VYSOKÁ ŠKOLA EKONOMICKÁ V PRAZE

Fakulta financí a účetnictví

katedra bankovnictví a pojišťovnictví

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Game Theory Analysis of Options

Equity value triggering bank run

Autor diplomové práce:	Bc. Petr Marek
Vedoucí diplomové práce:	prof. Ing. Karel Janda, M.A., Dr., Ph.D.
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Čestné prohlášení

Prohlašuji, že jsem diplomovou práci na téma „Game Theory Analysis of Options- Equity value triggering bank run“ vypracoval samostatně a veškerou použitou literaturu a další prameny jsem řádně označil a uvedl v přiloženém seznamu.

V Praze dne 24.5.2020

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Petr Marek

Poděkování

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Abstrakt

Tato diplomová práce se zabývá odvozením kritické hodnoty vlastního kapitálu banky, při které vkladatelé začnou vybírat svá depozita a tím spustí run na banku. Práce je rozdělena na teoretickou a aplikační část. V části teoretické je nejprve nastíněna základní problematika oceňování opcí a teorie her. Následně jsou obě tyto disciplíny zkombinovány a je představen teoretický model runu na banku. V aplikační části jsou použité tyto teoretické poznatky pro vymezení tří možných hodnot vlastního kapitálu a od toho odvíjejících se scénářů chování vkladatelů. Tento run na banku je zkoumán na příkladu populární české banky Air Bank a. s. V rámci nejpravděpodobnějšího scénáře je posléze proveden stress test likvidity, který simuluje potencionální run na Air Bank a. s.

Klíčová slova

Run na banku, likvidita, teorie her, oceňování opcí, pojištění vkladů

Abstract

This diploma thesis aims to derive the critical value of bank's equity, which triggers depositors to withdraw their deposits and by doing so trigger a bank run. Thesis is divided into theoretic and application part. In the theoretic part, the basics of option pricing and game theory are introduced. Afterwards, both disciplines are combined and the theoretical model of a bank run is presented. In the application section, these theoretical discoveries are used to calculate three possible equity values, which lead to the unique scenarios of depositors´ behavior. This bank run is examined on popular Czech bank named Air Bank a. s. Within the scope of the most likely scenario is eventually conducted liquidity stress test, which simulates a potential run on Air Bank a. s.

Key Words

Bank run, liquidity, game theory, option pricing, deposit insurance

Content

Introduction

Bank runs have been and still remain one of the most crucial threats to the stability of the financial system. This economic phenomenon was a defining feature of the modern global economic crisis in 2008 (Bernanke, 2010). Even though the economic literature on the subject of bank runs is sufficient, the question of how bank runs originated has been given very little attention. The answers to the questions of how bank runs occur and how to prevent them have relevant implications for policymakers, especially in times of economic distress.

Current economic literature on a topic of bank runs is primarily based on game theoretic framework of simultaneous coordination game (Diamond, 1983). The framework is built on idea that players have an incentive to take the same action as other players in the game. Therefore, depositors withdraw their deposits from the bank if they expect, that other depositors will withdraw as well. If the players decide to withdraw at the same time, bank run instantly happens.

This thesis will adopt a model of bank run, introduced in 'A game theory analysis of options' by Alexandre Ziegler (Ziegler, 2004). In the first chapters of this thesis, the theoretical framework of options and game theory will be described, and later implemented into the study of bank runs. The theoretical part of this thesis will address fundamental topics such as: at what exact value of assets and equity bank runs happen, and the optimal level of a bank´s capital.

In the final chapter of this thesis, we will determine the four most important factors affecting the susceptibility of Czech commercial banks to a possible bank run and apply them to the real world case of the popular Czech bank Air Bank a. s. We will also examine the effect of bank-run contagions as they relate to deposit insurance, which helps to increase depositors' trust in the financial system, and by doing so, preventing bank run contagions. At the end of this thesis, we will apply our findings from the theoretical part to estimate the equity value, which triggered Air Bank´s a. s. clients to withdraw their deposits. Then we will simulate a possible bank run, using a liquidity stress test, which is similar to stress tests used by some European supervisory authorities.

1 Options and their pricing

Options are financial derivatives that serve as a contract between two counterparties, in which the option holder (buyer of an option) has the right to buy or sell underlying assets in set time or during set time. On the other hand, the writer of an option (seller of an option) is obligated to sell or buy the underlying asset for given price.

Options can be divided into two types: call and put. Holders of *call options* have the right to buy an underlying asset for a define price in a set time. However, holders of *put options*, have the right to sell an underlying asset for a define price in a set time. In financial terms, we do not use the terms define price or set time. In financial terms, we use the terms strike price and maturity.

Options can be divided into several groups according to date in which option can be exercised. The two main categories are European and American options. The difference between the European and the American option is in time, when the option can be exercised. European options can be exercised only on maturity, but American options can be exercised on any day before the maturity day.

The main feature of options is that it gives to an option holder the right. The seller on the option has not such a right. This right is what distinguishes options from forwards and futures. In forward and futures contracts, the holder is always obligated to buy or sell an underlying asset for strike price (Chriss, 1996).

1.1 Option premium

The right for an option holder to choose, whether he is buying or selling underlying asset for strike price or choosing not to buy or sell is not for free. On the other hand, the option writer must sell or buy the underlying asset, if option is exercised, for the strike price. Option holders must pay for this privilege. This price is called *option premium*. The premium includes two parts: intrinsic value and time value.

Intrinsic value is a value that holder would receive if he exercised the option immediately, therefore it is given as a difference between the strike price and the spot price. Three types of options are recognized in terms of strike price and spot price difference:

- In the money option. Call options are in the money, when the spot price is higher than the strike price. In a case of put options, the spot price must be lower than the strike price, in order to be in the money. In the money options have positive intrinsic value and time value.
- Out of the money option. Call option is out of money, when spot price is below strike price and vice versa for put option. Out of money options have intrinsic value equal to zero.
- At the money option. When the spot price is equal to strike price, both options are at the money. The intrinsic value of at the money option is also equal to zero (Witzany, 2013).

The idea is that holder never exercises an option if he can buy the underlying asset cheaper on spot market or sell on the market for higher price. For the calculation of intrinsic value, we need to know current spot value of underlying asset, strike price and type of option. For a holder of a call option, intrinsic value can be calculated as follows:

$$\max(0, S_t - K) \tag{1.1}$$

For put option

$$\max(0, K - S_t) \tag{1.2}$$

The equations above (1.1 and 1.2) are called payoff function. The maximum in the function represents the right to choose, S_t represents spot price at time t and K strike price.

Time value is the amount that is required by the option seller for undergoing the risk, that he may end up with substantial loss. The time value of an options is also positively correlated to the option maturity. However, it can be observed that the value exponentially decays over time.

Time value isn't determined by equations as intrinsic value but is influenced by other different factors. Besides that, few of them can't be quantified so they need to be approximated. The Black-Scholes model is one of the models which tries to approximate time value of an option. The Black-Scholes model is the topic of following chapters. Time value can be affected by these factors:

- Current (spot) price of the underlying asset. If there is an increase in a spot price, price of a call option is also increased. When the spot price decreases, the value of put option increases and vice versa.
- Strike price. Time value of an option increases the more the option is in the money and vice versa.
- Time to maturity. Effect of time to maturity is not clear. In case of European option, it depends on the relationship between forward and spot prices (normal or inverted market). With longer time to maturity, the call option value is increasing if the spot prices are expecting growth.
- Volatility.
- Interest rate. Rising domestic interest rates lead to rising asset values (Hull, 2012)

1.2 The Black-Scholes Model

Black-Scholes model was firstly introduced by Fischer Black and Myron Scholes in 1973 and even today Black-Scholes model remains as one of the most popular option pricing models. Model is derived from partial differential equations, but the final formula is not that hard to understand.

1.2.1 Itô's lemma

Let the price of a stock option be a function of underlying stock's price and time. Arguably, the price of any derivative is a function of stochastic variables price and time. In 1951 mathematician K. Itô explained the behaviour of functions of stochastic variables. The result is Itô's lemma (Hull, 2012).

Asume that the value of variable x follows Itô process

$$dx = a(x,t)dt + b(x,t)dz \tag{1.3}$$

where a and b are functions of x and t and dz is Wiener process with mean equal to 0 and with standard deviation equal to 1. Drift rate of x is a and variance b^2. It can be shown that a function G of x and t follows the Itô's process

$$G = \left(\frac{\partial G}{\partial t} + \frac{\partial G}{\partial x} a + \frac{1}{2} \frac{\partial^2 G}{\partial x^2} b^2\right) dt + \frac{\partial G}{\partial x} b \, dz \qquad 1.4$$

Where dz is the identical Wiener process as in previous equation (1.3), which means that G follows an Itô's process too. With a drift rate of

$$\mu = \frac{\partial G}{\partial t} + \frac{\partial G}{\partial x} a + \frac{1}{2} \frac{\partial^2 G}{\partial x^2} b^2 \qquad 1.5$$

and a variance rate of

$$\left(\frac{\partial G}{\partial x}\right)^2 b^2 \qquad 1.6$$

1.2.2 Assumption for Black-Scholes Differential Equation

Black-Scholes formula is the element of a model of a financial market. On this market has to exist at least one risky and one risk-less asset. Model has other relatively stern assumptions:

- Asset price follows GBM (geometric Brownian motion) with constant drift rate and constant volatility.
- Risk-free interest rate is a known function of time over the life of an option.
- Markets are efficient and stock follows lognormal random walk.
- Asset pays no dividends.
- No arbitrage possibilities.
- No transaction cost or taxes.
- Short selling is permitted, and assets are divisible.

Some of these assumptions can be skipped in order to fit the model to the real world's conditions (Hull, 2012).

1.2.3 Lognormal property of stock prices

Itô's lemma can be used to derive the process which is followed by $\ln S$ when stock price S follows geometric Brownian motion. Now define

$$G = \ln S \tag{1.7}$$

Since partial derivations of G equals to

$$\frac{\partial G}{\partial t} = 0 \qquad \frac{\partial G}{\partial S} = \frac{1}{S} \qquad \frac{\partial^2 G}{\partial S^2} = -\frac{1}{S^2} \tag{1.8}$$

from equation (1.4), it follows that process followed by G is

$$dG = \left(\mu - \frac{\sigma^2}{2}\right) dt + \sigma\, dz \tag{1.9}$$

Since mean (μ) and variance (σ) are constant, equation shows that, $G = \ln S$ follows a generalized Wiener process with a constant drift rate $\left(\mu - \frac{\sigma^2}{2}\right) dt$ and constant variance σ^2. It indicates that changes in $\ln S$ between starting time 0 and future time T are normally distributed, with mean $\left(\mu - \frac{\sigma^2}{2}\right) T$ and with variance $\sigma^2 T$ (Hull, 2012). This leads to

$$\ln S_T - \ln S_0 \sim \phi \left[\left(\mu - \frac{\sigma^2}{2}\right) T, \sigma^2 T\right] \tag{1.10}$$

or

$$\ln S_T \sim \phi \left[\ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right) T, \sigma^2 T\right] \tag{1.11}$$

Where $\phi(m, v)$ is standard normal distribution with mean value m and variance v. S_0 is defined as spot price of stock at starting time 0 and S_T is defined as stock price at future time T. The mean of $\ln S_T$ is $\ln S_0 + \left(\mu - \frac{\sigma^2}{2}\right) T$ and the standard deviation is $\sigma\sqrt{T}$ (Hull, 2012).

Equation (1.11) shows that stock price at time T is lognormally distributed when the natural logarithm of stock prices is normally distributed. Variable with a lognormal distribution can take any value between zero and plus infinity. This implies that a stock's price at time T, given stock's price today, is lognormally distributed. The standard deviation is $\sigma\sqrt{T}$ which is proportional to the square root of how far into future it is looked. For lognormal distribution unlike for normal distribution mean, median and mode are not the same. It can be shown, that expected price of a stock in time T equals to:

$$E(S_T) = S_0 e^{\mu t} \qquad 1.12$$

1.2.4 Derivation of the Black-Scholes Differential Equation

From assumption above it is certain that price of an underlying asset follows a geometric Brownian motion:

$$dS = \mu S dt + \sigma S dz \qquad 1.13$$

We can use Ito's lemma, if we assume that the price of a call option *f* is a function of variables S and t. Using:

$$df = \left(\frac{\delta f}{\delta t} + \frac{\delta f}{\delta S}\mu S + \frac{1}{2}\frac{\delta^2 f}{\delta S^2}\sigma^2 S^2\right) dt + \frac{\partial f}{\partial S}\sigma S dz \qquad 1.14$$

The discrete version of geometric Brownian motion:

$$\Delta S = \mu S\, \Delta t + \sigma S\, \Delta z \qquad 1.15$$

The discrete version of call option price *f*

$$\Delta f = \left(\frac{\delta f}{\delta t} + \frac{\delta f}{\delta S}\mu S + \frac{1}{2}\frac{\delta^2 f}{\delta S^2}\sigma^2 S^2\right) \Delta t + \frac{\partial f}{\partial S}\sigma S\, \Delta z \qquad 1.16$$

Wiener process *dz* is the only source of uncertainty in this equation (1.16). Portfolio of derivative and stock can be created to eliminate this source of uncertainty. The portfolio consists of short one derivative and long amount $\frac{\partial f}{\partial S}$ of stocks. Value of synthetically created portfolio is

$$\Pi = -f + \frac{\partial f}{\partial S} S \qquad 1.17$$

The change of portfolio value in the short time interval is

$$\Delta\Pi = -\Delta f + \frac{\partial f}{\partial S}\Delta S \qquad 1.18$$

Substituting discrete versions of geometric Brownian motion and call option price into equation above gives

$$\Delta\Pi = \left(-\frac{\delta f}{\delta t} - \frac{\delta f}{\delta S}\mu S - \frac{1}{2}\frac{\delta^2 f}{\delta S^2}\sigma^2 S^2\right)\Delta t - \frac{\partial f}{\partial S}\sigma S\, dz + \frac{\partial f}{\partial S}\mu S\,\Delta t + \frac{\partial f}{\partial S}\sigma S\, dz \quad 1.19$$

This equation can be simplified to

$$\Delta\Pi = -\frac{\delta f}{\delta t}\Delta t - \frac{1}{2}\frac{\delta^2 f}{\delta S^2}\sigma^2 S^2 \Delta t \quad 1.20$$

The only source of uncertainty dz - Wiener process is truly eliminated and so the portfolio must be riskless during time Δt. If the portfolio is riskless, then its return must equal to risk-free return. If the rate of return should be higher than risk-free rate of return then, there would be an arbitrage opportunity to borrow money and buy a portfolio. If the rate of return should be lower than risk-free rate of return then there would be again arbitrage opportunity to short the portfolio and buy risk-free securities. Thus:

$$\Delta\Pi = r\Pi\Delta t \quad 1.21$$

where *r* represents risk-free interest rate. After substitution into previous equations we obtain:

$$\left(-\frac{\delta f}{\delta t} - \frac{1}{2}\frac{\delta^2 f}{\delta S^2}\sigma^2 S^2\right)\Delta t = r\left(-f + \frac{\partial f}{\partial S}S\right)\Delta t \quad 1.22$$

So that

$$\frac{\delta f}{\delta t} + \frac{\partial f}{\partial S}rS + \frac{1}{2}\frac{\delta^2 f}{\delta S^2}\sigma^2 S^2 = rf \quad 1.23$$

Final equation is famous Black-Scholes differential equation. This equation has many different solutions depending which derivative is defined with *S* as underlying variable. Particular solution for derivative price is obtained when correct boundary conditions are used. Boundary conditions closely specify values of the chosen derivative at the boundaries of probable values of *S* and *t*. European call options has following boundary conditions:

$$f = \max(0, S - K) \qquad \text{when } t = T \qquad 1.24$$

For European put option:

$$f = \max(0, K - S) \qquad \text{when } t = T \qquad 1.25$$

Portfolio that is used to derive Black-Scholes differential equation does not have to be riskless all the time. Portfolio is riskless only for an infinitesimally brief period of time. When *S* and *t* are changing, the $\frac{\partial f}{\partial S}$ is changing too. In order to keep the portfolio permanently riskless, it is necessary to often change relative proportions of stock and derivative in the portfolio (Witzany, 2013).

