs. In any information set, the belief about the history that has occurred should be derived from the players' strategies using Bayes' rule.

Structural Consistency

Given the situation that all the players will not have complete information about the other players, we may wish to require that other players' beliefs could be acquired from some alternative strategies using Bayes' rule. The constraints on the beliefs can be named as structural, as they do not depend on players' equilibrium strategy or their payoffs.

Common Beliefs

Game theory concepts also take into account information asymmetries; that is, every player is assumed to analyze the situation in the same way. If we look at this in the concept of subgame perfect equilibrium, we can say that all the players' beliefs about the plans of other players are the same (Fudenberg and Tirole, 1991).

2.8. Bayesian Games and Equilibrium

The model of the Bayesian game is closely related to the strategic game and occurs in the positions where some of the parties are not certain about the characteristics of some other parties (Osborne and Rubinstein, 2008). The Bayesian game evolves around two characteristics: the set of players and sets of actions. We model the uncertainties of players about each other by introducing a set W of possible "state of nature," that is, the possible characteristics of all participants. Each player has some belief about the state of nature given by the probability measure p on W. In this model, the players' information about the state of nature is introduced by signaling functions, given that the player gives a signal about observing other players before choosing his/her action. The model allows players to have different prior beliefs and is used in situations in which the state of nature is a profile of parameters of the players' possible types, and the player is uncertain about what other players know.

When it comes to the Bayesian equilibrium, authors Osborne and Rubinstein (2008) stated that each player knows its real type and does not need to plan in a hypothetical event where he/she is some other type. In any given state, if a player wishes to determine best-response action, the player should hold a belief in what other participants would do in other states, that the player might not have perfect information about. Moreover, the formation of a player's belief can also be depending on the action that the player would choose given other states, which would lead to imperfect information towards other players. So, if we try to define the concept of Nash equilibrium for a Bayesian model, we could say that it is player's formal belief in the state when he/she receives the signal taken into account other players' actions, and he/she chooses the best action available. Moreover, the existence of Bayesian equilibrium is a consequence of the Nash existence theorem.

2.9. Subgame-Perfect Equilibrium

A subgame is a part of the extensive form game. Subgame is said to be proper if a player who moves first knows the actions that have been played by others, which led to that point. Game theory offers a way for selecting the reasonable Nash equilibria in sequential move games by using the concept of subgame perfect equilibrium. The subgame-perfect equilibrium requires strategies to be rational. Furthermore, the subgame-perfect equilibrium is a set of strategies that form a Nash equilibrium on every proper subgame, and a subgame-perfect equilibrium is always a Nash equilibrium. Another component of subgame-perfect equilibrium is that it rules out any empty threat in any sequential game. If we compare the subgame-perfect equilibrium with Nash, we could say that Nash equilibrium requires players to be rational only on the part of the game tree where equilibrium is reached. On the other hand, the subgame-perfect equilibrium requires rational behavior in all parts of the game tree. So, all players should move by their best response strategies as anything else is ruled out (Nicholson and Snyder, 2010).

When we talk about the games, we could indicate that, at the beginning of the game, the players do not know each other's types, and the start of the game does not show a well-defined subgame until the players' prior beliefs are specified, and moreover, we cannot test whether the continuous strategies specify Nash equilibrium.

The complications caused by incomplete information can be easily observed in signaling games, such as a leader-follower game, where the leader is the only party with complete information. When the leader makes the first move, the follower has a chance to observe the leader's action, but not his/her type, before choosing his/her own action. One example illustrating this is Spence's (1974) famous model of the job market. In this model, the leader is the employee who has a level of education and knows his/her own productivity. Consequently, the firm is presented as a follower who can observe the employee's education level but not productivity. Based on the given information, the firm then decides what wage to offer the employee. The subgame perfection in this model demonstrates that for any given education level, the firm should offer a "reasonable" pay, meaning it should be consistent with the equilibrium in the continuation game. So, now the reasonable wage offered by the firm will mainly depend on the firm's beliefs about the worker's productivity, which on turn can depend on the given level of education. If this is the equilibrium that has a positive probability, then the distribution of worker's productivity can be calculated using Bayes' rule. Bayes' rule cannot compute this distribution of productivity if the equilibrium probability is equal to zero, the reasonable wage will depend on the distribution specified. Thus, in order to extend subgame perfection, one of the critical steps is to specify how players update their beliefs about their opponents, after an observation which has a probability of zero (Fudenberg and Tirole, 1991).

2.10. Perfect Bayesian Equilibrium

Osborne and Rubinstein (2008, p 25), explained that perfect Bayesian equilibrium arises from combining the ideas of subgame perfection, Bayesian equilibrium, and Bayesian inference. Strategies should yield to Bayesian equilibrium in any continuation game given the prior beliefs of the players, which require to be updated by following Bayes' law whenever needed.

Sequential equilibrium is similar but enforces stricter rules on how the players' beliefs should be updated. Both of the concepts are identical in a way that they place weak restrictions on beliefs in a 0-probability event. What is vital in the game is that one player's actions can reveal information to the other players, and this information can refer to anything that one player observes, and the other one did not, including also the player's past actions.

A simple example illustrated by authors, Osborne and Rubinstein (2008), can be observed in *figure 4*. In this game, player 1 has three actions to choose from L, M, and R. If the player chooses to play L, then the game will end with a payoff (2, 2).

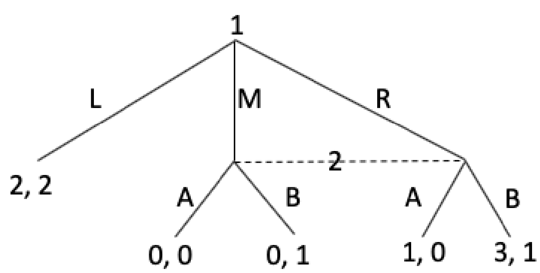


Figure 4: Subgame-Perfect Equilibrium Game, source: Osborne and Rubinstein, 2008

If player 1 chooses to play M or R, then player 2 can choose between A and B, and player 2 does not know whether player 1 chose M or R (that is why there is a dashed line for player 2). If player 1 chooses M and player 2 chooses A, then the payoff is (0, 0). The payoff for (M, B) is (0, 1), the payoff for (R, B) is (3, 1), and the payoff for (R, A) is (1, 0).

The game has two-pure strategy Nash equilibria, that is (L, A) and (R, B), both of which are subgame perfect. If player 1 deviates and does not play L, then optimal strategy for player 2 would be to play B.

Sequential equilibrium is defined for general games and does not eliminate the equilibrium of (L, A) in *figure 4*. Sequential equilibrium emphasizes the formation of players' beliefs and is equivalent to perfect Bayesian equilibrium in a class of signaling games (Osborne and Rubinstein, 2008).

So, the perfect Bayesian equilibrium is simply a set of strategies and beliefs which are optimal, and beliefs are obtained from equilibrium strategies and actions using Bayes' rule. It might be helpful to differentiate between strategies and beliefs. Beliefs are always consistent with strategies, assuming the latter ones are optimal.

Already in 1991, Fudenberg and Tirole provided a distinction between sequential equilibrium and perfect Bayesian equilibrium by considering a multi-stage game of incomplete information with more than two types per player and more than two periods. If we have a look at *figure 5*, we can see that the figure presents a situation where player 1 has three possible types: *a*, *b, c*. At the time (t), Bayesian assumption from previous play leads to the conclusion that player 1 must be type *c*. Equilibrium strategies are shown in the parentheses and indicate that for type *a*, the optimal strategy is to play µ2, for type *b* to play µ3, and for type *c* to play µ1.

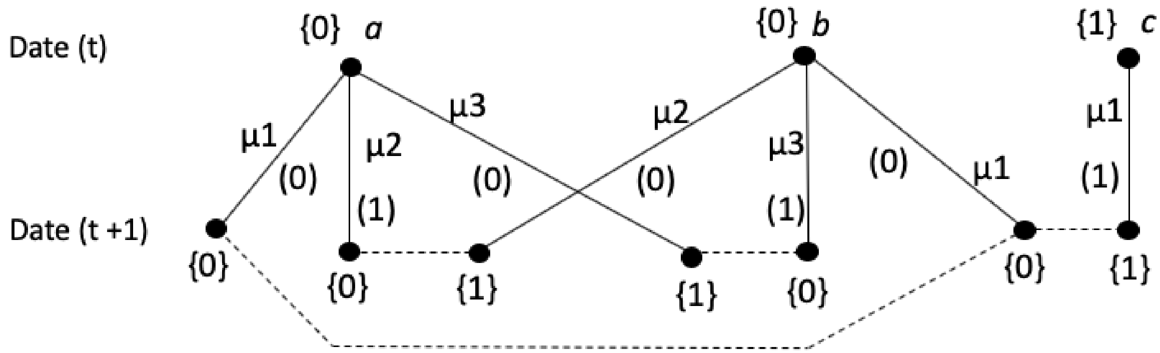


Figure 5: Multi-stage Game with Incomplete Information; source: Fudenberg and Tirole, 1991

Since the first two types (*a* and *b*) have a probability of 0, player 2 expects to see player 1 play *c*. So, what should be our belief if player 2 sees player 1 playing one of the other two options? The beliefs for each type are given in the braces. If player 2 sees µ2, concludes that he/she is facing type *b*, while µ3 is taken as a signal that the player is of type *a*. Moreover, since the perfect Bayesian equilibrium places no constraints on the belief of a player who has deviated from the initial strategy, *figure 5* is compatible with perfect Bayesian equilibrium and cannot be part of the sequential equilibrium (Fudenberg and Tirole, 1991, p 71).

3. Asymmetric Information

The work of authors Jehle and Reny (2011) in their book about *Advanced microeconomic theory* served as a base for practical part and personally the concepts and topics motivated the author in construction of own game models. This chapter about asymmetric information is based on the ideas of the mentioned authors and focuses on the concepts of several insurance-game models.

A state in which various agents possess diverse information is referred to as asymmetric information. The participants' strategic opportunities are unquestionably related to the distribution of information across economic agents. Furthermore, those opportunities that arise in the markets with asymmetries lead to inefficient market outcomes, otherwise known as market failures. When information is freely available, meaning there are no asymmetries involved, there is a unique competitive equilibrium where firms earn zero expected payoffs, and consumers are fully satisfied (Jehle and Reny, 2011, p 416).

To better illustrate the idea of asymmetric information, we will make a comparison from two significant theorems of finance literature and economics, which is the Modigliani-Miller theorem and the first fundamental theorem of welfare economics. The first welfare theorem suggests that in a competitive economy, prices would adjust in order to efficiently and optimally allocate all the resources in the Pareto sense. The critical component holding this theorem involves the idea of no uncertainties and characteristics of the products, such as prices, should be equally observed by all the agents. When these assumptions fail to hold, that leads to the idea of asymmetries in the market, and resources cannot be any longer efficiently allocated. Government interventions in order to regulate monopolies or alleviate the effect of externalities are not sufficient to restore optimality, as the government itself does not have access to all the information in the market and is also an imperfect institution (Nicholson and Snyder, 2010).

In a similar way from the literature of finance, the Modigliani-Miller theorem suggests that the firm's value is not dependent on its financial structure (Brealey, Myers, and Allen, 2011, p 618). Based on the idea of asymmetric information, the optimal financial structure within the firm shifted from fiscal considerations towards aligning the interests of employees and managers with the stakeholders.

3.1. Adverse Selection

If we consider a situation where participants in the market are about to make a mutual trade and some participants from the same group do not have full information, it refers to the idea of adverse selection. The theory on adverse selection also states how institutions; such as banks and insurance companies screen the customers with the use of collaterals and deductibles (Rothschild and Stiglitz, 1976), and how sellers provide product warranties by signaling to consumers about the quality of their products, or how students acquire higher levels of education in order to signal their future employers about their skills (Spence, 1976). Agents holding the contracts can also reveal their private information through self-selection. If we take the example of microfinance institutions, provided contracts can be in the form of high-interest rates and the possibility of more extended repayment periods or lower interest rates and shorter repayment periods.

3.2. Moral Hazard

On the contrary, in the cases when asymmetric information occurs after the agreement between individuals, the situation is known as moral hazard. Moral hazard situations usually are analyzed in the principal-agent model, where a principal hires an agent to perform a given task. The agent either accepts or denies the offer and usually is the party with private (hidden) information. In case the agent accepts the offer and takes action, this action is non-observable from the principal's side and is known as hidden action. Compared with adverse selection, moral hazard takes into account future asymmetries and therefore addresses the incentives problem. Hart and Grossman (1983), Mirrlees (1999), Holmstrom (1979), were the main contributors to the principal-agent paradigm. The same framework is widely used to address issues not only in the scope of economics but also in corporate finance if we consider quality control within the firms that is precisely the idea of elimination of asymmetries between managers and employees by observable actions; such as bonus schemes which are the tools designed for aligning the objectives of employees with stakeholders.

3.3. Information and the Efficiency of Market Outcomes

Asymmetric information can have a significant effect on the efficiency properties of market outcomes. In order to make things simple, authors Jehle and Reny (2011) presented interactions through one single market: the market for the insurance. Similar model will be applied throughout the practical part of this thesis when constructing one of the game models for microloan market.

Authors refer to symmetric information when the insurance companies are able to identify each consumer's accident probability, meaning that there are no asymmetries involved. When each participant in the market has full information, there will exist only one competitive equilibrium, under which all insurance companies will earn zero expected profits, and all the consumers will be fully insured. The competitive outcome under symmetric information gives each consumer higher utility and provides that the insurance company's expected profits are equal to zero. Under conditions of asymmetric information, market failures arise in the insurance market, and some opportunities for Pareto improvements go unrealized.

On the whole, when the insurance companies raise the price of the insurance, the expected utility for the consumer from buying that insurance falls, and the expected utility from not insuring stays the same. In this scenario, some consumers might switch to competitors or quit buying insurance, but there will still be people who will continue to get insurance despite increased prices because they know that expected losses from not doing so, are more significant. These consumers are considered to have high accident probabilities. Therefore, whenever the price of the insurance rises, the pool of consumers continuing to purchase is considered riskier on average. And this is the idea of adverse selection, which inclines to harm expected profits. It should not be unforeseen that insurance markets would adjust their strategies in order to cope with adverse selection (Jehle and Reny, 2011, p 420).

3.4. Signaling

If we imagine a situation where the insurance companies continue to raise their prices, there will be a proportion of consumers, who are low-risk type, that would wish to communicate to the company about their state of Nature. This type of attitude is assigned to signaling behavior by authors Jehle and Reny (2011). Moreover, in real-life situations, usually, this is how it works; companies can identify among consumers based on the insurance policies they choose.

In *figure 6*, extensive form game, otherwise referred to as insurance signaling game represented by the authors Jehle and Reny (2018, p 386); illustrates a game between two consumer types (low-risk/high-risk) and one insurance company. We assume that there are two accident probabilities, $\pi 1$ and $\pi 2$, where $0 < \pi 1 < \pi 2 < 1$. Consumers who have accident probability $\pi 1$ are known as low-risk consumers and the ones with accident probability $\pi 2$ as high-risk. Consumers distinguish themselves from others by choosing different insurance policies.