1.2.5 Black-Scholes Formula

Black-Scholes formula is the most famous solution to the differential equation introduced in previous section (1.23). The formula is used to calculate prices only of European style call and put options. Price of a call option according to Black-Scholes formula is:

$$c = S_0 N(d_1) - K e^{-rt} N(d_2) \qquad 1.26$$

And price of a put option:

$$p = K e^{-rt} N(-d_2) - S_0 N(-d_1) \qquad 1.27$$

Where d_1 and d_2 are defined as:

$$d_1 = \frac{ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \qquad 1.28$$

$$d_2 = \frac{ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \qquad 1.29$$

Function *N(x)* represents the cumulative probability function of the standard normal distribution. Stock price volatility is represented by σ.

One possible approach, when deriving the Black-Scholes formula, is to use the risk-neutral valuation. Thus, the expected value of a call option at maturity *T* in risk-neutral world yields

$$\hat{E}[\max(0, S_T - K)] \tag{1.30}$$

where $\hat{E}$ represents the expected value of an option in a risk-neutral world. Suppose a call option on a non-dividend paying stock with a maturity *T*. Price of a call option can be expressed using this argument multiplied (discounted) by risk-free interest rate.

$$c = e^{-rt}\hat{E}[\max(0, S_T - K)] \tag{1.31}$$

Suppose that S_T under the stochastic process assumed by Black-Scholes is lognormal. Also, from previous equations expected value of stock in risk neutral world

$$\hat{E}(S_T) = S_0 e^{rt} \tag{1.32}$$

Combining previous equations (1.31 and 1.32) yields

$$c = e^{-rt}[S_0 e^{rt} N(d_1) - KN(d_2)] \tag{1.33}$$

Or

$$c = S_0 N(d_1) - Ke^{-rt} N(d_2) \tag{1.34}$$

where

$$d_1 = \frac{ln\left[\hat{E}(S_T)/K\right] + \sigma^2 T/2}{\sigma\sqrt{T}} = \frac{ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \tag{1.35}$$

$$d_2 = \frac{ln\left[\hat{E}(S_T)/K\right] - \sigma^2 T/2}{\sigma\sqrt{T}} = \frac{ln(S_0/K) - (r + \sigma^2/2)T}{\sigma\sqrt{T}} \tag{1.36}$$

2 Game Theory basics

Game theory is a branch of economic theory that studies multi-personal decision problems through mathematical models. These interactions between players are called games. Every agent tries to maximize their winning price, which in economic theory is utility. Individual participants of the game are called players or agents and have often contradictory interests. It was introduced by John von Neumann and Oskar Morgenstern. They defined the game as any interaction between two or more agents. Game is defined by the set of rules with every possible move, which agents can make and with the outcome to every agent's moves. Game theory can be applied to many real-life situations from rock-paper-scissor game to company strategies (Gibbons, 1992).

Decision of one player affects not only his payoff, but his action can also affect the payoff of other players. Which means that payoffs of all players are mutually affected. Game theory assumes that a player is intelligent when he has perfect information about the game and acts in order to maximize his payoff. In other words, an intelligent player is the same as rational customer in economic theory.

The first step in analysis of any game or conflict is to start with a specification of a model, which is used to describe the game. Very simple, model structure may lead into ignoring important aspects of the game that is subject to a study. On the other hand, a complicated model structure can obstruct analysis by obscuring fundamental issues. In game theory we can find 2 ways how to describe the game. These two approaches have common goal, which is to define mathematical model of a decision situation (Myerson, 1991):

- Normal form.
- Extensive form.

2.1 Normal form

Normal game form or strategic form is a simple way how to represent the game. To define a game in a normal form it is only needed to specify the number of players playing the game, establish possible available actions to each player and payoffs resulting from the player's choices.

The game in normal form is defined by three components (Osborne, 2004)

- number of players
- possible available actions for each player
- payoffs resulting from the player's choices

In normal game players make their decisions simultaneously. No player is a making decision, which is a response to other players' action. Each players' goal is to maximize his utility. Players are aware of the structure of the game, meaning that every player knows the other player's possible actions and from them resulting payoffs.

Example: The most famous game is *The Prisoner's Dilemma*. In this game 2 suspects of crime are arrested by the police, but police lack evidence. Police need at least one suspect to confess. Suspects are in separated cells. There are four possible outcomes from this situation. If neither of them confess, then both suspects will be charged with minor offense and sent to jail for 1 month. If both of them confess, then both suspects will go to prison for 6 months. If one of them confess and other does not then the suspect who confessed will be set free and the other one will go to jail for 9 months, where 6 months are for the crime and additional 3 month for obstructing justice (Osborne, 2004).

Players 2 suspects.

Actions Each player has two possible actions {Quiet, Confess}.

Preferences for 1st suspect (from best to worst): (Confess, Quiet) (he is set free, because 2nd suspect stayed quiet), (Quiet, Quiet) (sentenced for 1 month), (Confess, Confess) (he gets 6 months in prison), (Quiet, Confess) (he gets 9 months in prison, because suspect 2 confessed). Preferences for 2nd suspect (Quiet, Confess), (Quiet, Quiet), (Confess, Confess) and (Confess, Quiet).

The Prisoner's dilemma can be represented in a table. Payoff functions represent 1st suspect's preferences. For 1st suspect function u_1 is defined for which

$$u_1(Confess, Quiet) > u_1(Quiet, Quiet) > u_1(Confess, Confess) > u_1(Quiet, Confes)$$

After specification $u_1(Confess,\ Quiet)$ = 0, $u_1(Quiet,\ Quiet)$ = -1, $u_1(Confess,\ Confess)$ = -6, $u_1(Confess,\ Quiet)$ = -9. Similarly, for 2nd suspect $u_2(Quiet,\ Confess)$ = 0, $u_2(Quiet,\ Quiet)$ = -1, $u_2(Confess,\ Confess)$ = -6, $u_2(Confess,\ Quiet)$=-9. Taking these representations into account, the game can be illustrated in figure below (tab. 1). In table below rows conform to player 1 potential actions and columns correspond to probable actions of player 2. Numbers in boxes represent player's payoffs corresponding to chosen action, with 1st payoff listed first (Osborne, 2004).

Tab. 1: Prisoner´s dilemma

		Suspect 2	
		Quiet	Confess
Suspect 1	Quiet	-1, -1	-9, 0
	Confess	0, -9	-6, -6

Source: Author´s compilation

2.1.1 Nash equilibrium

Nash equilibrium is used to find an optimal strategy of players in the game. We mentioned that every player is an intelligent (rational) and he is always choosing the best option. When playing a game, the best option always depends on different player´s choices. When a player is choosing an action, he needs to keep in mind actions of other players (Myerson, 1991).

Definition of Nash equilibrium *"The action profile a* in a strategic game with ordinal preferences is a* ***Nash equilibrium*** *if, for every player i and every action a_i of player i, a* is at least as good according to player i's preferences as the action profile (a_i, a^*_{-i}) in which player i chooses a_i while every other player j chooses a^*_j. Equivalently, for every player i,*

$$u_i(a^*) \geq u_i(a_i, a^*_{-i}) \text{ for every action } a_i \text{ of player } i$$

where u_i is a payoff function that represents player i's preferences." (Osborne, 2004)

In other words, if the player plays his Nash equilibrium strategy, he chooses the best strategy with respect to other players. He has no motivation to leave this strategy, but if he would leave then he is not increasing his utility.

Example: Let’s return to *The Prisoner’s Dilemma.* From the table (tab. 1) it can be observed that (Confess, Confess) is the unique Nash equilibrium. When player 2 wishes to *Confess*, player 1’s best action is to choose *Confess* rather than *Quiet* (table shows that *Confess* yields player 1 a payoff of -6, but on the other hand, *Quiet* yields him a payoff of -9), and considering that player 1 chooses to *confess* the best option for player 2 is to choose *confess* instead of choosing to stay. Therefore, no other possible set of actions than (Confess, Confess) is a Nash equilibrium (Osborne, 2004).

If both prisoners decide to stay quiet, then action set (Quiet, Quiet) does not satisfy the definition of Nash equilibrium. If player 2 remains *Quiet,* player’s 1 payoff to *Confess* exceeds his payoff to *Quiet*. Furthermore, when player 1 stays *Quiet,* player’s 2 payoff to *Confess* is also higher that 1st player´s payoff to stay *Quiet*. Player 2 wants to deviate as well as player 1. In order to show that a set of actions is not a Nash equilibrium, it is not necessary to study actions of player 2, if it is established that player 1 wants to deviate. To show that action set is not a Nash equilibrium is enough if at least one player wants to deviate (Osborne, 2004).

Action set (Confess, Quiet) does not satisfy the definition of Nash equilibrium, because if player 1 chooses to *Confess,* player’s 2 payoff to *Confess* yields higher payoff than *Quiet.*

Action set (Quiet, Confess) does not satisfy the definition of Nash equilibrium, because if player 2 chooses to *Confess*, player’s 1 payoff to *Confess* exceeds his payoff to choose action *Quiet*.

2.2 Extensive form

The main disadvantage of normal form is that it does not count with a sequential structure of decision-making process. When normal form is used, it is presumed that player´s actions have been set and cannot be further changed overtime. On the other hand, games in extensive form show the consecutive structure of decision making, which allows the player to alter his actions as events unfold.

Similarly, to normal form, we need to specify the number of players and their preferences. In addition, it is also needed to specify order in which players take their actions and what possible actions they have in given point of time. Hence, the sequence of all possible actions is set, as well as the player choosing a possible action at each point of time. The sequence of actions is called terminal history.

The game in extensive form is defined by four components (Gibbons, 1992)

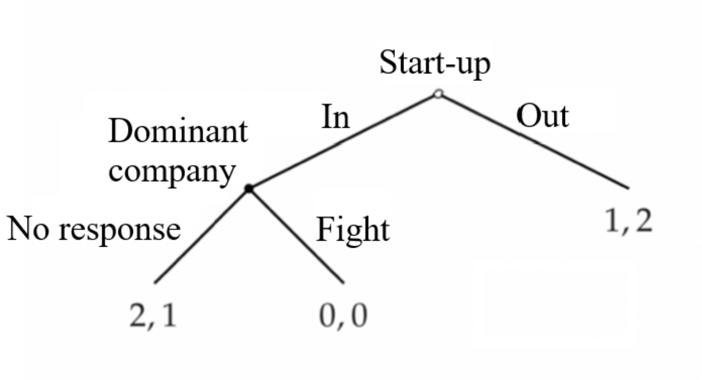
- set of players
- terminal histories (set of sequences)
- player functions
- player preferences

Example: *Entry game*: Imagine 2 companies. Company 1 is a dominant company on given market. Company 2 is a Start-up company and is considering entering into the market. Suppose that the best outcome for start-up is to enter the market, with no response from the dominant company. The worst outcome for start-up is that the dominant company decides to fight for their share of a market. For the dominant company, the best outcome is that the new company does not decide to enter the market and the worst outcome is that the start-up company enters the market and there is fight for share on the market. The game can be modelled in extensive form (Myerson, 1991).

Players	Dominant company and Start-up
Terminal histories	(*In, No response), (In, Fight*), and *Out*.
Player function	$P(\emptyset) = Startup$ and $P(In) = Dominant\ company$
Preferences	Start-up preferences are expressed by the payoff function u_1 for which $u_1(In,\ No\ response) = 2$, $u_1(Out) = 1$, $u_1(In, Fight) = 0$ and dominant company preferences are represented by a payoff function u_2 for which $u_2(Out) = 2$, $u_2(In,\ No\ response) = 0$ and $u_2(In, Fight) = 0$ (Osborne, 2004)

This game can be represented by a diagram. The start of the game is represented by a small circle at the top of the diagram below (Fig. 1). Label indicates that start-up starts the game and chooses its action ($P(\emptyset) = Startup$). Possible starting actions are represented by branches In and Out. Branch (action) In leads to a black dot, where the dominant company's turn starts and the company needs to take an action ($P(In) = Dominant\ company$). Dominant company choices are *No response* or *Fight*. Start-up payoffs are listed first.

Fig. 1: Entry game



Source: Author´s compilation

It seems clear that the start-up will enter the market and dominant company will later choose not to respond. Start-up can reason his choice that if he enters then dominant company will not respond to his action, because doing nothing is better than fight. Taking this into account, how dominant company will react, the start-up is better of entering the market.

Induction that the start-up makes is called *backward induction.* Each time a player in a game needs to make an action, he thinks of every possible action and every subsequent action that the players (including himself) will rationally make. Therefore, player always chooses the one action that will result in the most preferred terminal history (Osborne, 2004).

Backward induction cannot be applied to every extensive form game. For example, let consider a game, analogous to previous one in which dominant company has different terminal history, where action set (In, *Fight)* yields 1 instead of 0 ($u_2(In, Fight) = 1$). In this modified game, if the start-up decides to enter the market, dominant company is indifferent about their choice, whether to choose *No response* or *Fight.* When using, backward induction, start-up do not know what the dominant company is going to do, this result leaves the start up in insecurity, which path to take. There is another problem in which backward induction cannot tell what to do. This problem is in games with infinitely long history, because these games have no end from which backward induction can be started. (Gibbons, 1992)

2.2.1 Nash Equilibrium

As in a normal form of a game, it is possible to define an equilibrium solution in an extensive form game too..Assume that, when every player exactly knows the other player's responses, then that player has no incentive to alter his behaviour.

Definition of Nash equilibrium of extensive game with perfect information *"The strategy profile s^* in an extensive game with perfect information is a Nash equilibrium if, for every player i and every strategy r_i of player i, the terminal history $O(s^*)$ generated by s^* is at least as good according to player i's preferences as the terminal history $O(r_i,\ s^*_{-i})$ in which player i chooses r_i while every other player j chooses ${s^*}_j$. Equivalently, for each player i,*

$$u_i\big(O(s^*)\big) \geq u_i\big(O(r_i, s^*_{-i})\big) \textit{ for every strategy } r_i \textit{ of player i,}$$

where u_i is a payoff function that represents player i's preferences and O is the outcome function of the game." (Osborne, 2004)

The key in finding the Nash equilibrium of an extensive game form is to list every player strategy. Then, solve the extensive game as a normal form game in a way that normal form game of extensive form game is constructed.

Example: Entry game. Start-up has 2 possible actions *In* and *Out* and again, the dominant company also has two possible strategies *No response* and *Fight*. Table below (tab. 2) represents Entry game in the form of normal game. It can be observed that this normal form game has 2 possible Nash equilibria in pure strategies: (*In, No response)* and *(Out, Fight)*. Clearly, the backward induction yields the same result as one Nash equilibrium. Therefore, it is proven that both methods are suitable.

Tab. 2: Entry game

		Dominant company	
		No response	Fight
Startup	In	2, 1	0, 0
	Out	1, 2	1, 2

Source: Author´s compilation

The second possible equilibrium is when start-up chooses an action *Out* – not to enter the market. Start-up´s strategy not to enter is optimal, because the dominant company action strategy is to fight. Dominant company action to fight is optimal given the start-up company choice. This leads to a conclusion that, none of these two players are able to increase its payoff when choosing a different possible action, when taking opponent strategy into account.