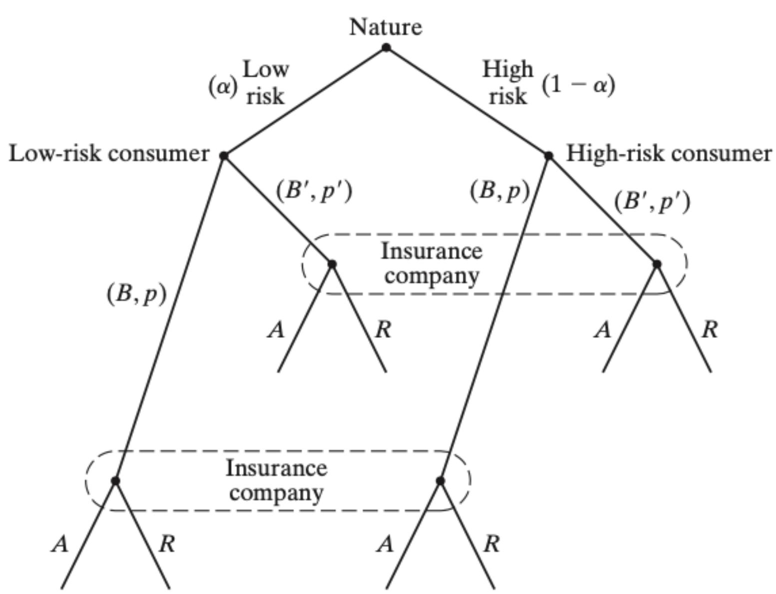


Figure 6: Insurance signaling game; source: Jehle and Reny, 2011

Nature moves first and identifies which type of consumer will propose the insurance company. The α illustrates the probability of low risk consumer being chosen and (1-α), the probability of high-risk one. The player who moves second chooses the policy (B, p) with the benefit of B≥0, and the insurance company pays if he/she has an accident, and the player pays the insurance company a premium whether or not he/she incurs accident. The insurance company moves last and does not know which consumer-type was chosen by Nature; on the contrary, the insurance company knows which insurance has been chosen by the player. In the end, the insurance company either agrees to accept the consumer's terms or rejects them.

Authors believe that once the policy is accepted, the insurance company forms some type of beliefs about the consumer's accident probability. Next, we wish to determine the pure strategy sequential equilibria for this game. But on the other hand, it is known that the idea of sequential equilibria required the game to be finite and as demonstrated in *figure 6*, participants can choose any of a continuum of policies. However, whenever the consumers choices are restricted to finite set policies, the game becomes finite, as well, meaning that every estimate satisfying Bayes' rule also satisfies the consistency condition. Thereupon, in every finite version of insurance signaling game, the sequential equilibrium exists if and only it is sequentially rational and meets the parameters of Bayes' rule.

Nonetheless, the idea of purchasing less insurance does not mean that accident will not occur. In this scenario, insurance companies take different measures to identify the consumer type. For example, low-risk consumers still can signal their truthful type by demonstrating willingness to accept a decrease in policy benefits for a smaller compensation in premium reduction, while on the contrary, high-risk consumers would not. Authors propose that for this type of signaling to be effective, risk types should have different marginal rates of substitution between benefit levels, B, and premium, p.

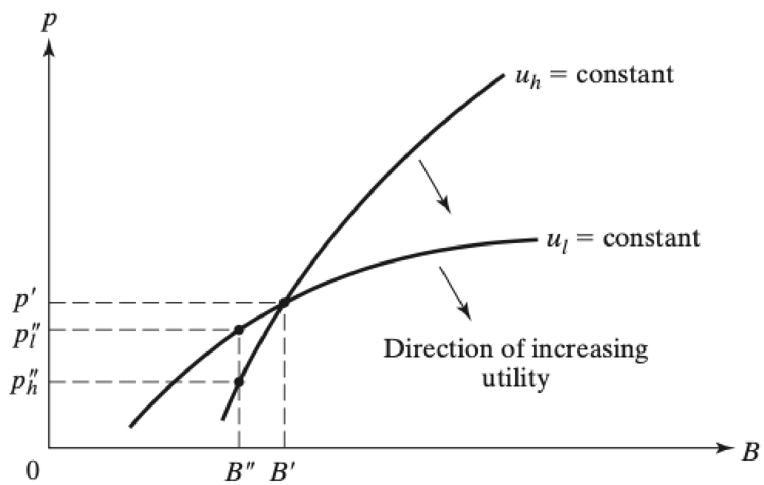


Figure 7: Single-crossing property; source: Jehle and Reny, 2011

In *figure 7*, u_l(B, p) and u_h(B, p) are continuous, differentiable, concave; increasing in B and decreasing in p. MRS_l(B, p) < MRS_h(B, p) for all (B, p) which is known as single-crossing property and implies that indifference curves for two consumer types intersect at most once. Figure also shows that different consumer types have different rate of marginal substitution between benefit levels, B and premiums, p when presented with the same policy. The steep indifference curve, u_h, refers to high-risk consumer and the flatter indifference curve represent the low-risk consumer, u_l.

Given policy is denoted by (B', p') and figure also shows that the low-risk consumer has willingness to accept a reduction in benefit to the point B" for a compensating reduction in premium leading to $p"_l$ which later should be reduced to $p"_h$ to keep the high-risk consumer as well off. This reduction in benefit is less costly for consumer as he/she knows its truthful, low-risk, type. The insurance company will accept any (B, p) policy where p > π1(B), as the policy will yield to economic profits and will reject any policy where p < π1(B) for low-risk consumers, as the company will make losses in this scenario. And the insurance company will be indifferent towards accepting or rejecting the policy at the point where p= π1(B) as it will lead to zero economic profits. Same analysis goes for high-risk type; p > π2(B) accept, p < π2(B) reject, p= π2(B) indifferent (Jehle and Reny, 2011, p 389).

3.5. Separating Equilibria

If we consider game with two possible consumer types then pure strategy sequential equilibrium is either separating or pooling.

In a separating equilibrium, we assume that two risk type of consumers, chosen by Nature, will propose different policies and based on that the insurance companies would be able to identify among them. We take into consideration also the fact that two risk types could feign their identity by behaving as other type according to equilibrium. The critical factor to keep in mind about separating equilibrium is, that we assume no participant should have interest to mimic the behavior of the other and thus, report its truthful type (Jehle and Reny, 2011, p 394).

It is assumed that the policies Ψ_l= (B$_l$, p$_l$) and Ψ_h= (B$_h$, p$_h$) are proposed, respectively, by low- and high-risk type where policy Ψ_l maximizes the low-risk consumer's expected utility, the policy Ψ_h maximizes the high-risk consumer's expected utility. They are accepted by the insurance company in the separating equilibrium if Ψ_l≠ Ψ_h= (L, π2L), or in written words at equilibrium consumer's expected utilities are different and they equal the firm's expected loss, and u$\binom{c}{h}$ ≡, $u_h(\Psi_h) \geq u_h(\Psi_l)$. When policy Ψ_l is proposed, the insurance company knows it faces the low-risk consumer. Therefore, the insurance company's beliefs satisfy Bayes' rule and, at the same time, the insurance company is maximizing its expected profits. Similarly, the equilibrium Ψ_h= (L, π2L) is accepted because the insurance company believes that it faces high-risk type consumer and, in this case, the expected profits from the policy are equal to 0 (π2L- π2L=0). All the remaining policy proposals (B, p) lead to the insurance company to believe that it faces high-risk consumer. Then the company's expected profits from obtaining such policy are p- π2B and, these policies are only accepted if they yield to non-negative expected profits based on beliefs. So, the insurance company maximizes its profits based on beliefs at any given policy (p, B) and, consecutively, consumers are choosing policies which maximize their utilities. *Figure 8* illustrates this situation where the high-risk consumer obtains the policy $\Psi\binom{c}{h} \equiv$ (L, π2L) and, the low-risk consumer has the policy Ψ_l= (B$_l$, p$_l$), shown in the figure as the shaded region.

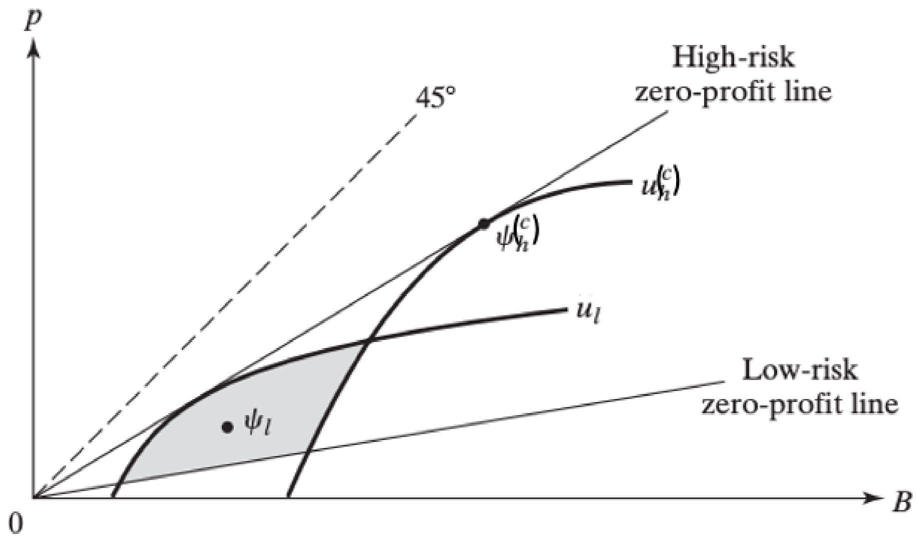


Figure 8: Potential separating equilibria; source: Jehle and Reny, 2011

In separating equilibrium in which insurance companies accept the proposed policies, the high-risk policy chosen by the high risk-type of consumer must be $\Psi\binom{c}{h}$ and the low-risk policy chosen by the low-risk type, should be Ψ_l in the shaded region and MRS_l(0, 0) > π1.

The important feature of a low-risk policy is that each is above the low-risk zero-profit line in order to guarantee acceptance by the insurance company and, above high-risk consumer's indifference curve in order to ensure high-risk will not mimic low-risk one, and below the indifference curve with utility u_l to ensure that low-risk consumer will also not act as a high-risk one. In the *figure 8*, the shaded region is always non-empty given the fact that MRS_l(0, 0) > π1, and even when, MRS_l(0, 0) ≤ π2. Therefore, pure strategy separating equilibrium always exists.

Due to this fact the policy proposals act as signals in a way that it does indeed make it possible for consumers to distinguish among them as either low- or high-risk type. As high-risk consumers have the same policy $\Psi\binom{c}{h}$ in each separating equilibrium, and also gain the same utility, the equilibrium outcome ($\Psi_l, \Psi\binom{c}{h}$) is Pareto efficient among separating equilibria and leads to zero profit for the insurance company. Even in the situations such as, that low-risk consumer receives no insurance because of the asymmetries under competitive equilibrium, authors strongly believe that the low-risk consumer still can get insured and market efficiencies can improve when signaling is possible (Jehle and Reny, 2011, p 400).

3.6. Pooling Equilibria

In separating equilibrium different types of consumers choose different policies and, by that, insurance companies are able to identify among them. In some way, consumers separate themselves from one another. On the contrary, in pooling equilibrium both consumer types choose the same policy. Therefore, the insurance companies cannot identify among consumer types by simply observing the policy they chose. So, the low-risk consumer might be treated as the high-risk one and vice versa. It makes more sense to say that it is more likely that the high-risk type will mimic the low-risk consumer type. The behavior of the insurance company matters as the company has no information about consumer's accident probability. Consequently, if the proposal is (B, p) then accepting will lead the insurance company to have expect profits equal to p – (α*π1 + (1- α) *π2) *B where, α Is the probability that the consumer is low-risk type. The policy will be accepted if p > (α*π1 + (1- α) *π2) *B, rejected if p < (α*π1 + (1- α) *π2) *B, and the insurance company will be indifferent whether to accept or reject if p = (α*π1 + (1- α) *π2) *B. The set of policies (B, p) satisfying pooling zero-profit line, as shown in *figure 9*, will play an essential role for the analysis of pooling equilibria. Such policies lie through the origin which is called the pooling zero-profit line (Jehle and Reny, 2011, p 397).

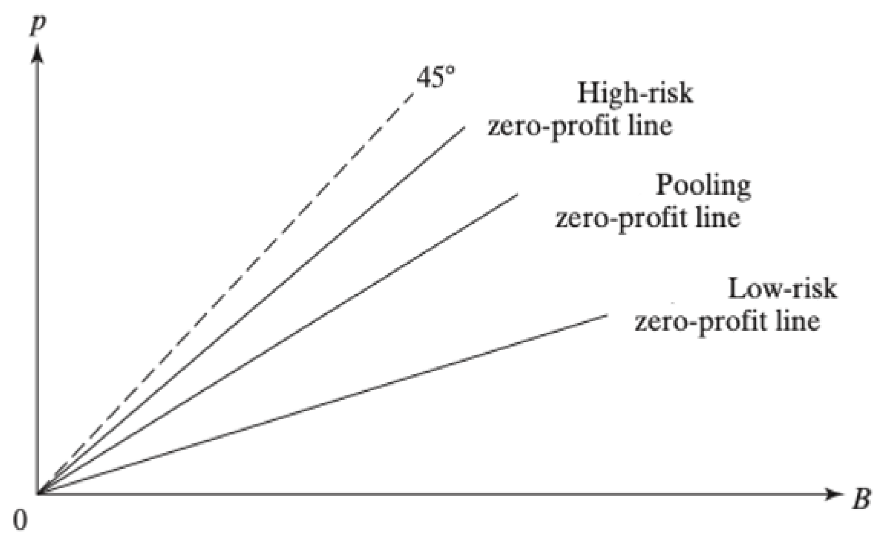


Figure 9: Pooling zero profit line; source: Jehle and Reny, 2011

As described by authors some characteristics of pooling equilibrium include that the policy Ψ'= (B$'$, p$'$) is the outcome in pooling equilibrium if u_l(B, p) ≥ u_l, $u_h(B, p)$ ≥ u$\binom{c}{h}$ and p ≥ (α*π1 + (1- α) *π2) *B. When the equilibrium policy Ψ' is proposed, the Bayes' rule will require the insurance company to keep its beliefs unchanged as the proposal can be made by both risk types and, given the beliefs, it is profit-maximizing to accept Ψ', because it leads to non-negative expected profits. As beliefs satisfy the Bayes' rule meaning, the insurance company is maximizing expected profits, no matter on policy proposal by consumer, and, simultaneously, two consumer types are maximizing their utility given the company's strategy. So, by proposing Ψ', the consumer obtains policy Ψ'. If (B, p) deviates from Ψ', meaning (B, p) $\neq \Psi'$, the consumer will end with policy (0, 0) as insurance company would reject the proposal. And the consumer will obtain policy (B, p) if insurance company accepts the proposal. As shown in *figure 10*, potentially there can be many pooling equilibria. The set of pooling equilibrium is affected from the probability, α, meaning that consumer is low-risk.