2.2.2 Subgame perfection equilibrium

Let's set a notion about equilibrium that models a sturdy stable state. Hence, it is required that every player's strategy is optimal. Players´ strategy needs to be optimal at every possible time of the game. (Myerson, 1991)

In order to define the concept, the notion of subgame needs to be defined. The subgame is the part of the game that remains after one terminal history already occurred.

Definition of subgame *"Let* Γ *be an extensive game with perfect information, with player function P. For any nonterminal history. For any nonterminal history h of* Γ, *the subgame* Γ*(h) following the history h is the following extensive game.*

Players	*The players in* Γ.
Terminal histories	*The set of all sequences h' of actions such that (h, h') is a terminal history of* Γ.
Player function	*The player P(h, h') is assigned to each proper subhistory h' of terminal history.*
Preferences	*Each player prefers h' to h'' if and only if he prefers (h, h') to (h, h'') in* Γ*."* (Osborne, 2004)

It is required that the subgame still follows the terminal history ∅ set at the start of the game. Other subgames are defined as *proper subgames.* There exists a subgame for every step of the game, thus the number of nonterminal histories equals to number of subgames.

Example: Entry game. From the two possible Nash equilibria in the Entry game, we want to choose a subgame perfect equilibrium. The Nash equilibrium *(Out, Fight)* is not a subgame perfect equilibrium, because when start-up company chooses action *Out*, then the dominant company's strategy *Fight* it is not an optimal decision. In this subgame, for dominant company, better action is to choose *No response* rather than *Fight*. Thus, the better action for dominant company, when start-up chooses to enter the market, is to do nothing. Hence, the subgame perfection is the Nash Equilibrium (*In, No response)*. (Osborne, 2004)

3 The Method game theory analysis of options

Game theory analysis of options is a theoretical attempt to bring game theory and option pricing into accord. This methodology was introduced by Alexander Ziegler in a publication Game Theory analysis of options (Ziegler, 2004), which uses arbitrage-free values for payoffs, discounted to the present time. These payoffs are later inserted into strategic games between players.

3.1 Three-step procedure

The model can be divided into three steps

1. At the start, the game is defined. Both players', actions are set. All choices and from them, resulting payoffs are specified.
2. The Second step involves evaluating the future uncertain players' payoffs. These payoffs are valued by option pricing theory. For every player, potential actions are used in the valuation formulas as parameters.
3. The final step is to solve the game. Players' optimal strategies are determined by using subgame perfection or backward induction (Ziegler, 2004).

The game theory analysis of options is a special application of game theory that instead of maximizing expected utility is maximizing the value of an option. Prioritizing maximization of option value over the expected utility provides arbitrage free value of payoffs with time value of money and price of risk taken into account. Author of this method sees the biggest advantage of his method in separation of the valuation problem (2nd step) from strategic interactions (3rd step). More complex problems are solved by minimization and maximization of the value of the option. The analysis often leads to finding first-order condition for minimum or maximum in the value of the option at each stage of the game.

For the clearer picture of how this method works, let's assume that player 1 chooses an action A and right after him player 2 chooses strategy B. Arbitrage free payoff function for chosen action for player 1 is $G(A, B, S, t)$ and for the second player a payoff function is

$H(A, B, S, t)$. Players' actions are comprised of selecting one of the parameters in the differential equation with its boundary conditions in order to maximize the value of payoffs.

At last second player chooses, action B to maximize his payoff sets,

$$\frac{\partial H(A, B, S, t)}{\partial B} = 0 \qquad 3.1$$

This is true only if action B is not a boundary solution. Thus, $\bar{B} = \bar{B}(A, S, t)$ yields the best strategy for player 2, which is also contingent on the player 1's action A. It can also depend on the underlying asset's value and time.

When player 1 wants to maximize his payoff G, he needs to anticipate player 2's subsequent action $\bar{B}$. Then:

$$\frac{dG(A, \bar{B}, S, t)}{dA} = \frac{\partial G(A, \bar{B}, S, t)}{\partial A} + \frac{\partial G(A, \bar{B}, S, t)}{\partial \bar{B}} \frac{d\bar{B}(A, S, t)}{dA} = 0 \qquad 3.2$$

This fact leads to player 1's optimal strategy $\bar{A} = \bar{A}(S, t)$, which depends only on the underlying asset's value and time. Then:

$$\frac{\partial G(A, \bar{B}, S, t)}{\partial \bar{B}} \frac{d\bar{B}(A, S, t)}{dA} = 0 \qquad 3.3$$

This shows indirect effect of player 1 action that is influenced by player 2's optimal action $\bar{B}$. This issue is the core of the backward induction, since when player 1 is making his decision he needs to anticipate what player 2's action will be.

3.2 When is the Method appropriate?

As mentioned above, game theory analysis of options uses value of an option as a proxy for each player's expected utility. This assumption may raise two fundamental questions:

The first question is: is it appropriate to use option value as a proxy value to expected utility? Option value is risk-adjusted present value of payoff. In other words, it is uncertain payoff to the current present value. There is a monotonic increasing relationship between the player's utility and option's value. Obviously, a monotonic increasing function doesn`t have to be linear. This fact leads to a situation when any utility maximization action done by a player causes the maximization of the option value and vice versa. Thus, the answer to 1st question is "yes".

The second question is: When is the option's value correct? The option's value is correct only if the time information structure of the game and players' preferences are coincident with uncertainty adjustment and time. In the economic theory there were specified conditions for the option pricing techniques. The main condition is that underlying's asset price density is lognormal. Therefore, the conditions for the Black-Scholes model must hold.

In situations where price density function isn't lognormal, classical option pricing techniques cannot be used. When this situation happens, the Game theory analysis of options is still applicable. Only with a difference that option prices have to be given by means of other option pricing techniques. In cases when other option pricing techniques cannot be used, then classical methods with the assumption of lognormality are used. The results obtained are only considered as approximate. Certainly, optimal strategies can be sensitive to the distribution of the underlying's asset value.

In order to use the method Game theory analysis of option, it is not necessary to have a tradeable underlying asset, which needs to be driven by the same source of uncertainty dB_t as tradeable asset. Brennan and Schwartz (1985) show that we can create a portfolio, which replicates the underlying asset. It is clear, that this method is not applicable in practice, but the method can be used as an approximation to asset, which allows us to closely replicate the value of the underlying asset at existing moderate costs (Ziegler, 2004).

3.3 What problems are suitable?

The methodology proposed my Ziegler (Ziegler, 2004) is mainly suitable for situations where it is difficult to evaluate players' utilities directly.This can be caused by various reasons.

First of possible reasons can be presence of uncertainty. Even when uncertainty is a significant determinant for future payoffs, option pricing techniques come with a useful way of performing the risk adjustment needed to value payoffs properly. The method is also very useful in situations where it is necessary to adjust the payoffs due to presence of some exogenously defined risk. Overall, this method is highly suited to analyse situations in which players are taking risk.

Second possible reason can be time, because we may not know when a payoff will be received. Thus, when time of payoff is not clearly specified, the payoffs cannot be easily discounted. It can be dependent on players´ decisions or it can be driven by exogenous uncertainty. Due to option pricing, the Method is highly suited to value payoffs occurring at random time.

The third possible reason can be that there are option value present in players´ payoffs. Payoff structure can be a nonlinear function of asset value thus, players have the possibility to make an optimal decision that affects their future payoffs. The method is highly suited to analyse real options. (Ziegler, 2004)

4 Bank runs

Banks provide loans, which cannot be sold very quickly at a high price. On the other hand, banks accept demand deposits, which can be withdrawn at any point of time. This discrepancy of liquidity between their liquid liabilities (deposits) and illiquid assets (loans) has caused problems for banks in situations, when the great deal of depositors wanted to withdraw their money at the same time. This situation is called a bank run. Over the years, banks have established procedures to stop runs. Governments in order to prevent bank runs from happening instituted the deposit insurance. Diamond and Dybvig (1983) developed a seminal model, which explains why banks are issuing deposits, which are more liquid than their assets. They explained why the banks are threatened by possible bank runs. The model was widely accepted, and it is used to understand the nature of banks run and show how banks should behave, in order to prevent them.

Diamond and Dybvig (1983) claim that banks have an important role as liquidity providers. Investors in demand for liquidity will prefer investing via a bank, rather than hold assets directly. Investors who require liquidity are not sure about their future consumption, which means that they are not sure how long they will hold the assets. As a result, investors are more willing to liquidate their assets on several possible dates, rather than on one single date. The bank run starts when too many depositors choose to withdraw their deposits at the same time.

4.1 The model

Let's consider a bank with two depositors: depositor A and depositor B. Both depositors have a deposit of X_0 money units at starting time t_0. Assume that bank´s equity holders add $x \geq 0$ units of capital, for each money units deposited in a bank at starting time. Bank uses these deposits to invest in risky assets, which value S follows geometric Brownian motion (4.1.) with initial price $X_0 = S_0$.

$$dS_t = \mu S_t dt + \sigma S_t dB_t \qquad 4.1$$

Thus, the total bank's asset value at time t equals to

$$2(x + 1)S_t \quad 4.2$$

Let's consider two interest rates. A risk-free interest rate r and interest rate r^*, which is paid on deposits, where $r > r^*$. Hence, the value of depositors´ claim at time t equals to

$$X(t) = X_0 e^{rt} \quad 4.3$$

In addition, assume that each depositor has a right to withdraw all his deposits at any time without a notice. If the outcome of depositors´ decision to withdraw his money, is a situation in which bank has to liquidate its assets, bank can liquidate its asset for proportional cost $\alpha > 0$.

Structure of a game between depositors and a bank is shown in a figure below (Fig. 2). At first bank equity holders need to decide how much capital x they will provide to the bank per unit of deposits. In the second step of the game, depositors are making a choice whether deposit their money or not. Then, bank management decides which investment strategy they are going to choose. At last depositors decide if it is the right time to run on a bank. They compare the face value of deposits $X(t)$ and the total value of bank´s assets S_t. If a bank run occurs, bank liquidates its assets and payoffs are realized (Ziegler, 2004).

4.2 Depositors deposit and run decision

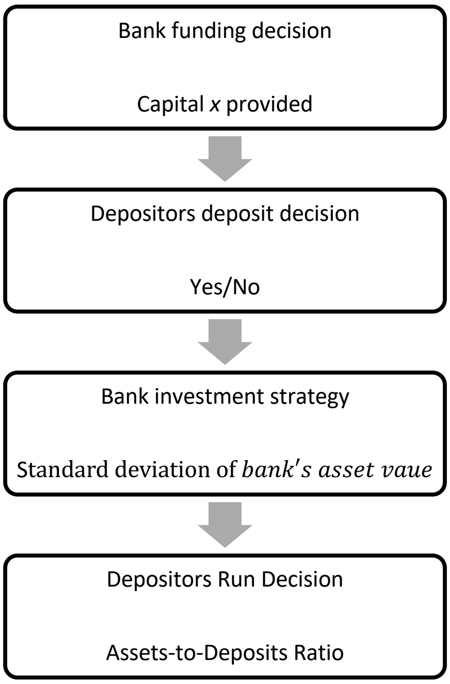
The main goal in this section is to analyse the first two steps of the model. It is necessary to clarify when depositors choose to run on a bank and how the bank should prevent bank runs from happening.

The vital question to ask when solving the game is: When the bank runs occur? To find answer for this question, 2 player game will be modelled. Total asset value, with liquidation costs taken into account, is:

$$2(x + 1)(1 - \alpha)S_t \quad 4.4$$

Instead of writing down the pay-off matrix and finding Nash equilibria out of this matrix, we simply consider the pay-offs for the players.

Fig. 2: Structure of the game between bank and depositors.



Source: Author´s compilation based on Ziegler (2004).

Pay-off function for a depositor, who withdraws first is

$$min[2(1+x)(1-\alpha)S_t, X(t)] \quad 4.5$$

Pay-off function for a depositor, who withdraws second is

$$max[0, 2(1+x)(1-\alpha)S_t - X(t)] \quad 4.6$$

With a 2-player game modelled, it is clear that a bank is exposed to a bank run, as soon as both players want to withdraw their funds first. This situation using previously defined pay-offs can be expressed as

$$min[2(1+x)(1-\alpha)S_t, X(t)] > max[0, 2(1+x)(1-\alpha)S_t - X(t)] \quad 4.7$$

Both pay-offs, when withdrawing first or withdrawing second are functions of asset value S_t. For the players, it is an advantage to withdraw first only when:

$$\frac{X(t)}{(1+x)(1-\alpha)} > S_t \qquad 4.8$$

Or

$$X(t) > (1+x)(1-\alpha)\,S_t \qquad 4.9$$

This condition says that a bank run can occur as soon as the value of bank assets net of liquidation costs is lower than the face value of money deposited in a bank.

Additional question to ask is, does the number of players affects the conditions for a run? To find an answer for this question, 3 player game will be modelled. Let, the total value of bank´s assets net of liquidation costs equals.

$$3(x+1)(1-\alpha)S_t \qquad 4.10$$

Payoff for a depositor, who withdraws first:

$$min[3(1+x)(1-\alpha)S_t, X(t)] \qquad 4.11$$

Payoff for a depositor, who withdraws second:

If $X(t) < 3(1+x)(1-\alpha)S_t - X(t)$ then:

$$max[0, 3(1+x)(1-\alpha)S_t - X(t)] \qquad 4.12a$$

Or if $X(t) > 3(1+x)(1-\alpha)S_t - X(t)$ then:

$$min\,[3(1+x)(1-\alpha)S_t - X(t), X(t)] = 3(1+x)(1-\alpha)S_t - X(t) \qquad 4.13b$$

Payoff for a depositor, who withdraws third:

$$max[0, 3(1+x)(1-\alpha)S_t - 2X(t)] \qquad 4.14$$

Analogous to 2-player game, a bank is vulnerable to a bank run, if all depositors want to withdraw their deposits first. As in the 2-player game, every payoff is a function of asset value S_t. For every player in the game, it is an advantage to withdraw first only if asset value net of liquidation is lower than the face value of deposits (4.9). The number of depositors (players) does not change the conditions for a run, therefore, in the later chapters we will work only with the 2-player game model.

$$X(t) > (1+x)(1-\alpha)\, S_t$$

It should be emphasized that, depositors run decision when the value of bank assets net of liquidation costs becomes lower than face value of deposits (4.9) has a major impact on the depositor's decision whether deposit or not to deposit their money in a bank in the first place. This equation (4.9) has an implication on the bank's funding policy too. In a situation when the initial bank capital x is not set high enough by the bank's shareholders, then the conditions for run can be met at initial time. If this situation occurs, depositors won't be willing to deposit their money in a bank.

When determining the amount of capital x, so that depositors are willing to deposit their money with a bank, the amount of capital x must be equal to a condition, where run on the bank won't occur. In that situation, value of bank assets net of liquidation costs is required to be higher than the face value of money deposited in a bank.

$$\frac{X(t)}{(1+x)(1-\alpha)} < S_t \qquad 4.15$$

Or

$$X(t) < S_t(1+x)(1-\alpha) \qquad 4.16$$

The no-run condition has to be satisfied also at initial time when $S_0 = X_0$. Then the previous conditions can be rewritten to:

$$(1+x)(1-\alpha) > 1 \qquad 4.17$$

or

$$x > \frac{\alpha}{1-\alpha} \qquad 4.18$$

This means that the depositors will be willing to deposit their money if the bank's shareholders agree to provide $\frac{\alpha}{1-\alpha}$ money units for each money unit deposited to compensate expected liquidation costs. Banks in economic theory come with an important role as providers of capital. The main goal isn't to finance investment in real assets, but the main purpose is to cover liquidation costs if the run occurs in order to encourage depositors to provide their own funds available for investment (Ziegler, 2004).