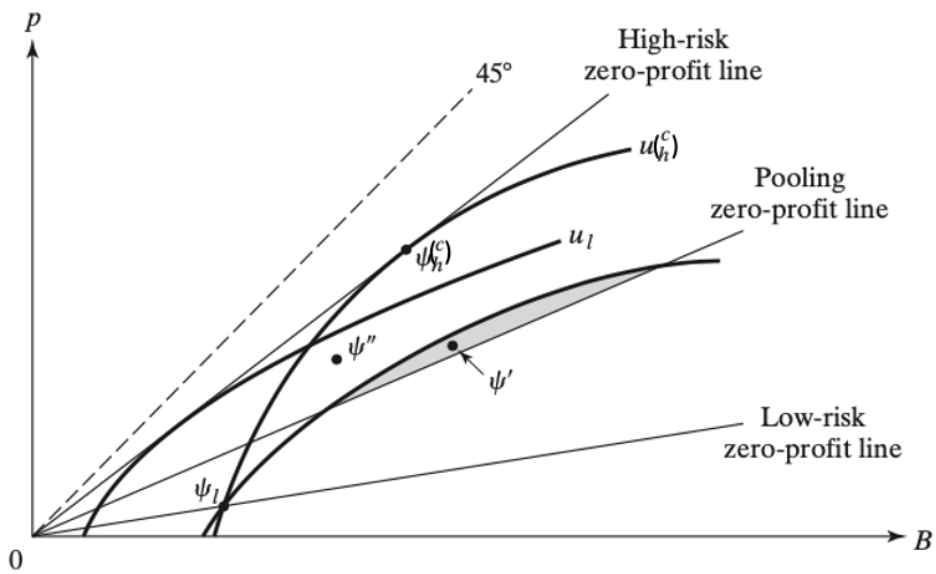


Figure 10: Pooling equilibrium outcome may dominate separation by making both consumers better off; source: Jehle and Reny, 2011

As α fails, the shaded area in the *figure 10* becomes smaller and smaller, because the slope of pooling zero-profit line increases and everything else in the figure remains the same. The shaded area eventually is gone. So, if the probability indicating that a consumer is high-risk type is adequately high, no pooling equilibria will exist.

As α increases, the region of the shaded area becomes bigger and bigger, given that the slope of pooling zero-profit line decreases. And when α becomes sufficiently high, there exists pooling equilibria that makes both consumers better off than they would be under every separating equilibrium, even low-risk type. The reason that the low-risk type would benefit, as well, is because it would be costly for him/her to separate himself/herself from high-risk consumer.

In order for this to be effective, low-risk consumer should choose a policy that high-risk type would not prefer to $\Psi\binom{c}{h}$. This factor reduces the choice of low-risk consumer and also decreases utility below the level that the consumer would get given no high-risk type existed. When α is undoubtedly high, the equilibrium is pooling, and there is a high probability that high-risk type is not present. As shown in *figure 10,* the pooling equilibrium outcome is $\Psi_l = \Psi_h = \Psi'$, where $\Psi_h = \Psi\binom{c}{h}$ and the shaded region is preferred by both types, in contrast with, equilibrium outcome $\Psi_l = \Psi_h = \Psi''$which is not preferred (Jehle and Reny, 2011, p 400).

4. The Characteristics of Microfinance Industry in Armenia

Since its independence in 1991 and until the global financial crisis (GFC) in 2008-2009, Armenia was considered a success story among the transition economies. Over those years, the country indicated a record of high growth, sustained macroeconomic achievements, low inflation, falling poverty, economic stability, and modest external debt. The macroeconomic performance led to the development of the banking sector as a result of liberalizing the economy. Armenia was one of the former Soviet economies, which led to so-called "first-generation reforms." Nowadays, a decade after GFC, the Armenian economy finds itself in a big contrast compared to the previous state (The World Bank, 2019).

Under the old Soviet planning system, Armenia had a chance to develop a modern industrial sector by supplying textile, manufactured goods, machine tools in exchange for energy and raw materials. Since then, Armenia has switched to small-scale agriculture. As in regards to trade, Armenia has only two open trade borders with Iran and Georgia. As a result of Armenia's ongoing conflicts with its neighboring countries, borders with Azerbaijan and Turkey have been closed since 1991 and 1993, respectively. Armenia joined the World Trade Organization in January 2003. Even though the country has been taking measures in improving tax and customs administration, the anti-corruption measures were always a challenging area to fight with. In general, Armenia needs to take additional economic reforms in order to raise the economic growth and improve employment opportunities, especially given the closed borders with neighboring countries; Turkey and Azerbaijan.

Armenia's geographic isolation, ubiquitous monopolies, narrow export base made the country particularly vulnerable in the global markets. Armenia is heavily dependent on Russia's governmental support as most of the infrastructure is Russian-owned, especially the energy sector. Armenia remains interested in pursuing closer ties with the EU, as a result of which the country signed a Comprehensive and Enhance Partnership Agreement with the EU in November 2017 (Moody's Analytics, 2019).

As of data related to 2018, Armenia has a total population of 2.9 million, GDP of current US $12.4 billion, and GDP per capita of current US $4.238. Despite all the obstacles, Armenia is still trying to grow and prosper; as a result of which at the end of 2018, a new government has been formed by following a new five-year program endorsed by parliament in February 2019. The Government has brought renewed regulations and commitments to good governance, mainly including anti-corruption efforts, accountability, and transparency. Following the robust expansion of 7.5 percent in 2017 and 5.2 percent in 2018, annual economic growth continued to stay strong in the first half of 2019 by expanding 6.8 percent. When it comes to inflation rates, annual inflation fell from 2.5 percent in 2018 to 1.8 percent in August 2019 (The World Bank, 2019).

As a result of low inflation, the country had the opportunity to gain a steadier economic growth and reduction in poverty levels. The purchasing power parity (PPP) measured at the lower-middle-income poverty line of US $3.2/day (2011) has fallen by 10.8 percent in 2018 and is expected to reach roughly 7 percent by 2021. Regarding real wages, it grew by 1.5 percent in 2018 and continued to grow in 2019. However, when we look at the unemployment rate, that remains high at 21.9 percent in the first quarter of 2019 (The World Bank, 2019).

When it comes to the economy, which expanded by 6.8 percent in 2019, growth was mainly supported by private consumption as a result of increased wages, up to 3.5 percent, and consumer lending, up to 40 percent. Armenia experiences the growth of recent trends, with expected GDP to rise by 5.5 percent in 2019, which will help to keep the inflation low and attract more investments. Global growth still remains one of the downsides due to trade tensions, as well as rising debt and risk to emerging markets.

Generally, the concentrated effort to restructure the country's economy is towards sustainable growth, but the growth path still remains one of the biggest challenges that will require an efficient government, more reliable infrastructure, higher investments in human capital, and better connectivity. On the positive side, the government has a strong commitment to improving the business environment and, especially in reducing poverty along with macro-fiscal policies, which gives a chance for an active response from the private sector.

4.1. Banking System of Armenia

The banking system has been growing during the past years, but still lacks essential efficiency and is beyond most of its peers. As of March 2017, the concentration of commercial banks operating in the Republic of Armenia accounts for 17 in total. Having 528 branches in Armenia and Nagorno Karabakh, out of which 237 are located in the capital city, Yerevan. The total amount of employees in the commercial banks reaches 11,175. As of March 2017, the total amount of outstanding loans grew by 17.8 percent, which leads to an increase in the baking income. The banking system had a total income of AMD 106 billion (198 million euros), which exceeded the income of the period 2016 by 10.3 percent (KPMG, 2018).

There are 33 credit organizations in Armenia, and the private sector credit to GDP reached about 38 percent in 2012, compared to Europe and Central Asia (ECA) median of 41.9 percent (Khachatryan and Avetisyan, 2017). Lending is mainly concentrated in the capital, Yerevan, as most of the adult population resides there. Yerevan accounts for 66 percent bank lending and 56 percent for credit organizations. According to WEF's Global Competitiveness Report, Armenia ranks 51st among 144 countries and, thus, is ahead among most of its peers. (The World Bank, 2017).

4.2. Microfinance Institutions in Armenia

Microfinance institutions were designed as a poverty alleviating tool by providing loans to low-income clients for self-employment. The main purpose of microfinance institutions is to serve the part of the population, which is considered "un-bankable". The notion of un-bankable or poor often implies to people who have limited collateral, no official credit history and are spread across a rural geography (Khachatryan and Avetisyan, 2017). Microfinance institutions proliferated in the region of Eastern Europe and Central Asia (ECA) during the 1990s. The World Bank, The German Development Agency (GTZ), and the United States Agency for International Development (USAID) were among the first to provide microfinance institutions (MFI) with capital supply.

MFI is a relatively young and developing sector in Armenia. Microloans in Armenia became popular right after independence in 1991. At first, they have been used as an instrument for poverty alleviation. From 2001 government started gradual institutional development in the field, and the last five years are characterized by the consolidation of small credit organizations into bigger ones. In 2015 microloans accounted for USD 871 million (or 8.2% of Armenian GDP). The number of active borrowers in the same year was 240,000 people (8% of the Armenian population), which makes this industry interesting for the analysis (Mix Market, 2015).

Starting from 1995, MFIs have been growing quite rapidly, in both terms such as volume and territorial coverage. The conducted research by Khachatryan and Avetisyan, in the scope of microfinance development in Armenia (2017), stated that, at that time, 33 credit organizations provide various microfinance products and services to the consumers. The authors also argued that the existence of so many players is not optimal for such a small economy, like Armenia, as it makes the market less efficient and supervision more challenging.

Nowadays, according to studies conducted by international organizations, there is a rising demand for microfinance services among the poor population and the start-ups in all regions of Armenia. Armenia has a significant rate of poverty among its population, with an estimated rate of 23.5% as of 2018. Even though this number has been decreasing over the years with a fall of 2.2% points compared to 2017, and of 4.1% points compared to 2008 (27.6%), there is still a significant part of the population living in destitution (Asian Development Bank, 2018). And for this reason, poverty alleviation stays among the top priorities for the Armenian government. Other existing challenges in the Armenian economic sector include; low per capita income, low level of financial intermediation, income inequality, and high unemployment rates. Financial institutions offer their support in order to tackle the development and provide assistance to the government of Armenia, including MFIs.

The financial capacity of the Armenian banking system still remains shallow. The main existing weaknesses include; high-interest rates (usually 18-19%), minor role of the capital market, insurance, and many other non-banking services which still have a place for improvement (Khachatryan and Avetisyan, 2017).

Other challenges include the reductions in taxes and customs in order to promote private growth and to encourage the companies to move to the formal sector.

4.3. Lending Practices

Microfinance sector in Armenia is distinctive when it regards to group lending and individual lending practices. In their research paper Khachatryan and Avetisyan (2017) showed that, at the beginning microfinance institutions were mainly concentrated in group lending in Armenia which did not show good results. As a result of conducted survey done by MFIs under the study showed that people seeking loans would create artificial groups as there was no possibility of individual loans. It was considered absolutely necessary to apply more customer-centered approach and shift from group lending to individual loans. Many microfinance institutions; such as Nor Horizon or ACBA, were giving group loans based on the following rationale: large groups were divided onto sub-groups and each sub-group had the responsibility to repay the loan. Both institutions at the end decided to end the group lending because it did not serve its real mission and was mainly abused by the citizens (Khachatryan and Avetisyan, 2017).

Nowadays, MFIs in Armenia primarily offer individual loans to entrepreneurs and small businesses engaged in agriculture, manufacturing, and services and group loans to self-employed entrepreneurs engaged in trade. MFIs were designed in the country for two main purposes, including a reduction in poverty and unemployment. As unemployment rates have been very high in the country, self-employment was the next best alternative solution. And as the bank interest rates have been quite high and required collateral, MFIs were designed as a more flexible tool in order to provide equal opportunities and help individuals in transition economies.

4.4. Problems of the Sector

One of the biggest problems of the microfinance sector in Armenia is the unfair fees. Compared to banks, which only need to make a lump-sum payment in order to open a branch, other companies have to make that payment annually (Khachatryan and Avetisyan, 2017). This factor makes it not only very troublesome for many microfinance institutions and companies to open their branches, but also small players don't see any regional opportunities for growth. One method that microfinance institutions use to avoid the fees is through branchless banking. Another problem is that companies can buy and sell foreign exchange currency only through banks. And as most of the companies consolidate investments in foreign currency, mainly US dollars, and give out loans in Armenian drams, these companies heavily depend on banks to exchange their currency.

MFIs also experience limit on the annual interest rates. The interest rate cap requirements should not exceed double the banking rate, which results in annual interest rate no higher than 24 percent which might not even cover the cost of lending. As a result, many MFIs create artificial fees that they charge to borrowers, such as monthly fee, front office fee, consulting and many others in order to ensure satisfactory revenue.

Another obstacle that MFIs face is the regulations. According to the Law on Credit Organizations, microlending is considered a commercial activity; meaning that its main purpose is to grant loans to individual entrepreneurs and legal entities. However, another principle of microfinance institution lies in giving loans to individuals whose business is unregistered. Currently, Central Bank of Armenia pressures the microfinance institutions through regulations which limits their economic development and leads to diverge from original social mission.

As a consequence, MFIs have limited abilities in regards to providing services to their clients which does not represent a sustainable business model for the country.

The competitive advantage in the microfinance sector should lie in the ability to create a long-term relationships and closer ties with the borrowers. The most successful practices are achieved where the reputational factor plays a significant role and results in fulfillment of the lending requirements (Khachatryan and Avetisyan, 2017).

5. Analysis of the Lending Practices at Firm X

5.1. Overall Description of the Firm X

In this thesis, as for the practical part, author analyzes Firm X, which is one among the 33 microfinance institutions in Armenia. Firm X started in Armenia in 2013 and till now expanded its operations in the capital and in other regions, in total having five branches that provide micro and small commercial loans. The vision of the company is to make loans accessible for everyone and gain a loyal client base for whom the company can be a right partner and maintain long-lasting relationships. The mission of the company is to provide a personalized approach to consumers by providing flexible and convenient services.

The firm X provides loans without collateral, loans with collateral, online loans, and loans secured by another security, meaning that another individual backed the obligation of the applicant. Consumers can apply for the loans both; in the company branches and online by phone or computer. The actual loans are provided through the branch network of the company. Loans are provided only in Armenian currency (AMD).

While providing loans, the company not only takes into account the previous financial performance of the clients but also considers facts, such as; the sector people work in, the marital status, number of family members and kids, working households within the family, and many others. For example, past statistics show that most trustworthy customers come from educational and medical sectors, married couples are more reliable than single ones, and if the couple is married, it's also taken into account the number of kids in the family.

Many employees in Armenia can work with unregistered salaries, and while applying for loans, the company always considers the factor of registered/unregistered salaries. Workers with unregistered salaries might get higher interest rates than those with registered ones. Also, the maturity of the loan will differ as well, taking into account if salaries earned are registered or not.

In general, the company uses the length of the loan as one of the main tools for risk mitigation; workers with unregistered salaries get higher interest rates and shorter time to close the loans. If, meanwhile, the client is acting responsibly and closing monthly payments on time, the interest rate can be reconsidered and lowered for future payments.

Firm X also provides loans online and is using a SaaS platform for fraud prevention in order to mitigate the risk for the company from consumers who apply online for the loans. Based on behavioral biometry and analysis of the data from online user sessions, the platform extends the risk engine and spots bad actors in real-time. In particular, the platform shows the time spent on each question while applying for online loans and based on metrics calculates if the client is trustworthy; if the client spends more than 7 seconds on a name and surname part, it can mean that the person is trying to apply for a loan under someone else's identity.

So, tools like these, past experience, and knowledge helps the firm to mitigate the risk and provide loans with fair conditions.

5.2. Descriptive Statistics of Firm's X Loan Portfolio

Firm X provided author with the data of all the past and present contracts. Data has been analyzed based on statistics of all contracts, statistics for non-active/active contracts, statistics by gender, statistics for overdue loans and statistics for active/non-active loans based on gender.

Following the below analysis, an official exchange rate of 527.98 AMD/Euro will be used as of 01.01.2020 provided by the Central Bank of Armenia.

Starting with statistics of all contracts if we have a look at *appendix 1* we can see that in total there were 17358 contracts, with an average amount of loan being slightly less than 350,000 AMD (663 Euro) and standard deviation being equal to approximately 620,000 AMD (1174 Euro); data shows that provided loans varied between the minimum amount of 10,000 AMD (19 Euro) and maximum amount of 18mln AMD (34,093 Euro). The data also indicates that 25% of the consumers borrowed less than 107,000 AMD (203 Euro), 50% less than 200,000 AMD (379 Euro), and 75% less than 350,000 AMD (663 Euro). If we look at the interest rate, the mean was 17.5%, reaching the maximum at 24%. And if we compare it with a real interest rate with a mean reaching almost 87% and 75% of consumers resulting in about 66% real interest.

Data also indicates that gender split is approximately equal, resulting in 56% male and 44% female applicants. The average age of the applicants is around 39 years, starting with the youngest applicants aged 17 and up till 77 years old. The average maturity of the loan is around 795 days with a standard deviation of 459 days, resulting in 25% of the applicants having maturity less than 547 days, 50% less than 731 days and 75% with less than 1095 days, the maximum maturity was reached at 3654 days.

If we direct our attention to the overdue figure, which is one of the most relevant for our study purposes, the company had 51% overdue loans, which accounts for more than half of the total loan portfolio and gives interesting aspects and area for study.

Out of all contracts, 21% are still active, and 79% are already closed.

We are shifting our attention to active contracts accounting for 21% of the total portfolio resulting in 3685 active contracts (*appendix 2*). The current average balance for those contracts is approximately 347,000 AMD (657 Euro), and a standard deviation of roughly 660,000 AMD (1250 Euro). The majority of the active applicants (75%) have less than 340,000 AMD (644 Euro). For all the active contracts the average value of the collateral is approaching 636,000 AMD (1204 Euro) with a standard deviation of roughly 3.5mln AMD (6629 Euro) with the spread of 0 (loans without any collateral) and 61mln AMD (115,535 Euro) between the minimum and maximum values.

Data also stipulates an average contract amount of 442,000 AMD (837 Euro), varying between the minimum amount of 20,000 AMD (38 Euro) and reaching the peak at 9mln AMD (17,046 Euro). The average interest rate accounts for 16%, reaching its maximum at 24%. Consequently, the real interest rate results in about 65%, with 75% of applicants paying less than 64%, 50% less than 55%, and 25% less than 42%. As for the gender, approximately 57% of the active applicants are male and the rest female. Average age accounts for around 40 years, with a minimum age of the applicant starting from 19 years old and reaching maximum at age 77. It's interesting to point out that compared to non-active contracts, the percentage of overdue active contracts increased, reaching 58% overdue compared to the previous figures of overdue in closed contracts (49%, *appendix 3*).

Shifting our focus onto statistics based on gender, for both active and non-active borrowers, we can point out some interesting insights. The data indicates that the number of male borrowers is higher than the number of female ones, and correspondingly the number of the contract amount is bigger for males than for the females (*appendix 4*). The interest rates for both genders are namely the same, and the average age of males is around 37 years, meanwhile for females about 40 years.

The overdue amounts are also roughly the same, approximately 50% overdue in both portfolios. Now, we will analyze the statistics separately for active/non-active loans based on gender type. Statistics for non-active loans based on gender is presented in *appendix 5*. Among non-active contracts, there were 7547 male and 6126 female contracts with an average amount of 354,500 AMD (671 Euro) and 282,000 AMD (534 Euro), respectively. The interest rate for both genders was mainly the same equaling 18%, which resulted in a real interest rate of 90% for males and, jointly, 95.72% for females. The average age of male borrowers was 37 years and for females 41 years. On average the overdue figure accounted for 48% among all male non-active contracts, and 50% among female non-active contracts. Most of the loans were secured for both genders, resulting in 76% secured loans for males and 74% secured loans for females.

Figure 1 presents the same idea within the pie charts for active/not active contracts. Among non-active loans there was almost a fair distribution of overdue figure, but in active loans it can be noticed that the proportion of overdues increased for both; males and females, resulting in 59.6% and 55.7% respectively.

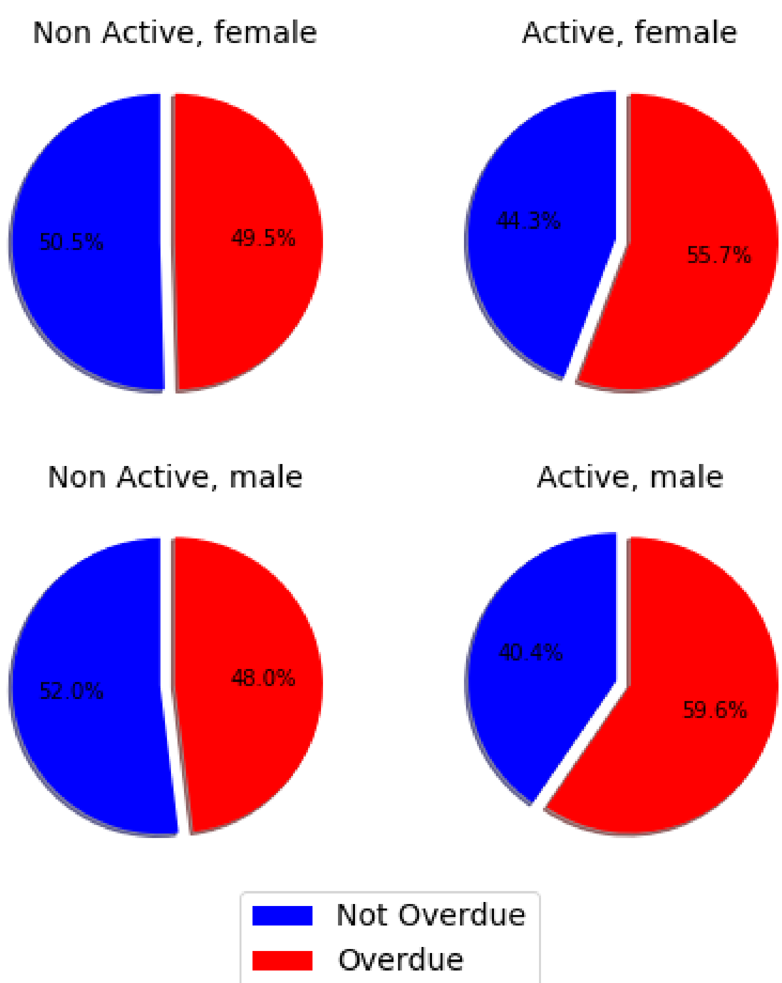


Figure 1: Proportion of overdue for active/non-active loans based on gender; source: author

Active contracts based on gender, shown in *appendix 6*, present the same information, there are 2092 active male and 1593 active female contracts. The average amount of loan is around 387,000 AMD (733 Euro) for males and 295,000 AMD (559 Euro) for females with an interest rate of 16.6% and 15.6%, respectively. The real interest rate is 66.5% for males and 63% for females. Average age of borrowers is around 39 years for males and 42 years for females. For both genders 88% of loans were secured. Overdue figures compared to past loans is increased, reaching almost 60% overdues in male present contracts and over 55% for female contracts, *figure 1* presents the same information in form of charts, as well.

Bringing our attention to the overall statistics for overdue loans between males and females, as presented in *appendix 7*, we could point out that the contract amount is more or less the same with an average amount of 330,000 AMD (625 Euro) for males and the average of 360,000 AMD (682 Euro) for females. The interest rate slightly varies with a rate of 16.7% for males and around 18% for females, and on the contrary, the real interest rate is 93% for males and 80% for females. The mean age of the borrowers for overdue loans accounts for 40 years for males and 38 years for females. The overdue loans for both parties are secured by another security, with respectively 78% for males and 78% for females. In the overall portfolio, overdue male loans account for 18%, and females' overdue loans account for 24%, resulting in 8568 overdue contracts with respect to males and 8790 overdue contracts with respect to females.

As can be concluded from all the data, overdue loans account for a big part of the portfolio and definitely are the area that needs to be studied and improved.

5.3. Statistical Analysis of the Factors Influencing the Quality of Loan Portfolio

Furthermore, in the analysis of the firm's loan portfolio, it was essential to find out whether the variables are independent of one another, or they have some dependence. For that purpose, the data has been organized onto five contingency tables, which represent the distribution of borrowers between two different categories. In general, contingency tables show the observed frequencies for two variables, arranged by rows and columns. Assuming that the two variables of study in the contingency tables are independent, we can use the contingency table to find the expected frequency for each cell. The contingency tables have been constructed in the following way: by active/non-active loans and overdue/not overdue, by gender-active loans and overdue/not overdue, by gender-non-active loans and overdue/not overdue, by secured-active loans and overdue/not overdue and finally, by secured-non-active loans and overdue/not overdue.

If we put our focus on *table 1*, we can see that there are two rows; non-active and active, and two columns; not overdue and overdue. Then we can easily see that there were 7016 people, from non-active loans, who did not get overdue their loans in the past and 6657 people, from non-active loans, who overdue their loans in the past. From active loans, we have 1552 people without overdue and 2133 people overdue.

Active	not overdue	overdue	All
non-active	7016	6657	13673
active	1552	2133	3685
All	8568	8790	17358

Table 1: Distribution of loans by variables "Active" and "Overdue"; source: author

With the same logic, we can analyze the remaining contingency tables. *Table 2* indicates what proportion of active males and females have overdue. We can see that out of 1593 female contracts, 706 was not overdue, and 887 was overdue. And as for active 2092 male contracts, 1246 was overdue and 846 not overdue. The overall data indicates that out of all 3685 active contracts; there was a considerable proportion of overdue; resulting in 2133 overdue and not overdue accounting for 1552.

Gender	not overdue	overdue	All
female	706	887	1593
male	846	1246	2092
All	1552	2133	3685

Table 2: Distribution of loans by variables "Active by Gender" and "Overdue"; source: author

In *table 3*, figures indicate non-active loans by gender out of which there were 6126 female and 7547 male contracts. As for females, there was almost an equal proportion of overdue (3035) and not overdue (3091) in past loans. A similar analysis goes for males resulting in 3925 not overdue and 3622 overdue past contracts.

Gender	not overdue	overdue	All
female	3091	3035	6126
male	3925	3622	7547
All	7016	6657	13673

Table 3: Distribution of loans by variables "Non-active by Gender" and "Overdue"; source: author

Moving on with *table 4*, we can see the figures for active secured loans and the proportions of overdue and not overdue. In total, there are 425, not secured-active loans and 3260 secured and active loans. Out of those, 145 loans are neither secured nor overdue, and 280 are not secured but overdue. If we have a look into secured loans, we can see that out of those, 1407 were not overdue and 1853 were overdue.

Secured	not overdue	overdue	All
not secured	145	280	425
secured	1407	1853	3260
All	1552	2133	3685

Table 4: Distribution of loans by variables "Active Secured" and "Overdue"; source: author

Continuing with the last table, which is *table 5*, figures indicate non-active secured loans and overdue/not overdue. In total, there were 3389 non-active not secured loans and 10284 non-active secured loans. Out of those two, 1765 past contracts were not secured and not overdue, and 1624 contracts not secured, but overdue. Out of 10284 past secured contracts, 5251 were not overdue, and 5033 contracts were overdue.

Secured	not overdue	overdue	All
not secured	1765	1624	3389
secured	5251	5033	10284
All	7016	6657	13673

Table 5: Distribution of loans by variables "Non-active Secured" and "Overdue"; source: author

The next question is whether there is a significant difference in the share of overdue loans between past loans and current loans. The method used for analyzing this is called the "Chi-square test." A chi-square independence test is used to test the independence of two variables. Using a chi-square test, we can determine whether the occurrence of one variable affects the probability of the occurrence of the other variable (Larson and Farber, 2012).

The test is applied when we have two categorical variables in the single population and used to determine if there is a significant association among two variables. For example, loans can be classified by gender (male or female) and being overdue or not. We could use a chi-square test for independence to test whether gender is related to loan becoming overdue. We use chi-square test when the variables under study are categorical and when the data is displayed in the contingency tables as already performed above. Next step just requires from us to state the null hypothesis which demonstrates that knowing one variable does not help to know the other variable, otherwise said, two variables are independent of one another. And consecutively, the alternative hypothesis shows that variables are dependent. Alternative hypothesis helps to confirm that the knowing the level of one variable can help to predict the level of the other variable.

In *table 6*, we can see the "P-values" and "Chi-squared statistics" for each of the above five contingency tables. The first row indicates results for the Chi-squared test, and the second row indicates the P-values. The use of p-values in statistical hypothesis testing is widespread in many fields of research such as physics, finance, economics and so forth. And p-value is also used in the context of null hypothesis testing in order to assess the idea of statistical significance of evidence.

	Active and overdue	Gender and overdue (active only)	Gender and overdue (nonactive only)	Secured and overdue (active only)	Secured and overdue (nonactive only)
χ^2	98.21	5.58	3.25	12.61	1.06
p-value	< 0.001	0.2326	0.5164	0.0134	0.9002

Table 6: P-values and Chi-square Test for Independence; source: author

The rejection of null hypothesis typically involves comparing P-values which demonstrates the probabilities of two variables are independent of one another. For example, the first column represents the idea whether loan getting overdue is independent of active loans. So, the first column is presenting the variables active and overdue with the probability (p-value) is $2.4 * 10^{-20}$ which is a minuscule probability and means that active loans are more likely to be overdue than the past loans.

Figure 2 demonstrates the same idea by visualization charts. Pie charts present the proportion of overdue/not overdue among present and past contracts. It can be easily observed that the proportion of overdue loans has increased for active contracts (57.9%) by comparison to past contracts, where overdue loans accounted for 48.7% of the overall portfolio.