4.3 Bank's equity value

The main goal in this chapter is to establish an analytical formula for evaluation of the bank's equity, which is going to help with later analysis of initial funding motivation faced by the bank's investors. Taking into consideration previous depositor's run decision and shareholders initial investment, it is time to value bank equity using option pricing theory.

We assume that equity holders provided capital share x, which is necessary in order not to trigger depositors to run on a bank. Volatility of bank´s asset value is represented by σ and equity's holders´ recapitalization share by w. More importantly, we make additional assumptions:

- *Assumption 1:* Bank runs are happening very swiftly and investors aren't able to provide new required capital for a bank. Especially in a situation, when the bank run is a possibility, or it is happening. In this state the recapitalization share w is equal to zero ($w = 0$).
- *Assumption 2:* When conditions for a bank run are satisfied (4.8), bank run immediately takes place. Bank runs occurs as soon as the value of bank assets net of liquidation costs is lower than the face value of money deposited in a bank:

$$\frac{X(t)}{(1+x)(1-\alpha)} > S_t$$

  When the bank run takes place, bank has to be liquidated and equity holder's payoff is equal to zero.

- *Assumption 3:* When the bank run does not occur, and bank is willing to liquidate its current projects bank is able do so, but at a proportional variable cost of β, where $\beta < \alpha$ manifests the common fact that bank's assets can be sold at a higher price, when bank is not in desperate need of money. Liquidation can happen slowly and intentionally. In case of a bank run, bank needs to liquidate quickly, which triggers a fire sale (Ziegler, 2004).

With these three additional assumptions in mind, let´s return to our model. Bank with two or more depositors holds a perpetual down-and-out call option on asset value net of liquidation costs,

$$S_t(1+x)(1-\beta) \qquad 4.19$$

with a time-varying strike price of $X(t)$ and knockout price, which is dependent on the depositor's run decision.

$$K(t) = S_t(1+x)(1-\beta) \qquad 4.20$$

Subtituing S_t

$$K(t) = \frac{X(t)}{(1+x)(1-\alpha)}(1+x)(1-\beta)$$

$$K(t) = \frac{(1-\beta)}{(1-\alpha)}X(t) \qquad 4.21$$

Now, we need to set C_∞, which denotes the value of perpetual down-and-out call option. So we have

$$C_\infty((1+x)(1-\beta)S_t, K(t)) \qquad 4.22$$

Make the change in variables

$$V = \frac{S_t(1+x)(1-\beta)}{X(t)} \qquad 4.23$$

And define

$$F(V) = \frac{C_\infty}{X(t)} \qquad 4.24$$

F satisfies the ordinary differential equation

$$\frac{1}{2}\sigma^2 V^2 F'' + (r-r^*)VF' - (r-r^*)F = 0 \qquad 4.25$$

Boundary condition is

$$F\left(\frac{1-\beta}{1-\alpha}\right) = 0 \qquad 4.26$$

Then, the solution is

$$F(V) = V - \left(\frac{1-\beta}{1-\alpha}\right)^{1+2\frac{r-r^*}{\sigma^2}} V^{-2\frac{r-r^*}{\sigma^2}} \qquad 4.27$$

Let define γ^*

$$\gamma^* = 2\frac{r-r^*}{\sigma^2} \qquad 4.28$$

Then

$$F(V) = V - \left(\frac{1-\beta}{1-\alpha}\right)^{1+\gamma^*} V^{-\gamma^*} \qquad 4.29$$

Value of perpetual down-and-out call option can be defined if original variables are substituted into equation

$$F(V) = \frac{C_\infty}{X(t)}$$

$$V - \left(\frac{1-\beta}{1-\alpha}\right)^{1+\gamma^*} V^{-\gamma^*} = \frac{C_\infty}{X(t)}$$

$$C_\infty = VX(t) - \left(\frac{1-\beta}{1-\alpha}\right)^{1+\gamma^*} V^{-\gamma^*}X(t)$$

$$C_\infty = S_t(1+x)(1-\beta) - \left(\frac{1-\beta}{1-\alpha}\right)^{1+\gamma^*} \left(\frac{S_t(1+x)(1-\beta)}{X(t)}\right)^{-\gamma^*} X(t)$$

$$C_\infty = (1+x)(1-\beta)\left(S_t - \left(\frac{X(t)}{(1+x)(1-\alpha)}\right)^{\gamma^*+1} {S_t}^{-\gamma^*}\right) \qquad 4.30$$

Value of the perpetual down-and-out call option C_∞, which also represents the value of bank's equity, when depositors are willing to run on the bank equals to asset value net of liquidation costs, which is represented by $S_t(1+x)(1-\beta)$ minus the losses we expect when the run occurs. These losses are equal to the discount, which results from the knock-out feature of the option. This discount feature can be described intuitively by rewriting equity value using the definition of S_t

$$C_\infty = (1+x)(1-\beta)\left(S_t - \overline{S_t}\left(\frac{S_t}{\overline{S_t}}\right)^{-\gamma^*}\right) \quad 4.31$$

Equation shows that discount factor is only the value of assets, which are lost when bank run occurs represented by $(1+x)(1-\beta)\overline{S_t}$, times the term $\left(\frac{S_t}{\overline{S_t}}\right)^{-\gamma^*}$ which represents the risk-neutral probability of the bank run appearance, which is adjusted for the time value of money (Ziegler, 2004).

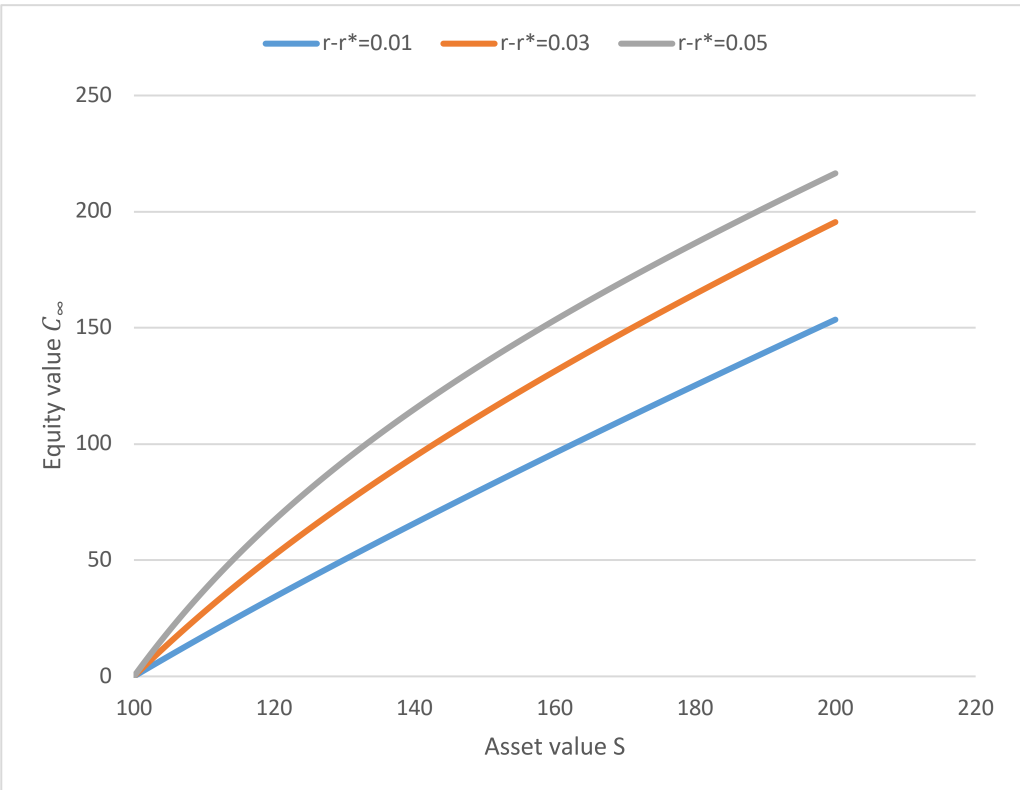
Figure below (Fig. 3) gives an example of the dependence of bank equity value C_∞ on asset value S for different values of deposit spread. Let's use following parameter values: $X = 100, \alpha = 0.2, \beta = 0.05, x = 0.25$ and $\sigma = 0.2$. Let's examine the dependence on three different scenarios:

a) Deposit spread equals to 0.01

b) Deposit spread equals to 0.03

c) Deposit spread equals to 0.05

Fig. 2: Dependence of equity value C_∞ on asset value S – variable deposit spread



Source: Author´s compilation based on Ziegler (2004)

From Figure above (Fig. 3), it can be observed that equity value is increasing with a wider deposit spread. This fact is intuitive, because the wider the deposit spread $r - r^*$ the more profitable the bank and therefore the higher value of bank's equity.

Next figure (Fig. 4), gives an example of the dependence of bank equity value C_∞ on asset value S for different values of standard deviation of asset value σ. Let's use parameter values: $X = 100, \alpha = 0.2, \beta = 0.05, x = 0.25$ and $r - r^* = 0.02$. Again, examine the dependence on three different scenarios:

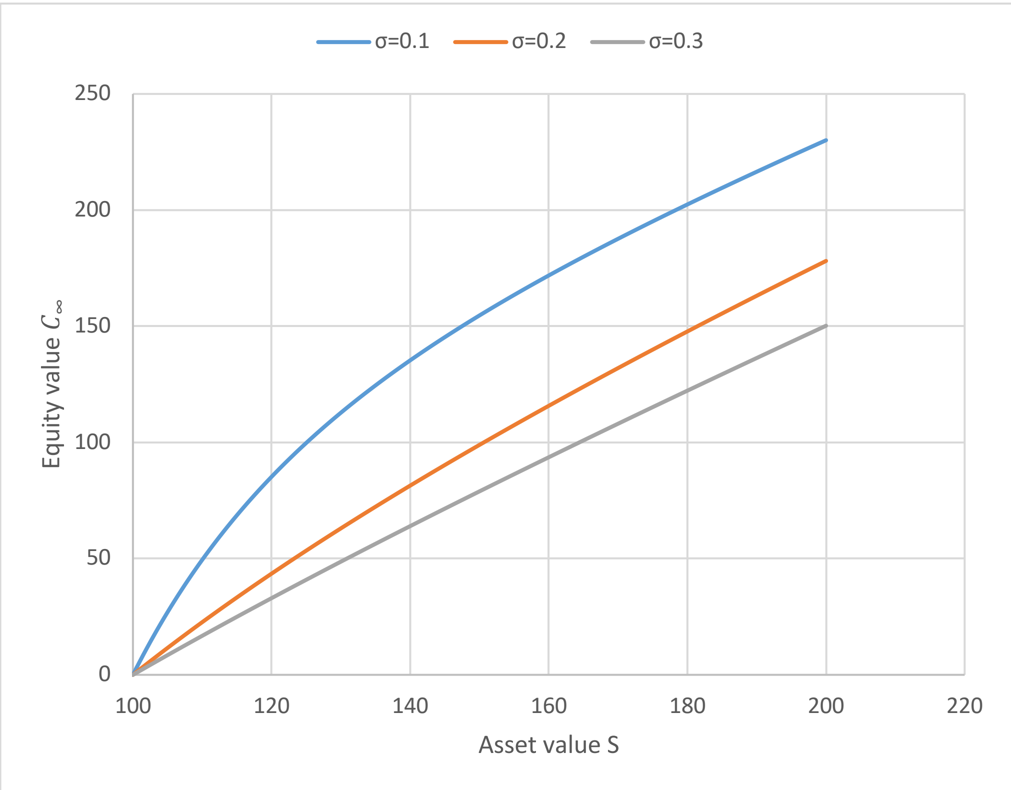
a) asset risk σ to 0.1

b) asset risk σ to 0.3

c) asset risk σ to 0.3

It can be observed that equity value is decreasing with higher volatility of the asset value. Bank is trying to minimize the volatility.

Fig. 3: Dependence of equity value C_∞ on asset value S – variable standard deviation of asset value



Source: Author´s compilation based on Ziegler (2004)

4.4 The Bank investment motivation when the bank run is a possibility

Once the bank's equity is valued and depositor's run decision is taken into consideration, the bank's investment incentive can be analyzed. Vital question that needs be answered is "How does the possibility of a bank run affect investment strategy of a bank?". Let's take into consideration that the bank's shareholders can't recapitalize the bank in a situation when asset value is dramatically falling. Then, bank's equity value is given by previously derived equation (4.29)

$$C_\infty = (1+x)(1-\beta)\left(S_t - \left(\frac{X(t)}{(1+x)(1-\alpha)}\right)^{\gamma^*+1} S_t{}^{-\gamma^*}\right)$$

Where γ^*

$$\gamma^* = 2\frac{r-r^*}{\sigma^2}$$

Partial derivate of this equation with respect to γ^* yields

$$\frac{\partial C_\infty}{\partial \gamma^*} = (1+x)(1-\beta)\left[- S_t{}^{-\gamma^*}\left(\frac{X(t)}{(1+x)(1-\alpha)}\right)^{\gamma^*+1} \ln\left(\frac{X(t)}{(1+x)(1-\alpha)}\right) - \left(\frac{X(t)}{(1+x)(1-\alpha)}\right)^{\gamma^*+1} S_t{}^{-\gamma^*}\ln(S_t)\right] \quad 4.32$$

Simplifying:

$$\frac{\partial C_\infty}{\partial \gamma^*} = -S_t{}^{-\gamma^*}(1+x)(1-\beta)\left(\frac{X(t)}{(1+x)(1-\alpha)}\right)^{\gamma^*+1}\left(\ln\left(\frac{X(t)}{(1+x)(1-\alpha)}\right) - \ln(S_t)\right) \quad 4.\ 33$$

This equals to

$$\frac{\partial C_\infty}{\partial \gamma^*} = -S_t{}^{-\gamma^*}(1+x)(1-\beta)\left(\frac{X(t)}{(1+x)(1-\alpha)}\right)^{\gamma^*+1}\left(\ln\left(\frac{\frac{X(t)}{(1+x)(1-\alpha)}}{S_t}\right)\right) \quad 4.34$$

Partial derivativ of the equity value with respect to γ^* is positive since the bank run doesn't occur until value of the bank's assets net of liquidation cost is lower than the face value of money deposited in a bank (4.14).

$$S_t > \frac{X(t)}{(1+x)(1-\alpha)}$$

So that

$$\ln\left(\frac{\frac{X(t)}{(1+x)(1-\alpha)}}{S_t}\right) < 0 \qquad 4.35$$

Since we assume that $r > r^*$ we have

$$\frac{\partial \gamma^*}{\partial \sigma^2} = -\frac{\gamma^*}{\sigma^2} < 0 \qquad 4.36$$

which yeilds

$$\frac{\partial C_\infty}{\partial \sigma^2} = \frac{\partial C_\infty}{\partial \gamma^*}\frac{\partial \gamma^*}{\partial \sigma^2} < 0 \qquad 4.37$$

Expression (4.36) demonstrates that, if assets are fairly priced, then shareholder's preferred action is to reduce asset volatility as much as possible. That means setting $\sigma = 0$. In other words, the higher possibility of a bank run forces bank to reduce its asset´s volatility. Bank's shareholders do not choose the level of the asset value at which it is liquidated, because it is driven by depositors run decision. The possibility of a bank run thus disciplines the bank's investment strategy (Ziegler, 2004).