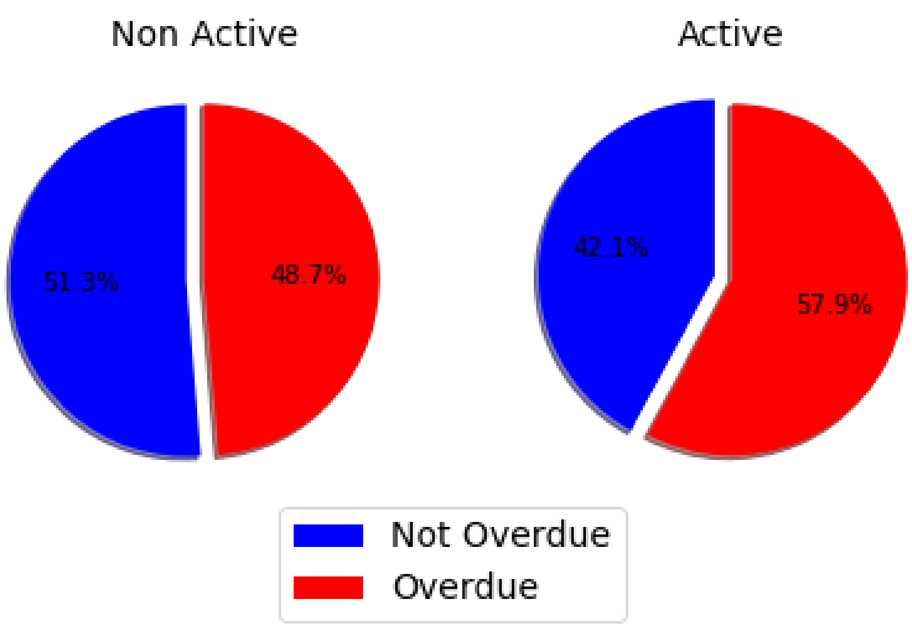


Figure 2: Distribution of overdue/not overdue loans for active/non-active contracts; source: author

Further on, if we put focus back on *table 6*, we can see that four more tests were conducted in order to find out which variable influences the likelihood of the loan being overdue. Variables studied included, starting from the second and third columns, showing active and non-active contracts by gender and overdue figure, and continuing with fourth and fifth columns representing the active and non-active secured contracts and relationship with overdue.

We can see that P-values by gender (active/non-active) are large enough, meaning that gender does not influence the likelihood of the loan being overdue, and leads to the conclusion that variables are not associated. But if we have a look at the P-value for active secured loans, in fourth column, we can examine a very small probability of 1.34%, meaning that loans that are secured are more likely to be overdue. In this scenario, we have the problem of moral hazard, meaning that once the loan is secured, borrowers do not act responsibly as they know they do not carry the whole responsibility for the loan. The same conclusion can be extracted from the below pie charts which help to visualize the data.

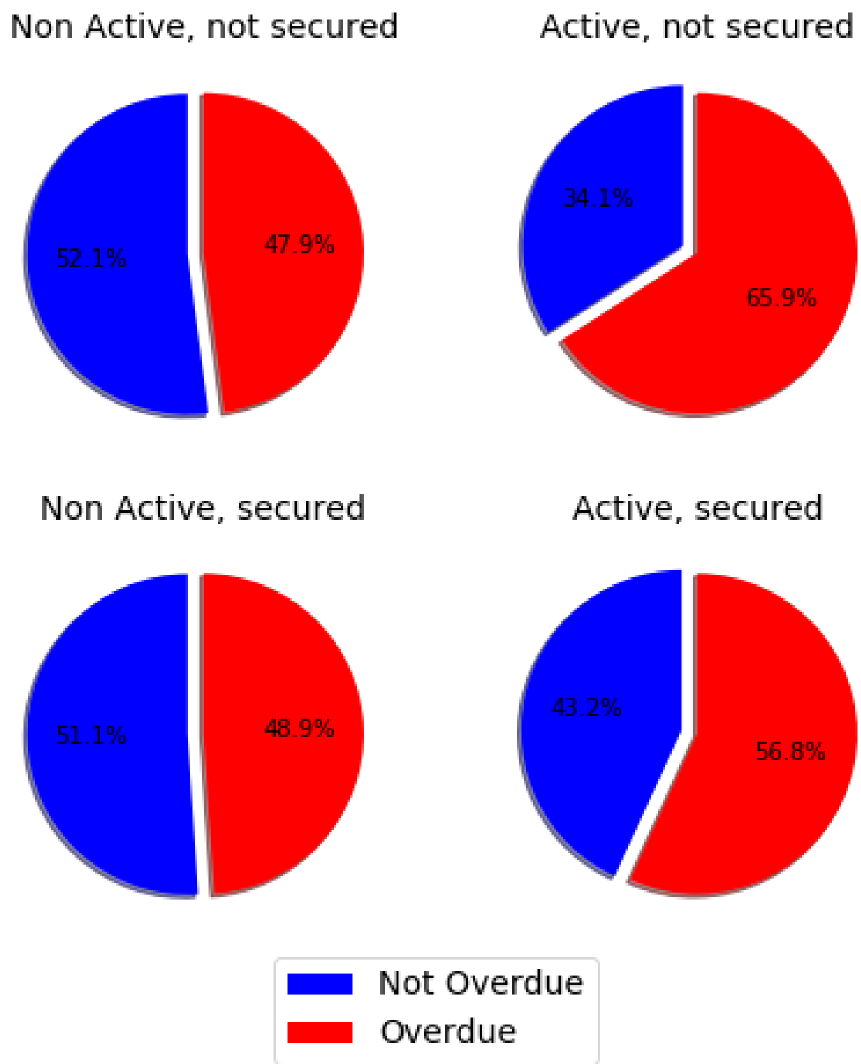


Figure 3: Distribution of overdue/not overdue loans among active/non-active and secured/not secured contracts; source: author

Pie charts, shown in *figure 3*, represent the idea of the loan being overdue or not overdue among active/non-active contracts and secured/not secured contracts. By showing the distribution of overdue, it is easily visible that for active contracts, the loans that were secured resulted in a respectively higher number of overdue (56.8%) compared to past loans where overdue among secured contracts accounted for 48.9%. In general, we see that the overdue figures drastically grew for present contracts compared to the past ones which gives an exciting area to study and improve.

Further on to have a better analysis of the other factors affecting the overdue loans, histograms have been constructed by the author in order to show the distribution of overdue figures across the age groups and based on interest rates. The first three histograms focus on the distribution of overdue figures across the age groups; x-axis indicating the age of the borrowers and y-axis showing the number of the loans. *Figure 4* shows the distribution of overdue/not overdue for all the loans in the portfolio. The data for all the loans does not give much insight as we do not see any concentration of overdue figures based on the age in the histogram but more of an evenly distribution.

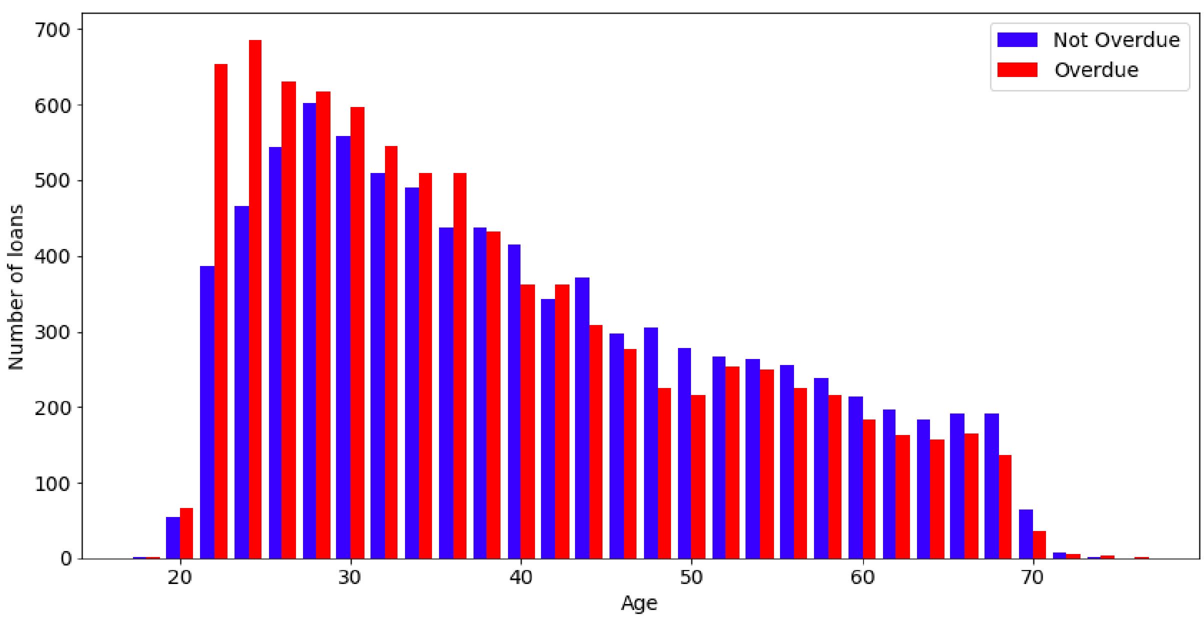


Figure 4: Age distribution among all the loans; source: author

Figure 5 displays the circulation across the age groups between overdue/not overdue for past loans. The data for past loans also does not provide an interesting observation to study the overdue figures because, again, there is no concentration of overdue among the age groups as it is mainly present among all the age groups.

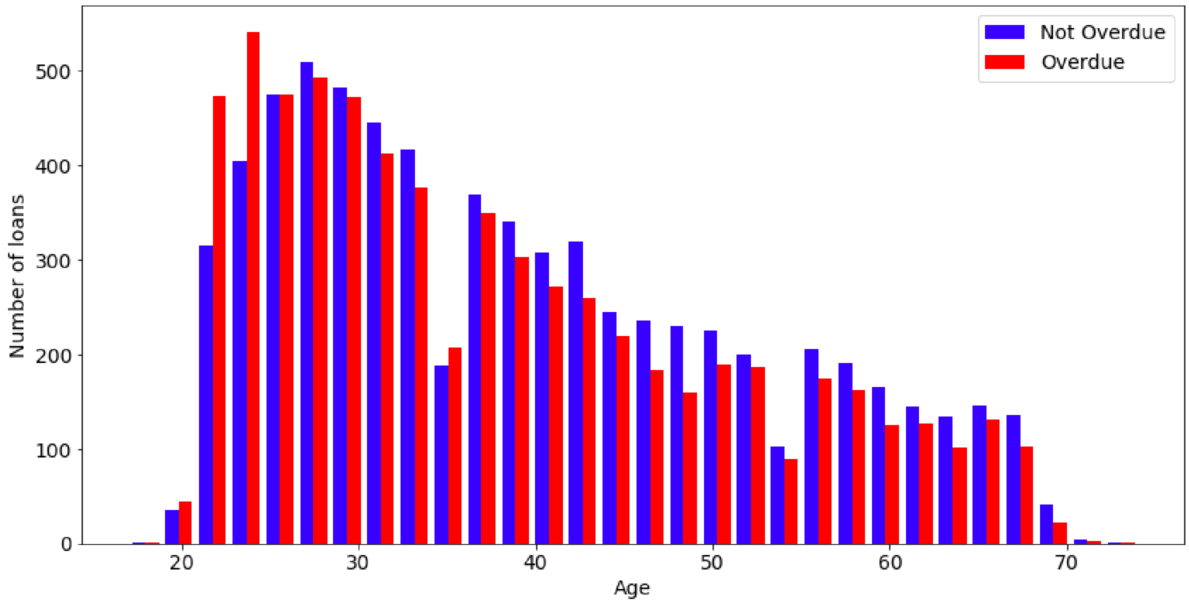


Figure 5: Age distribution among non-active loans; source: author

Finally, *figure 6* presents us with more interesting insights by showing the allocation of overdue/not overdue based on the age groups for all active loans. As we saw from the previous two histograms, no compelling correlation was observed between the age groups and the overdue figure. But *figure 6*, gives us the opportunity to discover something new by showing that overdue loans are mainly concentrated among 20-40 age groups. This is an additional factor that can be taken into consideration when analyzing the borrower.

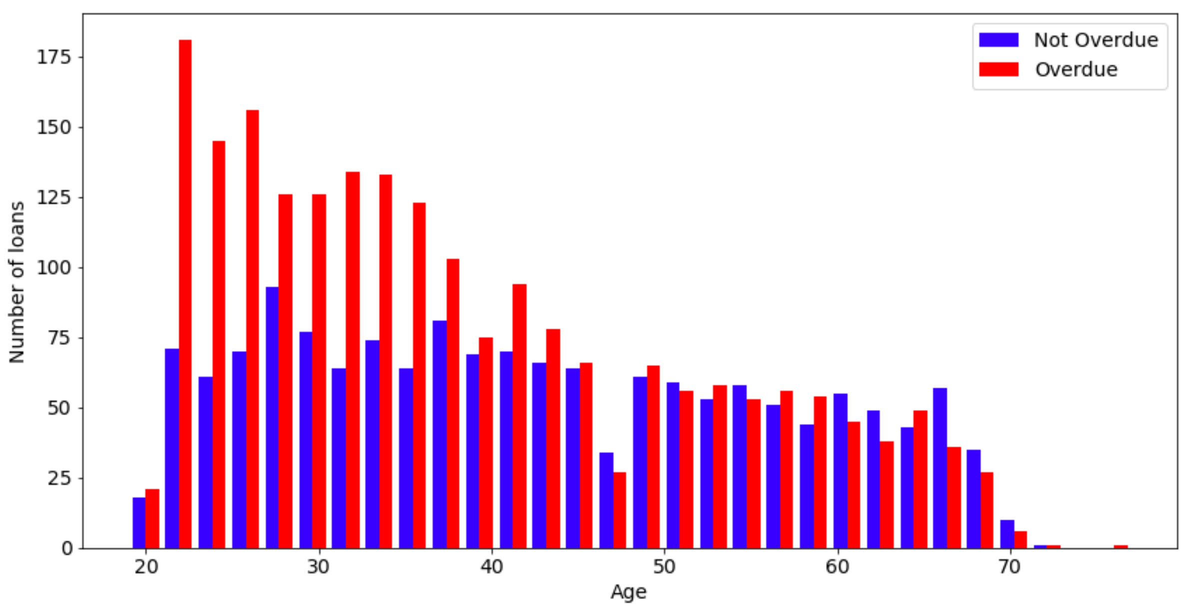


Figure 6: Age distribution among active loans; source: author

In general, microcredit initiatives are of great advantage to those who are not able to back it with collateral. However, MFIs that require neither joint liability nor collateral tend to have unusually high-interest rates, which seems unfair towards those applying for it. On the one hand, the micro-finance programs tend to be more focused on social incentives; such as loss of reputation, risk of failure, but on the other hand, they neglect more essential factors such as; interpersonal trust towards their applicants.

For this exact reason, furthermore, two other histograms have been constructed, but this time showing the relation between the interest rate and overdue figure. *Figure 7* presents the statistics for non-active contracts which shows that with increasing interest rate, overdue figures increase, as well.

In the scope of discussion with Firm X, it was also confirmed that the firm charges higher interest rates for short-term lending which can be one of the reasons leading towards the loan becoming overdue.

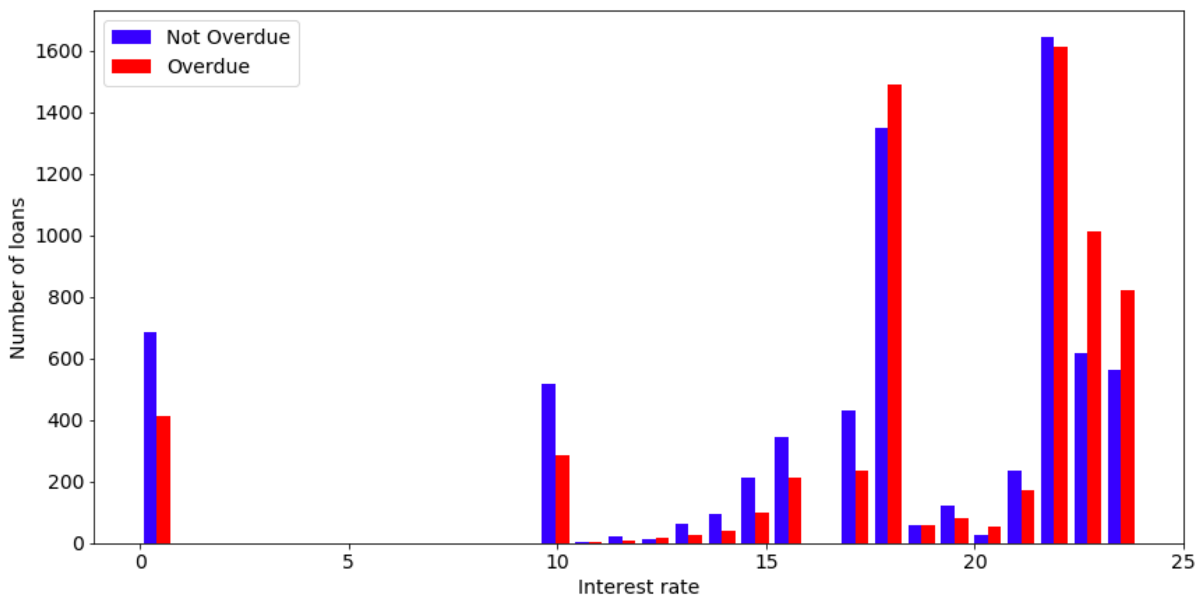


Figure 7: Overdue figures based on interest rate for non-active loans; source: author

If we look at the history of microfinance movements starting from the late 1970s, we can state that high microloan interest rates were criticized widely even back then. Nevertheless, during the past years, the criticism strengthened even more, leading to many countries to consider interest rate ceiling. One of the reasons for high-interest rates can also be all the public attention MFIs were receiving during past years, which increased the scope of the industry and raised high rates on the clientele.

As it was explained from Firm X's perspective, the main reason for high microfinance interest rates is due to the fact that modest rates of return achieved in most small-scale businesses are insufficient to cover the debt service at such rates. Of course, a better way is needed to design development projects for microfinance institutions, which will eventually result in lower interest rate spreads.

In similar way, *figure 8* was constructed to show the same connection between overdue and interest rate but for active contracts. We can notice that for present contracts the overdue figure increased even more with an increasing interest rate compared to past contracts.

Even though, we can neither claim nor prove that high interest rate causes the loans to be overdue but we can indeed argue that overdue figures and high interest rates are interconnected.

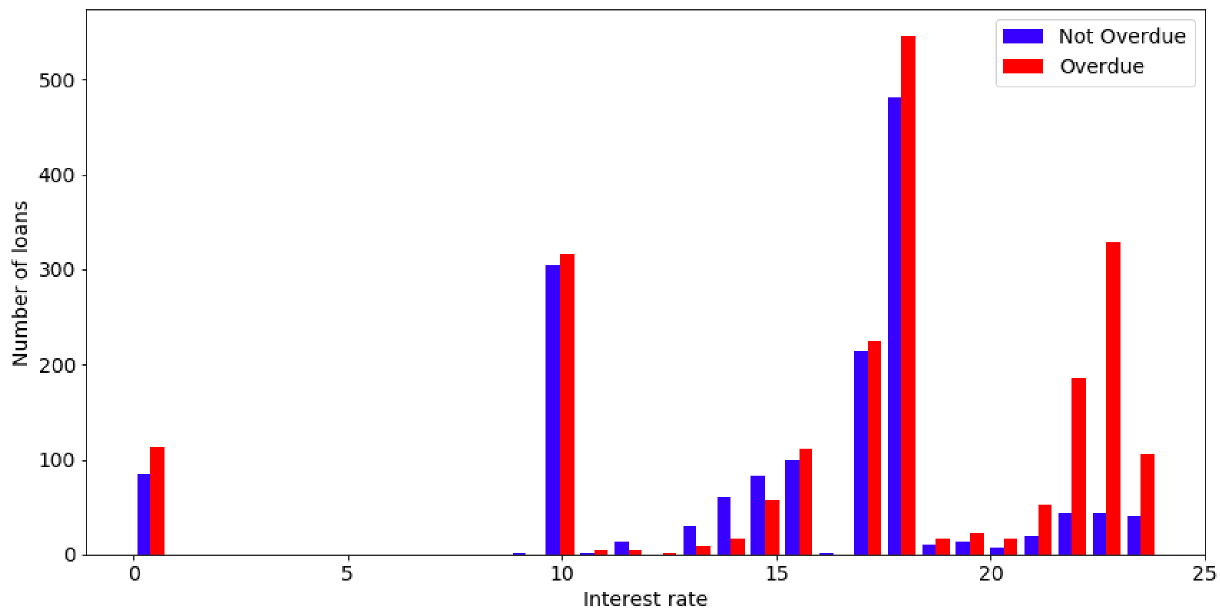


Figure 8: Overdue figures based on interest rate for active loans; source: author

The process of interpreting the data is very crucial as it tells whether or not the data is useful. T-tests can help to determine if the difference between the expected values and given set of values is significant. In other words, the independent t-test or Student's t-test is an inferential statistical test that determines whether there is statistically significant difference between the means in two unrelated groups. Student was the pseudonym first time used by W.S. Gosset in 1908 in his published paper about the t-distribution based on his empirical findings on the length and height of the left middle finger of the criminals in the local prison. T-test compares the mean of one sample against the mean of another sample (Chapman and Feit, 2014). And the essential point is that it compares the mean for precisely two sets of data, in our case as presented in *table 7,* it compares the means among pairs such as active/non-active, active female vs active male and so on. In the table below we also have the p-values for each pair; t-values and p-values are inextricably connected. In this thesis, t-test was performed in order to understand if there is a significant difference between population means. The greater the value of T, the higher chances exist that there is significant difference. The bigger the absolute value of t-value, the smaller will be the p-value and more abundant evidence against the null hypothesis, showing that there is statistically significant difference.

	Active vs Non -active	Active Female vs Male	Non-Active Female vs Male	Active Not Secured vs Secured	Non-Active Not Secured vs Secured
Mean 1	0.58	0.56	0.50	0.66	0.48
Mean 2	0.49	0.60	0.48	0.57	0.49
p-value	2.2E-23	0.0183	0.0713	0.0003	0.3026
t-value	10.01	-2.36	1.80	3.67	-1.03

Table 7: Independent samples T-Test; source: author

In *table 7*, when we look at p-values we can see two very low figures. In the first column, when we compare the means between active vs non-active contracts, we can see that probability (p-value) is almost zero which is very small probability and also, consecutively higher t-value (10.01), proving that there is significant difference between share of overdue loans in the active and non-active portfolios. The second smallest p-value occurring in the table is the probability of 0.03% showing that there is significant difference between the shares of overdue figures in active not secured and active secured contracts.

The test confirms that all the above constructed beliefs made about the borrower are proven to be accurate, showing that active loans and active secured loans are more likely to be overdue than compared to non-active and active not-secured loans.

5.4. Game Design

The strategic interactions between lender and borrower have been studied by Chadha (2015, p 214) in her book about *Game theory for managers* where the efficient outcome is the Nash equilibrium of the game. The author presents a simple game between two parties, where the joint payoffs are highest when the outcome is {Lend, Effort}, meaning when the lender lends, the borrower should put effort into the project and repay it on time. Chadha suggests that if the individual incentives of the contracting parties are aligned with the joint incentives, then it is possible to act according to the terms of the contract where the desired outcome is a Nash equilibrium of the game. Author concludes that for such contract to work, the payoff from "lend" should be higher than the payoff from "not lend" for the lender and the payoffs from "effort" should be higher than that from "no effort" so that the borrower does not have an incentive to deviate (Chadha, 2015, p 215).

Further on, Basilgan and Christiansen (2014, p 282) presented a lender-borrower game by considering approach of Prisoners' Dilemma in the banking sector. They followed the symmetric

2x2 Prisoners' Dilemma with ordinal payoffs and constructed a game between a lender and a borrower with two possible cases of cooperation. The borrower is regularly paying and the lender is happy about that or the borrower defaults and some cooperation should be established. Payoff matrix shows the ordinal payoffs as the exact amounts are not known; therefore, they can only be compared with respect to being more or less. Authors conclude that *"defaulting is strictly dominating strategy when the present value of the collateral mortgage is less than the present value of the exposure at default (i.e. outstanding loan)"* and when the present value of the mortgage is more than the present value of the exposure at default, then cooperation becomes strictly dominating strategy. And if the present value of the collateral mortgage is equal to the present value of the exposure at default then defecting may be weak strategy as a dominating strategy (Basilgan and Christiansen, 2014, p 284).

5.5. Description of the Simultaneous-Move Game

Similarly, the models were adopted based on the conducted statistical analysis within the scope of this thesis, and an optimal game model between the lender and the borrower was designed.

We start the analysis of Lender-Borrower relations with the simple simultaneous-move game. There are two players: Lender and Borrower. Lender chooses to offer loan ("Loan") or not to offer loan ("No Loan"). Borrower chooses to cooperate and repay a loan fully ("Cooperate") or to default and go away with the full amount of loan ("Default").

We assume that borrower receives a utility from consumption (U_c) and this utility is larger than the amount of a loan plus interest ($L \times (1 + i)$), thus the borrower is rational and utility-maximizer. In order to simplify analysis, we also assume that if the borrower decides to "Default" then the lender only loses the amount of the loan ($-L$) and interest rate is ignored, presuming that interest rate does not make much difference for the lender. The payoff matrix for this simple game looks as presented in *table 8*.

Borrower / **Lender**	Cooperate	Default
Loan	$L \times i,\quad U_c - L \times (1 + i)$	$-L,\quad L$
No Loan	0, 0	0, 0

Table 8: Payoff-matrix for the simultaneous "Lender-Borrower" game; source: author

If $U_c - L(1 + i) < L$,

$$\text{Then, } U_c < L + L(1 + i)$$

And usually, $U_c < L(2 + i)$ is the most common case, indicating that the strategy "Default" is weakly dominant for the borrower, and the only Nash equilibrium is (No loan, Default).

This simple game provides an evidence that the borrower has strong incentive to default a loan.

In reality, if a borrower defaults, this may worsen his/her reputation and lead to further impossibility of obtaining new loans. Knowing this, we may extend this game into the repeated-form game.

Suppose for simplicity that this game can be repeated infinitely and the payoff from strategy profile (Loan, Cooperate) that the borrower obtains is a present value of infinite stream of payments:

$$PV = \frac{U_c - L \times (1 + i)}{r}$$

Where r is a impatience rate for the borrower.

Impatience was already defined by Koopmans (1960, p 287) as the decrease in the aggregate utility with respect to time. Some other authors, like Cheng and González-Vallejo (2014) use the term impulsivity instead of impatience. Effectively, Takahashi, Oono and Radford (2007) define impulsivity in intertemporal choice as a "strong preference for small immediate rewards over large delayed ones". The research article by Rambaud and Torrecillas (2016) focuses on the measurement of the impatience associated with a discount function and concludes that the impatience at a given moment is the corresponding instantaneous discount rate.

Continuing on with the game, we see that if the borrower defaults, the game stops and borrower leaves with the one-shot payoff L. Thus, the incentive to default for the rational borrower is possible only, when

$$\frac{U_c - L \times (1 + i)}{r} < L$$

$$U_c - L \times (1 + i) < Lr$$

$$U_c < Lr + Li + L$$

$$\frac{U_c}{L} - 1 < r + i$$

But we know that rational borrower receives a loan only if $U_c > L$ which means that $\frac{U_c}{L} > 1$ and in infinitely repeated game the borrower defaults only if $\frac{U_c}{L} - 1 < r + i$, which means that either interest rate or the impatience rate of the borrower is too large.

Suppose that U_c equals the price of a good which borrower wants to buy with the help of loan. The inequality means that the lender should never finance 100% of this price because in this case the borrower always has an incentive to default. Rather the maximum amount of the loan must be equal to:

$$L < \frac{U_c}{1 + r + i}$$

Thus, the larger is the interest rate (i) or impatience of the borrower (r), the larger must be the difference between an amount of loan and price of a good that is funded by this loan.

And if the interest rate is observable, the borrower's impatience rate is not. To some extent we may attribute this variable to the age of borrower, assuming that younger people are less patient than the adults, which goes in line with the analysis of our loan portfolio, where younger borrowers, aged 20-40, tend to default more often than older ones.

Moreover, the research article written by the professors of Harvard and Michigan universities Chabris, Laibson and Schuldt (2016, p 6) about the intertemporal choice, suggests that most choices require decision-makers to trade-off cost and benefits at different points in time. And the decisions with consequences in multiple time periods are referred to as intertemporal choices. Authors mention that the theory of discounted utility is the most widely used framework for analyzing intertemporal choices, and provide an empirical study which links the estimated discount rates for monetary rewards to various individual behaviors and traits. The most interesting variable for our purpose of study is the variable age which appears to increase across the lifespan, with the young showing markedly less patience than middle-aged and older adults. The study suggests that the discount rate findings in children > young adults > older adults which corresponds to our analysis of increasing impatience rate being observed among the borrowers of younger age.

Of course, lender may influence the outcome with the help of on-going supervision over the borrower. This may lead us to the other form of game between these two parties as discussed below.

5.6. Description of the Supervision Game

Game theory was presented as a good starting point in the study of interactions and strategic decision making in Smojver's (2016) paper while examining the relationship between banks and banking supervisors. The relationship is evaluated from game theoretic perspective and agent-based model of banks and supervisors is created. The paper presents an inspection game, a special class of noncooperative games, in which one side, known as inspector verifies whether the other side (inspectee) abides by certain rules. The basic structure of the inspection game shows the relationships between payoffs for the bank and for the supervisor. Based on payoffs, the bank and the supervisor will always have a reason to change their strategies, therefore, equilibrium strategies are mixed strategies. The payoff matrix is presented in *figure 9* with probability of bank violating the rules marked as (p), and probability of supervisor performing supervision as (q).

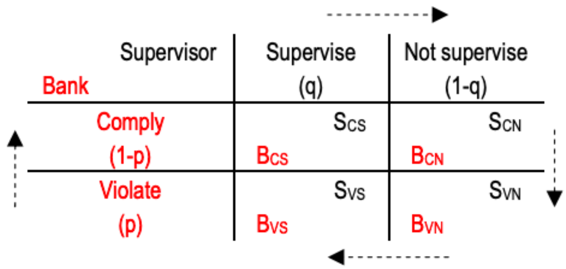


Figure 9: Payoff matrix; source: Smojver, 2016

Author incorporates also Tsebelis's (1990) idea who has shown that the optimal mix of pure strategies which led to some surprising conclusions, such as increasing or reducing penalty for observed violations of rules will not influence the frequency of violations in equilibrium. And this is an important finding as it is usually intuitively assumed that higher penalty will act as a deterrent and will decrease the amount of violations. Tsebelis also concluded that increasing penalty leads to lowering of probability of supervision (Smojver, 2016).