Result (4.36) yields to a justification of a demandable debt, which solves the problem of risk-shifting in a way which allows depositors to withdraw their deposits as soon as the value of bank's investments drops below pre-specified value. Calomiris and Kahn (1991) show that demandable debt can be represented as an incentive scheme to discourage the bank's management from fraud. Depositors' right to withdraw very early impels them to closely monitor the bank. More than that, this effect illustrates the disciplining effect that the bank runs have.

Because of the fact, that only bank's equity holders are exposed to the risk, and the reduction of risk result (4.36), the bank will decide to invest its funds into low-risk assets. The bank may have to invest everything into risk-free assets. Now, we will compute the value of equity in this case. Let assume that the risk-free asset can be liquidated at zero cost and consider B_0 as an initial price of a risk-free asset and $B(t)$ as a price of risk-free asset in time t. Then

$$B(t) = B_0 e^{rt} \qquad 4.38$$

Taking capital x into consideration, then total asset value at time t equals to

$$(1+x)B(t) = (1+x)B_0 e^{rt} \qquad 4.39$$

In this risk-less scenario, depositors never run on a bank. Because bank's equity value only depends on the bank's equity holder's liquidation strategy. Equity holders' pay-off if they choose to liquidate the bank at time *t* equals to

$$L(t) = (1+x)B(t) - X(t) \qquad 4.40$$

Where $X(t)$ equals to

$$X(t) = X_0 e^{r^* t} \qquad 4.41$$

Then

$$L(t) = (1+x)B_0 e^{rt} - X_0 e^{r^* t} \qquad 4.42$$

Presuming, that amount initially invested in the risk-free assets equals to the amount deposited initially $X_0 = B_0$, then the equity holder's pay-off can be written as

$$L(t) = X_0 \left((1+x) - e^{rt} - e^{r^* t}\right) \qquad 4.43$$

Then, the present value is

$$L_0(t) = X_0 \left((1+x) - e^{(r^* - r)t}\right) \qquad 4.44$$

To determine the bank's equity holders' optimal liquidation strategy t, we computethe partial derivative of present value with respect to t.

$$\frac{\partial L_0(t)}{\partial t} = 0 + X_0\left(0 - (r^* - r)e^{(r^* - r)t}\right) \qquad 4.45$$

which simplifies as

$$\frac{\partial L_0(t)}{\partial t} = (r - r^*)X_0 e^{(r^*-r)t} > 0 \quad 4.46$$

Equation (4.45) shows that the bank's equity holders are never going to choose an option, which results in liquidating the bank. To justify this statement let's think about interest rates. The bank from its activities earns an interest rate r on risk-free assets, but on deposits bank pays an interest rate r^*. It is clear that $r > r^*$ is profitable. The longer bank keeps this interest rate differential, the larger the present value of bank's cumulative profits. Now, it is clear why the bank's shareholders won't liquidate the bank, therefore the value of equity is given by

$$L = \lim_{t\to\infty} L_0(t) = \lim_{t\to\infty} X_0 \left((1 + x) - e^{(r^*-r)t}\right) \quad 4.47$$

$$L = X_0(1 + x) \quad 4.48$$

Equation (4.47) determines the value of equity only when the bank invests everything into risk-free assets. At initial time, when bank is making its investment decision

$$L > C_\infty$$

$$X_0(1 + x) > X_0(1 + x)(1 - \beta)\left(1 - \left(\frac{1}{(1 + x)(1 - \alpha)}\right)^{\gamma^*+1}\right) \quad 4.49$$

This condition (4.48) shows a situation, in which bank's shareholders are seeking to maximize the value of the bank's equity. They will decide to invest everything into risk-free assets resulting in a situation in which the bank run never occurs.

4.5 Bank's funding decision

Now let's go back in the game and analyse bank's equity holder's decision at initial time. Our goal in this chapter is to determine what exact amount of capital x should be provided to maximize shareholder's expected profit.

At initial time when the bank is founded, shareholders agree to provide capital only if the value of equity at current time is greater than the capital cost xX_0. Shareholders motivation in this case is that expected profit G from this investment must be positive (Ziegler, 2004).

$$G = C_\infty - xX_0 \qquad 4.50$$

If we consider that asset risk is positive and $X_0 = S_0$, then the expected profit from intermediation is given by

$$G = (1+x)(1-\beta)\left(X_0 - \left(\frac{X_0}{(1+x)(1-\alpha)}\right)^{\gamma^*+1} {X_0}^{\gamma^*}\right) - xX_0 \qquad 4.51$$

Simplifying

$$G = X_0\left(1 - \beta(1+x) - \frac{(1-\beta)(1+x)}{\big((1+x)(1-\alpha)\big)^{\gamma^*+1}}\right) \qquad 4.52$$

Expected profit G is positive as long as

$$1 - \beta(1+x) - \frac{(1-\beta)(1+x)}{\big((1+x)(1-\alpha)\big)^{\gamma^*+1}} > 0 \qquad 4.53$$

Simplifying

$$\big((1+x)(1-\alpha)\big)^{\gamma^*+1} > \frac{(1-\beta)(1+x)}{1-\beta(1+x)} \qquad 4.54$$

Condition can be rewritten as

$$\gamma^* > \frac{\ln\big((1-\beta)(1+x)\big) - \ln\big(1-\beta(1+x)\big)}{\ln\big((1+x)(1-\alpha)\big)} - 1 \qquad 4.55$$

Condition (4.54) means that, if expected profit from funding the bank has to be positive, The deposit spread must exceed a certain value, which for a given capital x is proportional to the instantaneous variance σ of the bank's asset value. Remembering that γ^* equals to

$$\gamma^* = 2\frac{r - r^*}{\sigma^2}$$

We get:

$$2\frac{r - r^*}{\sigma^2} > \frac{\ln\big((1-\beta)(1+x)\big) - \ln\big(1-\beta(1+x)\big)}{\ln\big((1+x)(1-\alpha)\big)} - 1 \qquad 4.56$$

Implying a minimum deposit spread proportional to variance

$$r - r^* > \frac{\sigma^2}{2}\left(\frac{\ln\big((1-\beta)(1+x)\big) - \ln\big(1-\beta(1+x)\big)}{\ln\big((1+x)(1-\alpha)\big)} - 1\right) \qquad 4.57$$

The minimum deposit spread is proportional to the variance of asset value σ^2, because the discount factor in the equity value, which arises in case of a bank run, is given by

$$(1+x)(1-\beta)\left(\frac{X(t)}{(1+x)(1-\alpha)}\right)^{\gamma^*+1} S_t^{-\gamma^*} \qquad 4.58$$

When volatility changes, then the value of this expression (4.57) changes as well. The main idea is simple, when σ rises, then the probability of bank run for a given deposit spread $r - r^*$ increases as well. The opposite effect can be compensated through a higher deposit spread.

Given an example of the dependence of expected profit from funding the bank on deposit spread $r - r^*$, for different values of σ. Let's use parameter values: $X = 100, \alpha = 0.15, \beta = 0.05, x = 0.25$. Again, examine the dependence on three different scenarios:

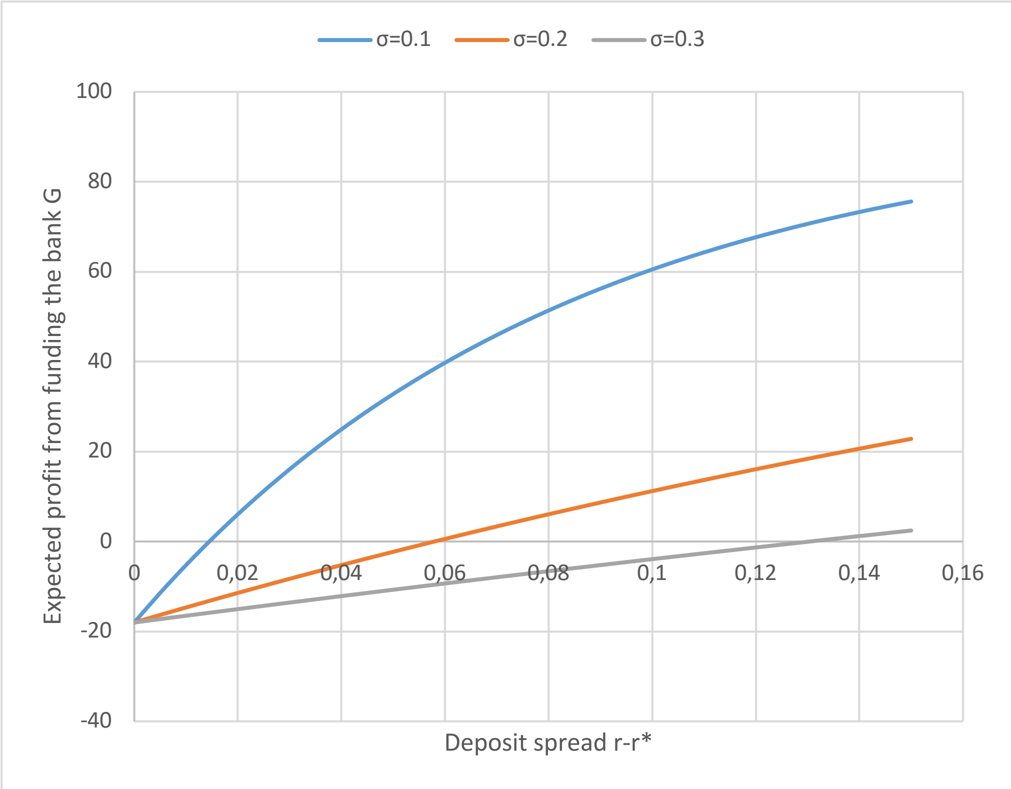
a) asset risk σ equals to 0.1

b) asset risk σ equals to 0.2

c) asset risk σ equals to 0.3

Figure (Fig. 4) shows, that for low values of deposit spread $r - r^*$, it is not profitable to fund a bank. When interest rate spread increases so does the bank's equity value. As shown in the figure, if the interest rate spread exceeds certain value, then funding of the bank becomes profitable for its shareholders. The critical spread value in which funding becomes profitable depends on volatility σ. The higher the σ, the higher the required deposit spread.

Fig. 4 Dependance of expected profit from funding the bank on deposit spread



Source: Author´s compilation based on Ziergler (2004)

4.6 Optimal level of capital

Partial derivative of shareholder's expected profit (4.51) with respect to the funding share x yields

$$\frac{\partial G}{\partial x} = X_0\left(-\beta - \frac{(1-\beta)\big((1+x)(1-\alpha)\big)^{\gamma^*+1} - (1-\beta)(1+x)(1-\alpha)(\gamma^*+1)\big((1+x)(1-\alpha)\big)^{\gamma^*}}{\big((1+x)(1-\alpha)\big)^{\gamma^*+1}\big((1+x)(1-\alpha)\big)^{\gamma^*+1}}\right) \tag{4.59}$$

Simplifying yields

$$\frac{\partial G}{\partial x} = X_0\left(-\beta + \frac{\gamma^*(1-\beta)}{\big((1+x)(1-\alpha)\big)^{\gamma^*+1}}\right) \tag{4.60}$$

Now, let's set this expression equal to zero

$$X_0\left(-\beta + \frac{\gamma^*(1-\beta)}{\big((1+x)(1-\alpha)\big)^{\gamma^*+1}}\right) = 0 \tag{4.61}$$

This leads to

$$(1+x)(1-\alpha) = \left(\gamma^* \frac{1-\beta}{\beta}\right)^{\frac{1}{\gamma^*+1}} \qquad 4.62$$

Solving for x leads to

$$x = \frac{\left(\gamma^* \frac{1-\beta}{\beta}\right)^{\frac{1}{\gamma^*+1}}}{1-\alpha} - 1 \qquad 4.63$$

Let´s denote $\bar{x}$ as shareholders´ optimal initial investment capital, which is given by the equation (4.62). Now, let's solve how the initial investment depends on the magnitude of liquidations costs α, β and riskiness of bank's asset and interest rate spread.

At first, examine how liquidation cost α affects the optimal initial investment. To do so, let´s compute partial derivative of optimal initial investment with respect to liquidation costs in the case of a run.

$$\frac{\partial \bar{x}}{\partial \alpha} = \left(\gamma^* \frac{1-\beta}{\beta}\right)^{\frac{1}{\gamma^*+1}} \frac{1}{(1-\alpha)^2} > 0 \qquad 4.64$$

If proportional liquidation cost in the event of a bank run α increases, then the equity holders are forced to provide additional capital to a bank.

Now, let's examine the effect of β, proportional liquidation cost if no run occurs. We compute the partial derivative of optimal initial investment with respect to liquidation cost β.

$$\frac{\partial \bar{x}}{\partial \beta} = -\frac{1}{\beta^2} \frac{\gamma^*}{(1-\alpha)(\gamma^*+1)} \left(\frac{\gamma^*(1-\beta)}{\beta}\right)^{\frac{\gamma^*}{\gamma^*+1}} > 0 \qquad 4.65$$

Increase in β, has the opposite effect than increase in α. With higher liquidation costs β shareholders' capital commitment should lower. Increase in α and β has the negative effect on the profitability of a bank. Shareholders can reduce the negative effect of α by increasing optimal capital $\bar{x}$, thereby reducing the probability of a bank run. On the other hand, in case of β, shareholders are choosing to reduce optimal initial investment $\bar{x}$ (Ziegler, 2004).

Now, let's examine the effect of changes within the deposit spread, and the overall riskiness of bank's assets on the optimal capital. Again, let´s compute the partial derivative of the optimal initial investment with respect to the instantaneous standard deviation.

$$\frac{\partial \bar{x}}{\partial \gamma^*} = \frac{1}{(1-\alpha)(\gamma^*+1)}\left(\frac{1}{\gamma^*} - \frac{\ln\frac{\gamma^*(1-\beta)}{\beta}}{\gamma^*+1}\right)\left(\frac{\gamma^*(1-\beta)}{\beta}\right)^{\frac{\gamma^*}{\gamma^*+1}} \qquad 4.66$$

The equation (4.65) shows that it is impossible to say what exact effect the change in the deposit spread or asset risk will have on optimal level of capital $\bar{x}$. Increase in the deposit spread or reduction of the asset volatility causes two conflicting effects. With higher deposit spread and lower volatility bank is more profitable, so shareholders would be willing to invest more money. But, on the other hand this effect decreases the probability of bank run, which causes reduction in optimal initial investment. The determination of which effect will dominate can only be done by case-by-case basis. Let's substitute initial capital (4.62).

$$x = \frac{\left(\gamma^*\frac{1-\beta}{\beta}\right)^{\frac{1}{\gamma^*+1}}}{1-\alpha} - 1$$

into expected profit (4.51)

$$G = X_0\left(1 - \beta(1+x) - \frac{(1-\beta)(1+x)}{\left((1+x)(1-\alpha)\right)^{\gamma^*+1}}\right)$$

It yields

$$G = X_0\left(1 - \beta\frac{\left(\gamma^*\frac{1-\beta}{\beta}\right)^{\frac{1}{\gamma^*+1}}}{1-\alpha} - \frac{(1-\beta)\frac{\left(\gamma^*\frac{1-\beta}{\beta}\right)^{\frac{1}{\gamma^*+1}}}{1-\alpha}}{\left(\left(\gamma^*\frac{1-\beta}{\beta}\right)^{\frac{1}{\gamma^*+1}}\right)^{\gamma^*+1}}\right) \qquad 4.67$$

Simplifying we obtain

$$G = X_0\left(1 - \frac{\beta}{1-\alpha}\left(\gamma^*\frac{1-\beta}{\beta}\right)^{\frac{1}{\gamma^*+1}} - \frac{1-\beta}{1-\alpha}\left(\gamma^*\frac{1-\beta}{\beta}\right)^{\frac{1}{\gamma^*+1}-1}\right)$$

$$G = X_0 \left(1 - \frac{\beta}{1-\alpha} \frac{1+\gamma^*}{\gamma^*} \left(\gamma^* \frac{1-\beta}{\beta} \right)^{\frac{1}{\gamma^*+1}} \right) \qquad 4.68$$

To exactly state when the bank will be profitable or not, depends on whether the expression (4.69) is positive. The bank's profitability is dependent on three factors. The first one is γ^*, which measures how the deposit spread $(r - r^*)$ compares with instantaneous variance of a bank's asset value. Second factor is α, which measures proportional liquidation costs in the event of a run, and the last third factor β, measures proportional liquidation costs, if no bank run occurs. The higher the first factor γ^* and the lower the liquidation factors α and β, the higher the profit (Ziegler, 2004).