Similar idea of inspection game was discussed in the article by Avenhaus (2004) suggesting that in order to design an optimal inspection scheme, one should assume that illegal action is executed strategically. The paper deals with random control of passengers using public transportation system and gives concrete advice what effort the inspector should spend in order to achieve his objectives through game theoretic approach. In the conducted game, on one side is the inspector with the frequency of controls being his strategic variable, and on the other side is the passenger choosing between paying (legal behavior) and not paying the ticket (illegal behavior). Then the payoffs for two players are considered in four possible outcomes and represented in two-person game. From the payoff matrix Avenhaus concludes that players have an incentive to cyclically deviate from chosen strategy, thus there is no equilibrium pure strategy. Therefore, the inspector will control with probability (p), and the passenger will behave legally with probability (q). As a conclusion, game-theoretic model gives an advice how frequently passengers using public transportation system should be checked (Avenhaus, 2004).

Now, we can incorporate above ideas in our game theoretic model by assuming, that the lender decides whether to supervise or not. Supervision costs c_S. The borrower may comply or violate the loan agreement. If the borrower violates and the lender finds that, the borrower pays a fee (Fee). The payoff matrix looks as follows, presented in *table 9*.

Lender \ Borrower	Comply (1-p)	Violate (p)
Supervise (q)	$L \times i - c_S, \quad U_c - L(1+i)$	$L \times i - c_S\,, \quad U_c - L(1+i) - Fee$
Not Supervise (1-q)	$L \times i, \quad U_c - L(1+i)$	$-L, \quad L$

Table 9: Payoff-matrix for the "Supervision game"; source: author

This game does not have a pure-strategy Nash equilibrium, thus, both players will be mixing and playing their strategies with some probabilities. Denote the probability of a supervision as q, and the probability of a violation as p.

In order to find the probabilities, we equalize expected payoffs of playing both strategies. First, we do this for Lender:

$$EP_L(Supervise) = EP_L(Not\ Supervise)$$

$$L \times i - c_S = L \times i \times (1-p) - L \times p$$

$$-c_S = -L \times i \times p - L \times p$$

$$p = \frac{c_S}{L(1+i)}$$

Next, we equalize expected payoffs of Borrower:

$$EP_B(Comply) = EP_B(Violate)$$

$$U_c - L(1+i) = (U_c - L(1+i) - Fee)q + L(1-q)$$

$$U_c - L(1+i) - L = (U_c - L(1+i) - Fee - L)q$$

$$q = \frac{U_c - L(1+i) - L}{U_c - L(1+i) - L - Fee}$$

$$q = \frac{2 + i - \frac{U_c}{L}}{2 + i + \frac{Fee}{L} - \frac{U_c}{L}}$$

Since $0 < q < 1$ in a mixed-strategy, we must put a restriction on the term $\frac{U_c}{L}$. To better understand the possible restrictions on this term, we build the following *table 10*:

	Numerator	Denominator	q
$1 < \frac{U_c}{L} < 2 + i$	+	+	< 1
$2 + i < \frac{U_c}{L} < 2 + i + \frac{Fee}{L}$	–	+	< 0
$\frac{U_c}{L} > 2 + i + \frac{Fee}{L}$	–	–	> 1

Table 10: The relationship between the probability of supervision (q) and the term $\frac{U_c}{L}$; source: author

Thus, the only possible interval consistent with the mixed-strategy is $1 < \frac{U_c}{L} < 2 + i$. As we assumed that the rational borrower would take a loan only if $U_c > L$, we can drop the left inequality.

The mixed-strategy Nash equilibrium of this game is such that Lender decides to supervise with probability $q = \frac{2+i-\frac{U_c}{L}}{2+i+\frac{Fee}{L}-\frac{U_c}{L}}$ (given that $\frac{U_c}{L} < 2 + i$) and Borrower decides to violate with probability $p = \frac{c_S}{L(1+i)}$.

The analysis of equilibrium gives us two important insights. First, the probability of violation does not depend of the level of Fee, but it depends on the relative size of supervision cost to Lender. Thus, in case if Lender has larger cost of supervision, the probability of violation may be decreased with the decrease of a loan amount. This insight is vital for the loans without collateral, or for the loans given to people without a constant job income.

Further, the probability of supervision decreases with the ratio $\frac{Fee}{L}$. This makes sense, since the larger the relative amount of fee is, the lower is an incentive of Borrower to violate.

Finally, we need to know how the probability of supervision depends on the interest rate (i) and the term $\frac{U_c}{L}$. We take partial derivatives and obtain:

$$\frac{\partial q}{\partial i} = \frac{\left(2 + i + \frac{Fee}{L} - \frac{U_c}{L}\right) - (2 + i - \frac{U_c}{L})}{\left(2 + i + \frac{Fee}{L} - \frac{U_c}{L}\right)^2} = \frac{\frac{Fee}{L}}{\left(2 + i + \frac{Fee}{L} - \frac{U_c}{L}\right)^2} > 0$$

$$\frac{\partial q}{\partial\left(\frac{U_c}{L}\right)} = \frac{-\left(2+i+\frac{Fee}{L}-\frac{U_c}{L}\right)+(2+i-\frac{U_c}{L})}{\left(2+i+\frac{Fee}{L}-\frac{U_c}{L}\right)^2} = \frac{-\frac{Fee}{L}}{\left(2+i+\frac{Fee}{L}-\frac{U_c}{L}\right)^2} < 0$$

Thus, probability of supervision increases with (i) and decreases with $\frac{U_c}{L}$. The intuition suggests that higher interest rate means riskier borrower, who needs to be controlled more often. And higher ratio of $\frac{U_c}{L}$ means that the borrower contributes more to the purchase, assuming that U_c is the utility consumer receives from obtaining the loan for specific purpose, thus the less control over him is necessary. So, we can say that the probability of supervision (q) depends on three terms: by being negatively correlated with $\left(\frac{U_c}{L}\right)$ and $(\frac{Fee}{L})$ and positively correlated with (i).

We can combine the restriction on $\frac{U_c}{L} < 2+i$ with the restriction that $L < \frac{U_c}{1+r+i}$ from the previous repeated game to obtain:

$$\frac{U_c}{2+i} < L < \frac{U_c}{1+r+i}$$

If the amount of loan falls below the left boundary, then Lender never supervises and Borrower always prefers to violate the loan agreement. If the loan exceeds the right boundary, Borrower prefers to announce a Default and stop the repeating game.

By taking above restriction, Firm X can calculate the boundaries for the specific loan if it obtains the value of corresponding U_c from client. We can assume that the client needs loan to buy a television which costs 300,000 AMD, meaning that if we assign value to client's utility it will be equal to the price of the television and (i) is the interest rate provided by firm, so it is known. If we presume that i=17% and r=10%, we can calculate the boundaries of loan $\frac{300{,}000}{2+0.17} < L < \frac{300{,}000}{1+0.1+0.17}$, meaning that the amount of loan should be greater than 138,249 AMD and not exceed 236,220 AMD. Supervision can only take place if the firm knows what the client needs, thus, it will be essential for the firm to assign value to client's utility.

5.7. Description of the Signaling Game

Let us now focus on the last model game presented as signaling game between the borrower and the lender. As already discussed, in incomplete information games, one player knows more information than the other, and in our case, the player with more information is the borrower as he knows its true "type". In signaling games with incomplete information, the more informed player; the borrower, has to decide whether to provide security for his loan or not, and the firm has to decide whether the signal was strategically chosen or not. Let us have a deeper look into the game which is presented in *figure 10*.

Players: Borrower and Lender

The sequence of the game:

1. Nature chooses a type of a borrower: "Credible" (C) or "Non credible" (NC) with probability (1- α) and α, respectively.

2. The borrower knows his type and decides whether to provide security (S) or not (NS).

3. The lender does not know the type of the borrower, but observes his action (S or NS) and decides whether to provide a loan (L) or not (NL).

4. We assume that the credible borrower (C) fully pays a loan and interest on time no matter if the loan is secured or not.

5. We assume that a non-credible borrower (NC) may decide whether to pay a loan (Pay) or not to pay (Not pay) depending on his payoff.

Payoffs:

If the lender provides a loan to the credible borrower, who secured a loan:

– The creditor receives I_S AMD as an interest

– The borrower receives ($Profit - I_S - E_C$) AMD, which is a profit (utility) from using a loan minus interest (I_S) minus expenses to provide a loan security (E_C)

If the lender provides a loan to the credible borrower, who did not secure a loan:

– The creditor receives I_{NS} AMD as an interest

– The borrower receives ($Profit - I_{NS}$) AMD, which is a profit (utility) from using a loan minus interest on unsecured loan (I_{NS}). We assume that $I_{NS} > I_S$ due to the fact that secured loans are less risky than unsecured ones and thus have lower interest rate.

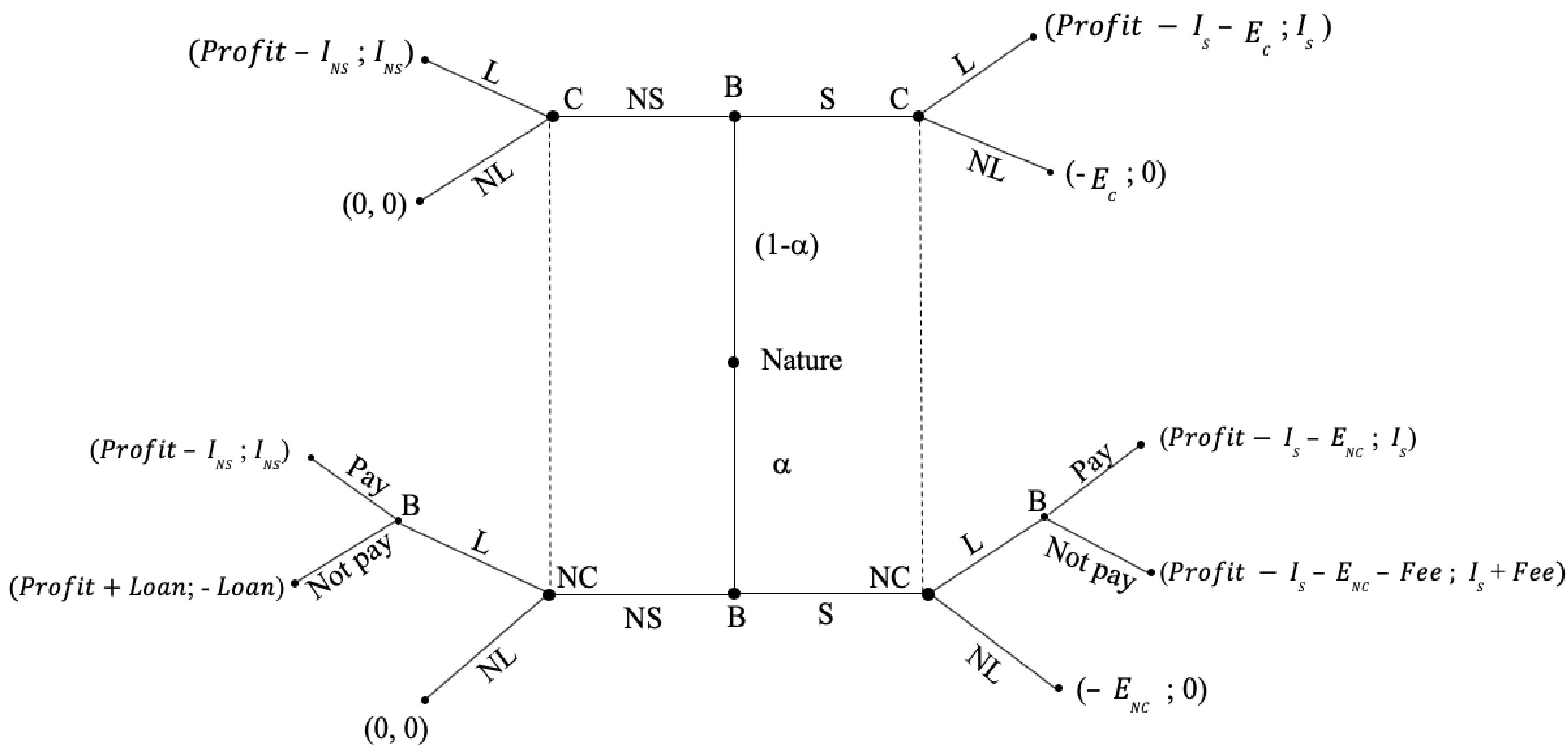


Figure 10: Signaling Game between Lender and Borrower; source: author

If the lender provides a loan to the non-credible borrower, who secured a loan, and the borrower pays the loan back:

– The lender receives I_S AMD.

– The borrower receives ($Profit\ -\ I_S - E_{NC}$) AMD, which is a profit (utility) from using a loan minus interest (I_S) minus expenses to provide a loan security (E_{NC}). We assume that $E_{NC} > E_C$ due to the fact that it is easier for credible borrower to obtain a security.

If the lender provides a loan to the non-credible borrower, who secured a loan, and the borrower does not pay:

– The lender receives ($I_S + Fee$) AMD, where fee is charged from the borrower according to the loan agreement.

– The borrower receives ($Profit\ -\ I_S - E_{NC} - Fee$) AMD.

If the lender provides a loan to the non-credible borrower, who did not secure a loan, and the borrower pays:

– The creditor receives I_{NS} AMD

– The borrower receives ($Profit - I_{NS}$) AMD.

If the lender provides a loan to the non-credible borrower, who secured a loan, and the borrower does not pay:

– The lender loses the amount of loan ($-Loan$) AMD

– The borrower receives ($Profit\ + Loan$) AMD because without security it might be very difficult for the lender to claim the loan back even in case of a lawsuit.

To analyze this game, we start with possible candidate for separating equilibrium:

The credible borrower chooses "Secure" thus signaling that he is credible, while non-credible borrower chooses "Not secure".

If credible borrower chooses to "Secure" then the dominating strategy for the lender is to choose" Loan", because its payoff Is, is strictly larger than 0.

The borrower will have an incentive to deviate to "Not secure" if $Profit\ -\ I_{NS} > Profit - I_S - E_C$. Thus, to prevent this incentive, the lender must set $I_{NS} >\ I_S\ +\ E_C$.

If non-credible borrower chooses to "Not Secure", then the expected payoff for the lender if he chooses "Loan" is $P_{pay} \times I_{NS} + (1 - P_{pay}) \times (-Loan)$ and 0 if he chooses "Not Loan". It is apparent, that for the non-credible borrower without a loan security "Not pay" is a dominating strategy, thus $P_{pay} = 0$ and $0 > (-Loan)$, which means that the lender does not provide a loan.

If the borrower deviates to the strategy "Secure", he may obtain positive outcome, which means that the borrower has an incentive to deviate.

Using the similar reasoning we can arrive to the conclusion that the credible borrower will not choose "Not secure" while non-credible borrower chooses "Secure". This means that there is no separating equilibrium in this game.

Now consider a pooling equilibrium where the borrower chooses to "Secure" no matter what type he is.

If the lender obtains a signal "Secure" he does not know what type of borrower is it. There is a probability α that it is a Not credible borrower and a probability (1- α) that it is a Credible borrower.