Bank will try to set γ^* as high as possible by reducing asset deviation σ. Optimal initial investment, $\bar{x}$ can be determined by taking the limit as $\gamma^* \rightarrow \infty$ of expression (4.61) which yields:

$$(1+x)(1-\alpha) = \lim_{\gamma^* \to \infty} \left(\gamma^* \frac{1-\beta}{\beta} \right)^{\frac{1}{\gamma^*+1}}$$

$$(1+x)(1-\alpha) = e^{\lim_{\gamma^* \to \infty} \left(\frac{\ln\left(\gamma^* \frac{1-\beta}{\beta}\right)}{\gamma^*+1} \right)}$$

$$(1+x)(1-\alpha) = e^{\lim_{\gamma^* \to \infty} \left(\frac{1}{\gamma^*} \right)} \qquad 4.69$$

Then

$$(1+x)(1-\alpha) = e^0 = 1 \qquad 4.70$$

Expression (4.69) states, that the value x, which maximizes the shareholders' profits is set as high as the depositors only agree to deposit their money into the bank. Bank's intention is to invest everything in an almost risk-free asset. Investor's decision to fund a bank is profitable and can be confirmed with

$$\lim_{\sigma \to 0} G = \lim_{\gamma^* \to \infty} X_0 \left(1 - \frac{\beta}{1-\alpha} \frac{1+\gamma^*}{\gamma^*} \left(\gamma^* \frac{1-\beta}{\beta} \right)^{\frac{1}{\gamma^*+1}} \right)$$

$$\lim_{\sigma\to 0} G = X_0\left(1 - \frac{\beta}{1-\alpha}\lim_{\gamma^*\to\infty}\frac{1+\gamma^*}{\gamma^*}\; e^{\lim_{\gamma^*\to\infty}\left(\frac{\ln\left(\gamma^*\frac{1-\beta}{\beta}\right)}{\gamma^*+1}\right)}\right)$$

$$\lim_{\sigma\to 0} G = X_0\left(1 - \frac{\beta}{1-\alpha}\right) \qquad 4.71$$

This expression is positive only, if

$$\alpha + \beta < 1 \qquad 4.72$$

That means, that the both liquidation costs must not be too high. Both these liquidation costs are relevant for the overall bank's profitability (Ziegler, 2004).

5 Case study: Air Bank a. s.

In final chapter of this thesis we will focus on the Czech bank Air Bank. Air Bank was chosen due to the fact that the computation of equity value triggering a bank run examined in the previous part of this thesis is well suited for retail banks, which fund their business from deposits collected from individual depositors. Air Bank complies with this requirement, since deposits comprise 98 % of bank's liabilities as is depicted in a simplified balance sheet from the third quarter of 2019 below (tab. 3)

Tab. 3: Air Bank´s balance sheet to 30.9.2019

<table>
<tr><td colspan="2">Air Bank´s balance sheet</td></tr>
<tr><td>Liquid assets (24 bn. CZK)</td><td rowspan="2">Deposits (104 bn. CZK)</td></tr>
<tr><td rowspan="2">Loans (88 bn. CZK)</td></tr>
<tr><td>Equity (8,8 bn. CZK)</td></tr>
<tr><td>Total 115 bn. CZK</td><td>Total 115 bn. CZK</td></tr>
</table>

Source: https://www.airbank.cz/file-download/informace-o-air-bank-k-30-9-2019-cast-1.xlsx

The model used to compute equity value triggering bank run also requires, in order to compute one of the model's variables, an initial investment made by shareholders when founding the bank. Due to this fact, it is very convenient to use Air Bank, because the bank was founded in 2011 and all necessary documents are easily accessible online. Air Bank is one of the youngest banks in the Czech banking market and one of Air Bank's main principles is to provide low to zero banking fees for common banking services like maintaining a bank account. Since its foundation in 2011, the bank has been trying to simplify banking to a level which can be easily understood by a customer without a business education.

From the beginning, one of the Air Bank's main goals has been to build a stable base of loyal clients. The management of Air Bank has been successful in achieving this goal, and has even surpassed expectations. The Chairman of the Board, Michal Strcula, claimed in the annual report from 2016 (Air Bank, 2016) that for the first five years of business they had planned for half the yearly increase in clients, compared to actual yearly increase. Air Bank reported that in 2019 they managed to increase the number of clients from 673 thousand to 788 thousand clients (Air Bank, 2019).

The issue of bank runs is always more prominent during crises than during economic booms. According to the IMF global economic outlook, (IMF, 2020) the Covid-19 pandemic will have a much bigger impact on the global economy than the financial crisis of 2017 and 2018. In order to protect lives, widespread closures of economies and lockdowns are required to slow the spread of the virus. Therefore, the health crisis has had a critical impact on global economic activity. Workplace closures are disrupting supply chains, and with that comes lower productivity. Fear of contagion, lay-offs, and declines in income are slowly increasing the uncertainty, forcing people to spend less, which then triggers further business closures and job losses. This shutdown of a significant portion of the economy will take its toll on global economic growth. The IMF expects (IMF, 2020) a sharp contraction of 3 % on global economic growth in 2020. The Czech Republic should, according to the IMF report, expect a contraction of 6,5 %. With the contraction of economic growth, business closures, and declines in income, unemployment rates can rise, thus increasing the risk of widespread defaults. From history, we know that in a times of economic distress, the probability of bank runs increases.

The Czech National Bank has already proposed a recommendation (CNB, 2020) for Czech banks to pause their dividend payments in order to prevent capital vulnerability. Also, the Czech National Bank has increased the weekly number of monetary operations in order to provide liquidity to banks. This means that banks have more possibilities to increase their liquid position. Instead of one weekly monetary repo operation, there are three possible week-days to buy liquidity for fixed repo rate without any markup.

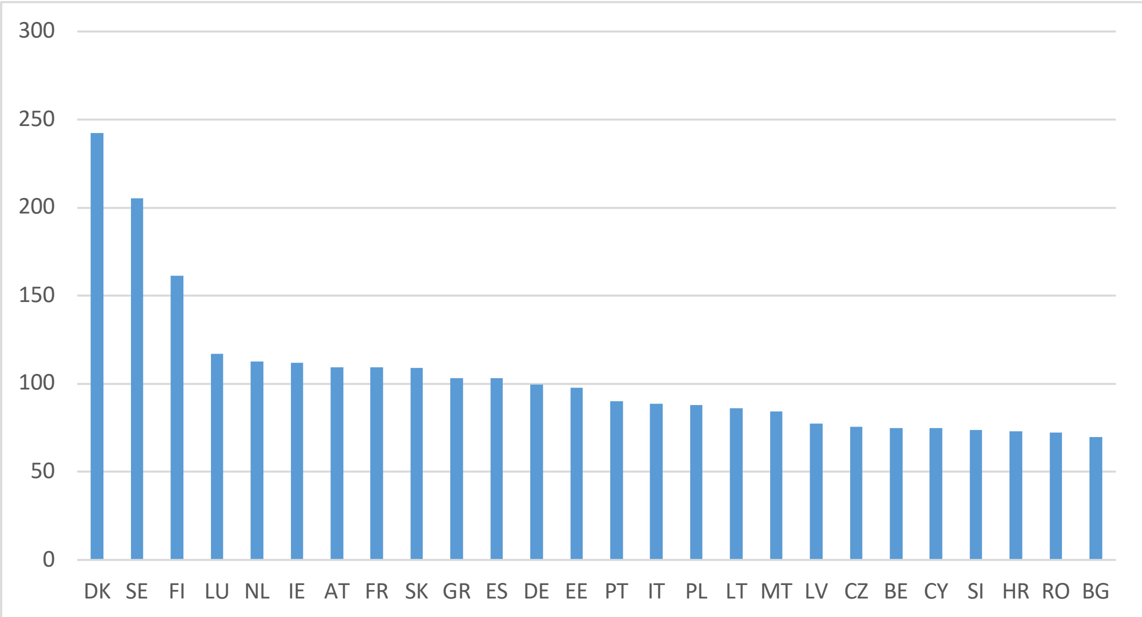
Even though there have been no sudden bank runs within the past few months, according to The Wall Street Journal (Ackerman, 2020) a few branches of U.S. banks and credit unions near New York and Seattle are reporting depositors making big withdrawals, sometimes reaching more than 100,000 U.S. dollars. These withdrawals appear to be motivated by recent financial-market confusion over the coronavirus pandemic.

5.1 Factors affecting the susceptibility of Czech banks to a bank run

The Czech banking system is characterized by relatively high liquidity. The ratio of loans to deposits is one of the lowest in the European Union (Fig. 5). The rest of bank assets are held in the form of short-term interbank deposits, government bonds, and deposits in the central bank. Therefore, the Czech National Bank absorbs liquidity rather than providing it (Klepkova Vodova, 2017).

The study done by Klepkova Vodová and Stavárek (2017) on factors affecting the susceptibility of commercial banks in the Visegrad group to a bank run shows that the susceptibility of Czech banks is primarily determined by two macroeconomic factors out of seven possible economic factors and two bank-specific macroeconomic factors out of seven possible bank specific factors.

Fig. 5: Loan to deposit ratio – EU 2019 Q4



Source: ECB Statistical Data Warehouse; http://sdw.ecb.europa.eu/reports.do?node=1000003329&fbclid=IwAR0-8Iz4MrhlOEXSWMQdf7xipj2C96jQzdN9rKvf-MaEV0PMXo9KUdXuu-Y

According to Klepková Vodová and Stavárek (2017), the two most significant macroeconomic factors affecting the susceptibility of bank runs in the Czech Republic are unemployment rate and interest rates on loans. Unemployment rate can be regarded as an indicator for the overall health of an economy, therefore a higher values of unemployment rate increase vulnerability of banks to a run. The second macroeconomic factor, interest rates on loans, is connected with bank's profitability. The higher the interest rate, the higher the bank's profitability from lending. Furthermore, with higher accumulated profits from lending, a bank is better prepared to withstand a crisis.

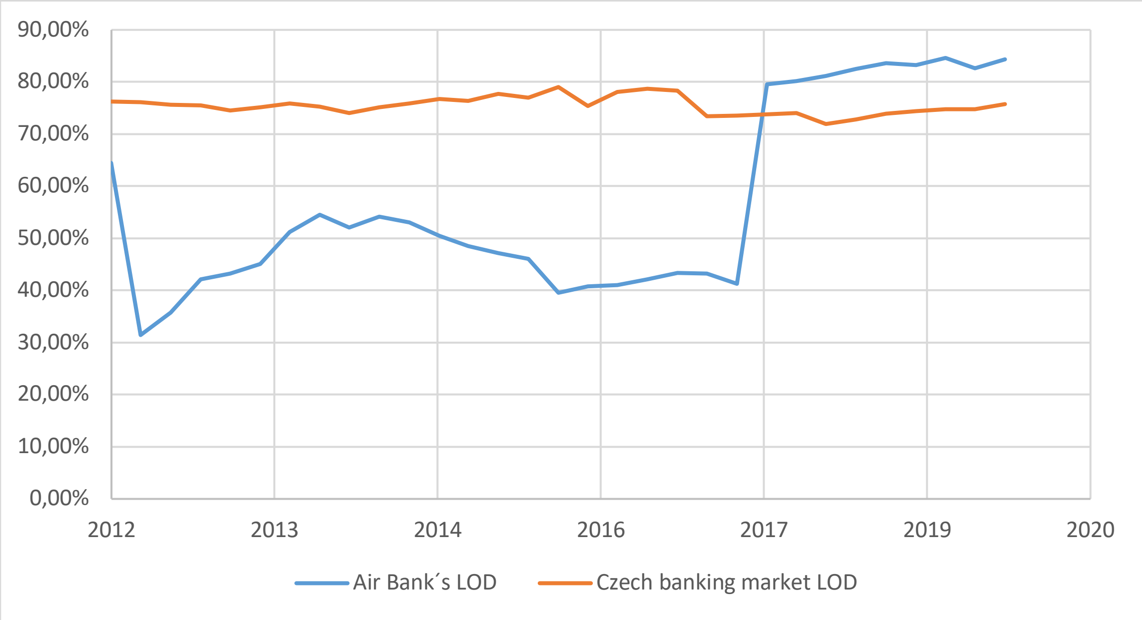
Without a doubt, it is certain that the coronavirus pandemic will cause an increase in the unemployment rate worldwide. The United States already reports that in the first week of April more than 6,6 million of Americans lost their jobs, and with this trend in every state, economists predict that the unemployment rate can reach 15%. (Rushe, 2020). In the Czech Republic the analysts predict that unemployment rate could even reach 10% from current 3%. This rapid increase in unemployment rate increases the susceptibility to and probability of a bank run.

The Czech National Bank has already decreased the 2W repo rate two times in March 2020 (CNB, 2020). From 2.25 % in February to 1.75 % in the middle of March and again to 1 % at the end of March (CNB, 2020). It is certain that the decrease in the 2W repo rate will affect the interest rate on newly provided loans, and therefore increase the susceptibility of banks to possible runs.

The two most significant bank specific factors affecting the susceptibility to bank runs are connected with bank's liquidity and profitability. As an indirect measure of a bank's liquidity, the share of loans to deposits is used. This ratio compares illiquid assets with liquid liabilities. Therefore, the lower the ratio, the more liquid the bank is and the more able to withstand the crisis. Return on assets is the factor most linked with profitability. Klepková Vodová and Stavárek (2017) suggest using this variable lagged by two years, which means that in the case of sudden withdrawals, safer banks are those which were financially stable in the past.

It has been observed that for past seven years the ratio of share of loans in deposits (LOD) in Czech banks was stable around 75% (Fig. 6). On the other hand, Air Bank's LOD value until end of 2016 was much lower than the Czech average, making the bank more liquid than the average Czech bank. Since 2017, Air Bank's LOD has exceeded the Czech average by almost 10%, which makes Air Bank more vulnerable to a possible bank run than the average Czech bank.

Fig. 6: LOD comparison

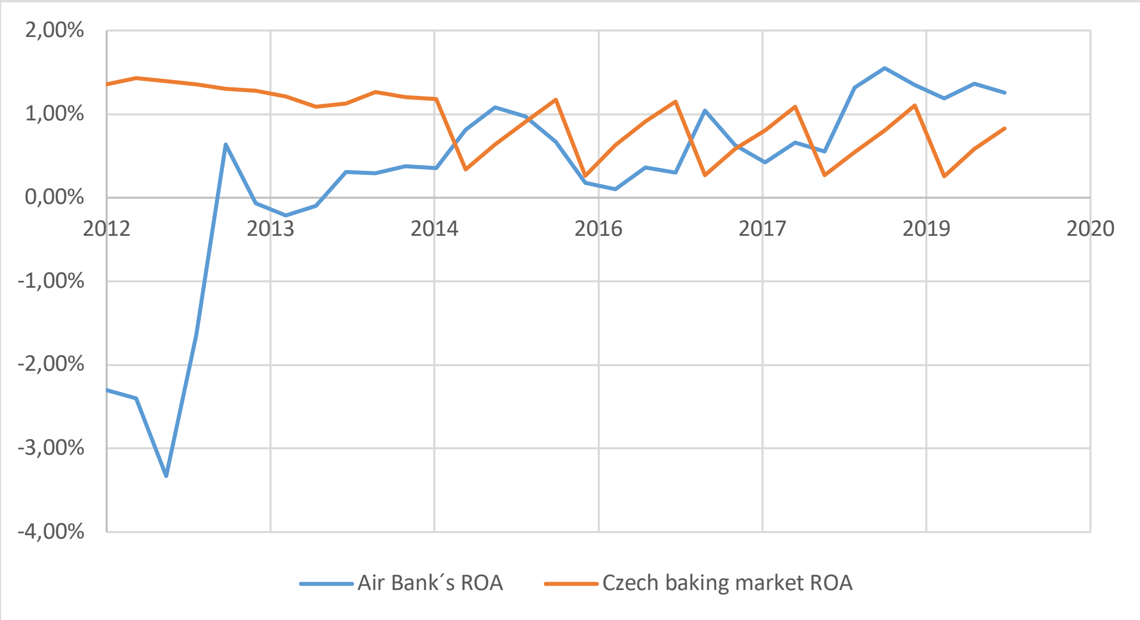


Source: Air Bank a.s., CNB

The second bank specific factor that affects the susceptibility of Czech banks to a possible bank run is return on assets. From figure below (Fig. 7) it can be observed that since mid 2017 Air Bank's return on assets has been higher than average in the Czech banking sector. Having the return of asset variable lagged by two years shows that Air Bank should be less susceptible to a potential bank run.

In Air Bank's case, there is a delicate balance between profitability and liquidity. At the end of 2016, Air Bank lowered its high liquidity buffer and started providing more loans. This action worsened Air Bank's liquidity position, which resulted in an increase in the share of loans to deposits. On the other hand, with a time lag, it boosted Air Bank's profitability. However, it may be far more difficult for a bank with a worse liquidity position due to higher lending activity to withstand the impact of a crisis than for a bank with lesser lending activity and a larger buffer of liquid assets (Klepkova Vodova, 2017).

Fig. 7: ROA comparison



Source: Air Bank a. s., ECB

5.2 Bank run contagion

A bank run contagion is a situation in which the liquidity risk is spreading within a banking system. Many policy makers and academics view a bank run contagion as a pivotal source of systemic risk in the banking sector (Brown et al., 2016). Contagion of deposits was documented during the Great Depression in the United States (Calomiris and Chen, 1997) or more recently in emerging markets (Iyer, 2012). A recent study by Iyer and Peydro (2011) shows that deposit withdrawals in distressed bank can affect other depositors in similar banks to withdraw their deposits. This study shows that this situation is highly contagious, especially when banks in the same region have interbank exposures to the distressed bank.

The ECB (Brown et al., 2016) experiment, which is built on a two-person coordination game, tries to capture a panic-based bank run. They examine under which conditions a bank run contagion could be triggered. Their setup is that in each bank there are two depositors, each with two options: withdraw their deposits or keep them in the bank. In the ECB (Brown et al., 2016) experiment, they divided depositors into two groups: leaders and followers. Followers closely observe the leader’s decision before they make their own decision.

The result of this study suggests that panic-based withdrawals are very contagious especially between economically related banks. Thus, a bank run at one bank makes a bank-run more probable at the other banks. According to a study done by ECB (Brown et al., 2016) panic contagion is not only an irrational psychological effect. The study suggests that depositors´ beliefs are affected by important economic events. The result does not definitely indicate that the bank runs occur at random. They suggest that the probability of a panic bank run increases with more economically related information. This information doesn’t necessarily have to change the incentives to withdraw deposits.

5.2.1 Deposit insurance

Deposit insurance is a widely adopted policy that promotes financial stability in the financial sector and which tries to prevent contagious bank runs. However, as a side effect, it also encourages banks and depositors to take on excessive risk.

Many papers have been written that document the economic benefits of deposit insurance. DeLong and Saunders (2011) studied depositor behavior in the United States after the implementation of deposit insurance and came to the conclusion that depositors are less susceptible to withdraw their deposits from financially weaker banks, if they know that their deposits are insured. Martin, Puri and Ufier (2017) studied the effects of deposit insurance on the inflows and outflows of distressed banks. The result of their study was that the deposit insurance reduces the volume of withdrawals, and thus provides the liquidity to banks. Angier, Demirguc-Kunt,and Zhu (2014) show that adopting deposit insurance enables greater competition in the banking market. With a deposit guarantee, smaller banks could attract more clients, and with that increase competition in the market.

On the other hand, the existence of deposit insurance brings the problem of moral hazard. With deposit insurance, banks can take on riskier investments because profits are captured only by the bank, but the losses are shared through a deposit insurance fund. From depositor’s side, if they know that their deposits are protected, then the depositors will much bother with monitoring bank´s overall health. Calomiris and Chen (2018) suggest that the adoption of deposit insurance might have caused an increase in loan to asset and debt to equity ratios. Angier, Demirguc-Kunt, and Zhu (2014) also prove that systems with a more generous deposit insurance policy experienced lower bank risk and greater stability of the financial system during the global economic crisis. But, during non-crisis years they registered higher bank risk and lower stability of the financial system.

In the Czech Republic, deposits made by natural persons or legal persons are insured to a maximum of 100,000 Euro, or 2.6 million Czech Crowns. In Air Bank's case, the ratio of deposits per client, when taking 788 thousand clients and face value of deposits 104,862 million Czech Crowns into account, equals to 133,073 Czech Crowns per client. Therefore, an average depositor has deposited 133,073 Czech Crowns within Air Bank. Obviously, in reality, almost all depositors have deposited more or less within Air Bank than this average. However, this average shows that, on average, deposits per client form only 5% of deposits insured per client. Taking this simplification into consideration, depositors shouldn't run on Air Bank at any given time or in any given situation, because their deposits are fully covered by deposit insurance. The information about median rather than average deposit or some more detail information about the size distribution of deposits would be preferable, in the case such data would be publicly available.

Goldstein and Pauzner (2005) show that depositors are driven by their beliefs and emotions. A bank run can strongly influence beliefs of observing depositors from different banks. This theory further emphasizes the theory of noisy public information, which can trigger bank panics. According to Yier and Jensen (2016), in times of crisis depositors are more likely to reallocate their deposits to a big systemically important banks because they believe that big banks are more likely to be bailed out.

The greatest enemy of deposit insurance in the Czech Republic is lack of interest. According to a survey done by the Czech Banking Association (Česká Bankovní Asociace, 2019) in 2019 37% of respondents have no idea about the insurance of their deposits. 11% of respondents even confidently claimed that their deposits were not insured at all. The study says that this notion about deposit insurance was held by respondents older than 65 as well as university students. On the other hand, a lack of awareness can be found within groups with the lowest education, suggesting that this may also be correlated with lower financial literacy.

5.3 Air Bank’s equity value triggering bank run

The conventional role of equity capital within a business is to ensure that the company will survive, even when they encounter an unexpected loss. During the economic crisis trigerred by coronavirus in 2020 the whole economy is facing an economic upheaval, one that could potentially be more critical than what we witnessed during the global financial crisis. The coronavirus pandemic is a different kind of shock than the previous one in 2008. Never before in modern history after 1920 have economies had to immediately shut down. From one week to the next, many workers have lost their jobs, and with that their pay checks. In the blink of an eye hotels, restaurants, and airports have all been emptied. Now businesses and consumers must face sudden losses in income and potential bankruptcies. Pressure on the banking system is increasing, and a higher percentage of defaults is forthcoming (IMF, 2020).

In this section we will mention cornerstones on which the model of equity value triggering a bank run is built. Then, we will use this methodology on the Air Bank a.s. case and and we will estimate at what level of equity depositors may choose to withdraw their deposits and start a bank run.

5.3.1 Theoretical model

The equity value triggering bank run is based on a model introduced in 'Game theory analysis of options' by Ziegler (2004). The model integrates a probability of a bank run with a bank equity valuation. Bank´s equity is valued as a down-and-out barrier call option, because down-and-out call option can disappear prior to its maturity, when the underlying´s asset price reaches given point. In our case, when the conditions for a bank run are satisfied.

The model considers two representative depositors. Both depositors have initially deposited X_0 units of money into the bank. At the same time equity holders added for each monetary unit deposited by depositors additional $x > 0$ units of capital into bank. It is assumed that both depositors have a right to withdraw their full amount of deposits from the bank without giving prior notice. Therefore, the bank additionally needs to take liquidation costs in the event of a run into account. The total asset value with liquidation costs equals (4.4), where S_t represents the value of bank´s assets and α proportional liquidation costs in the event of a run.

$$2(x + 1)(1 - \alpha)S_t$$

Pay-off function for depositor, who withdraws first (4.5) equals

$$min[2(1 + x)(1 - \alpha)S_t, X(t)]$$

Pay-off function for depositor, who withdraws second (4.6) equals

$$max[0, 2(1 + x)(1 - \alpha)S_t - X(t)]$$

The bank is exposed to a bank run, as soon as both depositors decide to withdraw first. From pay-off functions, it can be derived that a bank run occurs as soon as the face value of deposits is higher than the total asset value net of liquidation costs (4.9) Having more than two depositors (players) do not affect the conditions for a run, therefore, the model can be applied to real life cases, where banks have thousands of depositors.

$$X(t) > (1 + x)(1 - \alpha)\, S_t$$

Before deriving the formula for equity value triggering bank run, it is necessary to make 3 additional assumptions

- As soon as the face value of deposits is higher than the total asset value net of liquidation
  $X(t) > (1 + x)(1 - \alpha)\, S_t$, a run immediately takes place.
- A bank run happens very swiftly and equity holders aren't able to provide new capital in order to prevent or stop a bank run.
- Bank can liquidate its project even if bank run isn´t happening, but at a proportional variable cost of β, where $\beta < \alpha$.

The bank with two depositors holds a perpetual down-and-out call option on asset value net of liquidation, with a time-varying strike price of deposits $X(t)$ and a knockout price $K(t)$ (4.17), which is dependent on each depositor's run decision.

$$K(t) = \frac{(1 - \beta)}{(1 - \alpha)} X(t)$$

The value of perpetual down-and-out call option (4.18) $C_\infty((1+x)(1-\beta)S_t, K(t))$ represents the value of the bank's equity when depositors are choosing to withdraw their deposits from the bank.

It may be shown that equity value triggering a bank run equals asset value net of liquidation minus the losses occurred when the expected when bank run occurs (4.26). Losses are equal to discount, which results from the knock-back feature of the option.

$$C_\infty = (1+x)(1-\beta)\left(S_t - \left(\frac{X(t)}{(1+x)(1-\alpha)}\right)^{\gamma^*+1} {S_t}^{-\gamma^*}\right)$$

where γ^* equals to

$$\gamma^* = 2\frac{r-r^*}{\sigma^2}$$

where r represents the risk-free interest rate, r^*represents interest rate paid on deposits and σ standard deviation of bank´s asset value.

The down-and-out call option represents the fact, that a bank can face an illiquidity event a long before the maturity of all bank´s issued debts. The illiquidity event is connected with the value of bank´s assets as it shown in the equation above (4.26). As Petey and Soula (2018) mention the framework proposed by Ziegler (2004) does not allow to differentiate between illiquidity and insolvency. However, the illiquidity can be a herald of insolvency.

5.3.2 Literature review

Model proposed by Ziegler (2004) already saw a few applications. Petey and Soula (2018) use this model to measure a bank exposure to liquidity risk. They apply the model to the sample of European banks from 2004 to 2014. They found that banks outside of Eurozone seem to be more affected on average by stressed liquidity conditions, than banks within Eurozone. Petey and Soula claim that their result might reflect the ability of European Central Bank to better manage the effects of systemic events on the money markets.

Zhang (2016) uses model proposed by Ziegler and analyses the game of bank runs using stochastic volatility model, which assumes that asset value volatility is driven by meanreversion Ornstein-Uhlenbeck process. He later uses formula for equity value triggering bank run and analyses initial funding and recapitalization decision.

Zhang et al. (2020) combine option pricing and game theory to obtain formula for the value of bank equity with bank run risk. Unlike Ziegler (2014), Zhang et al. focus on liquidity ratios, which are derived from the perspective of bank´s shareholders. On Chinese listed banks Zhang et al. show the gap, between the optimal liquidity ratios and current ones and explain time-series and cross-sectional changes in liquidity and credit risk of banks.

5.3.3 Data extraction

The equation for computing equity value which triggers a bank run has eight variables as its parameters:

1. bank's asset value S,
2. face value of deposits $X(t)$,
3. risk-free interest rate r ,
4. interest rate paid on deposits r^*,
5. σ as standard deviation of bank's asset value S,
6. capital provided by equity holder at initial time x,
7. α as proportional liquidation costs in the event of a run,
8. β as proportional liquidation costs if no run occurs.

A bank's asset value S may be easily obtained directly from a company's balance sheet for Q3 of 2019. Thus, Air Bank's asset value was 115,815 million Czech Crowns in 2019.

As in the case of asset value, the face value of deposits $X(t)$ can also be directly obtained from a company's balance sheet. The face value of money deposited in the bank equals to 104,862 million Czech Crowns.

In this model, we will use a yield on a Czech 10-year government bond as a proxy for a risk-free interest rate r. In April 2020, the yield on 10-year Czech government bond was 1.39% p.a.

To offset the interest rate paid on a deposit r^*, we will extract deposit structure from balance sheet data (tab. 4).

Tab. 4: Client´s deposits in Q3 2019

	Deposited mil. CZK	Int. Rate
Current accounts	23 382	1,00 %
Savings accounts	76 480	1,50 %
Total	104 862	

Source: https://www.airbank.cz/file-download/informace-o-air-bank-k-30-9-2019-cast-1.xlsx

With data from Table 4 we compound proportional interest rate, which yields 1.36% p.a. The standard deviation σ of asset value S was calculated over quarterly published asset value from Air Bank's foundation from 2011 until the present day, and rebased into percentage equivalent. The standard deviation yields 41.75 %.

Using interest rate differential $r - r^*$ and standard deviation σ variable γ^* can be computed. Using equation (4.27)

$$\gamma^* = 2\frac{r - r^*}{\sigma^2}$$

$$\gamma^* = 2\frac{1.39\% - 1.36\%}{0{,}4175^2}$$

Variable γ^* equals to 0.0029 and it's used in later calculations.

In order to compute parameter x, initial capital and initial deposits must be known. Air Bank entered the market on 22nd of November, 2011. From their 2011 balance sheet, we find that initial capital was 500 million Czech Crowns and deposits totalled 2,223 million Czech Crowns. With these two values, parameter x equals to 22.39%.