The expected payoff of the Lender from choosing "Loan" is $(1-\alpha) \times I_S + \alpha\ (P_{pay} \times I_S + \left(1-P_{pay}\right) \times (I_S + Fee) = (1-\alpha) \times I_S + \alpha \times \left(P_{pay} \times I_S + \left(1-P_{pay}\right) \times I_S + \left(1-P_{pay}\right) \times Fee\right) = I_S + \alpha \times \left(1-P_{pay}\right) \times Fee$. This amount is always positive; hence the Lender will choose "Loan". Moreover, for the borrower the dominating strategy is to "Pay", because $Profit - I_S - E_{NC} > Profit - I_S - E_{NC} - Fee$, which results in fact that $P_{pay} = 1$ and the expected payoff for the creditor is just $I_S > 0$

The last case if both credible and not credible choose "Not Secure".

The expected payoff of the Lender from choosing "Loan" is $(1-\alpha) \times I_{NS} + \alpha\ (P_{pay} \times I_{NS} + \left(1-P_{pay}\right) \times (-Loan) = (1-\alpha) \times I_{NS} + \alpha \times P_{pay} \times I_{NS} + \left(1-P_{pay}\right) \times (-Loan) = I_{NS} \times \left((1-\alpha) + \alpha \times P_{pay}\right) + \left(1-P_{pay}\right) \times (-Loan)$

In order to provide loan, this payoff must be larger than zero:

$$I_{NS} \times \left((1-\alpha) + \alpha \times P_{pay}\right) + \left(1-P_{pay}\right) \times (-Loan) > 0$$

$$\frac{I_{NS}}{Loan} > \frac{\left(1-P_{pay}\right)}{\left((1-\alpha) + \alpha \times P_{pay}\right)}$$

But we already saw that the dominating strategy for the not credible borrower is "Not pay", thus, $P_{pay} = 0$ and $\frac{I_{NS}}{Loan} > \frac{1}{1-\alpha}$. The expression on the right-hand side is larger than 1, but it makes no sense if $\frac{I_{NS}}{Loan} > 1$, thus the lender never provides a loan if the borrower does not secure it.

Thus, we have two pooling equilibria: Borrower plays {"Secure", "Pay"} and Lender plays "Loan" and Borrower plays {"Not secure", "Not Pay"} while Lender plays "Not Loan".

6. Conclusion

When we think about business strategy, we do not think only about one exact meaning. Some people can associate business strategy with customer satisfaction; some others will say the business strategy is about innovation. Other diverse opinions about business strategy will include being first to the market, reaching the right people, building value for customers and managing risks, establishing partnerships, being ahead of competitors, and many more. The underlying theme in all of these statements is "interactions." In any firm interactions with suppliers, business partners, competitors, as well as interactions with customers and different organizations inside the firm, play an integral role in the decision-making process. Thanks to globalization and access to many markets, firms are facing an even higher complexity of these interactions. Hence, most business situations can be modeled through a "game," where the payoffs of each participant depend on the actions of others (Erhun and Keskinocak, 2003).

Autor's goal in this thesis is to design an optimal mechanism through the game-theoretic approach that will help Firm X in the decision-making process with the borrower, ensuring that the loan will be repaid promptly and on time. Moreover, interactions between firms and borrowers are repetitive, and as long as both parties continue to benefit, the relationships are maintained. This means that the dependency of future actions on past outcomes plays a significant role in the choices that Firm X makes and the contracts it provides to its borrowers.

Since in micro-credit programs, loans are usually not backed with collateral; thus, success in terms of repayment rate cannot fully be explained through traditional rational choice theory. Firm X should pay particular attention to the loans which are given with security and motivate borrowers to follow their loan obligations. Interest rates should also be set in a way that will not discourage the borrowers from paying and will lead to high repayment rates. Further on, microfinance is an integral part of the growth strategy in Armenia and, in recent years, gained even more recognition. Thus, MFIs should strive to design an optimal scheme for borrowers, where they will not have an incentive to deviate and not pay the loan. For example, one of the solutions provided in this thesis aims to help the firm to identify the minimum and maximum boundaries of the loan amount corresponding to the client's utility.

Findings in this thesis indicate that there is a relationship between the loan repayment rate and security of the loan, which leads to the conclusion that the loan should be designed in such a way that it will enhance and motivate borrowers to pay on time. Moreover, constructed game-theoretic models help to identify supervision techniques that can be used while interacting with clients. It can be recommended for the firm to collect data on the utility of consumers (U_c) by observing the needs of the client applying for a loan. After obtaining data on U_c, it would be recommended that the firm empirically tests if the probability of default depends on the defined boundaries in one of the game models. In case of successful results, Firm X can estimate the optimal amount of the loan before issuing it in the future to its clients.

Another difficulty in the presented models refers to the derivation of the discount rate or impatience rate of the borrower (r), which can be as another recommendation to the firm to establish an appropriate rate (r). Several works of authors provide a base to believe that the rate can be connected with consumer's age, and the data in this thesis also provided motives to believe that younger age groups tend to be more volatile. This idea can be taken as a starting point for the firm by focusing on the younger group of consumers and imposing supervision upon them.

In the end, many powerful tools, such as optimization, simulation, decision analysis, can help managers in their decision-making process. We believe that the use of game theory with other managerial tools can significantly improve the manager's understanding of dynamics in business interactions and can lead to better quality and more informed decisions.

"Managers have much to learn from game theory- provided they use it to clarify their thinking, not as a substitute for business experience." (The Economist Newspaper Limited, London, June 15, 1996)

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Annexes

Appendix 1: Statistics of all contracts; source: author

	Contract Amount	Interest Rate	Real Interest Rate	Maturity	Age
count	17358	17358	17358	17358	17358
mean	347 541.77	17.58	86.90	794.91	38.91
std	620 399.98	6.27	349.40	459.16	13.19
min	10 000	0	0	6	17
25%	107 000	15.9	48.07	547	28
50%	200 000	18	56.96	731	36
75%	350 000	21.9	66.66	1095	49
max	18 000 000	23.9	10 000	3654	77

Appendix 2: Statistics of Active Contracts; source: author

	Present Balance	Percentage	Value of the Collateral	Contract Amount	Interest Rate	Real Interest Rate
count	3685	3685	3685	3685	3685	3685
mean	346946.63	12639.28	635817.08	442496.39	16.16	64.85
std	659246.01	53970.59	3578229.49	712854.05	5.60	88.22
min	96.8	-19158.2	0	20000	0	0
25%	79648.8	319	0	142300	13.9	41.84
50%	173304.3	1475.1	0	250000	17.9	54.95
75%	339304.6	5700.5	0	438000	18	63.66
max	8817992.2	1803833.1	61756800	9000000	23.9	2291.21

	Gender	Maturity	Age	Overdue	Secured
count	3685	3685	3685	3685	3685
mean	0.57	1064.64	40.10	0.58	0.88
std	0.50	554.17	13.80	0.49	0.32
min	0	15	19	0	0
25%	0	731	28	0	1
50%	1	1089	38	1	1
75%	1	1098	51	1	1
max	1	3654	77	1	1

** 1=male, 0=female, 1=overdue, 0=not overdue, 1=secured, 0=not secured, 1=active, 0=not active*

Appendix 3: Statistics of Non-Active Contracts; source: author

	Contract Amount	Interest Rate	Real Interest Rate	Gender
count	13673	13673	13673	13673
mean	321950.62	17.96	92.84	0.55
std	590444.54	6.38	390.79	0.50
min	10000	0	0	0
25%	106000	16.9	49.67	0
50%	200000	19.9	57.50	1
75%	320000	21.9	67.07	1
max	18000000	23.9	10000	1

	Maturity	Age	Overdue	Secured
count	13673	13673	13673	13673
mean	722.22	38.59	0.49	0.75
std	400.01	13.00	0.50	0.43
min	6	17	0	0
25%	540	28	0	1
50%	730	36	0	1
75%	732	48	1	1
max	3654	74	1	1

** 1=male, 0=female, 1=overdue, 0=not overdue, 1=secured, 0=not secured, 1=active, 0=not active*

Appendix 4: Statistics based on gender; source: author

	Contract Amount_x	Interest Rate_x	Real Interest Rate_x	Gender_x	Maturity_x	Age_x
count	9639	9639	9639	9639	9639	9639
mean	383779.36	17.69	85.28	1	807.87	37.34
std	695078.03	6.09	304.02	0	464.54	12.82
min	10000	0	0	1	6	18
25%	119450	16.9	49.81615	1	547	27
50%	215000	18	57.5184	1	731	34
75%	400000	21.9	66.8103	1	1095	45
max	18000000	23.9	10000	1	3654	77

	Overdue_x	Secured_x	Active_x	Contract Amount_y	Interest Rate_y	Real Interest Rate_y
count	9639	9639	9639	7719	7719	7719
mean	0.51	0.79	0.22	302290.55	17.43	88.92
std	0.50	0.41	0.41	508509.90	6.48	398.88
min	0	0	0	15000	0	0
25%	0	1	0	100000	15.9	47.45
50%	1	1	0	200000	18	55.91
75%	1	1	0	315000	21.9	66.58
max	1	1	1	10000000	23.9	10000

	Gender_y	Maturity_y	Age_y	Overdue_y	Secured_y	Active_y
count	7719	7719	7719	7719	7719	7719
mean	0	778.73	40.88	0.51	0.77	0.21
std	0	451.85	13.38	0.50	0.42	0.40
min	0	14	17	0	0	0
25%	0	546	29	0	1	0
50%	0	731	39	1	1	0
75%	0	1092	51	1	1	0
max	0	3654	73	1	1	1

** x=male, y=female, 1=overdue, 0=not overdue, 1=secured, 0=not secured, 1=active, 0=not active*

Appendix 5: Statistics of non-active loans based on gender; source: author

	Contract Amount_x	Interest Rate_x	Real Interest Rate_x	Gender_x	Maturity_x	Age_x
count	7547	7547	7547	7547	7547	7547
mean	354514.31	17.99	90.50	1.00	733.17	36.93
std	673291.41	6.23	339.94	0	403.93	12.56
min	10000	0	0	1	6	18
25%	110000	16.9	49.82	1	546	27
50%	200000	18.9	57.56	1	730	34
75%	350000	21.9	67.09	1	1081	44
max	18000000	23.9	10000	1	3654	74

	Overdue_x	Secured_x	Contract Amount_y	Interest Rate_y	Real Interest Rate_y	Gender_y
count	7547	7547	6126	6126	6126	6126
mean	0.48	0.76	281833.39	17.91	95.72	0
std	0.50	0.42	465595.62	6.57	445.55	0
min	0	0	15000	0	0	0
25%	0	1	100000	16.9	48.07	0
50%	0	1	190000	20.8	56.91	0
75%	1	1	300000	21.9	67.06	0
max	1	1	10000000	23.9	10000	0

	Maturity_y	Age_y	Overdue_y	Secured_y
count	6126	6126	6126	6126
mean	708.73	40.64	0.50	0.74
std	394.73	13.25	0.50	0.44
min	14	17	0	0
25%	366	29	0	0
50%	730	39	0	1
75%	731	51	1	1
max	3654	73	1	1

** x=male, y=female, 1=overdue, 0=not overdue, 1=secured, 0=not secured, 1=active, 0=not active*

Appendix 6: Statistics of active loans based on gender; source: author

	Present Balance_x	Percentage_x	Value of the Collateral_x	Contract Amount_x	Interest Rate_x
count	2092	2092	2092	2092	2092
mean	386640.48	15786.96	710182.82	489354.58	16.60
std	698213.35	67440.42	3971601.82	759412.95	5.42
min	1828.9	-19158.2	0	20000	0
25%	92112.5	452.48	0	160000	15.9
50%	186985.8	1822.1	0	270000	17.9
75%	383871.13	7217.58	0	500000	18.9
max	6194567.9	1803833.1	61756800	7800000	23.9

	Real Interest Rate_x	Gender_x	Maturity_x	Age_x	Overdue_x
count	2092	2092	2092	2092	2092
mean	66.45	1	1077.37	38.82	0.60
std	92.55	0	559.52	13.63	0.49
min	0	1	15	19	0
25%	48.07	1	731	27	0
50%	56.51	1	1094	35.5	1
75%	64.06	1	1101	49	1
max	2291.21	1	3654	77	1

	Secured_x	Present Balance_y	Percentage_y	Value of the Collateral_y
count	2092	1593	1593	1593
mean	0.88	294818.86	8505.61	538156.62
std	0.32	600509.98	27139.47	2982310.02
min	0	96.8	-3253.3	0
25%	1	64535.7	200.7	0
50%	1	145575.3	1054.8	0
75%	1	290078.7	4022.6	0
max	1	8817992.2	533156.6	36342000

	Contract Amount_y	Interest Rate_y	Real Interest Rate_y	Gender_y
count	1593	1593	1593	1593
mean	380960.09	15.59	62.75	0
std	641695.41	5.78	82.18	0
min	30000	0	0	0
25%	107000	10	40.41	0
50%	215000	16.9	52.64	0
75%	392160	17.9	63.19	0
max	9000000	23.9	1658.30	0

	Maturity_y	Age_y	Overdue_y	Secured_y
count	1593	1593	1593	1593
mean	1047.92	41.77	0.56	0.89
std	546.78	13.85	0.50	0.31
min	25	19	0	0
25%	731	30	0	1
50%	732	41	1	1
75%	1097	53	1	1
max	3654	71	1	1

** x=male, y=female, 1=overdue, 0=not overdue, 1=secured, 0=not secured, 1=active, 0=not active*

Appendix 7: Statistics of overdue loans; source: author

	Contract Amount_x	Interest Rate_x	Real Interest Rate_x	Gender_x	Maturity_x	Age_x
count	8568	8568	8568	8568	8568	8568
mean	331687.68	16.79	93.15	0.56	792.05	40.01
std	579076.33	6.54	382.29	0.50	482.34	13.25
min	10000	0	0	0	6	18
25%	105000	14.9	48.56	0	523.75	29
50%	200000	17.9	56.98	1	731	38
75%	350000	21.9	66.56	1	1095	50
max	14500000	23.9	10000	1	3654	74

	Overdue_x	Secured_x	Active_x	Contract Amount_y	Interest Rate_y	Real Interest Rate_y
count	8568	8568	8568	8790	8790	8790
mean	0	0.78	0.18	362995.44	18.34	80.79
std	0	0.42	0.39	657853.18	5.89	313.93
min	0	0	0	20000	0	0
25%	0	1	0	120000	16.9	48.07
50%	0	1	0	210000	19.9	56.92
75%	0	1	0	350000	22.7	66.84
max	0	1	1	18000000	23.9	10000

	Gender_y	Maturity_y	Age_y	Overdue_y	Secured_y	Active_y
count	8790	8790	8790	8790	8790	8790
mean	0.55	797.71	37.85	1	0.78	0.24
std	0.50	435.39	13.05	0	0.41	0.43
min	0	12	17	1	0	0
25%	0	548	27	1	1	0
50%	1	731	35	1	1	0
75%	1	1091	47	1	1	0
max	1	3654	77	1	1	1

* *x=male, y=female, 1=overdue, 0=not overdue, 1=secured, 0=not secured, 1=active, 0=not active*