One of the hardest parameters to determine, when using only publicly know data, is the proportional cost in the event of a run α. From the previous chapter, it is clear that a run will occur as soon as the face value of deposits is higher than the asset value net of liquidation (4.9).

$$X(t) > S_t(1 + x)(1 - \alpha)$$

Solving for α yields

$$\alpha > 1 - \frac{X(t)}{S(1+x)} \tag{5.1}$$

Using equation (5.1) above yields that α must not be higher than 26%. In another case, a bank run would have already occurred. From the previous chapter, it is also clear that at the initial time shareholders provide capital which covers potential liquidity losses (4.69). Thus,

$$(1+x)(1-\alpha) = 1$$

$$\alpha = \frac{x}{x+1} \tag{5.2}$$

According to (5.2) in 2011 proportional liquidity costs in the event of a run (α) must had been equal to 18.29%. Since 2011 Air Bank lowered its liquidity buffer and started providing more loans. Assuming that, the lower liquidity buffer leads to the higher liquidation costs in the event of a run. From these two conditions, we can estimate that parameter α is between 18.29% and 26%. Within this interval, we will set multiple parameter values of α. We are going to use four values of parameter α each for every one fifth within the interval.

Tab. 5: α parameter values

α_1	19.839 %
α_2	21.385 %
α_3	22.930 %
α_4	24.476 %

Source: Author´s calculation

Last parameter to be determined is β, which stands for liquidity costs in a situation in which no run occurs. To determine parameter β we will use equation (4.59)

$$x = \frac{\left(\gamma^* \frac{1-\beta}{\beta}\right)^{\frac{1}{\gamma^*+1}}}{1-\alpha} - 1$$

Solving for β yields

$$\beta = \frac{\gamma^*}{\gamma^* + \left((1+x)(1-\alpha)\right)^{\gamma^*+1}} \tag{5.2}$$

Numerically solving for each parameter α we get:

Tab. 6: β parameter values

β_1	0.295 %
β_2	0.301 %
β_3	0.307 %
β_4	0.314 %

Source: Author´s calculation

Also, α_i and β_i satisfies condition (4.70) for positive profit to shareholders from founding the bank.

$$\alpha + \beta < 1$$

5.3.4 Computation of equity value triggering bank run

In this part of thesis, we will apply our findings to Air Bank. Now it is time to utilize the parameters computed in the previous part, insert them into the equation, (4.26) and compute equity value which causes depositors to run on a bank.

$$C_{\infty_i} = (1+x)(1-\beta_i)\left(S_t - \left(\frac{X(t)}{(1+x)(1-\alpha_i)}\right)^{\gamma^*+1} S_t^{-\gamma^*} \right)$$

Numerically solving for each pair of α_i and β_i yields:

Tab. 7: Scenario cases of equitiy value triggering bank run

C_{∞_1}	10 924.10
C_{∞_2}	8 348.49
C_{∞_3}	5 630.39
C_{∞_4}	2 894.66

Source: Author´s calculation

C_{∞_i} gives us the equity value, which would trigger Air Bank's depositors to run on the bank. In our four scenario cases, the equity value triggering a bank run ranges from 2,894.66 million Czech Crowns to 10,924.10 million Czech Crowns. When increasing the variable of proportional liquidation costs in the event of a run (α), the equity value triggering a bank run (C_∞) decreases. As defined in section 4.3 of this thesis, equity value triggering a bank run equals the asset value net of liquidation costs minus the expected losses resulting from a bank run. Therefore, the higher the proportional liquidation costs in the event of a run, the lower the equity value triggering a bank run and vice versa.

The current value of Air Bank's equity is 8,803 million Czech Crowns. Therefore, it is obvious that the first solution of equity value triggering bank run (C_{∞_1}) yields unrealistic scenario, because the run would have already occurred. Due to the fact, that we are using only publicly known data, proportional liquidity costs in the event of a run (α) must be estimated. The equation (5.2) sets the value for the liquidity costs in event of a bank run (α) during the foundation of Air Bank in 2011. Thus, using the liquidity costs in the case of a bank run (α_1), which is the closes to the level of α in 2011, provide an unrealistic scenario and can´t be further used. It is necessary to use higher values of liquidity costs in the event of a run (α) than Air Bank´s foundation value in 2011.

Other solutions seem to be more realistic, because computed equity values triggering a bank run (C_{∞_i}) are lower than current equity value, but there are great differences between the values. Therefore, we will examine three cases C_{∞_2}, C_{∞_3} and C_{∞_4}. Because, C_{∞_2} is the closest value to current value of equity, we will speak of this version as pessimistic. C_{∞_3} will be an optimistic version and the last one will be the very optimistic version, because Air Bank would have to encounter a significant loss and almost two thirds of bank equity would have to disappear.

Pessimistic version

The pessimistic version yields that equity value triggering a bank run (C_{∞_2}) equals to 8,348.49 million Czech Crowns, which indicates that Air Bank's current equity value needs to drop by 454.51 million Czech Crowns in order to trigger a bank run. This sudden drop in equity value could be caused by, for example, a marginal unexpected loss.

The pessimistic version of equity value triggering a bank run is the closest to the current value of banks' equity. If the real values of liquidation costs are equal or close to values of α_1 and β_1 Air Bank should pay attention to their business model and focus on decreasing the level of risky assets on their balance sheet, such as loans, and increase the value of safe assets, such as government bonds. In doing so, Air Bank would decrease the risk of a bank run, but on the other hand, it would sacrifice its profitability. This measure is more suitable, especially in the crises triggered by 2020 coronavirus, when economists predict an economic crisis, which could equal The Great Depression.

Optimistic version

The optimistic version yields that equity value triggering a bank run equals (C_{∞_3}) 5,630.39 million Czech Crowns, which means that Air Bank would have to incur a loss of 3,172.61 million Czech Crowns. This loss can be caused by the default of more than 3.5% of loans provided to retail customers. This volume of defaults is slightly higher than the percentage volume of defaults during years of 2008 and 2009, which was around 3.2% (Singer, 2009). Thus, similar losses to the ones experienced during the Global financial crisis would almost trigger bank run.

This version seems more realistic than the previous pessimistic version. Taking deposit insurance into consideration, when all of the deposits are insured, a marginal unexpected loss, which would wipe nearly a half of bank´s equity, would definitely cause a panic among depositors. A group of depositors, who are unaware of deposit insurance, could cause a panic bank run. In that situation, it would be up to Air Bank and other banks to calm the panicking depositors.

Very optimistic version

The very optimistic version yields that equity value triggering a bank run (C_{∞_4}) equals 2,894.66 million Czech Crowns, which indicates that Air Bank´s current equity value needs to drop by 5,818.34 million Czech Crowns from the current 8,803 million Czech Crowns. This loss would raise questions about Air Bank´s financial health.

On the four presented versions of equity value triggering a bank run it can be shown, how liquidation costs α_i and β_i affect the final value equity triggering a bank run. Due to the unknown value of fire sales costs, until the crisis begins, banks can only predict the expected costs.

5.4 Simulating bank run

In this section we will determine the susceptibility of Air Bank a. s. to a hypothetical run. Some central banks or other supervisory authorities have developed their own liquidity stress test in order to simulate a possible bank run. For example, in the Netherlands, a model proposed by Van den End (2008), in Romania, a model suggested by Negrila (2010), and in Czech Republic, a model developed by Komárková et al. (2011).The disadvantage of these tests is that they can't be performed with publicly known information.

Therefore, we are going to use a less complex liquidity stress test in order to simulate a bank run. This model is proposed by Klepková Vodova (2015), and it is similar to a model used by the Austrian supervisory authority (Boss, 2007) or Slovakian (Jurča and Rychtárik, 2006) supervisory authority. This model uses the following liquidity ratios:

Share of liquid assets in total assets

$$LIA_B = \frac{A_{LI}}{S_T} * 100(\%) \tag{5.4}$$

Share of loans in total assets

$$LOA_B = \frac{A_{L0}}{S_T} * 100(\%) \tag{5.5}$$

Share of loans in deposits

$$LOD_B = \frac{A_{L0}}{X(t)} * 100(\%) \tag{5.6}$$

the above mentioned studies (Boss et al., 2007; Jurča and Rychtárik, 2006; Negrilla, 2010; Rychtárik, 2009) simulate the deposit withdrawal rate around 20%. Therefore, in this model we will simulate a bank run which results in the withdrawal of 20% of client deposits. This decrease is applied on total deposits within Air Bank, while not taking agreed maturities of different types of deposits into account. In order to compute the stressed values of ratios, we need to subtract 20% of client deposits and accordingly 20% of liquid assets, because the liquid assets are used to repay withdrawn deposits. The following equations show stressed modifications of previously mentioned liquidity ratios.

$$LIA_S = \frac{A_{LI} - 0.2 * X(t)}{S_T - 0.2 * X(t)} * 100(\%) \tag{5.7}$$

$$LOA_S = \frac{A_{L0}}{S_T - 0.2 * X(t)} * 100(\%) \tag{5.8}$$

$$LOD_S = \frac{A_{L0}}{0.8 * X(t)} * 100(\%) \tag{5.9}$$

5.4.1 Liquidity stress test

In this part, we conduct a hypothetical bank run simulating an outflow of 20% of client deposits as proposed in earlier section. We use values from the Optimistic scenario accompanied by balance sheet data from the third quarter of 2019. We assume that the decrease in equity value, which triggered a bank run, was primarily caused by defaults on loans, therefore the decrease in the bank´s equity value, is caused by an equal decrease in illiquid assets. Also, we are going to calculate the highest value of deposits that can be withdrawn from Air Bank, that are still covered by liquid assets. All the liquidity ratios and their stressed values can be found in table below (tab. 8).

Tab. 8: Liquidity ratios

Share of liquid assets in total assets	
LIA_B	20.69 %
LIA_S	3.15 %
Share of loans in total assets	
LOA_B	76.33 %
LOA_S	93.21%
Share of loans in deposits	
LOD_B	84.31 %
LOD_S	105.39 %

Source: Author´s calculation

Base value of share of liquid assets in total assets (5.4) equals to 20.69 %, therefore, for this ratio, it is valid that the higher the value, the better the liquidity position of a bank. According to the CNB financial report (ČNB, 2019) the average value of LIA in 2018 in the Czech Banking market was 41.2 %. The stressed ratio of liquid assets in total assets (5.7) is 3.15%, which indicates that despite the substantial decline in liquidity, Air Bank would be able to cover a 20% withdrawal of deposits.

Base value of share of loans in total assets (5.5) is 76.33 %. Higher ratio value indicates that bank focuses primarily on lending activity. Banks with lower ratio focus on interbank transactions or trading with securities. Stressed value of share of loans in total assets (5.8) is 93.21% which indicates that after 20% outflow of deposits Air Bank would still be able to cover already provided loans. As mentioned above, Air Bank's business lies in lending activity. Providing more loans is possibly more profitable for Air Bank, but the result of this scenario indicates that it is very important to accordingly balance the relationship between profitability and liquidity.

The share of loans to deposits (5.6) is an important ratio, which shows how many loans provided to retail customers are financed by client deposits. The base value of Air Bank's LOD has an average value of 84.31%. The stressed value of Air Bank's LOD (5.9) is 105.39%. The stressed value exceeds 100%, which means that Air Bank would need additional sources of funding, such as an interbank loan or an issuance of debt securities. Air Bank should focus on lowering this ratio, because client deposits are considered to be a more stable source of funding.

Our goal is also to find the maximum volume of client deposits which can be instantly withdrawn from Air Bank. This means finding the worst case scenario of client withdrawals which would dry out Air Bank's liquid assets. Using equation for stressed ratio of liquid assets in total assets (5.6) and solving for deposit withdrawals yields 22.85%. This indicates that if depositors withdraw more than 22.85 % of their deposits, Air Bank would be forced to fire sell its illiquid assets.

Conclusion

The aim of this thesis was to estimate the exact equity value which would trigger Air Bank´s depositors run on the bank, as well as to evaluate Air Bank´s a. s. susceptibility to a run. In order to estimate the value of equity which would cause a bank run, we have applied an approach consisting of a combination of option pricing and game theory proposed by Ziegler (2004).

In times of economic distress, the probability and sensitivity to a bank run increases, especially in a situation in which the IMF expects an economic crisis. According to the IMF´s study (IMF, 2020), the Corona virus pandemic will have a bigger impact than the Global financial crisis in 2008. During the first few months of the Corona virus pandemic, there have not been any major bank runs reported, but a few branches of banks in the United States near New York and Seattle have already reported that their depositors are making big withdrawals (Ackerman, 2020).

The susceptibility of Czech commercial banks to a potential bank run should increase as the unemployment rate rises from its minimum values with the upcoming crisis. An additional factor that increases susceptibility to a bank run is the lowering of interest rates on loans provided to customers. Due to the specific factors affecting Air Bank, their susceptibility to a run is unclear. Air Bank´s higher ratio of loans to deposits than the Czech banking market average makes the bank more susceptible to a run. On the other hand, in previous years Air Bank managed to have a higher return on assets than average in the Czech banking market, which indicates good financial stability.

A widely adopted policy of deposit insurance should help to promote the stability of the financial system and to ensure that the depositors trust the system. Deposit insurance truly reduces the risk of a bank run contagion and promotes competition in banking market. Unfortunately, one of the greatest issues with deposit insurance is the lack of interest by depositors. In a survey done by the Czech Banking Association from 2019 (Česká Bankovní Asociace, 2019) 37% of respondents had no idea about deposit insurance. Therefore, this ignorance could be one of the drivers for possible bank runs coinciding with the upcoming economic distress. Even though deposits are insured, in times of financial distress depositors are more willing to transfer their money into big systemically important banks. This is driven by their belief that big systemically important banks will be bailed out by the government. However Air Bank is definitely not a systemically important bank by any definition.

Computation of equity value that would trigger depositors to run on Air Bank a. s. yielded three possible scenarios. From these analyzed versions, it is clear that the equity value triggering a bank run is highly dependent on the variable liquidity costs in the event of a run (α). The versions were arranged from the most pessimistic to the most optimistic. The pessimistic scenario yielded that depositors would run on a bank as soon as the equity value decreased by 454,51 million Czech Crowns, which accounts for 5% of Air Bank´s current equity value. The optimistic scenario yielded that the decrease of equity which would trigger a bank run is equal to 3 172,61 million Czech Crowns. This amount of loss represents 36% of the bank´s current equity value. The last version is very optimistic, and yields that a decrease in equity of 5 818,34 million Czech Crowns causes a bank run, which is equal to 66% of Air Bank´s a. s. current equity value.

The three proposed scenarios mentioned in the previous paragraph enable Air Bank a. s. to think about their customers' behavior in uncommon situations, resulting from significant losses. The most likely scenario which causes a bank run is the optimistic one. The reason is that reported loss of more than 3 billion Czech Crowns interannually will not go unnoticed, neither in times of growth nor in times of depression. Year-to-year losses of 5% of a bank´s equity value is definitely alarming in a time of growth, however, during economic distress, minor losses are expected. A loss exceeding 5.5 billion Czech Crowns, which does not cause depositors to run on a bank, is only acceptable with very phlegmatic clients or clients acquainted with issues in the banking market.

The methodology proposed by Ziegler (2004), on which the computation of this thesis is calculated, is built for retail banks with a simple balance sheet structure where liabilities consist of only deposits. Another weakness of this model is liquidity costs. If we are using only publicly known data, they need to be estimated. Using publicly known data leads into multiple scenarios, as it is shown in this thesis. Nevertheless, Ziegler (2004) introduced a very interesting methodology, which tries to predict behavior of depositors when a bank´s equity is decreasing due to unexpected loss. Definitely, there is a need for more surveys and case studies to be undertaken. In these studies, it would be appropriate to use non-public bank specific data to estimate exactly the behavior of the model and precisely calculate all the variables used in it.

From a conducted liquidity stress test, which is similar to stress tests used by supervisory authorities in some European countries, it is obvious that Air Bank a. s. has enough liquid asset to withstand a significant outflow of deposits. However, after a simulated bank run, the ratio of loans to deposits (LOD) would exceed 100%. That means that even though the examined bank would survive a bank run, additional sources of financing would be necessary.

Resources

Books and articles

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